STRESSES

STATICALLY DETERMINED

JOHN CLAYTON TRACY

STRESSES
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STRESSES

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BY

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PREFACE

This book is intended to give a thorough working knowledge of the theory of stresses in statically-determined framed structures. Great pains have been taken to explain fundamental conceptions, principles, and methods so clearly and completely, that the student can master them easily and quickly by himself. Any time thus saved in the acquisition of a knowledge of fundamentals can be used to advantage — by the student in learning to apply those fundamentals — by the instructor in the supervision of the student's work.

FEATURES OF THE BOOK

1. An exceptionally comprehensive and thorough presentation of the fundamentals of statics, with a complete summary in CHAPTER X of those fundamentals.

2. The classification of all problems in statics and the corresponding problems in stresses into eight cases, with a standard solution for each case.

3. A general method of attack, used throughout the book, that enables one to analyze a problem and the forces involved, and to determine under which of the eight cases the problem falls. The standard solution for that case can then be applied.

It is believed that this method of attack simplifies and unifies not only the whole subject of statics, but the subject of stresses as well. As the student begins the study of each type of truss, he is confident that any problem it presents in the determination of stresses is not essentially different from problems he has already solved, that it must fall under one of eight cases (usually one of three), and that, as soon as he determines the case under which it falls, he has a standard solution. Moreover, he is using a method of attack which is typical of the scientific method of analysis that engineers should cultivate and use in all kinds of problems. The development of this method of attack in **PART I** and its use throughout **PART II** and **PART III** are perhaps the most distinctive features of this book.

4. The insistence throughout the book of a clear conception of the action of the forces involved in any problem.

Students are led to think of a force as something real with definite characteristics even though it is invisible, and to realize that all forces act in accordance with a few simple laws or principles. No problem is solved by mere substitution in formulas, and the use of formulas is discouraged until after the principles on which they are based are thoroughly understood.

5. The solution of a large number of problems to illustrate the practical application of principles and methods. In many of these illustrative problems only a skeleton analysis of the solution is given, in order that the method may stand out clearly, unobscured by the more or less routine work of calculation.

6. The introduction throughout the book of practical comments, notes, and suggestions, printed in smaller type so that they supplement but do not obscure general principles and methods.

7. The exclusion of all subject matter pertaining to design except that which is clearly helpful in the study of stresses.

The theory of stresses can be developed very simply and logically as a more or less exact application of mechanics. The introduction of extraneous subjects, however closely related to stresses they may be, tends to obscure the simple theory and to confuse the student. Moreover, by exclusion of such subjects, space is gained for a more thorough treatment of stresses. It is assumed that the student who is taking a course in stresses will also take related courses, such as those in mechanics of materials and in structural design.

PART I — STATICS

PART I is devoted to statics. The need in a book on stresses of including such a thorough treatment of a subdivision of mechanics may be questioned by those who have not had experience in teaching stresses; teachers know, however, that when a student has serious difficulties in the study of stresses, it is usually because of an inadequate working knowledge of statics. If the student is well prepared in statics, it will be necessary for him merely to review rapidly the subject matter of **PART I** in order

to become familiar with the method of analysis and attack used throughout this book; if he is not well prepared, the time required to master **PART I** will be well invested. The sweeping statement that "nothing is difficult if one is prepared" is almost literally true in the study of stresses. It is believed that there is danger of passing over **PART I** too hurriedly — of spending too little rather than too much time in reviewing statics. A good working knowledge of the fundamentals summarized in CHAPTER X is evidence of the student's fitness to proceed to the study of stresses.

PART II — STRESSES DUE TO DEAD LOAD

PART II begins with a description of bridges and roofs, in order that the student may become familiar with typical structures, and particularly in order that he may know how loads are determined and how these loads eventually reach the trusses. The numerous photographs of bridges and roofs are used later in various problems. An unusually complete chapter on external forces, including reactions, is followed by one on shears and bending moments due to dead load. The remainder of **PART II** is devoted to the three basic methods of determining stresses, namely, the algebraic method of sections, the algebraic method of successive joints, including the method of coefficients, and the graphic method of successive joints. There is a decided advantage in studying these methods in connection with dead-load stresses before beginning the study of live-load stresses. With no possibility of being confused by questions pertaining to the position of the load, the student soon realizes that the basic methods of determining stresses are merely special applications of the general methods of statics given in **PART I.** If, however, it is desired to study dead-load and live-load stresses simultaneously, this may be done by combining **PART II** with the first four chapters of **PART III.**

PART III — STRESSES DUE TO LIVE LOAD

PART III begins with a chapter on influence lines, followed by one on shears and bending moments due to live load. The study of stresses due to uniformly distributed live load is then completed before beginning the study of stresses due to systems of concentrated live loads, such as locomotive loadings. It is repeatedly emphasized that once a live load is placed in the correct position, it is assumed as static in that position, and that, therefore, the corresponding live-load stresses are determined by exactly the same basic methods as those used for dead-load stresses. Thus the student who has mastered **PART II** soon realizes that there is little new to learn in **PART III** except the principles that govern the placing of live loads.

In the two chapters on shears, bending moments, and stresses due to concentrated live loads, both Cooper's E-system and Steinman's M-system of locomotive loading are used, but a novel method of indicating locomotive wheel loads in various criteria and in the calculation of reactions, shears, and bending moments, makes the presentation of the theory independent of any particular system of concentrated loads.

Since in many of the problems in **PART III** it is required to determine dead-load stresses as well as live-load stresses, **PART III** constantly affords an opportunity to review **PART II.**

A more definite idea of the general plan and scope of this book may be gained by reading the **INTRODUCTION TO PART II,** page 85, the **INTRODUCTION TO PART III,** page 263, and the short introductory notes at the heads of the various chapters.

ACKNOWLEDGMENTS

In common with all structural engineers I owe much to other writers on stresses, particularly to American pioneers in the subject, such as Burr, Green, DuBois, J. B. Johnson, Marburg, Merriman, Jacoby, Swain, Turneaure, and Waddell. In writing this book, however, I have drawn for the most part on a long experience in teaching, during which I have learned much from my students — difficulties most common in studying stresses, numerous little ways of overcoming these difficulties, the necessity for clear-cut, unmistakable definitions and explanations, and the importance and possibility of a logical development which simplifies and unifies the whole subject.

I am indebted to many engineers and to members of the faculty of the Sheffield Scientific School of Yale University for valuable assistance in the preparation of this book. Mr. R. B. Allen and Mr. W. A. Allison, instructors in civil engineering, have made many of the drawings for the cuts

and have assisted in the preparation of illustrative problems. Mr. P. G. Laurson, Associate Professor of Engineering Mechanics, and Mr. J. N. Eckle, Assistant Professor of Civil Engineering, have read much of the manuscript and contributed many suggestions. Grateful acknowledgment is made of the exceedingly valuable help received from Mr. C. T. Bishop, Associate Professor of Structural Engineering. Besides contributing knowledge gained from a long experience in practice and teaching, Mr. Bishop has been of the greatest assistance in the preparation of the manuscript.

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John C. Tracy.

New Haven, Connecticut
May, 1929

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STRESSES DUE TO LIVE LOAD

To the Student:

Many engineering subjects, particularly those which involve design and construction, can not be mastered in college, because success in such subjects depends upon combining theory with the knowledge and judgment which come only from practical experience. Stresses, however, is a subject which is mainly theoretical. Your success in it will depend more upon ability to apply fundamental principles and methods than upon practical knowledge. There is not a great deal to remember, but much to understand. The easiest and best way to learn a principle or to acquire a method is not to memorize it, but to use it, understandingly, until it becomes a part of your mental equipment. Answers to problems in stresses are usually definite and exact, and hence, two engineers in solving the same problem should get the same results. At the end of a good course in stresses you should be able to determine stresses in ordinary structures with the same certainty, though perhaps not with the same facility, as would an experienced structural engineer.

A diminutive carpenter was once asked how it was that he could compete successfully with larger and stronger men. He replied: "Because I keep my tools *sharp*, know which to use, and how to use them." There are only a few simple fundamentals in the subject of stresses, but they are the tools that you must keep sharp and use to the best advantage if you are to work easily and efficiently. If you master these few fundamentals early in the course you should have no difficulty later on in the subject.

Stresses is a subject which may seem dry and uninteresting to you, particularly at first, but it has its advantages. You will find it a striking example of how a subject may be developed logically from a few simple fundamental principles. It will afford you an unusual opportunity for practice in the general scientific method of attack that is so essential in practically all engineering work. One of the most important rewards which you may expect from studying stresses is an increase in your power of analysis.

Any text-book, such as this, which is restricted to the fundamentals of a subject, will merely start you on your way. This book will probably take you as far in stresses as it is wise for you to go in college, unless you desire to specialize in structural engineering. In that case, it will be necessary for you to continue to study even long after graduation, and an extensive literature on the subject is open to you. One aim of this book is to enable you to understand easily anything which you may read on the subject of stresses, and to go as far as you like in the subject without further assistance from an instructor.

Finally, do not work blindly by rule, do not depend upon formulas, but gain as early as possible an independence in method and thought. The sooner you are able to work without help from this book the better the book will serve its purpose. Throughout the course maintain an attitude of questioning. Do not accept what an instructor says or what the book tells you as gospel truth. Constantly ask yourself "*Why?*" and do not rest satisfied until you *know* why. This is the spirit that will enable you to get the most out of the course.

J. C. Tracy.

STRESSES
STATICALLY DETERMINED

CHAPTER I

INTRODUCTION

1. **Stresses in Framed Structures.** A framed structure is one composed of straight members so joined together as to form a rigid framework. **External forces,** such as gravity and wind pressure, acting on such a structure cause **internal forces** in its members and these internal forces are called **stresses.** If a framed structure is to be in static equilibrium, each member should be designed to withstand the greatest stress it will ever be required to carry. The study of the principles and methods used in determining such stresses is a subject in itself quite distinct from, though closely related to, the subject of structural design and construction.

When the stresses in a structure can be determined by the principles of statics (a subdivision of mechanics) the structure is said to be **statically determinate.** In this book only such structures, as a rule, will be considered. "Stresses" thus restricted is, therefore, a subject in which the principles of statics are applied to framed structures.

2. **Common Framed Structures.** The most common framed structures are steel bridges and steel frameworks of buildings. When the floor system of a bridge or the roof of a building is supported by trusses, each of these trusses, though a part of the entire structure, is nevertheless a framed structure complete in itself. There are many types of framed structures, some of them constructed entirely of steel, some entirely of wood, and some partly of steel and partly of wood.

3. **Scope of the Book. Statics.** Those fundamental principles and methods of statics upon which the subject of stresses is based are explained in **PART I,** particular emphasis being given to the development of a method of attack which will be useful in solving problems in stresses. For the student who has had a thorough course in mechanics, **PART I** will be in the nature of a review.

4. **Dead-load stresses** are those caused by static loads or external forces, each of which always acts at a fixed point of the structure. In **PART II** methods of determining dead load stresses are explained.

5. **Live-load stresses** are those caused by moving loads, each of which may act at different points of the structure. The live load stress in any member usually differs for different positions of the moving load, but the stress used in designing the member is the **maximum live load stress** that can occur, and unless otherwise stated it is understood that live load stress means the maximum live load stress. It is necessary, therefore, to determine first of all the position of the live load which will result in the maximum stress. Once this position is known, the live load is assumed as static in that position, and the corresponding live load stress is determined by exactly the same methods as those used for dead load stresses. In **PART III** are explained the methods of determining the positions of live loads which will result in the required live load stresses. The methods of determining stresses given in **PART III** are practically the same as those given in **PART II.**

6. *Related subjects.* It is assumed that the student who is studying stresses either has had or will have courses in related subjects, such as

engineering mechanics, mechanics of materials, structural drafting, and structural design. Hence, except for the review of statics in **PART I,** the aim has been to confine this book strictly within the field of stresses in ordinary framed structures.

1. **Algebraic and graphic methods.** For every algebraic method in stresses there is a corresponding graphic method based on the same fundamental principles. In any given problem, the choice between the algebraic and the graphic method of solution will depend partly upon which can be applied in the shorter length of time and partly upon whether or not the graphic method is sufficiently precise. **Graphic statics** is often treated as a subject in itself. In this book, however, the algebraic and graphic methods are developed side by side, partly because each can be used to explain the other, but mainly because they are merely different methods of applying the same principles and should therefore be considered together. Both methods, moreover, are often used in the same problem.

2. NUMBERING OF FIGURES AND PARAGRAPHS. In this book each **figure** bears the same number as the page upon which it is found. When there are two or more figures on a page they are distinguished from each other by adding lower case letters, as for example, "Fig. 42(*a*)" and "Fig. 42(*b*)," for two figures on page 42. The **paragraphs** on each page are numbered in accordance with the order in which they occur on that page, beginning with the number "1". Cross references are made to both page and paragraph with a colon between, as for example, 124 : 1 for page 124, paragraph 1. When enclosed in parentheses (124 : 1) such a reference is usually to a principle, a method or a statement which the student does not need to consult if he already has it in mind; when not enclosed within parentheses the reference is to some part of the text which almost invariably should be consulted. Without such a distinction in cross references the student is likely either to waste time in looking up something which he already knows or carelessly to ignore the more important references.

3. ASSIGNMENTS. The thorough study of any subject involves much more than the study of any text-book on that subject; it means, among other things, that the student must broaden his viewpoint, as he proceeds in the subject, by supplementary reading and investigation. For example, the subject of secondary stresses does not come within the scope of this book, but the student should have some idea of what secondary stresses are, how they are caused, how they may be reduced to a minimum, and where in engineering literature the subject is treated. As a guide to such supplementary reading and investigation, assignments have been given at the ends of chapters throughout this book. Many of these assignments call for reports, which, if desired, may be presented by students as part of their class-room work.

4. So much relating to stresses has been published that it is feasible to suggest here only the general types of books, periodicals and other publications which may be consulted in connection with assignments, namely:

(a) Engineering periodicals, particularly the *Engineering News-Record.*

(b) Transactions of Engineering Societies, particularly the *Transactions of the American Society of Civil Engineers.*

(c) Books on mechanics, particularly those on engineering mechanics.

(d) Books on stresses.

(e) Structural engineering text-books and handbooks.

PART I — STATICS

CHAPTER II

FORCES

In this chapter are given certain fundamental conceptions of the action of forces upon and within framed structures.

1. **Stress.** In Fig. 3 a small body W is suspended by a rope C which is attached to a block B. A separate rope E is fastened to the block B and to a hook H in a wall. A horizontal stick F extends from the wall to the block B. The force of gravity acts on the body W and this body in turn exerts a vertical force downward on the rope C. This downward force is *external* to the rope C, i.e., it acts on the rope from without. The block B exerts an equal **external force** upward on the rope C. The body W and the block B may be said to pull in opposite directions on rope C and cause **internal forces** in this rope. In like manner, the block B and the hook H pull in opposite directions on rope E and cause internal forces in rope E. The block B and the wall G push in opposite directions against the stick F and cause internal forces in the stick. The internal forces in rope C are innumerable, but they may be considered parallel to the axis of the rope, and, therefore, they may be replaced by one **resultant force** acting along that axis. This force may be represented by a single straight line c (lower part of Fig. 3). This line is the line of action of the resultant internal force in the rope c. In like manner, e and f represent lines of action, respectively, of the resultant internal forces in the rope E and in the stick F. These resultant internal forces are called **stresses;** from now on the word " stress " should suggest at once force *within* a body. In the figure, the lines of action c, e, and f are drawn equal in length, respectively, to rope C, rope E, and stick F, but each line of action may be considered to be extended in length *without limit;* hereafter the line of action of a force will mean a straight line *of indefinite length* along which that force acts.

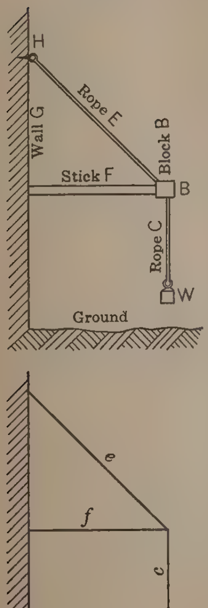


Fig. 3.

2. *Note:* In the explanation just given, the action of gravity on the ropes, the stick, and the block was ignored, and consequently the corresponding internal forces were ignored. The action of gravity on the stick F, for example, tends to bend the stick downward and cause certain stresses in the stick some of which are not parallel to its axis. Moreover, these stresses depend to some extent on how the stick is joined to the block and to the wall — whether these joints are hinged joints or rigid. Such stresses are called **secondary stresses.** They are usually negligible in small, light structures but may be important in large, heavy structures. It is not within the scope of this book to treat the subject of secondary stresses, and hence the term " stress " will be understood to mean the resultant stress (**axial stress**) acting along the axis of a member as explained above.

3. **Elements of a Force.** The elements of a force are:

(*a*) **Magnitude** (denoted by " M "),
(*b*) **Direction** (denoted by " D "),
(*c*) **Any point in its line of action** (denoted by " P ").

If any one of these elements of a force is unknown, the force itself is not fully determined.

4. **Magnitude** (M). Among English-speaking engineers, the magnitude of a force is usually expressed in pounds, less frequently in tons.

1. **Direction** (D) includes:

(*a*) The **inclination** of the line of action of the force (denoted by " A ").

(*b*) **Sense,** i.e., the direction of the force along the line of action.

2. Unless otherwise stated, the angle of inclination will mean, throughout this book, the acute angle which the line of action makes with the vertical. This angle A may be expressed in angular units or in linear units, as indicated in Fig. 4 (*a*).

3. The sense of a force is indicated by an arrow placed on or along the line of action. If either the inclination or the sense of a force is unknown, the direction is not fully known. Thus, in Fig. 4 (*a*), no arrow is shown and consequently it is impossible to say whether the force is acting upward or downward, to the right or to the left.

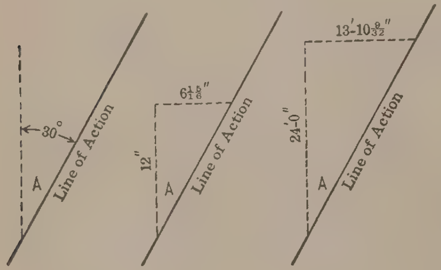


Fig. 4 (*a*).

4. *Note:* The student is cautioned against using the term *direction* in place of either *inclination* or *sense;* direction includes both. For example, if the inclination of the line of action of a force is known but the sense is unknown, it is incorrect to say that the direction is wholly unknown since the direction is in reality partly known.

5. *Note:* If a force is designated by two letters, one at each end of a definite portion of its line of action, the sense may be expressed by the order in which these letters are given. For example, the order *AB* means that the sense is from *A* to *B* as shown in the first line of Fig. 4 (*b*), whereas the order *BA* means that the sense is from *B* to *A* as indicated in the second line. This difference between *AB* and *BA* is often very important.

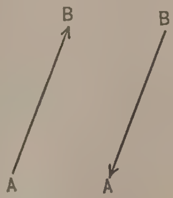


Fig. 4 (*b*).

6. **Point in the Line of Action** (P). To determine completely the line of action of a force, not only must the inclination of the force be known, but at least one point in its line of action must also be known. For example, the inclination of a force *B* may be given as 30° to the right of the vertical (Fig. 4 (*c*)), yet no point in its line of action may be known and hence the force could be acting in any one of an infinite number of lines of action inclined at 30° to the vertical parallel to *B*. If, however, one point, as *P*, is given in addition to the inclination, the line of action is completely determined since it must pass through *P*. An unknown inclination may frequently be determined by finding two points in the line of action.

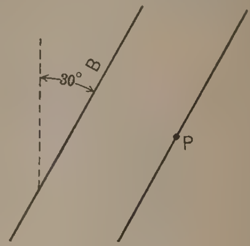


Fig. 4 (*c*).

7. *Note:* The term **point of application,** frequently used in mechanics to designate any point in the line of action, implies that particular point of contact at which one body acts upon another. For this reason the more general term **point in the line of action** is to be preferred.

8. **Line of Action** (denoted by " L "). The line of action of a force includes the element P and part of the element D, namely *inclination;* it does not include *sense*, the other part of D.

9. The line of action of a force may be known yet the direction be partly unknown since the sense may be unknown.

10. The direction of a force may be known yet the line of action be unknown since no point in that line may be known (as in Fig. 4 (*c*)).

11. **Unknowns.** If it is required to determine a force that is completely unknown, three **unknown elements** must be found, i.e., magnitude (M), direction (D), and a point (P) in the line of action. It will be shown later that these three elements may be determined algebraically by means of three different equations. Since an unknown direction includes two unknowns (inclination (A) and sense), it would seem as if in reality four unknown elements were involved (M, P, A, and sense), but sense may always be determined by the algebraic signs which result from the solution of the equations just referred to, so that the unknowns to be determined by equations are only M, A, and P; hence, only three equations are necessary. A fourth or separate equation is never required to determine sense, and for this reason sense is not considered as a separate element of a force but rather as a part of direction.

12. One element of a force may be known and the other two unknown. For example, P may be known, M and D unknown; or M may be known, but P and D unknown. In such a case only two equations are necessary to determine completely the force.

1. Two elements of a force may be known and only one unknown. For example, M and P may be known and D unknown. In such a case only one equation is necessary to determine completely the force.

2. *Note:* When only the line of action (L) of a force is given and the magnitude and sense are unknown, apparently there are two unknown elements, but in reality there is only one since sense is not an element that requires a separate equation. In other words, if a force is known to act in a given line, the value of M may be found by solving a single equation, the sense will be determined by the algebraic sign of that value, and thus the force itself will be completely determined.

3. **Tension and Compression.** Let a body W be suspended by a rope E which is attached to a body B (Fig. 5 (a)). Taking the rope by itself (Fig. 5 (b)), let the arrows B and W represent respectively the forces exerted on the rope by the body B and the body W. These forces act *away* from the rope, and the stress in the rope caused by these forces pulling in opposite directions is called **tension.** The arrows E and E' (Fig. 5 (c)) represent respectively the directions of the forces exerted on the body B and the body W by the rope, i.e., the force exerted by the rope on the object at either end is *away* from the object.

Fig. 5 (a).

Fig. 5 (b).

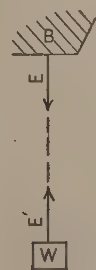


Fig. 5 (c).

4. *Question:* The arrows E and E' seem to indicate that the rope acts in opposite directions at the same time. How can this be true?

Answer: This is a catch question. The rope does not act in opposite directions on either one of the bodies. The two arrows represent the action of the rope on two different bodies.

5. *Question:* Let W and B in Fig. 5 (b) represent two men pulling on a rope, each with a force of 30 pounds. Is the tension in the rope 30 pounds or 60 pounds?

Answer: This question has puzzled many students though the answer is simple. The tension in the rope is 30 pounds. (Why?)

6. Let a body W rest on a post F which stands on a body C (Fig. 5 (d)). The arrows W and C (Fig. 5 (e)) represent respectively the forces exerted on the post by the body W and the body C. These forces act *toward* the post and the stress in the post is called **compression.** The arrows F and F' (Fig. 5 (f)) represent respectively the direction of the forces exerted on the body W and the body C by the post, i.e., the force exerted by the post on the object at either end is *toward* that object.

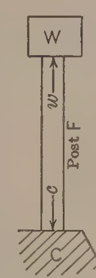


Fig. 5 (d).

Fig. 5 (e).

Fig. 5 (f).

7. **Tension Members and Compression Members.** A **truss** is a framed structure composed of straight members so joined as to form a rigid framework. The simplest form of truss is a triangle. A triangle is a rigid figure because its shape cannot be changed without changing the length of one or more of its sides. A rectangle is not a rigid figure, but may be made so by inserting a diagonal, thus dividing it into two triangles.

8. The triangle is the truss element, and each of the different types of standard trusses is an assemblage of triangles. A **member of a truss** is that part of the truss which extends between two **apices** or **joints.**

9. The body W (Fig. 5 (g)) is supported by a simple triangle-truss which rests on two supports A and B. The lines ac, cb, and ab (Fig. 5 (h)) represent the lines of action of the stresses in the members of the truss and the diagram is called a **truss diagram.** The arrows A, W, and B represent, respectively, the forces exerted on the triangle by support A, body W, and support B — all external forces to the truss. The stress in either of the inclined members is compression and the stress in the horizontal member is tension. An arrow on a line of action near a joint (Fig. 5 (h)) indicates the sense of the force exerted on that joint by the corresponding member.

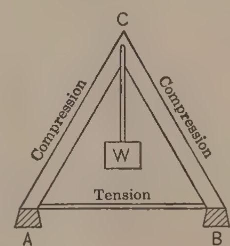


Fig. 5 (g).

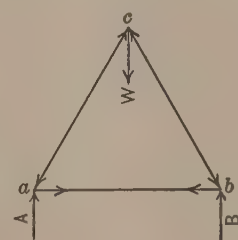


Fig. 5 (h).

Thus, for example, at joint b the horizontal tension member exerts a *pull* on the joint (acts away from the joint) and the inclined compression member exerts a *pressure* on the joint (acts toward the joint).

1. *Note:* A member may be conceived as acting upon the other members at a joint or upon the joint itself considered as a particle.

2. *Summary:* A tension member always acts away from (exerts a pull on) a body to which it is attached, whether that body is at one end or the other of the member.

3. A compression member always acts toward (exerts a pressure on) a body to which it is attached.

4. A truss diagram represents the lines of action of the stresses in the various members of a truss. For each member there is a line corresponding to the axis of the member, and this line represents the line of action of the resultant of all the longitudinal stresses in that member.

5. Any arrow near a joint of a truss diagram indicates the sense of the force which the corresponding member of the truss exerts on that joint — away from the joint if the member is in tension, toward the joint if the member is in compression.

6. The two arrows on any member of a truss diagram, one at either end, are opposite in direction, point toward each other if the member is in tension and away from each other if the member is in compression. (Why?)

7. *Remark:* Keep in mind the fact that an arrow at the end of a member in a truss diagram always indicates what that member is doing to some other body at that end, i.e., the member is either pulling (tension) or pressing (compression) on a joint or another member.

8. **Strain.** Structural materials, such as wood and steel, are elastic, hence members of a truss under stress are actually lengthened or shortened, though usually by an amount imperceptible to the eye. When forces external to a member tend to lengthen that member, the stress in the member is tension; when they tend to shorten the member, the stress is compression. The amount by which a member is lengthened or shortened is called **strain** and it is measured in units of length — usually in decimals of an inch. Strain is often defined in more general terms as the distortion of a body due to the action on that body of external forces. Distortion, however, is not exactly synonymous with strain since it implies change of shape. A sphere for example, compressed evenly into a smaller sphere, is not distorted, though the strain may be obvious. There is no word exactly synonymous with strain. Within the elastic limit, the strain in a member is proportional to the stress in that member.

ASSIGNMENTS

(1) Report on the meanings of the following terms: Ultimate strength; elastic limit; modulus of elasticity; yield point; stress-strain diagram; unit stress; working stress; factor of safety.

(2) Report on the difference between the two principal systems of measurement of force (1) the absolute system and (2) the gravitation system, and tell why the gravitation system is used in engineering computations.

(3) Report on "secondary stresses," how they are caused, how they may be reduced to a minimum, when they should be taken into account in designing.

CHAPTER III

A FORCE AND ITS COMPONENTS

This chapter is a review of the methods of resolving a force into two components and, *vice versa*, of finding a force from its components. These methods are merely simple applications of geometry and trigonometry and are exactly the same as those used in determining many other magnitudes, such, for example, as latitudes and departures in surveying. In stresses it is necessary that the student should be able to use these methods with accuracy and speed. It is assumed that he already knows the principle of the "parallelogram of forces," what a "component" of a force is, and the difference between a "resultant" and an "equilibrant."

1. **The Horizontal and Vertical Components of a Force.** Since a force is a vector quantity, it may be resolved into horizontal and vertical **components** just as any other vector may be. Thus in Fig. 7 (a) if the magnitude and direction of a force F are represented by the length and direction of the line ab, the **H component** F_H and the **V component** F_V will be represented respectively by the lines ac and ad. Hence the magnitudes of the components F_H and F_V may be determined by any method that will give the lengths of the lines ac and ad. There are three such methods in common use, namely, (a) the graphic, (b) the trigonometric, and (c) the geometric.

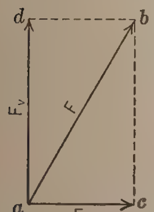


Fig. 7 (a).

2. *Graphic method:* Along the line of action of the force, or on a line parallel to it, lay off the magnitude of the force to any convenient scale, as, for example, the distance ef. Draw the horizontal and vertical projections F_H and F_V; the corresponding magnitudes may then be found by scaling the lengths of these projections.

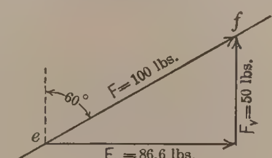


Fig. 7 (b).

3. *Illustration:* A force of 100 lbs. is inclined at 60° as shown in Fig 7 (b). To find the H and V components, lay off 100 lbs. anywhere on the line of action to any convenient scale as 1 inch = 10 lbs., draw the projections F_H and F_V and scale their lengths, $F_H = 8.66'' = 86.6$ lbs.; $F_V = 5.0'' = 50$ lbs.

4. *Note:* If it were necessary actually to plot the angle of inclination, the graphic method might involve more work than either of the algebraic methods. But an outline drawing of a truss or other structure can usually be made to scale quickly from its principal dimensions, and on such a drawing the inclination of each force is given by the inclination of the line that represents the corresponding member of the structure. (See outline of a derrick, page 8.) Simple as the graphic resolution of a force is, it is the basis of a graphic method of determining stresses which is extensively used by engineers.

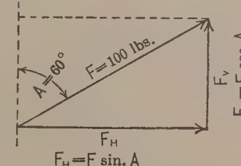


Fig. 7 (c).

5. *Trigonometric method:* When the angle of inclination is given in units of angular measure, the components may be found by trigonometry, as illustrated in the following example (Fig. 7 (c)):

$$A = 60°, \quad F = 100 \text{ lbs.}$$

$$\begin{aligned} F_H &= F \sin A & \qquad F_V &= F \cos A \\ &= 100 \times 0.866 & &= 100 \times 0.500 \\ &= 86.6 \text{ lbs.} & &= 50 \text{ lbs.} \end{aligned}$$

$$\text{Check } \sqrt{F_H^2 + F_V^2} = F = \sqrt{86.6^2 + 50^2} = 100$$

6. *Note:* In this book the angle A is usually understood to mean the acute angle which the line of action makes with the vertical, and therefore corresponds to the angle of bearing in surveying. Hence the H and V components of a force correspond respec-

tively to departure and latitude and may be found by exactly the same trigonometric methods; the only difference is that in stresses the magnitude is expressed in units of force while in surveying it is expressed in units of length. If the H and V components of several forces are to be found by the trigonometric method, standard tables of latitudes and departures may be used to advantage.

1. *Geometric method:* When the inclination of the line of action is given in units of linear measure (as in Fig. 8 (*a*)), the magnitudes of components may be found by proportion, thus: The force in one line is to a length on that line as the force in another line is to the corresponding length on that line, or,

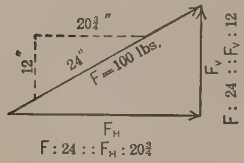


Fig. 8 (*a*).

Force	: Length	: :	Force	: Length
100 lbs.	: 24″	: :	F_H	: $20\frac{3}{4}''$
100 lbs.	: 24″	: :	F_V	: 12″

$$F_H = (100 \div 24) \times 20\tfrac{3}{4} = 86.5. \quad F_V = (100 \div 24) \times 12 = 50 \text{ lbs.}$$

Notice that the first setting of a slide rule is the same for both equations.

2. *Note:* In Fig. 8 (*a*) the slope of the line of action of *F* is expressed as $20\frac{3}{4}$ inches horizontal for every 12 inches vertical. It is the practice in structural drafting to express a slope in such a manner that one of the two dimensions is 12 inches and the other is *less* than 12 inches, regardless of whether the 12 inches corresponds to a horizontal or vertical line. Following this practice, the slope of *F* would be given as in Fig. 8 (*b*), i.e., $6\frac{15}{16}$ inches vertical to 12 inches horizontal.

The values of F_H and F_V, found by the geometric method, will be the same as those obtained above.

Force	: Length	: :	Force	: Length
100 lbs.	: $13\frac{7}{8}''$	: :	F_H	: 12″
100 lbs.	: $13\frac{7}{8}''$	: :	F_V	: $6\frac{15}{16}''$
F_H = 86.5 lbs.			F_V = 50 lbs.	

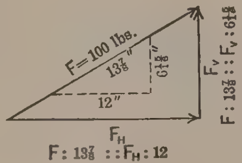


Fig. 8 (*b*).

3. *Note:* In engineering work the inclination of a strut, a rod, a rope, or some other part of a structure is seldom given in degrees or other units of angular measure, but by *dimensions* expressed in feet and inches. For this reason the geometric method is often used. When, however, there are several forces all having the *same* angle of inclination, it is better to use the trigonometric method even though the angle of inclination is given in linear units. The necessary trigonometric functions of the angle of inclination may easily be found by preliminary computations.

4. *Algebraic signs for H and V components.*

H components: Sense to the right, *plus;* sense to the left, *minus.*

V components: Sense upward, *plus;* sense downward, *minus.*

5. *Illustration:* In Fig. 8 (*c*) the algebraic signs of the different components, from inspection, are as follows:

$B_H = +$	$C_H = -$	$D_H = -$	$E_H = +$
$B_V = +$	$C_V = +$	$D_V = -$	$E_V = -$

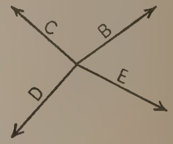


Fig. 8 (*c*).

Note: Notice that these signs correspond to those usually used for departures and latitudes in surveying.

Note: If a force slopes upward and to the left or downward and to the right, its two components cannot have the same algebraic sign.

Illustration: H and V components. In the outline of the derrick in Fig. 8 (*d*) three forces act upon the point *a*, namely rope *A*, boom *B*, and rope *C*. Rope *A* acts downward on *a* but upward on the weight *W*. (Tension, 6 : 2.) Since it is pulling straight

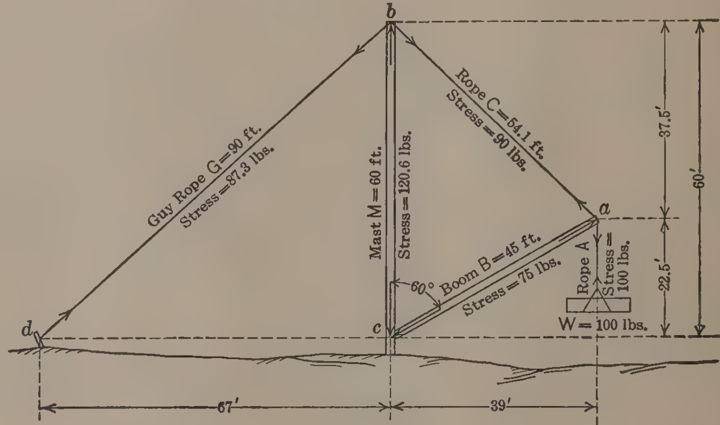


Fig. 8 (*d*).

down on *a*, it tends to move *a* neither to the right nor to the left, i.e., the force which the rope exerts on *a* has no horizontal component, and its vertical component is equal in magnitude to the force itself or 100 lbs.

Boom *B* acts upward and to the right on *a*, but downward and to the left on *c*. (Compression, 6 : 3.) The force which it exerts along its own line of action is 75 lbs.; the

H and V components of this force may be found by either the trigonometric or the geometric method.

Trigonometric	*Geometric*
$B_H = B \sin 60° = 75 \times .866 = 65$	$75 : 45 :: B_H : 39'$ or $B_H = 65$
$B_V = B \cos 60° = 75 \times .50 = 37.5$	$75 : 45 :: B_V : 22.5'$ or $B_V = 37.5$

Thus the boom B could be replaced at a by two members, one in a horizontal line through a exerting a force of 65 lbs. to the right, the other in a vertical line through a exerting a force of 37.5 lbs. upward.

Rope C acts upward and to the left on a, but downward and to the right on b. The H and V components of the force in the rope (90 lbs.) are best found by the geometric method since the angle of inclination is not given in angular measure.

1. *Problems in finding H and V components:*

(1) From the data given in Fig. 8 (d), check the algebraic signs and the magnitudes of the H and V components given in the table, using in each case the method designated in the column headed "method."

Member	*Stress*	*Method*	*H Component*	*V Component*
		Forces Acting on the Point a		
Rope A	100		0	−100
Boom B	75	Trigonometric	+65	+37.5
Rope C	90	Geometric	−65	+62.4
		Forces Acting on the Point b		
Rope C	90	Geometric	+65	−62.4
Mast M	120.6		0	+120.6
Rope G	87.3	Geometric	−65	−58.2
		Forces Acting on the Point c		
Mast M	120.6		0	−120.6
Boom B	75	Trigonometric	−65	−62.4

(2) Check the magnitudes of the components in Problem (1) by the graphic method of 7 : 2, first drawing the outline of the derrick and guy rope to some convenient scale, as 1″ = 20′, or 1″ = 16′.

(3) Given: A force of 500 lbs. acting upward and toward the left; angle of inclination is 42°. Required: The H and V components (trigonometric method).

(4) Given: A force of 1000 lbs. acting downward and toward the left; inclination is 12 ft. horizontal to 15 ft. vertical. Required: The H and V components (geometric method).

(5) Check by the graphic method the results obtained in the preceding problem.

2. **To Find a Force from its H and V Components.** There are two general cases, namely, (1) When both components are known; (2) when only one component is known. In the first case a point in the line of action of the required force must be known, and in the second case the line of action must be known.

I. WHEN BOTH COMPONENTS ARE KNOWN

3. *Graphic method:* Let P in Fig. 9 (a) be the only point of the line of action that is known. Through this point lay off, to any convenient scale, the magnitudes of F_H and F_V. F will be the diagonal of the rectangle of which F_H and F_V are two sides. Be careful that the components when laid off have the correct sense. The same result may evidently be obtained by merely constructing a right triangle.

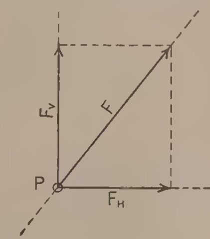


Fig. 9 (a).

4. *Illustration:* The H and V components of a force B in Fig. 9 (b) are respectively +10 lbs. and −16 lbs. A point P in the line of action is known. Find the force B. Through the known point P (Fig. 9 (b)) plot $B_H = +10$ lbs. and $B_V = -16$ lbs. This makes B to the right and downward; its magnitude, found by scaling, is 18.8 lbs.

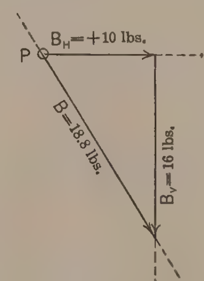


Fig. 9 (b).

5. *Algebraic method:* The magnitude may be found from $F = \sqrt{F_H^2 + F_V^2}$. The angle of inclination A may be found in slope per ft. $F_V : F_H : 12 : x$ or in angular measure from $\tan A = F_H \div F_V$.

6. *Illustration:* In Fig. 9 (b), $B = \sqrt{10^2 + 16^2} = 18.8$ lbs. The slope of B is found from the equation $\frac{10}{16} = \frac{x}{12}$, i.e., $x = 7\frac{1}{2}''$, and the slope of B is $7\frac{1}{2}''$ horizontal to 12″ vertical. If the inclination is desired in angular measure, $\tan A = 10 \div 16 = 0.625$, and $A = 32°$.

II. WHEN ONE COMPONENT AND THE LINE OF ACTION ARE KNOWN

There are three methods, graphic, trigonometric, and geometric; these are merely the converse methods of those explained on pages 7 and 8.

7. *Graphic method:* From some point P in the line of action (Fig. 10 (a)) lay off, to some convenient scale, F_H or F_V, whichever is given, and find the corresponding length of F along the line of action.

1. *Trigonometric method:*

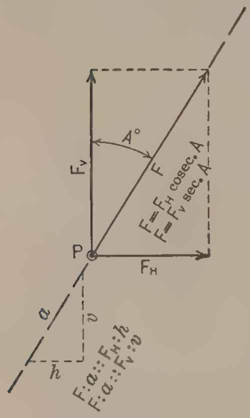


Fig. 10 (*a*).

$$F = F_H \div \sin A \text{ or}$$
$$F = F_H \times \operatorname{cosec} A$$
$$F = F_V \div \cos A \text{ or}$$
$$F = F_V \times \sec A$$

2. *Geometric method:* In Fig. 10 (*a*) let the inclination be given by the distances h and v (both expressed either in feet or in inches).

F (lbs.) : a (feet or inches) : : F_H (lbs.) : h (feet or inches)

$$F = (F_H \times a) \div h$$

F (lbs.) : a (feet or inches) : : F_V (lbs.) : v (feet or inches)

$$F = (F_V \times a) \div v$$

3. *Note:* All three methods are used throughout the subject of stresses. The simple trigonometric and geometrical relations of a force and its components, used in the last two methods and in the corresponding methods of 7:5 and 8:1, should not be regarded as formulas, but should be evident instantly from inspection of a figure in any given case. The sense of each component or the corresponding algebraic sign must always be taken into account.

4. **To Find One Component From the Other Component.** It is sometimes required to find an unknown component directly from a known component without determining the magnitude of the force itself. Thus, in Fig. 10 (*a*) to find F_H from F_V:

Trigonometric method: $F_H = F_V \tan A$

Geometric method: $F_H : h : : F_V : v$ or

$$F_H = F_V \times h \div v.$$

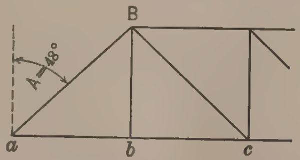


Fig. 10 (*b*).

5. *Problems in finding a force, given one or both of its components.*

(1) The line aB in Fig. 10 (*b*) is the end post of a truss. The angle of inclination is 48°. If the V component of the stress in aB is 60,000 lbs. downward at a, what is the magnitude of the stress? (Trigonometric method.) Is the member in tension or compression? (5:7.) Find the H component directly from the V component.

(2) If in Fig. 10 (*b*), the panel length $ab = bc$ is 21 ft., and the height of truss Bb is 28 ft., and if the V component of the stress in Bc is 44,000 lbs. downward at B, what is the magnitude of the stress in Bc? (Geometric method.) Is the stress tension or compression?

(3) Check by the graphic method the result obtained in the preceding problem.

(4) Given: The truss shown in Fig. 10 (*c*). If the V component of the stress in an inclined member is 36,000, what is the magnitude of the stress? (Geometric method.) What is the magnitude of the H component? (By two different geometric methods.)

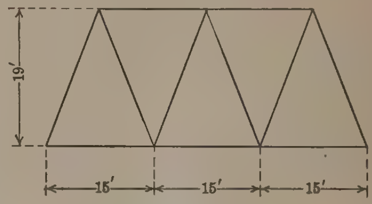


Fig. 10 (*c*).

(5) Check by the graphic method the results obtained in the preceding problem.

6. **Rectangular Components in General.** Any two components of a force that are at right angles to each other are called rectangular components. In stresses, H and V components are the rectangular components most used. Occasionally, however, it is advantageous to resolve a force into rectangular components that are neither horizontal nor vertical. Thus, for example, in Fig. 10 (*d*) the force F, making an angle of 30° with the vertical, is resolved (graphically) into two rectangular components F_X and F_Y, the axes X and Y making angles of 70° and 20° respectively with the vertical. The corresponding trigonometric and geometric methods are those already given in 7:5 and 8:1.

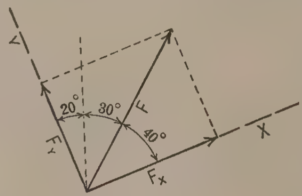


Fig. 10 (*d*).

7. *Note:* The terms X component and Y component are perfectly general and may be used in place of H component and V component respectively, since the X and Y axes may be respectively horizontal and vertical. Objection is sometimes made to the use of the terms H component and V component since they may be somewhat misleading. For example, a force that lies in a horizontal plane has no vertical component though it may have a Y component; in fact, both rectangular components of such a force are, strictly speaking, horizontal since they lie in a horizontal plane. Nevertheless, in such a case the terms H component and V component are commonly used in stresses even though they are inconsistent. The great majority of the problems in stresses, however, involve only forces which lie in a vertical plane, and for such problems the

terms H component and V component are more definite than are the terms X component and Y component.

1. **Oblique Components.** A force may be resolved into *any* two non-rectangular components by methods based on the principle of the parallelogram of forces. There are three such methods, graphic, trigonometric, and geometric, corresponding to the three methods already explained for rectangular components.

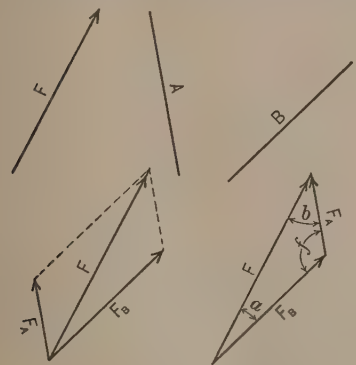


Fig. 11 (*a*).

2. *Graphic method:* It is required to find the components of a force F which are parallel respectively to two lines A and B. Construct the parallelogram with F as a diagonal (Fig. 11 (*a*)), thus obtaining the magnitudes of F_A and F_B, or, more simply, construct the triangle as shown in the figure.

3. *Trigonometric method:* When the angle of inclination of each force is known in angular measure, the triangle may be solved by trigonometry since the side F and two adjacent angles are known.

$$F : \sin f : : F_A : \sin a \qquad F : \sin f : : F_B : \sin b$$
$$F_A = (F \div \sin F) \times \sin a \qquad F_B = (F \div \sin f) \times \sin b$$

4. *Geometric method:* Let the lines in Fig. 11 (*b*) represent members of a framework with lengths as indicated, and let the force F in the diagonal = 100 lbs. The components of F parallel to A and B respectively may be found by proportion by the method already explained for rectangular components.

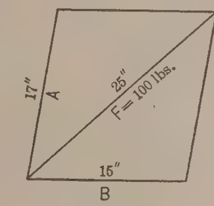


Fig. 11 (*b*).

Force : Length : : Force : Length
100 lbs. : 25″ : : F_A : 17″
$F_A = (100 \div 25) \times 17 = 68$ lbs.
100 lbs. : 25″ : : F_B : 15″
$F_B = (100 \div 25) \times 15 = 60$ lbs.

5. *To find F, given both oblique components, or one oblique component and the line of action of F:* The graphic, trigonometric, and geometric methods are the converse of those just given, and will be evident without further explanation.

Note: It is seldom that oblique components can be used to advantage in stresses.

ASSIGNMENT

(1) Report on the various proofs of the principle of the parallelogram of forces, — experimental and mathematical.

CHAPTER IV

EQUILIBRIUM OF A PARTICLE

The whole subject of stresses is based on three conditions which must be fulfilled if a body is to be in equilibrium. When the body is so small that it may be considered a particle, only two of these conditions of equilibrium must be satisfied. This chapter explains what these two conditions are and how they may be expressed algebraically (by equations) and graphically (by plane figures).

1. **Equilibrium of a Particle.** When several parts or members of a structure come together at a joint, it is desirable to have their axes intersect at a point. Thus, in Fig. 12 the axes of the two ropes and the stick meet at the point a. The forces in the members may then be represented by lines of action which meet at a point as shown in the lower part of the figure. Such forces are called **concurrent forces;** when the lines of action do not meet at a point the forces are **non-concurrent.** The point at which concurrent forces meet may be considered as a **particle,** i.e., the smallest conceivable body.

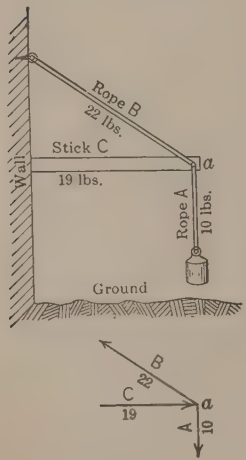


Fig. 12.

2. The axes of rope A, rope B, and the stick C are assumed to be in the same plane, hence the lines of action of the corresponding forces are in a plane (vertical). When the lines of action of any system of forces lie in the same plane, the forces are **co-planar.** In the subject of stresses, it is understood that forces are co-planar unless otherwise specified.

3. Any particle acted on by co-planar forces is said to be in **equilibrium,** if there is no motion of the particle with respect to each of two rectangular axes in the plane of the forces. Thus, for example, particle a moves neither up nor down, right nor left, with respect to the ground and the wall, and is therefore in equilibrium.

4. *Note:* Equilibrium does not mean that there is no motion but merely that there is *no change* of motion. The particle a is moving with the earth, but there is no change of motion. In the subject of stresses equilibrium usually means that a body has no motion relative to the earth. Motion may be of two kinds, **translation** and **rotation.** There can be, however, no rotation of a particle, hence *a particle is in equilibrium if there is no motion of translation.* To determine whether or not there is motion of translation it is best to assume two rectangular axes of reference. These two axes may be any two rectangular axes assumed at random, provided they lie in the plane of the forces that act on the particle. It is customary in stresses, however, to assume one axis horizontal and the other vertical, except in special cases.

Question: Is it essential that the axes of reference should be rectangular?

5. **Conditions of Equilibrium for a Particle.** Rope B is pulling on the particle a (Fig. 12) with a force of 22 lbs., while the stick C is pushing with a force of only 19 lbs., yet a has no motion horizontally. The H component of B must therefore be 19 lbs. to the left in order to equal the H component of C which is 19 lbs. to the right. (Rope A has no H component.) Similarly (since C has no V component) the V component of B must be equal to 10 lbs. upward in order to equal the V component of the rope A which is 10 lbs. downward. In other words, for the H components, $-B_H + C_H = -19 + 19 = 0$, and for the V components, $+B_V - A_V =$

+10 − 10 = 0. (Algebraic signs are in accord with the system of 8:4.) Thus, the algebraic sum of the *H* components is zero and the algebraic sum of the V components is zero.

1. Let the stick *C* in Fig. 12 be replaced by two sticks *E* and *F* (Fig. 13 (*a*)), the forces in ropes *A* and *B* remaining 10 lbs. and 22 lbs., respectively. If *a* remains in equilibrium, the sum of the H components of *E* and *F* must be the same as the force in the stick *C* (Fig. 12), i.e., 19 lbs., and the sum of their V components must equal zero since the V component of the stick *C* was zero. Hence:

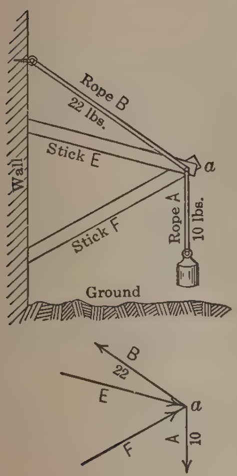


Fig. 13 (*a*).

H components:

$$-B_H + (E_H + F_H) = -19 + (19) = \mathbf{0}$$

V components:

$$+B_V + (-E_V + F_V) - A_V = +10 + (0) - 10 = 0$$

2. Thus, the algebraic sum of the H components of all of the forces is still zero and the algebraic sum of the V components is also zero. These two conditions would evidently hold true for any number of forces acting on *a*, provided *a* remains in equilibrium.

3. It is evident, also, that any particle acted upon by co-planar forces only, will remain in equilibrium as long as there is no motion with respect to each of any pair of axes, X and Y, in the plane of the forces. This means that the algebraic sum of the X components of all of the forces must equal zero, and that the algebraic sum of the Y components of all of the forces must equal zero; if either sum does not equal zero, there will be motion with respect to at least one of the two co-ordinate axes. These two conditions of equilibrium may be expressed both algebraically and graphically.

4. **Conditions of Equilibrium of a Particle Expressed Algebraically.** *The algebraic sums of the X components and of the Y components respectively must each equal zero or*

$$\Sigma X = 0 \qquad \Sigma Y = 0$$

The X and Y axes may be any two lines at right angles to each other in the plane of the forces. In stresses, these two axes are usually assumed one horizontal, the other vertical, hence

$$\Sigma H = 0 \qquad \Sigma V = 0$$

These two equations are known as the **resolution equations of equilibrium.**

5. *Question:* If any number of concurrent forces are in equilibrium, will the algebraic sum of all of their components parallel to any given line be zero?

6. **Conditions of Equilibrium of a Particle Expressed Graphically.** The forces acting in Fig. 12 are represented again in Fig. 13 (*b*). If the components of these forces are laid off in order to any convenient scale (units of force), they will form a rectangle (Fig. 13 (*c*)). One horizontal side will represent the H component to the right, the other horizontal side the H component to the left. One vertical side will represent the V component upward, the other the V component downward. This rectangle is the simplest possible graphic representation of the two conditions of equilibrium of a particle, ΣH = 0 and ΣV = 0.

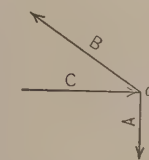


Fig. 13 (*b*).

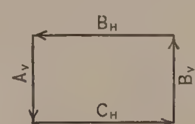


Fig. 13 (*c*).

7. **Force Triangle.** The length of the diagonal in Fig. 13 (*c*) evidently represents the magnitude of the force *B*. Hence if the three forces *A*, *B*, and *C* are laid off to scale (units of force) in any order, end to end and parallel each to its line of action, as in Fig. 13 (*d*), they will form a closed figure called a **force triangle.** The diagram which represents the actual lines of action (Fig. 13 (*b*)) is called a **space diagram.** There are three principles which should be kept in mind with respect to a force triangle, namely:

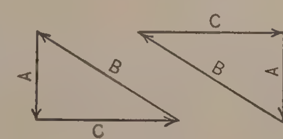


Fig. 13 (*d*).

(*a*). *The length of each line in a force triangle is a measure of the magnitude of the corresponding force in the space diagram.*

(*b*). *The arrows indicating sense all point in the same general direction around the triangle.*

(*c*). *Principles* (*a*) *and* (*b*) *are not affected by the order in which the forces are laid off.* (Both triangles in Fig. 13 (*d*) give the same results.)

1. *Note:* The rectangle (Fig. 13 (*c*)) is not often drawn in stresses though it is the only true graphic representation of the conditions of equilibrium, $\Sigma H = 0$ and $\Sigma V = 0$, applied to three forces. The force triangle was derived, however, from this rectangle and may, therefore, be said to be based upon the two equations $\Sigma H = 0$ and $\Sigma V = 0$.

2. *General case of a force triangle.* One of the forces in Fig. 13 (*b*) is horizontal and another is vertical; a more general case is that in which no one of the three forces is horizontal or vertical, as in Fig. 14 (*a*). If these forces are in equilibrium, they too will form a force triangle when laid off to scale, as is evident from an inspection of the lower part of the figure. The three principles given for the special triangle (Fig. 13 (*d*)) hold true for this force triangle also. The following fundamental principles are the basis of all graphic methods in statics:

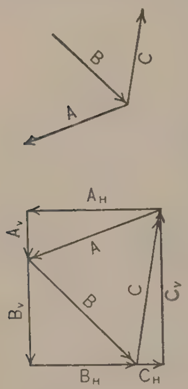


Fig. 14 (*a*).

3. *If three forces are in equilibrium, they may be laid off to scale to form a force triangle.*

4. *Conversely, if three concurrent forces laid off to scale form a force triangle, they must be in equilibrium.*

5. *In a force triangle, any one side represents the magnitude and direction of the* **equilibrant** *of the two forces represented by the other two sides; if the arrow (sense) for any side is reversed that side represents the* **resultant** *of the other two forces.*

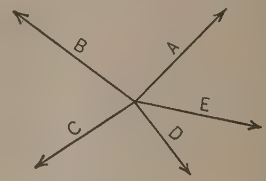
6. **Force Polygon.** Let the space diagram (Fig. 14 (*b*)) represent five concurrent forces in equilibrium. If these forces are laid off to any convenient scale (units of force) each parallel to its line of action and each beginning at the arrow end of the preceding force, there will be formed a closed figure called a **force polygon.** (Fig. 14 (*b*).) The figure must close because, from the two conditions of equilibrium,

$$\text{I.}\quad D_H + E_H + A_H - B_H - C_H = 0$$
$$\text{II.}\quad A_V + B_V - C_V - D_V - E_V = 0$$

FUNDAMENTAL PRINCIPLES PERTAINING TO FORCE POLYGONS

7. The following principles hold true for any number of concurrent forces in equilibrium:

8. *If a system of forces is in equilibrium, the forces will form a force polygon when laid off to scale (units of force) in any order, each parallel to its line of action and each beginning at the arrow end of the preceding force.* (This principle holds true for non-concurrent as well as concurrent forces.)



9. *Conversely, if any number of concurrent forces laid off to scale form a force polygon, these forces must be in equilibrium.* (This principle does not hold true for non-concurrent forces.)

10. *The arrows representing the sense of the forces all point in the same general direction around the polygon.*

11. *Any one side of a force polygon represents the* **equilibrant** *of all of the forces represented by the remaining sides. If the arrow on any one side be reversed so that it points in the opposite direction around the polygon from all of the other arrows, that side will then represent the* **resultant** *of all of the other forces.*

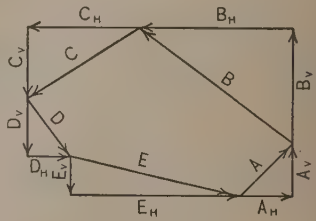


Fig. 14 (*b*).

12. *If from any apex of a force polygon a line be drawn across the polygon to any other apex, dividing the polygon into two parts, that line will represent the magnitude of the resultant and of the equilibrant of both sets of forces in the polygon with which it forms a closed figure.*

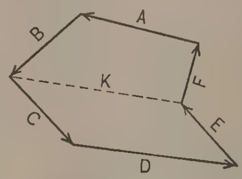


Fig. 14 (*c*).

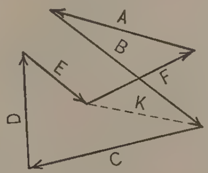


Fig. 14 (*d*).

Illustration of Principle: The force polygon may be one in which no two sides meet except at an apex or it may be one in which two sides cross each other. Both kinds are shown in Figs. 14 (*c*) and 14 (*d*), which correspond to two different systems of forces in equilibrium.

In both figures the broken line K represents the magnitude of both the equilibrant and the resultant of two sets of forces, i.e., F, A, and B on the one hand and C, D, and E on the other.

1. *Questions:* (1) If in Fig. 14 (*c*) K represents the equilibrant of F, A, and B, in what direction must the arrow on K point?

(2) Will the arrow point in the same or in the opposite direction if K represents the equilibrant of C, D, and E?

(3) What line, if drawn, would represent the magnitude of the equilibrant of D and E or of F, A, B, and C? If it represents the equilibrant of F, A, B, and C, in what direction will the arrow point?

(4) What line, if drawn, would represent the resultant of A, B, and C, and what would be the direction of its arrow?

(5) Answer all four questions just asked but refer to Fig. 14 (*d*) instead of to Fig. 14 (*c*).

2. *Analogous case in surveying.* A polygon formed by the transit lines of a survey is analogous to the force polygon. If the sum of the east departures equals the sum of the west departures ($\Sigma H = 0$), and the sum of the north latitudes equals the sum of the south latitudes ($\Sigma V = 0$), the survey is said to close, i.e., the lines, if plotted to scale, will form a closed figure or polygon.

3. *The order in which the forces in a force polygon are laid off is immaterial.* (This statement the student may prove for himself.) (13 : 7 (*c*).) They are usually laid off, however, in the order (either clockwise or counter-clockwise) in which they occur in the space diagram.

4. **Space Diagram and Force Polygon Compared.** The difference between a space diagram and the corresponding force diagram or polygon is important. The two diagrams may be compared as follows:

Space Diagram (Fig. 14 (*b*))	*Force Polygon* (Fig. 14 (*b*))
(*a*) The lines represent the actual lines of action.	(*a*) The lines are not the actual lines of action, but are parallel, respectively, to the actual lines of action.
(*b*) The lengths of the lines have no significance — they may all be of the same length.	(*b*) The length of each line represents the magnitude of the corresponding force.
(*c*) If any scale is used, it is merely for the purpose of plotting the inclinations of the different forces and it is in units of angular or linear measure.	(*c*) The scale used is in units of force, as for example, 1 inch = 100 lbs., or 1 inch = 5 tons.
(*d*) A space diagram, as its name implies, represents spatial relations or the positions of lines of action with respect to each other.	(*d*) The force polygon, as its name implies, represents magnitudes of forces as well as the directions of their lines of action.

5. **Special Cases of a Particle in Equilibrium.** There are four special cases of a particle in equilibrium. In each of these cases, the lines of action of the forces that hold the particle in equilibrium are so located with respect to each other that there is a fixed relation between the magnitudes of the forces, which may be expressed in the form of a special principle. These four special principles, used repeatedly in stresses, may be stated as follows:

6. *If two and only two forces act upon a particle, these two forces must have a common line of action, be equal in magnitude and opposite in sense.* (Why?)

7. *If there are apparently three and only three lines of action which meet in a point, and if two of these lines are in the same straight line, there can be no force in the third, and the forces in the first two lines must be equal in magnitude and opposite in sense.*

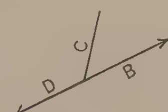


Fig. 15 (*a*).

By taking the X-axis parallel to the forces D and B in Fig. 15 (*a*) prove that there can be no force in C, and that forces D and B must be equal in magnitude and opposite in sense.

8. *If four forces act on a particle and these forces act in two and only two straight lines, any two that act in the same line must be equal in magnitude and opposite in sense.*

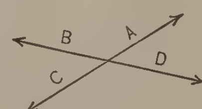


Fig. 15 (*b*).

Prove that in Fig. 15 (*b*), A must equal C and be opposite in sense, and that B must equal D and be opposite in sense.

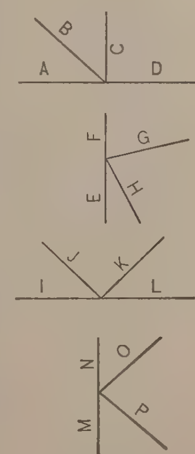


Fig. 15 (*c*).

9. *If four forces hold a particle in equilibrium and two act in the same horizontal line, the V components of the other two must be equal in magnitude and opposite in sense; similarly, if two of the four forces act in the same vertical line, the H components of*

the other two must be equal in magnitude and opposite in sense. If two of the four forces act in either the same horizontal or the same vertical line and the other two have equal inclinations, the two inclined forces must be equal in magnitude, but opposite in sense.

Prove that in Fig. 15 (*c*):

$$B_V = C_V \text{ and } G_H = H_H$$
$$J = K \text{ and } O = P$$

(Assume that the inclination of J is equal to that of K and that the inclination of O is equal to that of P.)

1. The last principle just given may be stated in a more general form as follows:

If four forces A, B, C and D hold a particle in equilibrium and the lines of action of any two as A and B are in the same straight line, the two components normal to that line of the remaining two forces C and D must be equal in magnitude and opposite in sense. If the lines of action of these last two forces C and D are equally inclined to the common line of action of the first two A and B, the last two forces are equal in magnitude.

2. **Use of the Resolution Equations of Equilibrium.** In writing any equation of equilibrium, it is well to enter all quantities (components or expressions for components) on one side of the equation and zero on the other.

3. When there is only one unknown component in a resolution equation, that unknown is given a plus sign. This is equivalent to assuming the component to act to the right if it is an H component or upward if it is a V component (8 : 4). If upon solving the equation the result is plus, the assumption was correct; if minus, the assumption was incorrect. Hence:

4. In solving $\Sigma H = 0$ a plus result indicates that the sense of the unknown component is to the right; a minus result indicates that the sense is to the left.

5. In solving $\Sigma V = 0$ a plus result indicates that the sense of the unknown component is upward; a minus result indicates that the sense is downward.

6. When there are two unknown components in each of the two resolution equations $\Sigma H = 0$ and $\Sigma V = 0$, these two equations must be solved as simultaneous equations. In that case the algebraic sign of an unknown component in either equation is not always assumed to be plus; on the contrary, the sign of each unknown component is assumed in accordance with what is thought to be the sense of the corresponding force. (Frequently the sense of this force is known.) If a result obtained by solving the equation is plus, the sense assumed for the corresponding force is correct; if minus, the sense is opposite to that assumed. Note that if a force acts upward and to the left, or downward and to the right, its two components cannot have the same algebraic sign.

ASSIGNMENT

Formulate and report the conditions of equilibrium of a particle where the forces which act on the particle are not all in one plane.

CHAPTER V

GENERAL METHOD OF ATTACK

The aim in this chapter is to develop a general method of attack which will simplify the solution of all problems in statics, particularly problems in stresses.

1. **Unknown Elements.** A body in equilibrium is acted on by a system of two or more external forces. For each of these forces, there are three elements, namely, magnitude (M), direction (D), and a point (P) in the line of action. One or more elements may be unknown at the beginning of a problem; but if more than three are unknown, the problem is statically indeterminate. Often it is indeterminate if more than two are unknown. These unknowns may be elements of one force or of two different forces, or of three different forces. For example, the magnitude, direction, and point in the line of application of one force may be unknown, in which case the force is wholly unknown; or the magnitude of one force and the magnitude and direction of another force may be unknown; or the magnitude of one force, the magnitude of a second force, and the magnitude of a third force may be unknown. A considerable number of such combinations of unknown elements is possible and the solution of a problem is determined by the number and combination of these unknowns.

2. **Classification of Problems.** Problems may be divided into three classes according to whether all the forces acting on the body in equilibrium are (1) **concurrent,** (2) **parallel,** or (3) **non-concurrent.** It will be shown later that in the subject of stresses there are only *eight* different types of problems which commonly arise, *two* in concurrent forces, *two* in parallel forces, and *four* in non-concurrent forces. Any given problem will fall usually under one of these **eight cases** according to the number and nature of the unknown elements. If a standard solution for each of these eight cases can be learned, and if it can be determined under which case any given problem falls, the whole work of solving problems is greatly simplified. To determine under which case a given problem falls, it will be necessary first to consider whether the system of forces is concurrent, parallel, or non-concurrent, and then to observe what elements are unknown. *The unknown elements will determine the case.*

3. *Note:* At first thought, parallel forces appear to be non-concurrent forces. If the lines of action of parallel forces were limited in length, these lines might be considered to be non-concurrent, but since the lines of action are infinite in length and are parallel, they meet at infinity, and hence, in a mathematical sense, they are concurrent. This is no mere academic question, because, as a matter of fact, in the solution of practical problems parallel forces cannot be treated as non-concurrent. For example, in a statically determinate problem in non-concurrent forces there can be as many as *three* unknown elements, whereas in a statically determinate problem in parallel forces there can be only *two* unknown elements. Problems in parallel forces also differ from problems in concurrent forces. For example, in problems in concurrent forces a point in any line of action *cannot* be an unknown element, whereas in problems in parallel forces a point in a line of action *can* be an unknown element. It is therefore advantageous to treat problems in parallel forces as belonging to a separate class from problems in either concurrent or non-concurrent forces.

4. **General Method of Attack.** In any general method of attack, whether it be a problem in stresses or a problem in any other type of engineering work, there are three main steps, namely: (1) To determine what is known, (2) to determine what is unknown or required, and (3) to solve the problem by working from the known to the unknown by some correct and efficient method. If the first two steps can be carried out correctly, the third step is usually simple and often largely mechanical.

In a problem in statics (or in stresses) the body in equilibrium is known

and certain forces that act on that body are known or partly known; certain other forces which act on the body are unknown or partly unknown. The method of working from the known to the unknown is by means of equilibrium equations or their graphic equivalents. The general method of attack, therefore, may be outlined as follows:

First: Determine the knowns. (*a*) The body in equilibrium; (*b*) the forces that are completely known and the known elements of forces that are partly known.

Second: Determine the unknowns. The forces that are wholly unknown or *the unknown elements of forces partly unknown.*

Third: Solve the problem. From the unknowns determine under which of the eight cases the problem falls, and use the standard algebraic or graphic solution for that case, whichever is the more efficient.

The first two steps are for the purpose of determining the exact nature of the problem. If they are carried out correctly they will usually render the third step largely mechanical since this step merely involves the use of equilibrium equations or their graphic equivalents. The remainder of the chapter will be devoted to practical suggestions for carrying out the three steps just outlined.

1. TO DETERMINE THE KNOWNS. (*a*) *To decide which is the body in equilibrium.* In problems in concurrent forces the body in equilibrium is invariably the particle at the intersection of the lines of action of all of the forces, but in problems in parallel forces or in non-concurrent forces it is not always obvious what body should be considered as in equilibrium. The body must be one on which at least one force of known magnitude is acting. Moreover, enough must be known of all the forces that hold it in equilibrium to render the problem determinate, i.e., there must not be more than two unknown elements if the problem is one in concurrent or parallel forces, or more than three unknown elements if the problem is one in non-concurrent forces. A clear conception of just what the body is that is being held in equilibrium is a most essential first step in attacking any problem in statics or stresses.

(*b*) *To decide which are the known forces or partly known forces.* It is not always easy to get a clear conception of the forces that hold a body in equilibrium. Most of the mistakes made in attacking a problem in statics are due to overlooking one or more forces that are acting on the body, or from assuming a force to act differently from the way in which it really does. A force that is practically always acting, though sometimes negligible, is that due to gravity, i.e., the weight of the body itself, and often this force may be represented by a vertical line of action through the center of mass of the body. The body in equilibrium will be in contact with other bodies and it will be necessary to determine as far as possible all the known elements of each of the forces which these other bodies exert. Thus, for example, if a bridge rests upon two abutments, one at each end, these abutments exert forces on the bridge which must be taken into account just as much as the weight of the bridge and any load which may come upon the bridge. It may be necessary to compute from known data some of the external forces on a structure such, for example, as those due to various kinds of loads, but this is merely a part of the work of determining the known forces. It is essential, first of all, to get clearly in mind what forces are acting, then if possible what are their lines of action, and finally what are their magnitudes. When a force is only partly known, the known elements should be noted. For example, it may be that the line of action (L) of a force or even a single point (P) in that line is all that is known of a force, but it is important to note that this much at least is known.

2. If the knowns can be determined correctly in any problem, that problem, in most cases, is practically solved, hence this part of the work should be done with the greatest care. Nothing is more helpful in determining the knowns than a good sketch which represents the body in equilibrium and the external forces which act on that body. While the first impulse of an experienced engineer is usually to make such a sketch, the inexperienced student, curiously enough, is apt to think he can solve the problem just as well without an adequate sketch or without any sketch at all. The result is that the student too often overlooks a force or makes some other mistake at the start which renders his subsequent work on the problem useless. The mere making of a carefully drawn sketch, step by step, usually helps to clear up the whole problem. So important is the sketch that suggestions for making it are given here in considerable detail.

3. *Suggestions for making sketches in order to represent the forces that hold a body in equilibrium:*

First: Make a sketch of the body in equilibrium as a **free body in space,** i.e., without any forces whatever acting upon it.

Second: If the weight of the body is to be taken into account, indicate the corresponding line of action and the magnitude of the force of gravity. (Sometimes the force of gravity is represented by a single vertical line drawn through the center of gravity of the body, sometimes by several vertical lines drawn from different points of the body, such points, for example, as the joints of a truss.)

Third: Observe carefully all bodies that are in contact with the body in equilibrium, and represent, as far as possible, the lines of action and the magnitudes of the forces that these bodies exert on the body in equilibrium. Be careful not to represent a line of action as known when it is really unknown; if a point (P) in a line of action is known but not the line itself, this unknown line may be represented tentatively by a **wavy line** drawn through the known point and in the general position in which it is thought the unknown line will lie.

Fourth: Represent other forces, if any, which may be acting on the body, such for example as forces due to wind.

Fifth: Show on the sketch all data that will be needed in solving the problem, such for example as known magnitudes, inclinations or slopes of lines of action and dimensions which will locate lines of action with respect to each other.

Sixth: The sketch may generally be made free-hand and should be as simple as may be consistent with clearness. In addition to this sketch it is frequently advantageous to draw a space diagram in which *the body in equilibrium and the bodies in contact with it are not represented* but only the known elements of the forces in equilibrium are shown, i.e., magnitudes, lines of action, and given points in unknown lines of action. Where such a space diagram is drawn, it may be better to show the data referred to in the fifth step on this space diagram instead of on the original sketch.

1. An arrow should be placed on each line of action when both the magnitude and sense of the corresponding force are known. It is usually better not to put an arrow on a line of action when the magnitude of the corresponding force is unknown even though the sense seems to be apparent. The omission of the arrow indicates that the magnitude is unknown, and moreover, helps to avoid mistakes which are too frequently made by wrong assumptions of sense. If the magnitude of a force is unknown, nothing is gained by assuming the sense.

2. **To Determine the Unknowns.** The second step in the general method of attack, namely that of determining the unknown elements of the forces that hold a body in equilibrium, can be easily carried out if the first step of determining the knowns has been completed correctly. For example, if in the first step a force B is seen to be completely known, force B will, of course, be disregarded when determining the unknowns; if the line of action (L) of a force C is all that is known of that force, then the magnitude (M) of force C will be all that is unknown; if a point (P) in the line of action of a force E is all that is known of that force, then the magnitude (M) and direction (D) will be the unknown elements of force E. Simple as this second step is, however, it is exceedingly important because from the unknowns will be determined the method of solution of the problem.

3. *Note:* When the direction (D) is an unknown element of a force, two unknowns are really involved, namely the *inclination* (A) of the line of action (L) and *sense*. The latter, however, does not require a separate equation since it is determined by algebraic signs. (4:11.) Hence in determining the direction (D) of a force, the first object should be to determine its line of action (L).

4. **Solution of the Problem.** The third step in the general method of attack is the actual work of solving the problem after it has been analyzed in the first two steps. Having determined from the unknowns under which case the problem falls, one has only to use the standard algebraic or graphic solution for that case. In using the equations of equilibrium or their graphic equivalents, it is most important to make sure that every force acting on the body in equilibrium is taken into account. The omission of a single force will vitiate the entire work. In order to guard against such mistakes and in order to work most efficiently, some good systematic method of procedure should be adopted in the analysis of problems.

5. **Written Analysis.** The form of written analysis which will now be outlined is useful not only as a part of the general method of attack in solving problems in statics, but also as a most effective method of explaining the solutions of different problems in stresses. It will be used throughout this book as a means of bringing the student back to fundamentals whenever he begins the study of a new type of problem. It is

based on the assumption that a problem is to be solved algebraically, but, modified slightly, it will serve also for problems that are to be solved graphically.

1. The written analysis consists of the answers to four questions, namely:

(1) What is the body in equilibrium?
(2) What are the knowns?
(3) What are the unknowns (and the corresponding case under which the problem falls)?
(4) What are the equilibrium equations for solving the problem?

2. The forces that hold the body in equilibrium will be represented by letters. In the statement of knowns a force which is wholly known will be represented by a single letter, but a force partly known will be represented by a letter followed by parentheses in which the known elements of that force are indicated. For example, B means that force B is wholly known, B(M) means that only the magnitude of B is known, B(D) means that only the direction of force B is known, B(P) means that only a point in the line of action of force B is known, B(L) means that only the line of action of force B is known, B(M and D) means that only the magnitude and direction of force B are known, and so on for all the combinations of known elements. A similar method is used for representing unknowns. The accompanying problem will serve to illustrate the form of analysis used.

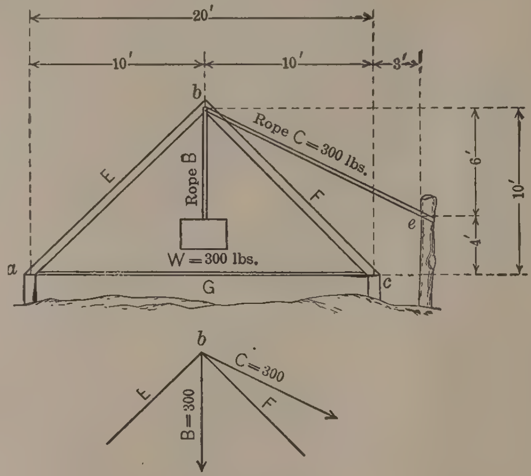
3. *Problem:* A triangular framework *abc* (Fig. 20) is supported at *a* and *c*. A body *W* weighing 300 lbs. is suspended by a rope *B* which passes over a pulley at *b* and is fastened at a point *e*. The size of the pulley and friction may be disregarded; this makes the stress in the portion of the rope marked *C*, 300 lbs., or equal to the stress in the portion marked *B*. Required: To determine by algebraic methods the stresses in members *E* and *F* and the forces exerted on the framework by the supports at *a* and *c*.

(*a*) *Analysis for determining the stresses in E and F.*

Body in equilibrium: Particle *b*.
Known: B, C, E(L), and F(L).
Unknown: E(M) and F(M) (Case B).
Equations: $\Sigma H = 0$ and $\Sigma V = 0$

$$\Sigma H = B_H + C_H + E_H + F_H = 0$$
$$\Sigma V = B_V + C_V + E_V + F_V = 0$$

The problem is one in concurrent forces since the size of the pulley and of the joint *b* are negligible. When in a problem in concurrent forces the unknowns are two magnitudes, the problem falls under Case B and the standard algebraic solution for that case is the use of the two equations



Space Diagram

Fig. 20.

ΣH = 0 and ΣV = 0 as will be explained later. Each of the four forces that act on *b* should be taken into account in writing these equations, as indicated in the equations shown in the analysis. These equations have been written tentatively with plus signs for *all* components; when real values for the components of known forces are substituted, each should be given its correct algebraic sign, and the equations may then be solved in order to determine the unknowns, as explained later. The main thing at present is to observe the general form in which the analysis is written and to understand its real purpose.

(*b*) *Analysis for determining the forces exerted on the framework at a and c.*

Let K and N (Fig. 21 (*a*)) represent the forces that the supports exert on the framework at *a* and *c* respectively.

Body in equilibrium: Framework *abc.*
Known: B, C, K(P), and N(P).
Unknown: K(M, D) and N(M, D). (*Indeterminate.*)

The analysis reveals the fact that there are four unknown elements. A problem in which there are four unknown elements is statically indeterminate. The analysis, therefore, shows, as any good method of attack should, that the problem cannot be solved unless some assumption is made concerning the forces K and N which the supports exert on the framework.

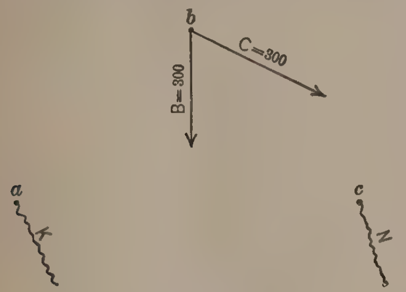


Fig. 21 (*a*).

1. *Note:* Much effort is sometimes wasted in attempting to solve, by methods of statics, problems that are statically indeterminate. An analysis, such as that just illustrated, should reveal at the start whether or not a problem is statically determinate.

2. *Breaking up a problem into several problems.* It is often necessary, as in the problem just analyzed, to subdivide a problem into two or more problems, each of which, complete in itself, will require a separate analysis. In such a case it will be necessary to solve first that problem which is independent of the others. The order in which the problems should be solved is usually evident.

3. *Illustration:* Given: A beam B supported on the ends of two beams A and C as shown. Required: The forces at a, b, c and d caused by a given load W on beam B. This problem will be subdivided into three problems.

First: It will be necessary first to consider beam B as the body in equilibrium since this is the only body on which any force of known magnitude is acting. In this problem the forces that beams A and C exert on beam B can be found.

Second: The beam A may now be considered as the body in equilibrium. The force exerted on beam A by beam B will be equal in magnitude but opposite in sense to the force that beam A exerts on beam B. In this problem the forces that supports a and b exert on beam A (down at a, up at b) may be found.

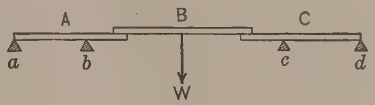


Fig. 21 (*b*).

Third: The beam C may now be considered as the body in equilibrium and the forces at c and d can be found.

Thus the main problem has been broken up into three problems, each complete in itself; for each there would be a separate analysis based on a different body in equilibrium.

4. *Note:* A large amount of practice in the solution of problems may be had by simply writing the analysis of each problem without any computation. It is well to spend considerable time in such practice before actually solving problems. This drill in the method of attack will save time in the end. Eventually it will become unnecessary in solving a problem to write out an analysis, but a *mental* analysis exactly similar to the written analysis should become a fixed habit in attacking any problem in stresses.

5. *Note:* The method of attack outlined in this chapter will be developed more in detail in the succeeding chapters. The student is urged to adhere to this method of attack and especially to adopt the form of analysis for algebraic methods. It is not the intent to compel all students to work in the same way; on the contrary, the aim is to develop freedom of method and confidence in the ability to solve problems. There are, however, certain reasons, which will soon become evident, why the same *general* methods of attack and of analysis should be used and why for the present the student should follow in detail those given in the next three chapters.

CHAPTER VI

CONCURRENT FORCES

The method of attack outlined in the preceding chapter is applied in this chapter to problems in concurrent forces.

1. **The Element P Eliminated.** In any system of concurrent forces at least one point in the line of action of each force is always known, namely, the point at which all lines of action intersect; consequently the determination of P as an element of a force is never involved in any problem in concurrent forces, and P, therefore, may be eliminated from consideration in any discussion of concurrent forces.

2. **Unknown Elements of Concurrent Forces.** Since P can never be an unknown element, only magnitude (M) and direction (D) remain as possible unknown elements in any system of concurrent forces. This means that no force is ever completely unknown in such a system since at least one element (P) is known.

3. **Problems Divided Into Cases.** A problem in concurrent forces is statically indeterminate if more than two elements of forces are unknown; these elements may be elements of the same force or of two different forces. There are, therefore, four and only four possible combinations of unknown elements, namely:

Case A: Magnitude and direction of the same force unknown.
Case B: Magnitude of one force and magnitude of a second force unknown.
Case C: Direction of one force and direction of a second force unknown.
Case D: Magnitude of one force and direction of a second force unknown.

4. Any problem in concurrent forces must fall under one of these four general cases; in any given problem it is merely necessary to determine what the unknown elements are in order to know under which case the problem falls. Using "M" and "D" for magnitude and direction respectively, the four cases may be expressed in tabulated form thus:

Case	*Unknown Elements*
A	M and D (of the same force)
B	M and M (of two different forces)
C	D and D (of two different forces)
D	M and D (of two different forces)

5. *Illustration:* In Fig. 22 three lines of action meet at the point a. Given: The force in the rope A. Required: To find the forces in the rope C and the boom B. Magnitudes of two different forces are required (M and M), hence the problem falls under Case B.

6. Assume the mast E, the guy rope G, and the rope C to be in the same plane, so that the forces at b are not only concurrent but also co-planar. The force in rope C having been found, it is required to find the forces in the guy rope and the mast. This problem also falls under Case B. (Why?)

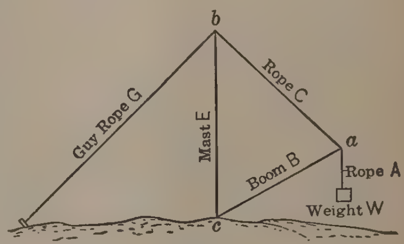


Fig. 22.

7. The forces in the mast and the boom having been found, it is required to find the force the ground exerts on the joint c. The line of action of this force (or reaction, as it is called) is not vertical as at first it might appear, but, on the contrary, its direction is unknown. The magnitude of the reaction is also unknown. Hence, the un-

knowns are magnitude and direction (M and D) of the same force (reaction), and the problem falls under Case A.

1. Suppose that it were required to replace rope G by a different guy rope in the same plane as G, but making a different angle with the mast at b. Suppose also that this angle is unknown, but that the force G must be of a given magnitude. The magnitude of the force in C having been previously found, it is required to find the force in the mast and the direction of the new guy rope. Unknown elements are magnitude (M) of one force and direction (D) of another force, and the problem falls under Case D.

2. Suppose that it were required to replace the rope G by two guy ropes in the same plane as G and the mast, but making unknown angles with the mast. Suppose the forces in rope C and in the mast at b are known, that the magnitudes of the forces in the two guy ropes are specified, but that the directions of these two ropes are to be determined. The unknown elements are two directions of different forces (D and D) and hence the problem falls under Case C.

3. *The two most common cases in concurrent forces* are Case A and Case B. The other two cases seldom occur in the subject of stresses. Thus, for example, it was difficult to devise problems in connection with the derrick, Fig. 22, which would fall under either Case C or Case D, and those given for these two cases are somewhat far-fetched and improbable. It will be seen later that all four cases correspond to the four cases of omitted measurements in surveying.

4. *Object of grouping problems into cases.* It is helpful to know that there are only four different types of problems in concurrent forces and, furthermore, that only two of these four cases are common in stresses. The standard method for solving problems that fall under a given case is easily learned, and then, once the case under which a problem falls has been determined from the unknowns, it remains merely to apply the standard solution for that case.

5. *Statically indeterminate problems.* A problem is statically indeterminate when it is impossible to determine by methods of statics *all* of the unknown elements. This does not mean, however, that *none* of the unknowns can be determined. For example, in the special case of 16 : 1, two of the four concurrent forces in equilibrium have a common line of action. If the magnitudes of these two forces and the magnitude of one of the remaining forces are unknown, the problem is indeterminate because there are three unknown elements, but it is possible, nevertheless, to determine one of these unknowns. (Which?)

6. **General Method for Solving Problems.** Since a problem in concurrent forces cannot involve more than two unknowns, only two equations are ever necessary to solve such a problem. *These two equations are the same for all four cases*, namely, the two resolution equations of equilibrium of a particle, $\Sigma H = 0$ and $\Sigma V = 0$ (or $\Sigma X = 0$ and $\Sigma Y = 0$). the article is the point at which the lines of action of the concurrent forces intersect. For every algebraic method there is a corresponding graphic method based upon the force polygon.

7. *Note:* In certain problems it may be advantageous to change the system of concurrent forces to a system of non-concurrent forces by a method to be explained later; if this is done, a third and entirely different equation of equilibrium can be used.

8. **General Method of Attack.** The general method of attack has been outlined in the preceding chapter, but in applying this method to problems in concurrent forces the following suggestions should be kept in mind:

9. *The body in equilibrium* is invariably the particle at the intersection of the lines of action of the forces. In stresses this particle is usually a joint of a structure where two or more members meet. If there are several such joints, attack one where the magnitude of at least one force is known and where there are not more than two unknown elements.

10. *Sketches and space diagrams.* Problems will occur for which a complete sketch will be advantageous, but more often a simple space diagram, which shows merely lines of action, will be all that is necessary. When the line of action of a force is unknown, it may be represented by a wavy line drawn approximately where the true line is supposed to be. Do not put an arrow on a line of action when the magnitude of the corresponding force is unknown. Dimensions, angles of inclination, known magnitudes of forces, and other known data involved in the solution of the problem may be indicated on the space diagram.

11. *An unknown* is always either a magnitude (M) or a direction (D). The first object in determining a direction (D) is to determine the line of action (L), i.e., to determine the angle of inclination (A) expressed either in units of angular measure or in units of linear measure. The other unknown part of direction, namely, *sense*, will not require a separate algebraic equation or its graphic equivalent. (Why?) There can be only two unknowns in a problem that is statically determinate, and it is usually obvious what these unknowns are.

12. *Method of solution.* When it has been determined what the unknown elements are, it will be evident under which of the four cases the problem falls. The method of solution, based upon the two resolution equations

of equilibrium, then becomes largely mechanical, whether it be algebraic or graphic.

1. **General Method of Procedure for Algebraic Solutions.** The following general method of procedure for problems in concurrent forces is recommended:

First: Make a good sketch or space diagram.

Second: Analyze the problem, setting down the knowns and unknowns as outlined in the method of written analysis (19 : 5).

Third: Indicate completely the two resolution equations of equilibrium. Make sure that all forces that appear among the " knowns " and " unknowns " are included in these equations, and then solve. *Do not indicate or include in these equations any trigonometric or geometric computations or any other work that would tend to obscure the two simple conditions of equilibrium.*

2. *Note:* The student is advised to write out, first of all, the equilibrium equations in general form, using letters to represent forces, omitting all values and giving a plus sign to each component as, for example:

$$\Sigma H = A_H + B_H + C_H + D_H = 0$$
$$\Sigma V = A_V + B_V + C_V + D_V = 0$$

The object in writing these preliminary equations is to make sure that every force is included, and this may be done by merely observing what forces were set down in the written statement of "knowns" and "unknowns" — it is not even necessary to look at the sketch or space diagram if the "knowns" and "unknowns" have been correctly indicated. The various components of the known forces may then be determined, and the value of each component with its correct algebraic sign may be entered in the equation in which it belongs. Very likely some of the components will be found to equal zero. Components of unknowns are given plus signs. An exception to this statement will be explained later (28:4). Enter all quantities on one side of the equation and zero on the other, in order to avoid mistakes in algebraic signs. (16:2 to 6.)

3. *Note:* The H and V components may be found by any one of the three methods explained in Chapter III. This work should not be undertaken until after the entire problem has been outlined, when it can be carried out more or less mechanically by the use of tables or of the slide rule or of both. The object of leaving all such computations until the end is two-fold: (1) in order to make sure that the general method of solution is correct, otherwise computations are wasted, and (2) in order to carry through like operations with undivided attention, thus gaining both speed and accuracy. It is well to keep all such work apart by itself, entering only the *final* value for each component just above or below the corresponding term in one of the preliminary equilibrium equations. The final step is the solution of the equations.

4. **General Method for Graphic Solutions.** The general method of attack is the same as that just explained for the algebraic solutions, except that in place of the equilibrium equations a force polygon is used. The data may be entered upon the preliminary free-hand sketch, which may be drawn as the first step. This sketch may frequently be omitted, but if it is not perfectly evident what forces are involved, the making of the sketch will often clear up the whole problem, while the sketch itself is a help in drawing the space diagram quickly and accurately.

5. *The space diagram:* The space diagram should be drawn carefully to scale. Upon the accuracy of this diagram will depend to a large degree the accuracy of the final results. Any inclined line of action in the space diagram that is short compared with the other lines may be laid off to a larger scale in order that the corresponding line in the force diagram may be drawn parallel to it with greater accuracy.

6. *The force polygon:* The scale used for the force polygon will depend upon the degree of accuracy required. If values are to be scaled to the nearest hundred pounds, then the smallest division on the scale should represent not more than fifty pounds. The scale will also depend somewhat on the magnitudes of the largest forces, and the limits of the paper. A scale of 1 inch = 100 lbs. or 1 inch = 1000 lbs. or 1 inch = 10,000 lbs. is convenient, but any decimal scale such as 1 inch = 500 lbs. or 1 inch = 300 lbs. may be used if a triangular scale similar to that used in mapping is at hand.

7. *Accuracy:* The usual precautions for accurate drawing should be observed, namely, fine lines, clear-cut intersections, and accurate use of instruments. Triangles and straight edges should be tested for accuracy.

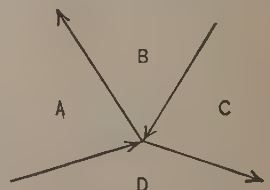
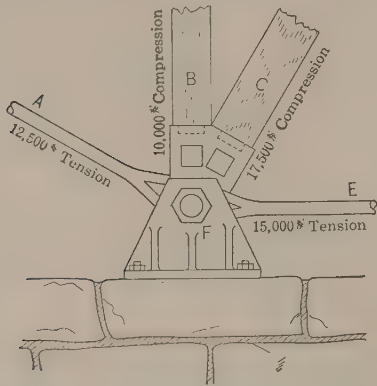


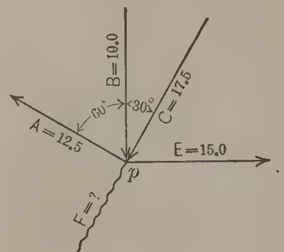
Fig. 24.

8. *Bow's notation:* When the graphic solution is used, the forces in the space diagram are usually designated according to a system known as Bow's notation. This consists merely in designating a force by two letters (or numbers) one on either side of its line of action instead of one at either end. Thus, for example, in Fig. 24 the force whose line of action lies between A and B is the force AB (or BA), the force whose line of action is between B and C is BC (or CB), and so on.

1. **Illustrative Problems in Cases A and B.** A typical problem in Case A (M and D of the same force unknown) and a typical problem in Case B (M and M of two forces unknown) will now be solved in order to illustrate the algebraic and graphic method for each of these two most common cases in concurrent forces. In each problem, the student should study the analysis first and the actual solution afterward, with a view of mastering the general method of attack which is practically the same for both cases. (See pages 26 to 29.)

ILLUSTRATIVE PROBLEM IN CASE A, — MAGNITUDE AND DIRECTION OF THE SAME FORCE UNKNOWN.





Space Diagram

Four members (A, B, C, and E) of a framework meet at a pin which is supported by a shoe F. The stresses in the four members (in thousands of pounds) and the angles of inclination are given in the space diagram in which the pin is shown as a particle p. Required: The force F which the shoe exerts on the pin.

Analysis.

Body in equilibrium: Particle p.
Known: A, B, C, and E.
Unknown: F (M and D) (Case A).
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.

Algebraic Solution.

$$\Sigma H = A_H + B_H + C_H + E_H + F_H = 0$$
$$= -10.8 + 0 - 8.8 + 15.0 + F_H = 0$$
$$+4.6^{\#} = F_H \longrightarrow$$
$$\Sigma V = A_V + B_V + C_V + E_V + F_V = 0$$
$$= 6.3 - 10.0 - 15.2 + 0 + F_V = 0$$
$$+18.9^{\#} = F_V \uparrow$$
$$19.5^{\#} = F = \sqrt{(F_H)^2 + (F_V)^2} = \sqrt{(4.6)^2 + (18.9)^2}$$
$$.243 = \text{Tan. } e = F_H \div F_V = 4.6 \div 18.9$$
$$13°\text{-}40' = e = \text{Angle of inclination of force } F. \nearrow$$

Comments

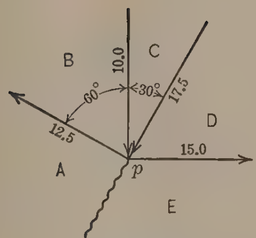
1. The direction of the arrow on any line of action in the space diagram indicates the sense of the corresponding force as it acts on the joint p, that is, toward the joint for compression, away from the joint for tension. The force E is represented by a wavy line since its line of action is unknown.

2. The algebraic solution was indicated completely by means of letters before numerical values were used. The values of components were then calculated (by slide rule) and inserted, each with its correct algebraic sign, underneath the corresponding terms in the equations. (24:3.)

3. When the equations were solved the algebraic sign of F_H was found to be plus, hence F_H acts toward the right; the algebraic sign of F_V is also plus, hence F_V acts upward. The force F, therefore, acts toward the right and upward; its magnitude is 19.5 lbs. and the inclination of its line of action is 13° 40′.

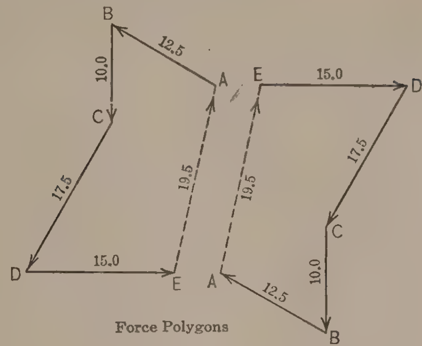
4. *Exercise:* Check by the trigonometric method the values of the components of the inclined forces.

GRAPHIC SOLUTION, CASE A



Space Diagram

Fig. 27 (*a*).



Force Polygons

Fig. 27 (*b*). Fig. 27 (*c*).

First: Draw the space diagram (Fig. 27 (a)). The inclination of each line of action must be plotted accurately and each line should be at least 3 or 4 inches long in order that the corresponding line in the force polygon may be drawn parallel to it with accuracy. Bow's notation is used. (24:8.)

Second: Draw as much as possible of the force polygon by plotting the known forces AB, BC, CD and DE (Fig. 27 (b)). Each line in this force polygon must be drawn accurately parallel to the corresponding line in the space diagram and its length must be laid off accurately to a convenient scale of force units. (14:6.)

Third: *To make the polygon close* it is necessary to connect E and A. This line EA therefore represents the force EA both in magnitude and direction. Since EA is an equilibrant, the arrow from EA must be in the same general direction around the polygon as the other arrows (14 : 10); hence EA acts upward and to the right as indicated.

Comments

1. The regular clockwise sequence of known forces in Fig. 27 (a) is AB, BC, CD, and DE; the regular counter-clockwise sequence is ED, DC, CB, and BA. If the force polygon is drawn to correspond to the clockwise sequence, the polygon shown in Fig. 27 (b) will be the result; if drawn to correspond to the counter-clockwise sequence, the polygon in Fig. 27 (c) will be the result. It is immaterial which of these force polygons is drawn since the magnitude and direction of the unknown force EA (or AE) is the same in both.

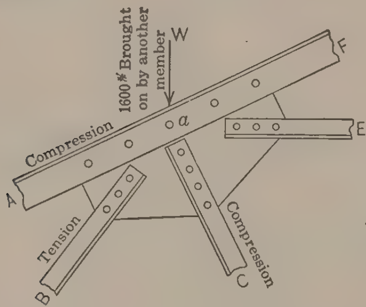
2. Each line in a force polygon is designated by two letters, one at each end. The sequence of these letters corresponds to the sense of the force which the line represents. For example, in Fig. 27 (b) the horizontal force is DE (sense from D to E) but in Fig. 27 (c) the horizontal force is ED (sense from E to D). (4:5.)

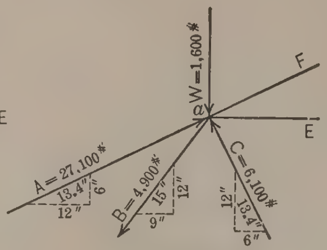
3. It is not essential that the forces be laid off in a force polygon in the sequence, clockwise or counter-clockwise, in which they occur in the space diagram. If they are laid off in any other order, as, for example, BC, DE, AB, and CD, a polygon of different shape will be the result, but the magnitude and direction of the unknown force EA (or AE) will still be the same. Nothing is gained, however, by laying off the forces in irregular order; on the contrary, if the forces are not laid off in the sequence, clockwise or counter-clockwise, in which they occur in the space diagram, the advantage which results from the use of Bow's notation is lost.

4. *Note: Case A* corresponds to the case in omitted measurements in surveying in which the length and bearing of one side are missing. The equations $\Sigma H = 0$ and $\Sigma V = 0$ correspond exactly to the method of calculating the bearing and length of the missing side by the method of latitudes and departures, and the graphic method just explained corresponds exactly to the method of plotting the missing side in mapping.

5. *Problem:* In place of the force polygon shown in Fig. 27 (b) or 27 (c) draw one in which the forces are laid off in the following order: BC, DE, AB, and CD. Observe whether or not the magnitude and direction of EA thus obtained are the same as those obtained in Fig. 27 (b). (Since in this problem Bow's notation cannot be used to advantage, it is suggested that each force be designated by a single letter.)

Illustrative Problem in Case B, — Magnitudes of Two Forces Unknown.





Space Diagram

The sketch shows a joint of a roof truss. The stresses in members A, B, and C and the slopes of all members are given in the space diagram. An external vertical load W of 1600 lbs. brought on by a member in the plane of the roof acts at the joint. Required: The forces exerted on the joint by the members E and F.

Analysis.

Body in equilibrium: Joint a.
Known: W, A, B, C, E(L), and F(L).
Unknown: E(M) and F(M) (Case B).
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.

Algebraic Solution.

$$\Sigma V = W_V + A_V + B_V + C_V + E_V + F_V = 0$$
$$= -1600 + 12\,130 - 3920 + 5460 + 0 + (6 \div 13.4) \times F = 0$$
$$\Sigma H = W_H + A_H + B_H + C_H + E_H + F_H = 0$$
$$= 0 + 24260 - 2940 - 2730 + E + (12 \div 13.4) \times F = 0$$
$$-12\,480^{\#} = 6/13.4 \times F = F_V \downarrow$$
$$27\,000^{\#} = F \swarrow \text{(Compression)}$$
$$+5\,790^{\#} = E = E_H \rightarrow \text{(Tension)}$$

Comments

1. In the space diagram arrows are placed on the lines of action of forces W, A, B, and C since the magnitudes of these four forces are known, but no arrows are placed on the lines of action of forces E and F because the magnitudes of these forces are unknown. (19:1.)

2. The algebraic solution was indicated completely by means of letters before numerical values were used. The values of components were then calculated (by slide rule) and inserted, each with its correct algebraic sign, underneath the corresponding terms in the equations. (24:3.)

3. Since E is horizontal its V component is zero, and hence F_V is the only unknown in the equation $\Sigma V = 0$. By expressing F_V in terms of F the value of F may be found directly from this equation. A more general case would be that in which both E and F are inclined to both co-ordinate axes. There would then be apparently *four* unknowns in the two equilibrium equations, namely, E_H, E_V, F_H, and F_V. But since E_H and E_V can each be expressed in terms of E and since F_H and F_V can likewise be expressed in terms of F, there are in reality only two unknowns, namely E and F. These unknowns can be determined by the method of simultaneous equations. It frequently happens, however, that one component of an unknown force is zero, as in the illustrative problem just explained, in which case the solution of the equations is simplified since it then becomes unnecessary to use the method of simultaneous equations.

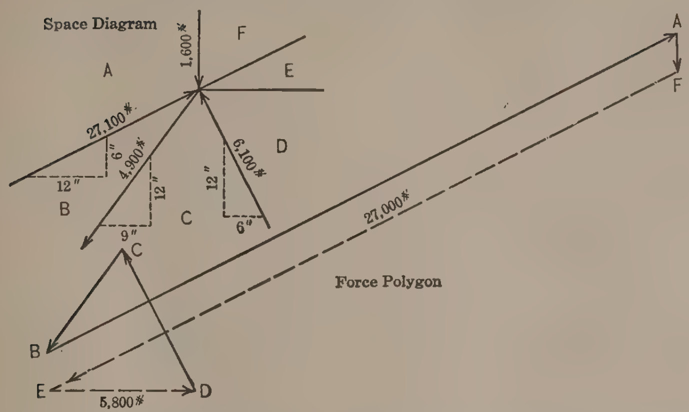
4. *Note:* In the case of simultaneous equations, the sense of each of the unknown forces must be assumed, and the corresponding algebraic signs for the H and V components of these two forces must then be inserted in the equations. If a result obtained by solving the equations is plus, the sense assumed for the corresponding force is correct; if minus, the sense is opposite to that assumed. Note that if a force acts upward and to the left or downward and to the right, its two components cannot have the same algebraic sign.

5. The method of expressing H and V components in terms of the forces themselves was explained on pages 7 and 8. (The geometric method was used.)

6. A problem in *Case B* is indeterminate when the two forces whose magnitudes are unknown have the same line of action or parallel lines of action.

7. *Problem:* Assume one co-ordinate axis (X) parallel to the lines of action of forces A and F and the other axis (Y) at right angles, i.e., parallel to the line of action of force C. Solve the problem by means of the two resolution equations $\Sigma X = 0$ and $\Sigma Y = 0$. (10:6.) It will be found that less work is involved in the use of these two equations than in the use of $\Sigma H = 0$ and $\Sigma Y = 0$. This is an illustration of the advantage that may be gained occasionally in using co-ordinate axes other than the H and V axes.

GRAPHIC SOLUTION, CASE B



First: Draw the space diagram. The inclination of each line of action must be plotted accurately and each line should be at least 3 or 4 inches long in order that the corresponding line in the force polygon may be drawn parallel to it with accuracy. Bow's notation is used. (24:8.)

Second: Draw as much as possible of the force polygon by plotting accurately the known forces *DC*, *CB*, *BA*, and *AF*. Each line in the force polygon must be drawn accurately parallel to the corresponding line in the space diagram, and its length accurately laid off to a convenient scale of force units. (14:6.)

Third: To make the polygon close, it is necessary to draw two lines DE and FE. Proceed as follows: There are two lines of action in the space diagram, *FE* and *ED*, for which the magnitudes are unknown. Choose either one, as for example that which lies between the letters *D* and *E*. One of these letters will be found in that portion of the force polygon drawn in (2), namely, *D*. Through this letter *D* in the force polygon draw a line of *indefinite length* parallel to *ED* in the space diagram. The apex *E* of the force polygon must lie somewhere in this line. Dropping this unknown for the time being, proceed in like manner with the other unknown which lies between *F* and *E*; one of these letters will also be found in the force polygon, namely, *F*. Through *F* in the force polygon draw a line parallel to *FE* in the space diagram. The apex *E* must lie in this line also, and hence the intersection of this line with the line drawn through *D* will be the apex *E* of the force polygon; the two unknown magnitudes are found by scaling the lengths of the sides *FE* and *ED* of the force polygon. To determine the sense of each, follow around the polygon in the same general direction as that indicated by the known forces (14:10); it is found that *FE* acts downward and to the left, or toward the joint (compression), and *ED* acts toward the right or away from the joint (tension).

Comments

1. The graphic method just explained is the most important method in graphic statics. It is used for finding stresses in trusses more than any other graphic method, possibly more than any algebraic method. Its practical application will be explained in Chapter XVI.

2. The method is based on the principle that if a system of forces is in equilibrium the forces will form a closed polygon (14 : 8). After the known forces are laid off to scale, the polygon is made to close by drawing two lines parallel respectively to the two lines of action in the space diagram for which the magnitudes are unknown. The somewhat detailed method of procedure given above for drawing these two lines is more important than appears at first. This will become evident in Chapter XVI. The principal things to remember are that one unknown is to be considered at a time (either one first), and that of the two letters denoting this unknown in the space diagram *one and one only* will already be on the force polygon.

3. In the illustrative example given, the two lines of action for which the magnitudes are unknown are *adjacent* in the space diagram. This is almost invariably the case in practical problems. The graphic solution for a problem in which the unknowns are not adjacent is the same as that just given except in the details of drawing the two missing sides of the force polygon. In such a case, Bow's notation is not as good as one in which each force is represented by a single letter or numeral.

4. *Note: Case B* corresponds to the case in omitted measurements in surveying in which the lengths of two sides of a polygon are missing. The equations $\Sigma H = 0$ and $\Sigma V = 0$ correspond exactly to the calculation of the two missing lengths by the method of latitudes and departures, and the graphic method just explained corresponds exactly to the method of plotting the two missing sides in mapping.

ASSIGNMENT

Be prepared to explain the algebraic and the graphic methods for each of the two remaining cases in concurrent forces, namely, *Case C* in which the unknowns are direction of one force and direction of a second force, and *Case D* in which the unknowns are magnitude of one force and direction of a second force.

CHAPTER VII

EQUILIBRIUM OF A RIGID BODY

The two conditions of equilibrium for a particle were explained in CHAPTER IV. The additional condition of equilibrium for a rigid body is explained in this chapter.

1. **AN ADDITIONAL CONDITION OF EQUILIBRIUM.** The two conditions of equilibrium of a particle expressed by the two equations $\Sigma X = 0$ and $\Sigma Y = 0$ are really equivalent to one condition, namely, that there shall be no **motion of translation.** (12 : 3.) When a rigid body is larger than a particle, there is an additional condition of equilibrium, namely, that there shall be no **motion of rotation.** The measure of the tendency of any force to produce rotation about any fixed point is the **moment** of that force, with respect to that point; how the magnitude and direction of such a moment is obtained is explained in the next article.

2. *Note:* The word motion is used in the restricted meaning explained in (12 : 4). A body can rotate and still be in equilibrium, as for example a pulley rotating about an axle, but in statics "no rotation" usually means no rotation of a structure as a whole or of one part of a structure about another part.

3. **MOMENT OF A FORCE.** The moment of a force always involves not only the force itself but also a center of moments in the plane of the force. Nothing can be stated with regard to the moment of any force until the position of the center of moments is known; how this center of moments is selected or why any particular point is chosen will become evident later on. For the present it will suffice to assume the center of moments at random. The moment of any force may be said to have both **magnitude** and **direction.**

4. *Magnitude of moment.* In Fig. 30 let F be a force and m the center of moments. The shortest distance l from the center of moments m to the line of action of the force F is called the **lever arm** of F with respect to m. The **magnitude of the moment** of F with respect to m is equal to the product of the magnitude of F by its lever arm l, or $F \times l$. This moment is a compound quantity, i.e., *force* multiplied by *distance;* as each of these component parts may be expressed in different units, the moment itself may be expressed in different units. The combinations which are most common are as follows:

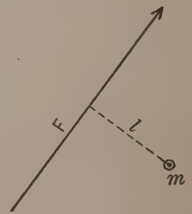


Fig. 30.

Force		*Lever Arm*		*Moment*
Pounds	×	Feet	=	Pound-feet.
Pounds	×	Inches	=	Pound-inches.
Tons	×	Feet	=	Ton-feet.

Henceforth the term *moment* should suggest instantly two ideas: (1) that it is a *product*, and (2) that the product is that of *force* × *distance*.

5. The term *moment* is used also to designate the *algebraic sum of the moments* of two or more forces about a common center of moments. What is really meant in this case is the *resultant* moment. The use of the term in two senses sometimes causes confusion.

6. *Note:* The term **pound-feet** is used in this book in preference to **foot-pounds** because the latter is a term employed in mechanics for an entirely different unit, namely, the unit of work. Moreover, in thinking of a moment it seems to be more natural to think first of the force and then of its lever arm, and this order (force-lever arm) is the order followed in expressing the unit of moment.

7. *Direction of rotation.* If a force tends to produce clockwise rotation about a given center of moments, the moment is called *plus;* if counter-

clockwise, the moment is *minus*. The algebraic sign depends therefore on the **direction of rotation,** and direction of rotation depends not only on the direction of the force that tends to produce rotation but also on whether the center of moments is on one side or the other of the line of action of that force. Thus, for example, in Fig. 31 (*a*) the algebraic sign of the moment of the force *C* will depend on whether the center of moments is on one side or the other of its line of action: if *n* is the center of moments the moment is **minus,** but if *o* is the center of moments the moment is **plus.** On the other hand, the algebraic sign of the moment of any force, as *E*, with respect to some fixed center of moments, as *p*, will depend on the *sense* of the force *E*; if *E* acts downward the moment is **minus,** but if *E* acts upward the moment is **plus.** Thus it is evident that neither the direction of a force nor the position of the center of moments can alone determine the direction of rotation; both must be taken into account.

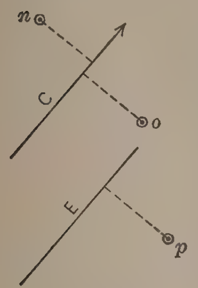


Fig. 31 (*a*).

1. *Note: Choice of algebraic sign:* The decision to call clockwise rotation plus and counter-clockwise minus, or *vice versa,* is purely arbitrary. Unfortunately no general agreement on this point has been reached among engineers, and both systems are in common use. Results are the same whichever system is adopted, provided the same system is used throughout a given problem. It is best, however, for all the students in a class to use one and only one system. The tendency among engineers seems to be toward calling clockwise rotation plus, and hence that practice is followed in this book.

2. *Note:* The fact that a moment of a force has both magnitude and direction, and can therefore be represented as a vector quantity, is often overlooked. The treatment of a moment of a force as a vector permits simple solutions of many problems involving two or more such moments.

3. *Illustrations of Moments of Forces.* The following moments will be evident from inspection of Fig. 31 (*b*):

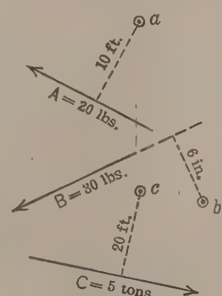


Fig. 31 (*b*).

Force	Center of Moments	Lever Arm	Moment
A = 20 lbs.	a	10 ft.	+ 200 lb.-ft.
B = 30 lbs.	b	6 in.	− 180 lb.-in.
C = 5 tons	c	20 ft.	− 100 ton-ft.

4. *If a force is replaced by any pair of components at any point P in its line of action, the algebraic sum of the moments of these two components (resultant moment), with respect to any center of moments is equal to the moment of the force itself with respect to the same point.* (*Prove.*)

5. **Moment of a Force Determined Graphically.** In Fig. 31 (*c*) let *F* represent a force whose magnitude is 70 lbs., let *m* represent a center of moments, and let the lever arm *l* be 7 ins. long. Then the moment of *F* with respect to *m* expressed algebraically is $F \times l = 70 \times 7 = 490$ lb.-in. To obtain the same result graphically, proceed as follows: Lay off *ab* parallel to *F* and equal in length to 70 lbs. (to any convenient scale). Assume any point *p* at random and draw *ap* and *bp*. The point *p* is called the **pole,** and the lines *ap* and *bp* are called **rays.** The lengths of all lines in the force diagram represent *magnitudes* of forces, and in this particular figure the perpendicular distance *pc* is found (by scaling) to represent 34 lbs.

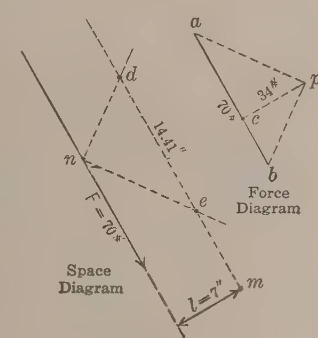


Fig. 31 (*c*).

From any point *n* in the line of action of *F*, draw *ne* parallel to the ray *ap*, and *nd* parallel to the ray *bp*. Through *m* draw a line parallel to *F*, intersecting the two lines just drawn in points *d* and *e*. All lines in the space diagram represent lengths in inches, and *de* is found (by scaling) to be 14.41 ins.

From similar triangles $l : de :: pc : ab$, or $ab \times l = pc \times de$. But $ab \times l$ is the moment of *F* with respect to *m*; hence $pc \times de$ is equal to that moment. Substituting the values as scaled, 34 lbs. × 14.41 in. = 489.94 lb.-in., as compared with the correct values 490 lb.-in.

6. It is evident that the graphic method would not be used for finding the moment of a single force, since in addition to drawing the diagrams it involves a multiplication as laborious as that of the algebraic method; but it is the basis, nevertheless, of an important graphic method used for certain problems that involve several forces.

1. **Moment of a Couple.** Two forces of equal magnitude but opposite in sense, with parallel lines of action, are called a **couple.** The tendency of a couple is always to produce rotation, and the measure of this tendency is the **moment of the couple.** The magnitude and direction of the moment for any given couple is always the same regardless of where the center of moments may be taken. For example, if in Fig. 32 (a) F is equal to B in magnitude, the moments of F and B with respect to points m, n, and o are:

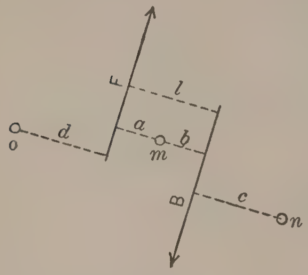


Fig. 32 (a).

Center of Moments	*Moment*	*Moment*
m	$+\mathrm{F}\times \mathrm{a} + \mathrm{B}\times \mathrm{b}$	$= +\mathrm{F}\times(\mathrm{a}+\mathrm{b}) = +\mathrm{F}\times \mathrm{l}$
		$= +\mathrm{F}\times \mathrm{l} = +\mathrm{B}\times \mathrm{l}$
n	$+\mathrm{F}\,(\mathrm{l}+\mathrm{c}) - \mathrm{B}\times \mathrm{c}$	$= +\mathrm{F}\times \mathrm{l} + \mathrm{F}\times \mathrm{c} - \mathrm{F}\times \mathrm{c}$
		$= +\mathrm{F}\times \mathrm{l} = +\mathrm{B}\times \mathrm{l}$
o	$-\mathrm{F}\times \mathrm{d} + \mathrm{B}\times(\mathrm{l}+\mathrm{d})$	$= -\mathrm{F}\times \mathrm{d} + \mathrm{F}\times \mathrm{l} + \mathrm{F}\times \mathrm{d}$
		$= +\mathrm{F}\times \mathrm{l} = +\mathrm{B}\times \mathrm{l}$

2. The perpendicular distance l between the two lines of action of a couple is called the *lever arm* or simply the **arm of the couple.**

3. *The magnitude of the moment of a couple about any point whatsoever in the plane of the couple is equal to the magnitude of either force multiplied by the length of the arm of the couple.*

4. *Note:* The moment of a couple is really the sum of the moments of two forces, and hence the term "moment of a couple" really means "resultant moment of a couple." (30:5.)

5. *If the forces of a couple are the only forces acting on a body that body cannot be in equilibrium.* (*Why?*)

6. **Relation Between Two Resultant Moments of the Same Parallel Forces.** In Fig. 32 (b) six parallel forces are represented. Let points m, n, and o be three different centers of moments. Through these three points, lines are drawn parallel to the given forces. The lever arm for any one of the forces is longer for the center of moments n than for the center of moments m by the distance d. Similarly the **lever arm difference** for centers of moments n and o is d′ and the lever arm difference for centers of moments m and o is d″. The algebraic sum of the moments of the six given forces with respect to any one of the centers of moments is the **resultant moment** of those forces with respect to that point. Let M, N, and O represent respectively the resultant moments of the six forces with respect to the points m, n, and o. Let S represent the algebraic sum of the six forces. Any one of the following relations between two resultant moments may easily be demonstrated.

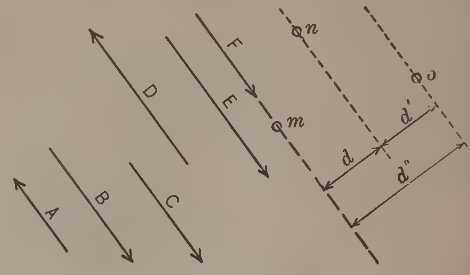


Fig. 32 (b).

$$\mathrm{N} = \mathrm{M} + \mathrm{S}\times \mathrm{d} \qquad \mathrm{O} = \mathrm{N} + \mathrm{S}\times \mathrm{d}' \qquad \mathrm{O} = \mathrm{M} + \mathrm{S}\times \mathrm{d}''$$
$$\mathrm{N} - \mathrm{M} = \mathrm{S}\times \mathrm{d} \qquad \mathrm{O} - \mathrm{N} = \mathrm{S}\times \mathrm{d}' \qquad \mathrm{O} - \mathrm{M} = \mathrm{S}\times \mathrm{d}''$$

7. *For any system of parallel forces the difference in magnitudes of any two resultant moments is equal to the algebraic sum of the forces multiplied by the lever arm difference for the two corresponding centers of moments.* Assuming that a resultant moment N is larger than a resultant moment M, the general principle may be expressed algebraically as follows:

$$N - M = S \times d.$$

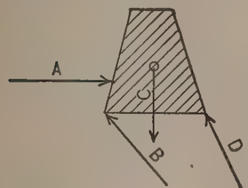
The most common use of the above principle is in obtaining the value of an unknown resultant moment from a known resultant moment. This is done by merely adding to or subtracting from the known resultant moment (according to which resultant moment is the larger) the product $S \times d$.

$$N = M + (S \times d) \quad \text{or} \quad M = N - (S \times d).$$

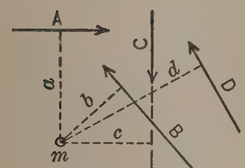
8. **Conditions of Equilibrium for Any Rigid Body.** In the sketch, Fig. 33, are shown four forces (A, B, C, and D) acting on a body. The corresponding space diagram is shown below the sketch. If the

body is to remain in equilibrium the same conditions of equilibrium given for a particle must be fulfilled, namely:

$$\Sigma H = A_H - B_H + C_H - D_H = 0$$
$$\Sigma V = A_V + B_V - C_V + D_V = 0$$



Sketch



Space Diagram

Fig. 33.

In addition, there must be no rotation about any point whatsoever in the plane of the forces. This means that, taking any point at random m as a center of moments, the algebraic sum of the moments of all the forces must equal zero, or:

$$\Sigma M = +A \times a - B \times b + C \times c - D \times d = 0.$$

The three equations of equilibrium just given correspond to the three conditions of equilibrium that are most used in statics. These three conditions may be expressed as follows:

I. *The algebraic sum of the horizontal components of all the forces that act on the body must equal zero.*

II. *The algebraic sum of the vertical components of all the forces that act on the body must equal zero.*

III. *The algebraic sum of the moments of all the forces that act on the body about any center of moments whatsoever in the plane of the forces must equal zero.*

If all three of these conditions are fulfilled with respect to *all* the external forces that act on any rigid body, that body must be in equilibrium.

1. *Note:* The first two conditions are not expressed in the most general form but in terms of the components (H and V) most used in stresses. It should be remembered that there can be no motion of translation if the algebraic sum of the components is zero for first one and then the other of *any* two axes, rectangular or non-rectangular.

2. *Conditions of Equilibrium Stated Differently.* Each of the following two statements of conditions of equilibrium is equivalent to a combination of the three statements previously given, though neither, at first thought, may appear to be so.

IV. *A rigid body is in equilibrium if the algebraic sum of the moments of all the forces that act on the body is zero for each of three centers of moments not in the same straight line.*

V. *A rigid body is in equilibrium if the algebraic sum of the moments of all the forces that act on the body is zero for each of two centers of moments and the algebraic sum of the components parallel to one axis is zero, provided that axis is not perpendicular to the line joining the two centers of moments.*

3. *Exercise:* Prove that each of the statements IV and V is an algebraic identity with statements I, II, and III combined.

4. **Conditions of Equilibrium of Any Rigid Body Expressed Algebraically.** Statements I, II, and III in the preceding article are equivalent to one combination of three equilibrium equations, statement IV is equivalent to another combination, and statement V is equivalent to still another combination. These three combinations may be expressed algebraically as follows:

$$\begin{array}{rl} \text{I} & \Sigma H = 0 \\ \text{II} & \Sigma V = 0 \\ \text{III} & \Sigma M = 0 \end{array} \qquad \text{IV} \left\{ \begin{array}{l} \Sigma M = 0 \\ \Sigma M = 0 \\ \Sigma M = 0 \end{array} \right. \qquad \text{V} \left\{ \begin{array}{l} \Sigma M = 0 \\ \Sigma M = 0 \\ \Sigma H = 0 \quad \text{or} \quad \Sigma V = 0 \end{array} \right.$$

In the first combination there are two *resolution equations* and one *moment equation*, in the second combination there are three *moment equations* (centers of moments must not be in the same straight line); and in the third combination there are two *moment equations* and one *resolution equation.* Since the second and third combinations may each be deduced from the first, the three equations of equilibrium given in the first are fundamental; the whole subject of stresses is built up largely on these three equations ($\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$) or their graphic equivalents.

5. *Two systems of algebraic signs.* In equations $\Sigma H = 0$ and $\Sigma V = 0$ the algebraic sign for any term depends upon the *sense* of the component in that term, i.e., whether in $\Sigma H = 0$ the component acts toward the right or toward the left or whether in $\Sigma V = 0$ the component acts upward or downward. In equation $\Sigma M = 0$, however, the algebraic sign for any term depends upon the *direction of rotation*, i.e., whether the moment represented by that term is clockwise or counter-clockwise.

6. *Note:* These two entirely different systems of signs are a source of confusion. To avoid this confusion, the student should always keep in mind, when using the equation

$\Sigma M = 0$, that the *sense* of a force cannot be determined by the algebraic sign without taking into account *direction of rotation*, which means that the position of the center of moments with respect to the line of action must always be considered. (30:7.)

1. **Use of the Moment Equation of Equilibrium.** A moment equation may be used to determine either one of the two elements magnitude (M) and point (P) in the line of action, but it cannot be used to determine the element direction (D).

2. In determining an unknown element by means of a single moment equation of equilibrium, it is often necessary to eliminate from that equation one or two other unknown elements. This is done by assuming the center of moments in such a position that the lever arm is zero for any force that should be eliminated from the equation. This means that if there is one force to be eliminated, the center of moments may be assumed almost anywhere in its line of action, but if there are two forces to be eliminated the center of moments must be assumed at the point of intersection of their lines of action. The most common cases in which a moment equation of equilibrium may be used are the following:

3. *When the unknown elements are two magnitudes.* Either magnitude may be determined by assuming a center of moments in the line of action that corresponds to the other magnitude.

4. *Illustration:* Five forces, A, B, C, E, and F, hold a body in equilibrium. (Fig. 34 (*a*).) The unknown elements are the magnitudes of the two forces C and F. The force C may be determined from a moment equation by assuming the center of moments in the line of action of F, thus eliminating F, and similarly the force F may be determined by assuming the center of moments in the line of action of C, thus eliminating C.

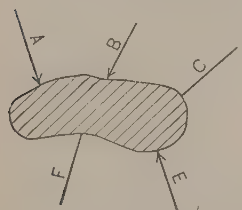


Fig. 34 (*a*).

5. *When the unknown elements are three magnitudes.* Any one of the three magnitudes may be determined if the two lines of action that correspond to the other two magnitudes intersect, since the point of intersection may be used as a center of moments.

6. *Illustration:* Six forces, A, B, C, E, F, and G, hold a body in equilibrium. (Fig. 34 (*b*).) The unknown elements are the magnitudes of the three forces C, F, and G. To determine F assume the center of moments at a, the intersection of C and G, thus eliminating those two forces. To determine G assume the center of moments at b, the intersection of F and C, thus eliminating those two forces. To determine C assume the center of moments at c, the intersection of F and G, thus eliminating those two forces.

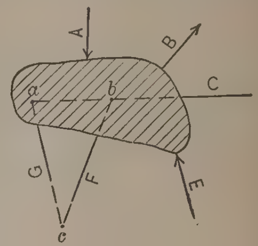


Fig. 34 (*b*).

7. *When the unknown elements are the magnitude of one force and the magnitude and direction of a second force.* All that is known of the second force is a point in its line of action. If this point is used as a center of moments the magnitude of the first force may be determined.

8. *Illustration:* Four forces, A, B, C, and E, hold a body in equilibrium. (Fig. 34 (*c*).) The unknown elements are magnitude of A and magnitude and direction of E. All that is known of E is p, a point in its line of action. By assuming this point as a center of moments the force E is eliminated from the equation and the magnitude of A may be determined. The magnitude of E cannot be determined from a moment equation because the line of action of E is not known.

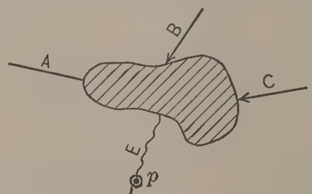


Fig. 34 (*c*).

9. *Algebraic sign.* When the magnitude of a force is to be determined from a single moment equation of equilibrium, the moment of the unknown force is given a *plus* sign in the moment equation. This is equivalent to assuming that the unknown force acts to produce *clockwise* rotation about the center of moments. If, upon solving the equation, the algebraic sign of the result is *plus*, the assumption was correct, if *minus*, it was incorrect, i.e., a *plus* sign for the result means that the sense of the unknown force must be such as to produce clockwise rotation, a *minus* sign means that the sense must be such as to produce *counter-clockwise* rotation.

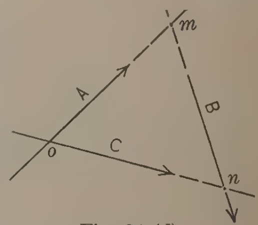


Fig. 34 (*d*).

10. **Three Non-parallel Forces Cannot be in Equilibrium Unless They are Concurrent.** Let A, B, and C be any three co-planar forces. If the center of moments is taken at the intersection of the lines

of action of any two of the forces, as at m, n, or o, the moments of each of the two forces whose lines of action pass through the point will be zero since the lever arm of each will be zero, but the moment of the third force will not be zero since its lever arm will not be zero. Therefore the algebraic sum of the moments of all three forces cannot equal zero, and the third condition of equilibrium is not fulfilled. In other words, if the center of moments is taken at the intersection of the lines of action of any two of the three forces, the third force will produce rotation about that point unless this third force also passes through the point, i.e., unless the forces are concurrent. Hence: *If three forces are in equilibrium their lines of action must meet in a point or be parallel.*

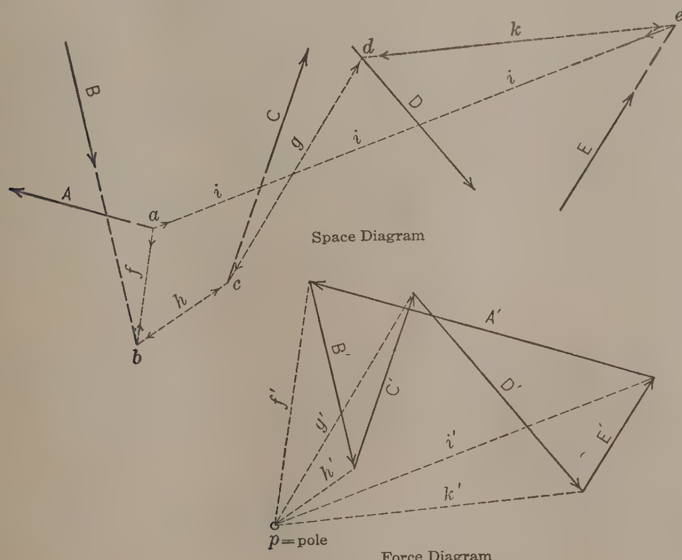


Fig. 35.

1. **Construction of an Equilibrium Polygon.** The construction of an equilibrium polygon is based on the following two fundamental principles:

(*a*) If three concurrent forces are in equilibrium, they may be laid off to scale to form a force triangle. (14 : 3.)

(*b*) Conversely, if the force polygon of a system of forces in equilibrium is a triangle, there are three and only three forces in the system, and the lines of action of these three forces must meet in a point. If three forces are in equilibrium they must be concurrent.

2. The construction of an equilibrium polygon is best explained by an illustrative example. In the space diagram, Fig. 35, the lines marked A, B, C, D, and E represent five forces. It is assumed that each of these forces is completely known, and that the five acting on a rigid body would hold it in equilibrium.

3. If these forces are laid off to scale in a force diagram, they will form a closed polygon which is the graphic equivalent of the equation $\Sigma H = 0$ and $\Sigma V = 0$. (14 : 8.) This polygon is composed of the lines marked A', B', C', D', and E' in the force diagram in Fig. 35; it was constructed by exactly the same methods as those used in drawing the force polygon for concurrent forces in (14:6). It remains to explain the construction of the equilibrium polygon *abcde* in the space diagram, which is the graphic equivalent of the equation $\Sigma M = 0$.

4. Let p in the force diagram be any point assumed at random. From the point p draw a line to each of the apices of the force polygon; these lines, f', g', h', i', and k', are called **rays,** and the point p is called the **pole.** Since these rays are lines in a force diagram, the length of each ray may be assumed to represent the magnitude of a force whose line of action is parallel to that ray.

5. From the fundamental principles stated at the beginning of this article, (1) forces represented by A', f', and i' may be in equilibrium; and if they are, (2) their lines of action (in the space diagram) must meet in a point. Hence, if from any point a in the line of action of A (in the space diagram) two lines f and i are drawn parallel respectively to f' and i' (in the force diagram), these lines may represent the lines of action of two forces; the point a may now be considered a particle held in equilibrium by three forces, A, f, and i, the magnitudes of which are given respectively by the lengths of the lines A', f', and i' in the force diagram. Following around the triangle in the force diagram beginning with A', the sense of which is known, it is seen that f', and therefore f, acts downward, while

i', and therefore i, acts upward. (14 : 10.) Arrows to correspond are put on f and i *near* a to indicate the sense of each force; it is important that these arrows be placed *near* the particle that is being held in equilibrium, for reasons which will become evident later on.

6. The process will now be repeated in order to indicate three forces that are holding in equilibrium a particle in the line of action of B, but instead of choosing *any* point at random in B the point b is selected, i.e., the point in which f drawn through a intersects B. Through this point draw two lines parallel respectively to the two rays that form a triangle with B'. It is found that f' is one of these rays, and the corresponding line f is already drawn; the other line h is drawn through b parallel to h'. The three forces B, f, and h (the magnitudes of which are given by B', f', and h' respectively) hold in equilibrium the particle b, and the sense of each force indicated by its arrow is found by following around the triangle $B'h'f'$, beginning with B', the sense of which is known. It is seen that at b the force f acts upward while at a it acts downward; the significance of this will be evident later.

1. The process is repeated for three more points: first for the point c where h intersects C, then for d where g intersects D, and finally for e where k intersects E. At each of these points the directions (including both inclination and sense) of the three concurrent forces must be the same respectively as those of the three forces in the corresponding force triangle; hence C, h, and g must have the same direction as C', h' and g' respectively; D, g, and k the same as D', g', and k'; and E, k, and i the same as E', k', and i'.

2. The last line drawn is i; when this line is drawn through e parallel to i', it will be found to coincide with i drawn through a at the beginning, thus forming a closed polygon *abcde*. The fact that this polygon closes is proof that the third condition of equilibrium $\Sigma M = 0$ is fulfilled. This may be shown in several ways; perhaps the simplest is to regard the polygon as composed of three sticks and two strings. The sides h, g, and k all have the arrows acting toward the ends and hence may be considered as sticks in compression; while the sides f and i have the arrows acting away from the ends and may therefore be regarded as a string in tension (6 : 6). Each of the particles a, b, c, d, and e is in equilibrium since the three forces acting at any one of these points form a force triangle in the force diagram and therefore fulfill the two conditions of equilibrium of a particle, i.e., $\Sigma H = 0$ and $\Sigma V = 0$. If each of the five points a, b, c, d, and e is in equilibrium the polygon as a whole must be in equilibrium and there can be no *rotation*. Hence the five forces A, B, C, D, and E, which are external to the polygon, do not produce rotation, i.e., *the algebraic sum of their moments about any point is zero.*

3. Summarizing with respect to the five given forces A, B, C, D, and E:
(*a*) *The force polygon, A', B', C', D', and E' shows that $\Sigma H = 0$ and $\Sigma V = 0$ are true of A, B, C, D, and E.*
(*b*) *The equilibrium polygon shows that $\Sigma M = 0$ is true of A, B, C, D, and E.*

4. *Note:* The *equilibrium polygon* is frequently called the **funicular polygon** and less often the **string polygon**. It is the basis of important graphic methods in stresses.

5. *An important check:* When any force polygon and the corresponding equilibrium polygon have been completed, they should be checked by inspection to see if the three lines at each apex of the equilibrium polygon (two sides of the equilibrium polygon and a line of action of a force) are parallel respectively to the three sides of a triangle in the force polygon (two rays and a side of the force polygon). This is also a useful principle to keep constantly in mind during the construction of the equilibrium polygon.

6. *Note:* Since the pole may be assumed at random, the inclination of any side of the equilibrium polygon may be determined in advance if desired. For example, if it were desired to have the side K of the equilibrium polygon horizontal, the pole p would be assumed in such a position that the ray K' would be horizontal.
Since the first side of the equilibrium polygon may be located at random, it may be drawn through any desired point.

7. **Force Polygon and Equilibrium Polygon — Summary of Principles.**

8. If a force polygon closes there can be no motion of translation, i.e., a force polygon is equivalent to two resolution equations of equilibrium.

9. If the equilibrium polygon closes there can be no motion of rotation, i.e., the equilibrium polygon is equivalent to a moment equation of equilibrium.

1. If a system of non-concurrent forces is in equilibrium both the force polygon and the equilibrium polygon must close.

2. Two sides of an equilibrium polygon and the line of action in the space diagram at which they intersect are parallel respectively to three sides of a triangle in the force polygon; and conversely, the side of a force polygon and the two rays that form a triangle with that side are parallel respectively to the corresponding force in the space diagram and the two sides of the equilibrium polygon that intersect in the line of action of that force.

3. The ray that is drawn to the intersection of any two consecutive forces in the force polygon is parallel to the side of the equilibrium polygon that extends between the two corresponding forces in the space diagram; and conversely, a side of an equilibrium polygon between two adjacent forces in the space diagram is parallel to the ray that is drawn to the intersection of the two corresponding forces in the force polygon.

4. If a system of forces is in equilibrium each side of an equilibrium polygon is the line of action of the resultant of all of the forces to the left or to the right of that side. (Prove from inspection of the force diagram.)

5. **Notation for a Center of Moments.** It is often desirable to indicate in a moment equation the point taken as the center of moments. The notation used for this purpose in this book is to affix a subscript letter to the symbol "ΣM." For example, $\Sigma M_a = 0$ means that the algebraic sum of the moments of all forces about a as a center equals zero; ΣM_E means that the point E is taken as the center of moments.

ASSIGNMENTS

(1) The intersection of any two sides of an equilibrium polygon is a point in the resultant of certain forces in the corresponding space diagram. Show what these forces are.

(2) Report on the method of drawing an equilibrium polygon through three given points.

(3) Prove Varignon's Theorem: "The moment of the diagonal of a parallelogram of forces equals the sum of the moments of the other two sides."

(4) Report on certain restrictions with respect to the choice of the centers of moments when any combination of equilibrium equations involves two or more moment equations (second and third combinations in 33 : 4).

CHAPTER VIII

PARALLEL FORCES

The method of attack outlined in CHAPTER V is applied in this chapter to problems in parallel forces.

1. **PROBLEMS IN PARALLEL FORCES** are understood to mean problems in which all of the forces involved have parallel lines of action. Thus, for example, if in Fig. 38 the parallel lines A, B, C, and F represent all of the forces that, acting externally on a body E, hold that body in equilibrium, any problem that involves only the forces A, B, C, and F is one in parallel forces.

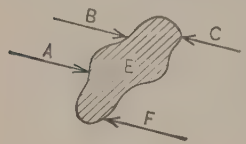


Fig. 38.

2. **THE ELEMENT D ELIMINATED.** In any system of parallel forces the angle of inclination for all forces is always known. *Sense* (a part of the element D) may be unknown for one or more of the forces, but sense is never treated as an unknown element. (4 : 11.) Consequently the determination of the *direction* (D) as an element of a force is never involved in any problem in parallel forces, and D may therefore be eliminated from consideration in any discussion of parallel forces.

3. **UNKNOWN ELEMENTS OF PARALLEL FORCES.** Since D can never be an unknown element, only magnitude (M) and point (P) in the line of action remain as possible unknown elements in any system of parallel forces. This means that no force is ever completely unknown in such a system, since at least one element (D) is partly known, i.e., the inclination is known.

4. **PROBLEMS DIVIDED INTO CASES.** A problem in parallel forces is statically indeterminate if more than *two* elements of forces are unknown; these elements may be elements of the *same* force or of *two* different forces. There are, therefore, four and only four possible combinations of unknown elements, viz.:

Case	*Unknown Elements*
A′	M and P (of the same force)
B′	M and M (of two different forces)
C′	P and P (of two different forces)
D′	M and P (of two different forces)

If there is more than one unknown P, a problem is indeterminate; this eliminates Case C′. Case D′ seldom arises in practical problems. Hence only Cases A′ and B′ will be considered.

5. *Note:* Henceforth it will be helpful to remember that the distinguishing characteristic of problems in concurrent forces is that P cannot be an unknown element, while the distinguishing characteristic of problems in parallel forces is that D cannot be an unknown. A characteristic common to both groups (concurrent and parallel) is that if there are more than two elements unknown a problem is indeterminate. In each group only two cases are common. Case A′ (parallel forces) corresponds to Case A (concurrent forces) except that P is an unknown instead of D. Case B′ corresponds exactly to Case B since the two unknowns in both cases are M and M of two forces.

6. **GENERAL METHODS FOR THE ALGEBRAIC SOLUTION OF PROBLEMS.** Only two equations are necessary in the solution of any problem in parallel forces since there can be only two unknowns in such a problem. *One of these equations must always be a moment equation of equilibrium.* ($\Sigma M = 0$.) The other equation will ordinarily be a resolu-

tion equation of equilibrium, most frequently $\Sigma V = 0$. The usual combinations of equations for the two common cases A' and B' are as follows:

Case A′. (M and P of the same force)
  Find M from $\Sigma V = 0$ or $\Sigma H = 0$
  Find P from $\Sigma M = 0$ (34:1)
Case B′. (M and M of two forces)
  Find one M from $\Sigma M = 0$
  Find the other M from $\Sigma V = 0$ or $\Sigma H = 0$.

1. The second M in Case B′ may also be found from a second moment equation used in place of a resolution equation.

2. In using a moment equation in Case A′ the center of moments may be taken in any convenient position. (If it is taken in the line of action of a known force, one multiplication will be saved.)

3. In using a moment equation to find one unknown M in Case B′, the center of moments should be taken in the line of action that corresponds to the other unknown M in order to eliminate this other unknown from the equation (34:3).

4. For each algebraic solution there is a corresponding graphic solution in which a force polygon is used in place of a resolution equation and an equilibrium polygon in place of a moment equation. These graphic methods of solution will be explained later.

5. **General Method of Attack.** In the general method of attack explained in *Chapter V* there are three steps: namely, (1) to determine the knowns; (2) to determine the unknowns; (3) from these unknowns to determine the case under which the problem falls and to use the standard algebraic or graphic solution for that case. This method of attack will now be applied to several typical problems and then practical suggestions will be given for using this method in attacking any problem in parallel forces.

6. *Problem* 1

A beam B of a uniform weight per linear foot rests upon two supports E and F (Fig. 39 (a)). Two bodies A and C, of known weights, rest upon the beam in the positions shown. It is required to determine the forces exerted on the beam by the two supports.

7. *First: Determine the knowns:*

(a) The body in equilibrium is the beam since this is the body on which all other bodies, including the supports, are acting.

(b) The forces that are completely known are the weight of the beam itself, represented by B', and the weights of the bodies A and C, represented by A' and C'. The supports E and F are bodies in contact with the beam and exert forces (reactions) on it. Since the other external forces, A', B', and C', are vertical and since the tops of the supports and the bottom of the beam are horizontal surfaces, the lines of action of E' and F' are vertical. The parallel forces E', A', B', C', and F' are the only forces acting on the beam.

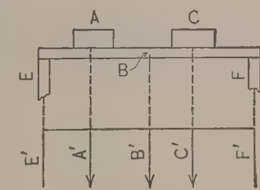


Fig. 39 (a).

Second: Determine the unknowns: Since the lines of action of E' and F' are known, the only unknowns are the magnitudes of these two forces.

Third: Solve the problem: The case in which the unknowns are two magnitudes is Case B′, and the equations for this case are $\Sigma M = 0$ and $\Sigma V = 0$.

8. The complete analysis as just outlined may be written as follows:

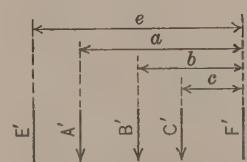


Fig. 39 (b).

Body in equilibrium: The beam.
Known: A', B', C', E'(L) and F'(L).
Unknown: E'(M) and F'(M) (Case B′).
Equations: $\Sigma M = 0 \quad \Sigma V = 0$.
$\Sigma M = E' \times e - A' \times a - B' \times b - C' \times c + F' \times 0 = 0$ (Solve for E') Fig. 39 (b).
$\Sigma V = E' - A' - B' - C' + F' = 0$ (Solve for F').

9. The graphic solution, to be explained later, involves an equilibrium polygon to correspond to $\Sigma M = 0$, and a force polygon to correspond to $\Sigma V = 0$.

10. In place of $\Sigma V = 0$ a second moment equation can be used, the center of moments being assumed anywhere in the line of action of E'.

1. *Problem 2*

A beam B (Fig. 40 (a)) of uniform weight per linear foot rests on two supports E and F. Two bodies A and C, of known weights, rest upon the beam in the positions shown. The force exerted by the support E is equal to two-thirds of the combined weight of A and C. It is required to find the position of the second support F and the force that it exerts on the beam.

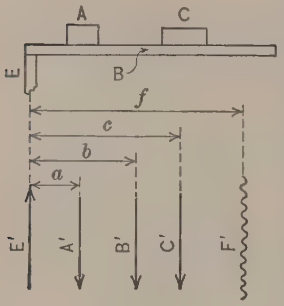


Fig. 40 (a).

The lines of action of the forces that act upon the beam, including B', the weight of the beam, are shown in the lower part of Fig. 40 (a). Since no point in the line of action of F' is known this line is represented by a wavy line.

2. The analysis may be written as follows:

Body in equilibrium: The beam.
Known: A', B', C', and $E' = \frac{2}{3}(A + C)$.
Unknown: F' (M and P) (Case A′).
Equations: $\Sigma V = 0$ and $\Sigma M = 0$

$$\Sigma V = 0: E' - A' - B' - C' + F' = 0 \text{ (Solve for } F')$$
$$\Sigma M = 0: -A' \times a - B' \times b - C' \times c + F' \times f = 0 \text{ (Solve for } f)$$

3. Notice that the object in determining the distance f was to locate a point (P) in the line of application of F' and that this distance could not have been found from a resolution equation — only a moment equation can be used for determining P.

4. The graphic method that corresponds to the algebraic solution of this problem involves a force polygon to correspond to $\Sigma V = 0$ and an equilibrium polygon to correspond to $\Sigma M = 0$.

5. *Problem 3*

A beam of uniform weight per linear foot rests upon three supports, one at each end and one in the middle. If the weight of the beam is W, what is the pressure on the beam exerted by each support?

6. Analysis:

Body in equilibrium: The beam.
Known: W and the lines of action of the three forces at the supports.
Unknown: Magnitudes of the three forces at the supports.
Equations: This problem is statically indeterminate since there are three unknowns.


Fig. 40 (b).

7. *Comment:* It is not uncommon for students, and sometimes more experienced men, to spend considerable time in attempting to solve by the methods of statics a problem that is statically indeterminate. A good method of attack or analysis should reveal the fact that a problem is indeterminate.

8. The above problem finds its counterpart in any beam resting on three supports regardless of how it may be loaded. Certain types of swing bridges, when closed, furnish similar problems. These problems, though statically indeterminate, may be solved by methods not included in statics.

9. **Practical Suggestions Concerning Problems in Parallel Forces:**

10. *The most common problems* in parallel forces are those in which all the external forces that act to hold the body in equilibrium are due directly or indirectly to the action of gravity and hence are vertical forces. Problems frequently arise, however, in which the parallel forces are not vertical, as, for example, problems in which forces are due to wind pressure (horizontal).

11. *Free body in space:* It is usually evident what body should be selected as the free body in space since it is the body on which all of the parallel forces act. It is not always clear, however, what these parallel forces are, and care should be taken not to omit any force or to represent it incorrectly in the sketch. Determine first whether or not the weight of the free body must be taken into account (sometimes it is negligible) and then observe what bodies are in contact with the body that is considered as the body in equilibrium. Any such body in contact usually exerts a force and this force must be determined and correctly represented.

12. *The sketch* may often be merely a space diagram in which only lines of action are represented; but all known magnitudes should be indicated, and all known spatial relations, such as distances of lines of action from each other or from some point of reference, should be shown. Do not

represent the sense of any force whose magnitude is unknown. When the sketch has been completed, study it to make sure that all known or partly known forces have been correctly indicated and none omitted.

1. **Summary of Principles for Problems in Parallel Forces.**

2. A problem is indeterminate if there are more than two unknowns.

3. M and P are the only elements that are ever unknown.

4. There are only two cases (A′ and B′) that commonly occur.

5. The equation $\Sigma M = 0$ must be used at least once in any problem.

6. The equation $\Sigma M = 0$ is always used to determine P and may be used to determine M.

7. A resolution equation is used to determine the unknown M in Case A′, and may be used to determine one unknown M in Case B′.

8. The combinations of equations ordinarily used are: For Case A′, $\Sigma V = 0$ and $\Sigma M = 0$; for Case B′, $\Sigma M = 0$ and $\Sigma V = 0$.

9. In choosing a center of moments, assume one in the line of action of a force, — in Case B′ in the line of action of a force that it is desired to eliminate from the equation.

10. The algebraic sign of a result from a resolution equation gives the *sense* of the force, but from a moment equation the algebraic sign gives the *direction of rotation*, from which the sense may be determined by inspection. (33:5.)

11. If the parallel forces are inclined to the horizontal and vertical it is often advantageous to assume a Y axis parallel to the direction of the forces and use $\Sigma Y = 0$ in place of $\Sigma V = 0$.

12. **Graphic Methods.** Since in the algebraic solution of a problem in parallel forces the moment equation must be used once, it follows that in the graphic solution the equilibrium polygon must be used once. (Why?)

13. A force polygon in the graphic solution will be one in which all sides lie in a straight line. (Why?) Such a polygon corresponds to a resolution equation.

14. Magnitude and sense can be determined from the force polygon but not from the equilibrium polygon. (Why?)

15. A point in a line of application may be determined from the equilibrium polygon (which corresponds to $\Sigma M = 0$) but not from the force polygon.

16. **Illustrative Problems.** The remainder of this chapter will be devoted to algebraic and graphic solutions of problems typical of those which come under Cases A′ and B′. These illustrative problems should be studied with a view of still further mastering the fundamentals which underlie the standard method of attack. (See pages 42 to 46.)

ILLUSTRATIVE PROBLEM IN CASE A′ — M AND P OF THE SAME FORCE UNKNOWN.

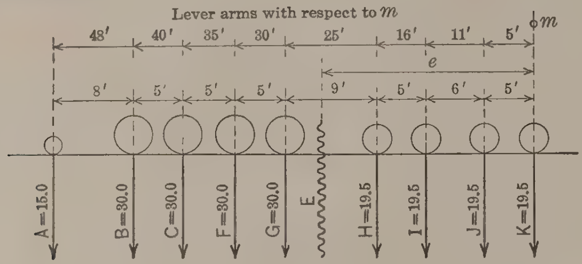


Fig. 42.

The forces exerted on a railroad rail by the wheels on one side of a locomotive and tender are indicated (in thousands of pounds) in Fig. 42. It is required to find the equilibrant (E) of these forces. For purposes of calculation the rail may be assumed as the body in equilibrium supported at one point only; incidentally this point will be in a vertical line through the center of mass of the locomotive — a line frequently required in the calculation of stresses due to locomotives.

Analysis

Body in equilibrium: The railroad rail.
Known: A, B, C, F, G, H, I, J, and K.
Unknown: E(M and P) (Case A′).
Equations: $\Sigma V = 0$ and $\Sigma M = 0$.

Algebraic Solution

$$\Sigma V = -A - B - C - F - G - H - I - J - K + E = 0$$
$$= -15 - 30 - 30 - 30 - 30 - 19.5 - 19.5 - 19.5 - 19.5 + E = 0$$
$$+213^{\#} = E \uparrow \text{ Magnitude and sense of } E.$$
$$\Sigma M_m = -A\times48 - B\times40 - C\times35 - F\times30 - G\times25 - H\times16 - I\times11 - J\times5 + E\times e = 0$$
$$= -15\times48 - 30\times40 - 30\times35 - 30\times30 - 30\times25 - 19.5\times16 - 19.5\times11 - 19.5\times5 + E\times e = 0$$
$$213\times e = 5244 \circlearrowright$$
24.6 ft. = e = Distance of any point (P) in the line of action of E from the line of action of K.

Comments.

1. In Case A′ both unknowns are elements of the same force; hence in using $\Sigma M = 0$ no other force must be eliminated and the center of moments may be assumed anywhere in the plane of the forces. If it is assumed in a line of action of a known force, one multiplication is saved. In the problem above, the center of moments m was assumed in the line of action of force K, thus making the algebraic signs of the moments of the given forces all the same (minus). Had the center of moments m been assumed in the line of action of force A, the signs would all have been positive; had it been assumed in the line of action of any of the other forces between A and K, some signs would have been plus and others minus. In any event, e is the perpendicular distance between the vertical line in which the center of moment m lies and the line of action of E.

2. The lever arms with respect to the center of moments m are indicated in the figure by a single line of dimensions or **extension figures**. This method, much used in structural drafting, is a good method of indicating several dimensions from the same starting point.

3. The solution of the problem including equations was indicated completely before the work of computation was begun. (Why?)

4. The algebraic sign for E, obtained from $\Sigma V = 0$, is plus; hence E acts upward. The algebraic sign for e, obtained from $\Sigma M = 0$, is plus (clockwise rotation); hence e must be measured to the left from the center of moments m since the force E, acting upward, must be to the left of m in order to produce clockwise rotation. Note the difference in the significance of algebraic signs, i.e., *sense* from a resolution equation, *direction of rotation* from a moment equation. (35:5.)

5. *Problem.* Check the position of the line of action of E by assuming the center of moments in the line of action of force F. The algebraic sign will determine whether the line of action of E is to the left or to the right of force F. (Would the algebraic sum of the moments of the given forces equal zero for a center of moments in the line of action of E?)

GRAPHIC SOLUTION, CASE A′

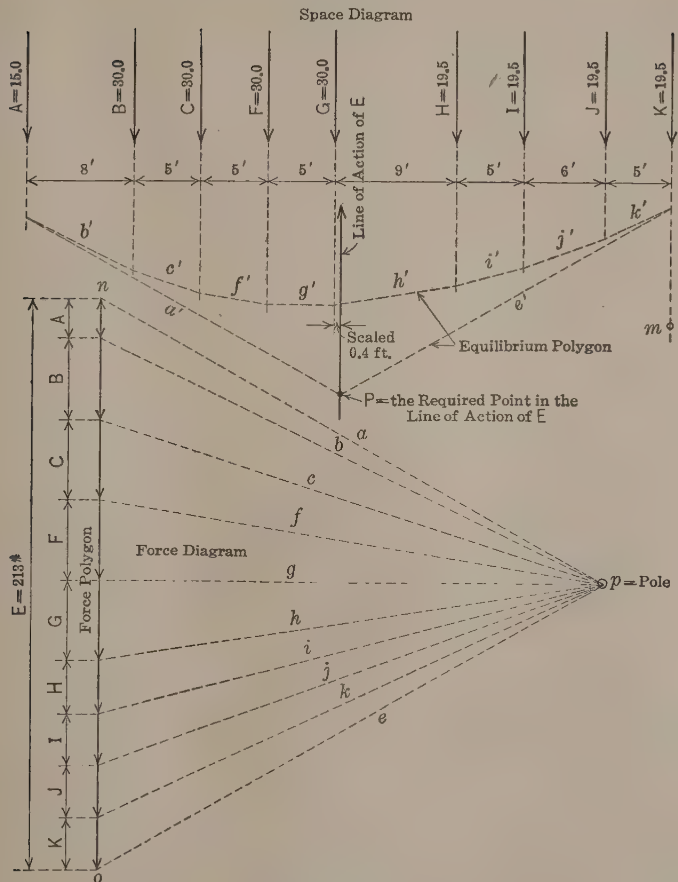


Fig. 43.

First: Draw the space diagram, consisting of the nine vertical lines of action of the nine given forces. The distances between these lines of action should be laid off accurately to any convenient scale.

Second: Draw as much as possible of the force polygon. Use any convenient scale. Since the nine given forces are all vertical, the nine corresponding sides, *A*, *B*, *C*, *F*, *G*, *H*, *I*, *J*, and *K* of the force polygon will lie in the same vertical line extending from *n* to *o*.

Third: To make the polygon close, the required equilibrant *E* must extend upward in a vertical line from *o* to *n*, i.e., it must equal the sum of the given forces. (This is equivalent to the use of $\Sigma V = 0$.)

Fourth: Assume a pole p and draw the rays *a*, *b*, *c*, *f*, *g*, *h*, *i*, *j*, *k*, and *e*.

Fifth: Draw the equilibrium polygon in order to determine a point *P* in the line of action of *E*. (This is equivalent to the use of $\Sigma M = 0$, the equation which must always be used in the algebraic method of determining the element *P*.) The line of action of *E* will be a vertical line through *P*.

Comments.

1. The pole *p* may, theoretically, be assumed anywhere, but it is well to assume it approximately in the position shown in order that the various rays and the corresponding sides of the equilibrium polygon may have reasonably good inclinations.

2. The method of constructing an equilibrium polygon is explained in 35:1. It may be started from any point in the line of action of any given force, but it is usually best to start it from a point in the line of action of the extreme left-hand force (*A* in this problem) or from a point in the line of action of the extreme right-hand force (*K*). During the construction of the equilibrium polygon, keep in mind the principle that three lines that meet at any apex of the equilibrium polygon must correspond to three sides of a triangle in the force diagram. (36:5.)

3. In the force diagram the equilibrant *E* and the rays *a* and *e* form a triangle; hence in the equilibrium polygon the line of action of the equilibrant *E* must pass through *P*, the intersection of the sides *a′* and *e′*, which are parallel respectively to rays *a* and *e*.

4. By choosing a new position for the pole, a new force diagram and a new equilibrium polygon will result, and a new *P* will be found; but this new *P* will be in the line of action of *E*, i.e., in a vertical line through the old *P*, and the magnitude of *E* in the new force polygon will be identical with that in the old one.

Illustrative Problem in Case B′ — M of One Force and M of Another Force Unknown.

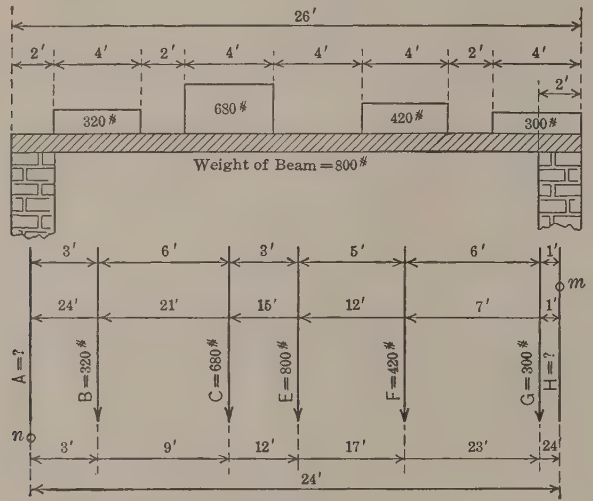


Fig. 44.

Given: A beam resting on two supports and loaded as shown in the sketch. Required: The pressure of each support on the beam, i.e., the **reaction** at each end of the beam. For purposes of calculation the length of the beam is taken as the distance from the center of bearing on one support to the center of bearing on the other support. The forces exerted by the supports on the beam are both vertical for reasons given in the problem on page 39.

Analysis

Body in equilibrium: The beam.

Known: A(L), B, C, E, F, G, and H(L).

Unknown: A(M) and H(M) (Case B′).

Equations: $\Sigma M = 0$ and $\Sigma V = 0$.

Algebraic Solution

$$\Sigma M_m = A\times 24 - B\times 21 - C\times 15 - E\times 12 - F\times 7 - G\times 1 - H\times 0 = 0$$
$$= A\times 24 - 320\times 21 - 680\times 15 - 800\times 12 - 420\times 7 - 300\times 1 - 0 = 0$$
$$A\times 24 = 29760 \curvearrowright$$
$$+1240^{\#} = A\uparrow = \text{Magnitude and sense of } A.$$
$$\Sigma V = +A_V - B_V - C_V - E_V - F_V - G_V + H_V = 0$$
$$= +1240 - 320 - 680 - 800 - 420 - 300 + H_V = 0$$
$$+1280^{\#} = H_V = H\uparrow = \text{Magnitude and sense of } H.$$

For check:

$$\Sigma M_n = A\times 0 + B\times 3 + C\times 9 + E\times 12 + F\times 17 + G\times 23 + H\times 24 = 0$$
$$= 0 + 320\times 3 + 680\times 9 + 800\times 12 + 420\times 17 + 300\times 23 + H\times 24 = 0$$
$$H\times 24 = -30720 \curvearrowleft$$
$$+1280^{\#} = H\uparrow = \text{Magnitude and sense of } H.$$

Comments

1. In the space diagram neither the sense of the reaction A nor that of the reaction G is indicated. While it is certain in this case that both reactions are acting upward, there are many cases in which a reaction acts downward. It is better to follow the general rule and not indicate the sense of a force whose magnitude is unknown.

2. Two combinations of equations are possible; the combination $\Sigma M = 0$ and $\Sigma V = 0$ involves less work than the combination $\Sigma M = 0$ and $\Sigma M = 0$. If a moment equation and a resolution equation are used, the results may be checked by a second moment equation, as indicated in the solution; if two moment equations are used, the results may be checked by a resolution equation.

3. In using equation $\Sigma M = 0$ to find either unknown, observe what the other unknown is and choose the center of moments anywhere in the line of action corresponding to that unknown. Thus if the magnitude of A is desired choose the center of moments in the line of action of H in order to eliminate H from the equation, and if the magnitude of H is required choose the center of moments anywhere in the line of action of A in order to eliminate A from the equation.

4. The lever arms with respect to any point m in the line of action of H are indicated by a single line of **extension figures**; in the space diagram the lever arms with respect to a point n in the line of action of A are given by a second line of extension figures.

5. The solution of the problem including equations was indicated completely before the work of computation was begun.

6. The value of $A \times 24$, obtained from $\Sigma M = 0$, has a plus sign; hence A must produce clockwise rotation about m in the line of action of H; to do this A must act upward. The value of H_V, obtained from $\Sigma V = 0$, is plus; hence H must act upward. Note the difference in the significance of the algebraic signs, i.e., *direction of rotation* from a moment equation and *sense* from a resolution equation. (33 : 5.)

GRAPHIC SOLUTION, CASE B′

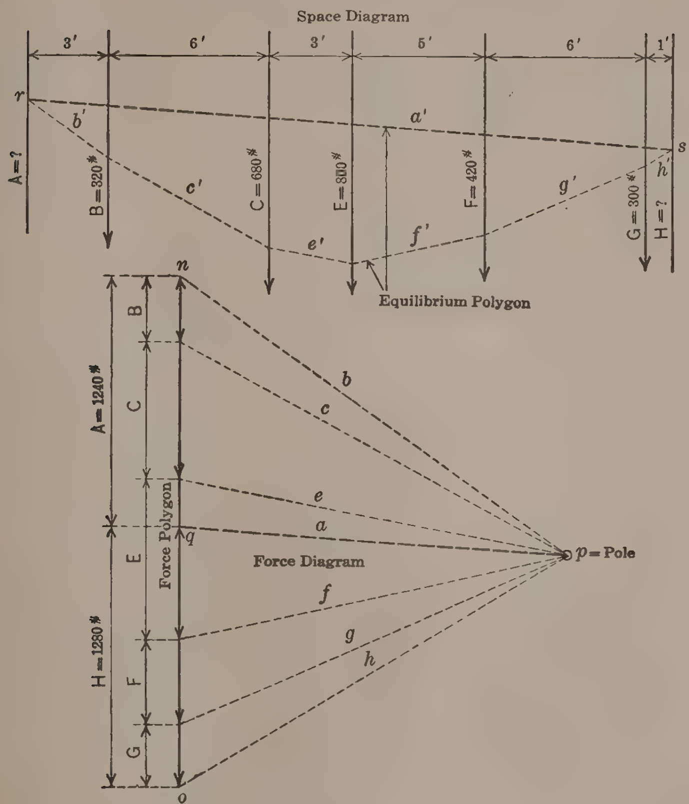


Fig. 45.

First: Draw the space diagram consisting of the lines of action of the five given forces B, C, E, F, and G, and of the two forces A and H whose magnitudes are unknown. The distances between these lines of action should be laid off accurately to any convenient scale.

Second: Draw as much as possible of the force polygon. Use any convenient scale. Since the five given forces are all vertical, the five corresponding sides B, C, E, F, and G of the force polygon will lie in the same vertical line extending from n to o.

Third: To make the force polygon close, the two reactions A and H, laid off one after the other, must extend from o to n; but the point q at which one of these reactions ends and the other begins cannot be determined until after the equilibrium polygon has been drawn.

Fourth: Assume a pole p and draw the rays b, c, e, f, g, and h.

Fifth: Draw the equilibrium polygon in order to determine the point q in the force polygon. Begin the equilibrium polygon at any point in the line of action of A, as, for example, r, and continue the construction of the polygon in the usual manner until a point s in the line of action of H is reached. (Or begin at any point in H, as s, and draw the polygon until r is reached.) Since the seven external forces that act on the beam are in equilibrium, the equilibrium polygon must close; (36:10); to make it close the remaining side must be drawn between the points s and r; hence the closing side is a'.

Sixth: Through the pole p draw a ray a parallel to the closing side a' of the equilibrium polygon. This ray a will intersect the vertical line from o to n in the point q. The line oq in the force diagram is one side of a triangle of which rays a and h are the other two sides; hence oq represents a force that acts in the equilibrium polygon at the intersection of a' and h', namely, the force H at the point s. (Why?) Consequently oq is the magnitude of the force H. The line qn in the force diagram is one side of a triangle of which the rays a and b are the other two sides; hence qn must represent a force that acts in the equilibrium polygon at the intersection of a′ and b′, namely, the force A at the point r. Consequently qn is the magnitude of the force A.

Comments

1. In the algebraic solution for *Case B′* it was necessary to use the moment equation $\Sigma M = 0$ first and the resolution equation $\Sigma V = 0$ second. Similarly in the graphic solution it is necessary to draw the equilibrium polygon (which corresponds to $\Sigma M = 0$) before the force polygon (which corresponds to $\Sigma V = 0$) can be completed.

1. The pole p may be assumed in any position that will result in reasonably good inclinations for the rays.

2. The method of constructing an equilibrium polygon was explained in 35:1. During its construction keep in mind the principle that three lines that meet at an apex of the equilibrium polygon must correspond to three sides of a triangle in the force diagram (36:5).

ASSIGNMENTS

(1) Be prepared to explain the algebraic and the graphic methods for *Case D′*, i.e., the case in which the two unknowns are the magnitude of one force and a point in the line of action of a second force.

(2) Prove that a system of parallel forces is in equilibrium if $\Sigma M = 0$ holds true for each of two points not in a line parallel to the forces.

(3) Prove that a problem in parallel forces is indeterminate if the two unknown elements are P and P.

CHAPTER IX

NON-CONCURRENT FORCES

The method of attack outlined in CHAPTER V is applied in this chapter to problems in non-concurrent forces.

1. **PROBLEMS IN NON-CONCURRENT FORCES** are understood to mean problems in which the forces involved taken as a whole are neither concurrent nor parallel.

2. *Note.* Parallel forces may be considered mathematically as concurrent, while in another sense they may be considered as non-concurrent. It is advantageous to regard such forces as neither concurrent nor non-concurrent and to treat problems in parallel forces as belonging to a separate group. (17:3.)

3. **UNKNOWN ELEMENTS OF NON-CONCURRENT FORCES.** In a problem in non-concurrent forces, any one of the elements M, D or P of a force may be unknown; hence in such a problem there may be as many as three unknown elements, and these elements may pertain to the same force, to two forces, or to three forces. In problems in concurrent or parallel forces a force is never wholly unknown; but in non-concurrent forces a force may be wholly unknown, since all three elements may be unknown. A problem in non-concurrent forces is statically indeterminate if there are more than *three* unknowns, whereas a problem in either concurrent or parallel forces is indeterminate if there are more than two unknowns.

4. **PROBLEMS DIVIDED INTO CASES.** There are twenty different combinations of unknowns possible in non-concurrent forces. Of these twenty different cases only three are of common occurrence; to these three may be added a fourth case in which there are only *two* unknowns. Practically all ordinary problems in non-concurrent forces fall under one or another of these four cases. In order to designate these four cases in a way that will at once distinguish them from cases in concurrent or parallel forces, Arabic numerals will be used instead of letters.

Case	*Unknown Elements*
1	M, D, and P of the same force unknown.
2	M and M of two forces unknown. (Corresponds to Cases B (22 : 4) and B′ (38 : 4).)
3	M of one force, M and D of another force unknown.
4	M, M, and M of three forces unknown.

Of the four cases given, the last two are by far the most common in problems in stresses.

5. *To Change Case 3 to Case 4 or Vice Versa.* In Case 3 the magnitude of one force and the magnitude and direction of a second force are unknown. A single point P in the line of action of the second force must be known; otherwise there would be four unknowns and the problem would be statically indeterminate. If at this point P the second force (whose M and D are unknown) is replaced by two components, as, for example, by its H and V components, the magnitudes M and M of these two components will then replace the unknown M and D of the second force, making three magnitudes unknown, i.e., M of the first force, M of one component of the second force, and M of the other component of the second force. Thus the case has been changed to Case 4. When the magnitudes of the two components of the second force have been determined, the magnitude and direction

of the second force may be found from these two components by the usual method. (9 : 2.)

1. Conversely, Case 4 may be changed to Case 3 by assuming any two of the three forces whose magnitudes are unknown to be replaced by their resultant acting through the point of intersection of the lines of action of the two forces. The unknowns will then be M of one force and M and D of the resultant of two other forces, i.e., Case 3. When the magnitude and direction of the resultant have been determined, the magnitudes of the two corresponding forces may be found from the resultant by the usual method. (11 : 1 or 7 : 1.)

2. *To Change a System of Concurrent Forces into a System of Non-Concurrent Forces.* Resolve any one of the forces into two components and conceive these two components to be acting at some point in the line of action of the force other than the point of concurrence. For example, let the four concurrent forces represented in Fig. 48 (*a*) be in equilibrium. Since the force A may be considered as applied at any point in its line of action, any two components of A may likewise be considered as applied at any point in the line of action. Let p in Fig. 48 (*b*) be a point selected at random in the line of action of A, and let A_H and A_V be considered as applied at p and replacing A. The system has now been changed from concurrent to non-concurrent forces. The process may be repeated, if desired, for any other force in Fig. 48 (*b*); if it were applied to all four, a system of eight non-concurrent forces would result. By changing a system of concurrent to non-concurrent forces, a moment equation can often be used to advantage in place of resolution equations.

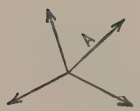


Fig. 48 (*a*).

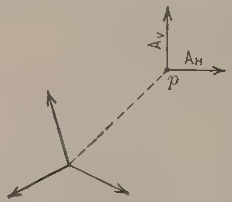


Fig. 48 (*b*).

3. General Methods for the Algebraic Solution of Problems. Problems in non-concurrent forces that are statically determinate may be solved algebraically by the use of one or more of the equilibrium equations $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$.

4. In order to determine what combination of these equations can be used for a given case, it is desirable to keep in mind the following guiding principles:

5. If there are three unknowns, at least one moment equation must be used; hence a moment equation is necessary in the solution of Case 1, Case 3, and Case 4, but not in Case 2.

6. A magnitude (M) may be determined by means of resolution equations or by a moment equation.

7. A moment equation must always be used to determine a point P in the line of action.

8. The element of direction (D) is best determined by resolution equations though it can be determined by using two moment equations.

9. The usual combinations of equations for the four common cases in non-concurrent forces are as follows:

10. *Case 1 (M, D, and P of the same force)*

Combination: $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$.

First: Find M and D from $\Sigma H = 0$ and $\Sigma V = 0$ exactly as for Case A in concurrent forces.

Second: Find P from $\Sigma M = 0$ (in accordance with principle 48:7).

11. *Case 2 (M and M of two forces)*

Combination: $\Sigma H = 0$ and $\Sigma V = 0$.

This case corresponds exactly to Case B of concurrent forces and may be solved by exactly the same method, namely by the use of $\Sigma H = 0$ and $\Sigma V = 0$.

12. *Case 3 (M of one force, M and D of a second force)*

Combination: $\Sigma M = 0$, $\Sigma H = 0$ and $\Sigma V = 0$.

First: Find M of the first force from $\Sigma M = 0$, assuming the center of moments at the given P of the second force.

Second: Find M and D of the second force from $\Sigma H = 0$ and $\Sigma V = 0$.

13. *Note.* It is frequently simpler to solve a problem in Case 3 by changing it to Case 4. (47:5.)

14. *Case 4 (M of one force, M of a second force, and M of a third force)*

There are three combinations of equilibrium equations that are in common use for the solution of Case 4.

First combination Case 4: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.

First: Find one magnitude from $\Sigma M = 0$.

Second: Find the other two magnitudes from $\Sigma H = 0$ and $\Sigma V = 0$.

1. This combination is best used when all forces involved in the problem are parallel to one or the other of two co-ordinate axes, as, for example, when all forces are either horizontal or vertical.

2. *Note.* When the first magnitude has been determined the problem has been reduced to one in two unknown magnitudes, i.e., Case 2, which is solved by means of two resolution equations.

3. Second combination Case 4: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$ or $\Sigma H = 0$.

First: Find one magnitude from $\Sigma M = 0$.

Second: Find a second magnitude from $\Sigma M = 0$. (Different center of moments.)

Third: Find the third magnitude either from $\Sigma V = 0$ or $\Sigma H = 0$, whichever can be used to advantage.

4. This combination is particularly advantageous when there is only one inclined force among all the forces involved in the problem and the magnitude of that inclined force is one of the three magnitudes required. In that case the magnitude of a horizontal or vertical force is determined from a moment equation while the magnitude of the inclined force is determined from a resolution equation.

5. Third combination Case 4: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma M = 0$.

First: Find M of the first unknown force from $\Sigma M = 0$ by taking center of moments at intersection of the lines of action of the second and third unknown forces.

Second: Find M of the second unknown force from $\Sigma M = 0$ by taking the center of moments at the intersection of the first and third unknown forces.

Third: Find M of the third unknown force from $\Sigma M = 0$ by taking the center of moments at the intersection of the line of actions of the first and second unknown forces.

6. This combination can be used to advantage only when the three lines of action corresponding to the three unknown magnitudes intersect each other, i.e., no two are parallel.

Suggestions for the use of moment equations:

7. In using a moment equation, choose the center of moments in such a position, if possible, as to eliminate all unknowns except the one to be determined by that equation (34 : 2). This may best be done in any given case by keeping in mind *all* of the unknowns. For example, in Case 3 let the unknowns be:

Force A : M; Force B : M and D.

It is evident that M of force A may be obtained from a moment equation by assuming the center of moments in the line of action of force B, thus eliminating the two unknown elements of force B, namely, M and D; but on the other hand, force B could not be obtained from a single moment equation by assuming the center of moments in the line of action of force A since two unknown elements of force B, namely, M and D, would be involved in the equation.

8. As another illustration, let the unknowns of Case 4 be:

Force A : M; Force B : M; Force C : M.

To find force M of A from a moment equation, assume the center of moments at the intersection of the line of action of B and C; to find M of B, assume the center of moments at the intersection of A and C; to find M of C, assume the center of moments at the intersection of A and B.

(When the lines of action of two of the forces whose magnitudes are unknown are parallel or do not intersect within reasonable limits, it is inexpedient to use a moment equation for finding the magnitude of the third force.)

9. The method just outlined of selecting a center of moments is used all through the subject of stresses. It is to be noted that if a moment equation contains two or three unknowns the values of all but one of these unknowns might be found by means of other equations and inserted in the original moment equation, thus making it possible to solve that equation for the value of the remaining unknown. The elimination of unknowns, however, by selecting the center of moments in their lines of action, avoids the use in the equations of values previously found and in addition usually lessens the amount of computation required.

10. In any moment equation, the moment of any inclined force may be replaced by the sum of the moments of its H and V components about the same center of moments. When the lengths of the lever arms of inclined forces must be determined, it is often advantageous to make the substitution just referred to, especially if the lengths of the lever arms of the H and V components can be determined more easily. The H and V components may be assumed as acting at any convenient point in the line of action of the force that they replace. (31:4.)

11. General Method of Attack. In the general method of attack explained in *Chapter V*, there are three steps, namely: (1) to determine the knowns; (2) to determine the unknowns; (3) from these un-

knowns to determine the case under which the problem falls and to use the standard algebraic or graphic method of solution for that case.

1. The first step (determining the knowns) consists of (*a*) selecting the body in equilibrium, and (*b*) representing by means of a sketch all of the known forces and all that is known of the partly known forces. When this has been correctly done, the problem is as good as solved. So important is this step that it has been discussed in detail in Article 18 : 3, and it will repay the student to re-read that article at this point. It is more difficult to determine the knowns in problems in non-concurrent forces than in problems in concurrent or in parallel forces. For this reason some of the suggestions given in Article 18 : 2 are repeated here in different form, and there are added other suggestions which pertain particularly to determining the knowns when forces are non-concurrent.

2. *Selecting the body in equilibrium.* The first thought in attacking any problem should be: "What is the body to be considered in equilibrium?"

First: It must be a rigid body; if it is part of a structure, that part must be in itself rigid.

Second: The body must be such that at least one of the forces acting upon it is of known magnitude. (Frequently this force is due to gravity acting upon the body itself.)

Third: The body must be such that in the entire system of forces that hold it in equilibrium there are not more than three unknown elements; otherwise the problem is statically indeterminate.

3. *Representing the knowns.* This is best done by making a sketch. The suggestions for making the sketch given on page 19 may be summarized as follows:

First: Begin by making a sketch of the free body.

Second: If the weight of the body is to be taken into account, indicate the corresponding force.

Third: Observe carefully all bodies that are in contact with the body in equilibrium, and represent as far as possible the lines of action and the magnitude of the forces exerted by these bodies.

Fourth: Represent other forces, if any, such, for example, as those due to wind.

Fifth: Show on the sketch all data needed for solving the problem.

4. Additional suggestions that are particularly applicable to the preliminary work of making sketches and determining forces in problems in non-concurrent forces will now be given.

5. *The sketch.* In complicated problems it may be well to make first a complete sketch showing not only the body in equilibrium, but also all other bodies in contact with it. The ultimate aim, however, is to show the lines of action of all forces acting on the body in equilibrium or as many such lines as are known. Any known point of an unknown line of action should also be shown, and the line of action itself represented by a wavy line. It is to be noted that the only use made of the body in equilibrium is in determining the spatial relations of the forces in the system. Once these relations are known, the body in equilibrium may be eliminated and there remains a space diagram pure and simple. Thus, for example, in Fig. 50 the rectangle $ABCD$ is assumed to be held in equilibrium by the forces F, R, W, and R' (line of action of R' unknown). Below, these four external forces are represented by themselves in a space diagram. The dimensions of the rectangle ABCD fix at least one point in each line of action, and thus help to determine the spatial relations of the lines of action with respect to each other. Any other body in Fig. 50, such as the irregular body indicated by broken lines, would serve as well, provided it could be used to determine the positions of four points (one in each line of action) with respect to each other, such, for example, as points A', D', G', and C. (Remember that the lines of action in the space diagram are indefinite in length.)

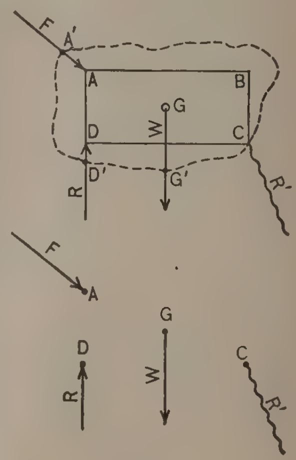


Fig. 50.

6. Whether the sketch is simply a space diagram or whether it is more than a space diagram, it should show linear dimensions and angles of inclination with such clearness and completeness that there can be no doubt as to the spatial relations of all forces involved in the problem.

7. *Determining forces:* The most difficult part of a problem often lies in forming a true conception of each and every force that is acting on the body in equilibrium. Begin by thinking of the body as a free body in space. This conception is one of the most valuable steps in any analysis of a problem. Usually the first force to look for is that due to gravity acting on the body. This is one of the few forces that are not caused by another body in contact with the body in equilibrium. (Sometimes this force is negligible.) The second step is to note what bodies are in contact with the body in equilibrium and to ascertain, if possible, the magnitude and line of action of the force exerted by each of these external bodies. The line of action will depend on one or more conditions of contact and it should not be assumed in any direction unless that direction is

known to be correct. For example, when a horizontal beam merely rests upon two supports, one at each end, and the other external forces are all vertical, as in Problem 1, 39:6, the lines of action of the forces exerted by the supports may be assumed as vertical. If, however, the beam is inclined and is fastened to either support or there is friction between that support and the beam, the line of action of the force exerted on the beam by the support may not be vertical even though it is caused in the last analysis by a vertical force (gravity) acting on the beam. For example, in Fig. 51 (*a*), an inclined beam *B* is fastened to each of two supports *C* and *E*. The upper surface of each support is fitted to the bottom of the beam. The forces that *C* and *E* exert on the beam are due to the weight of the beam, which is represented by the vertical force *W*; but although *W* is vertical, neither *C* nor *E* is vertical. The magnitude and direction of each is unknown and the problem of finding forces *C* and *E* is indeterminate (four unknowns).

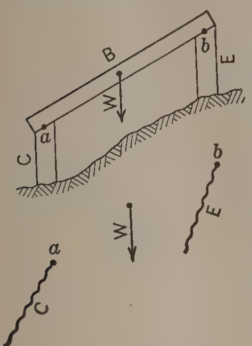


Fig. 51 (*a*).

1. Let it be assumed that the beam is fastened to the support *C* but not to the support *E* and that there is no friction at support *E*. The forces at *a* and *b* may each be resolved into two components, one parallel the other perpendicular to the axis *X* that is parallel to the longitudinal axis of the beam. (Fig. 51 (*b*).) E_X will equal zero and C_X will equal the force that is exerted by whatever holds the beam to the support *C*. The problem of finding the magnitudes of C_X, C_Y, and E_Y is determinate, since there are only three unknowns (Case 4).

2. A common mistake is the omission, due to oversight, of some force that should be included, or the failure to represent all that is known of a force that is not completely known, as, for example, a known point in an unknown line of action. After completing the sketch, it is well therefore to study it carefully to see that it represents correctly and adequately all of the knowns. Form this habit of checking the sketch. In the long run it will save much time.

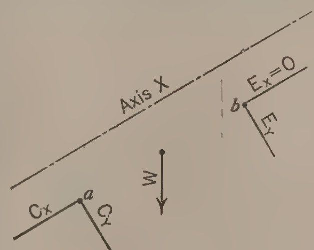


Fig. 51 (*b*).

3. *Written Analysis.* When the sketch or space diagram has been completed, the student will find it particularly advantageous in problems in non-concurrent forces to write out the analysis of knowns, unknowns, and equations, in the general form explained in Chapter V and already used in connection with concurrent and parallel forces. A corresponding mental analysis should eventually become a fixed habit in attacking any problem.

4. **Summary of Principles for Problems in Non-Concurrent Forces.**

5. A problem is indeterminate if there are more than three unknowns.

6. Any one of the three elements M, D, and P may be unknown.

7. There are only four cases that commonly occur: namely, Case 1, one force wholly unknown (M, D, and P); Case 2, two magnitudes unknown (M and M); Case 3, magnitude of one force, magnitude and direction of another force unknown (M, M and D); Case 4, three magnitudes unknown (M, M, and M).

8. Case 3 may be changed to Case 4 by replacing the force whose M and D are unknown by two components, usually its H and V components.

9. Case 4 may be changed to Case 3 by replacing any two forces whose magnitudes are unknown by their unknown resultant.

10. The equation $\Sigma M = 0$ must be used in each of the four common cases except Case 2.

11. The equation $\Sigma M = 0$ is always used to determine P and may be used to determine M.

12. Resolution equations may be used to determine M and D, but not to determine P.

The combinations of equations ordinarily used are: For Case 1, $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$; for Case 2, $\Sigma H = 0$ and $\Sigma V = 0$; for Case 3, $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$; for Case 4, one of three combinations, namely, (1) $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$, (2) $\Sigma M = 0$, $\Sigma M = 0$ and $\Sigma V = 0$, or $\Sigma H = 0$, (3) $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma M = 0$.

13. In using a moment equation, choose the center of moments in such a position as to eliminate all unknown elements except the one to be determined from the equation.

14. In any moment equation, the moment of any inclined force may be replaced by the sum of the moments of its H and V components about the same center of moments, the H and V components being assumed to act at any convenient point in the line of action of the force.

15. The algebraic sign of a result obtained from a resolution equation gives the *sense* of the force; but the sign of a result obtained from a moment equation gives the *direction of rotation*, from which the sense of the force may be determined by inspection.

1. **Graphic Methods.** The following guiding principles for the use of the force polygon and equilibrium polygon in problems in non-concurrent forces correspond to those given for the use of the equilibrium equations in such problems.

2. If there are three unknown elements, as in Cases 1, 3 and 4, the equilibrium polygon (corresponds to $\Sigma M = 0$) must be used at least once.

3. The element of magnitude (M) and the element of direction (D) are best determined by means of a force polygon (corresponds to $\Sigma H = 0$ and $\Sigma V = 0$).

4. The equilibrium polygon must always be used to determine a point P in a line of action.

5. The use of the force polygon and of the equilibrium polygon in the solution of problems in non-concurrent forces will be explained later by means of illustrative problems.

6. **Combination of Algebraic and Graphic Methods.** A combination of algebraic and graphic methods may frequently be used to advantage in solving problems. For example, in determining the stresses in a roof truss, it often is best to determinine the reactions by an algebraic method but to determine the stresses by a graphic method. Again, if a moment equation involves lever arms of unknown lengths, these lengths may often be scaled from a space diagram more quickly than they can be calculated, and with sufficient accuracy.

7. **Comparison of Problems in Non-concurrent Forces with Problems in Concurrent and Parallel Forces.** It has been shown that all problems in statics may be divided into three groups according to whether the forces involved are (1) concurrent, (2) parallel or (3) non-concurrent, and that in each of these groups there are characteristic fundamental principles. The following comparisons will serve to fix in mind these characteristic principles.

8. The element P is never unknown in a problem in concurrent forces, the element D is never unknown in a problem in parallel forces, whereas any one of the three elements M, D or P may be unknown in a problem in non-concurrent forces.

9. In a problem in either concurrent or parallel forces *no* force is wholly unknown, whereas in a problem in non-concurrent forces *one* force may be wholly unknown.

10. A problem in either concurrent or parallel forces is indeterminate when there are more than *two* unknown elements; a problem in non-concurrent forces is indeterminate when there are more than *three* unknown elements.

11. In problems in either concurrent or parallel forces only *four* combinations of unknowns (or four cases) are possible; in problems in non-concurrent forces *twenty* combinations of unknowns (or twenty cases) are possible.

12. In problems in either concurrent or parallel forces only *two* cases are of common occurrence; in problems in non-concurrent forces there are *four* cases of common occurrence.

13. Case A (concurrent forces), Case A′ (parallel forces), and Case 1 (non-concurrent forces) are alike in one respect, namely, that in each of these cases the unknown elements are all elements of the *same* force, and that force is as much unknown as it can be under the conditions stated in 52 : 8. The unknown elements of the force in the three cases are: Case A : M and D. Case A′ : M and P. Case 1 : M, D and P.

14. Case B (concurrent forces), Case B′ (parallel forces), and Case 2 (non-concurrent forces) are exactly alike in the elements that are unknown, since in each case these unknown elements are M and M of two forces.

15. Six of the eight cases of common occurrence have just been placed for the sake of comparison into two groups of three cases each, and in each group there is a case in concurrent, a case in parallel and a case in non-concurrent forces. The remaining two cases of common occurrence are both in non-concurrent forces, namely, Case 3 (unknowns: M of one force, M and D of another force) and Case 4 (unknowns: M, M, and M). Either of these cases can be changed to the other, but almost always if any change is made, it is from Case 3 to Case 4 and not, conversely, from Case 4 to Case 3. (47 : 5.)

16. A problem in concurrent forces (either Case A or Case B) is solved by means of the two resolution equations of equilibrium (or their graphic equivalent, the force polygon), and consequently no moment equation is necessary, whereas, a problem in either parallel or non-concurrent forces requires the use of at least one moment equation (or its graphic equivalent, the equilibrium polygon). (Case 2, non-concurrent forces, is an exception to the last statement since it is solved by two resolution equations.)

1. In any problem in stresses, the forces to be determined by methods of statics are usually the reactions exerted on the truss by the supports and the stresses within the truss. Most problems in determining reactions fall either under Case B′ or Case 4. (If a problem in reactions falls under Case 3, as quite frequently happens, it is usually best to change it to a problem in Case 4.) Most problems in stresses fall either under Case B or Case 4. It is evident, therefore, that Case B, Case B′, and Case 4 are the most important of all the eight common cases.

2. **Illustrative Problems.** The remainder of this chapter will be devoted to problems illustrating methods of solution for each of the four common cases in non-concurrent forces. The student should study the problem under any one of the four cases with a view of fixing in mind the distinguishing characteristics of that case. Do not allow such details as the calculation of components or the solution of equations to obscure the few important underlying principles. In the written analyses for algebraic methods, numerical values have purposely been omitted in some cases in order that there may be a bare outline of the work to be done. The most efficient method of solving a problem is first to prepare such an outline of the entire problem and then to carry out the mechanical work of computation. (24 : 2 and 24 : 3.)

Many of the illustrative problems are exactly like those which will be encountered in stresses (Parts II and III), and the student who acquires a thorough mastery of these typical problems will find comparatively little that is new in principle as he proceeds in the study of stresses in structures.

Illustrative Problem in Case 1. — Magnitude, Direction, and Point in the Line of Action of the Same Force Unknown. Given a beam suspended by four chains, A, B, C, and E, fastened to four points of support, a, b, c, and d, respectively, as shown in the sketch. The weight of the beam W is 800 lbs. and the stresses in the chains are: A = 900 lbs., B = 800 lbs.; C = 1200 lbs.; E = 700 lbs. Find M, D, and P of a sixth force X which must be acting on the beam if the beam is in equilibrium.

As a preliminary step in the solution of the problem the lines of action of these five forces must be located with respect to each other. One of the most convenient methods of doing this is to locate the points in which the five lines of action intersect a common reference line. The reference line chosen is the horizontal line through the points c and d. The lines of action of the five known forces, A, B, W, C, and E intersect this line in points h, g, f, c, and d, respectively. The distances cf, fg, and gh were calculated by geometry and found to be, respectively, 6.6 ft., 3.8 ft., and 18.2 ft. The space diagram was constructed as follows: The line of reference $h'd'$ was drawn to correspond to the line hd, and points g', f', and c' were plotted to correspond to points g, f, and c. Through h', g', f', c', and d' the lines of action A, B, W, C, and E were drawn parallel respectively to the corresponding lines in the sketch. The distances from d' to c', f', g', and h' were then indicated.

Each of the known forces was replaced by its H and V components, and each pair of components was assumed to act at the point at which the corresponding line of action intersects the axis $h'd'$. This causes all of the H components to lie in the line $h'd'$ as shown in the modified space diagram. The moment of each of these H components about d' as the center of moments will therefore be zero, and this fact facilitates the solution of the problem as will become evident later. Moreover, the forces that will be entered in the three equations of equilibrium used in the algebraic solution are the forces represented in the modified space diagram and not those that are represented in the first space diagram.

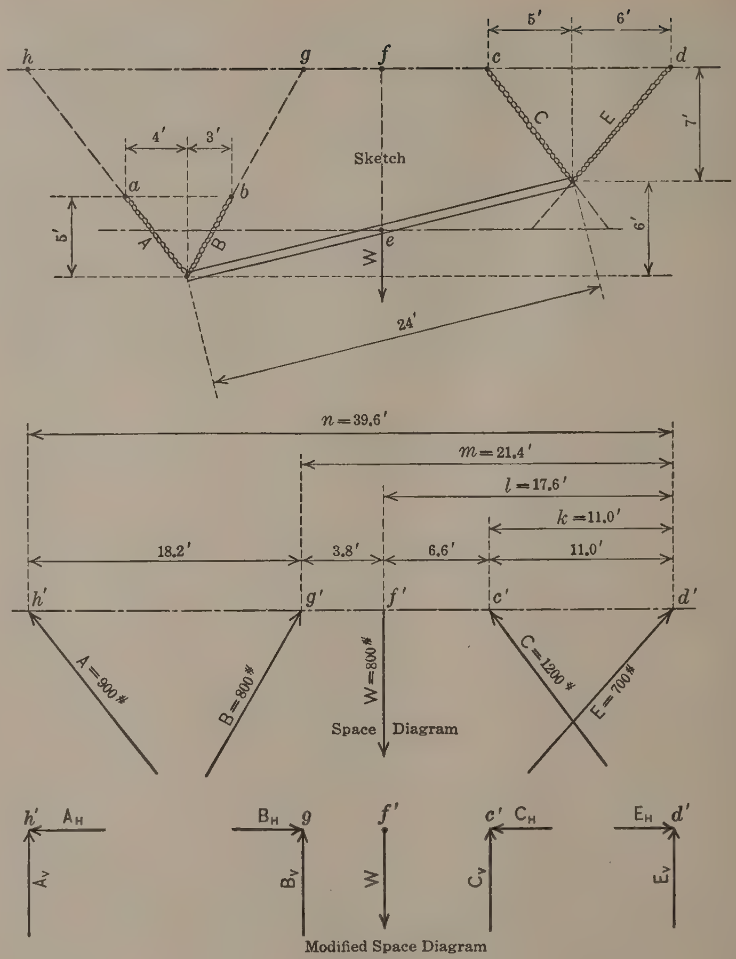


Fig. 54.

The analysis and the solution of the problem on the preceding page are as follows:

Analysis

Body in equilibrium: The beam.

Known: A, B, C, E, and W.

Unknown: X (M, D, and P). (Case 1).

Equations: $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$.

Algebraic Solution

$$\Sigma H = -A_H + B_H - C_H + E_H + W_H + X_H = 0$$
$$= -563 + 412 - 698 + 456 + 0 + X_H = 0$$
$$\Sigma V = +A_V + B_V + C_V + E_V - W_V + X_V = 0$$
$$= +703 + 686 + 977 + 532 - 800 + X_V = 0$$
$$+393^{\#} = X_H \rightarrow \text{Magnitude and sense of } X_H$$
$$-2098^{\#} = X_V \downarrow \text{Magnitude and sense of } X_V$$
$$2135^{\#} = X = \sqrt{(X_H)^2 + (X_V)^2} = \sqrt{(393)^2 + (2098)^2} = \text{Magnitude of } X.$$
$$.187 = X_H \div X_V = \text{Slope} \searrow = 393 \div 2098 = \text{Direction of } X$$
$$\Sigma M_{d'} = +A_V \times n + B_V \times m + C_V \times k - W \times l + X_V \times x = 0$$
$$= +703 \times 39.6 + 686 \times 21.4 + 977 \times 11 - 800 \times 17.6 + 2098 \times x = 0$$
$$18.7\text{ ft.} = x \text{ Measured from } d' \text{ to the left (Point P).}$$

Comments

1. The work of finding the magnitude and direction of the unknown force X by means of the equations $\Sigma H = 0$ and $\Sigma V = 0$ needs no explanation, since it is exactly the same in method as the algebraic solution for Case A on page 26. The student should check the H and V components of the known forces by the geometric method; he should also check the values of X_H and X_V by solving the equations. From the two components X_H and X_V the magnitude and the inclination of X may be determined as indicated.

2. A point P in the line of action of X was found from a moment equation in which the forces involved are those indicated in the modified space diagram. (A moment equation is always used to determine P. (51:11).) The point d' in this modified space diagram was chosen as a convenient center of moment, first because the lever arms with respect to this point are known for all of the V components, and second because the lines of action of all of the H components pass through this point and hence all of the H components will be eliminated from the equation. Any other point in the line of action of the H components could have been chosen as a center of moments, but, whatever point is used, the value of x will be the lever arm of X, i.e., it will be the horizontal distance from the center of moments to the point where the force X crosses the axis of reference in which lie the points h', g', f', c', and d'. The V components used in the moment equation are the same as those used in the equation $\Sigma V = 0$.

3. The significance of the algebraic sign of x deserves special attention. The significance of any algebraic sign obtained from a moment equation is *direction of rotation* (33 : 5). The minus sign of x means that the distance must be measured to the right or to the left of the center of moments, according to which of these two directions will locate the force X in such a position as to cause counter-clockwise rotation. Since X was found from $\Sigma V = 0$ to be downward, it must be to the left of the center of moments d' in order to cause counter-clockwise rotation. Hence the force X passes through a point P in the horizontal axis of reference, 18.7 ft. to the left of d'.

GRAPHIC SOLUTION, CASE 1

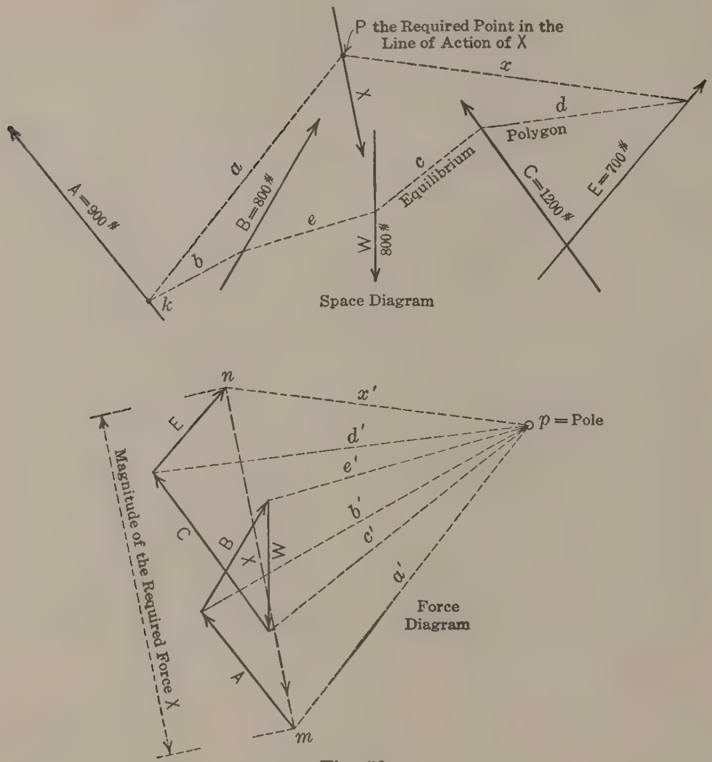


Fig. 56.

The analysis on the preceding page showed that there are three unknown elements, namely, M, D, and P of the force X. If the three principles governing the use of the graphic method 52:1 are kept in mind, it is evident from inspection of these unknowns, that:

(a) The equilibrium polygon must be used at least once, since there are three unknowns.

(b) The unknowns M and D may be determined by means of a force polygon.

(c) The unknown P can only be determined by means of an equilibrium polygon.

The graphic solution of the problem, which is similar to that of *Case A′* on page 43, is shown in Fig. 56. First the space diagram showing the five given forces, A, B, W, C, and E, was drawn carefully to scale. These forces were then laid off in order in a force polygon, beginning at the point m and ending at n. The closing line mn must correspond both in magnitude and direction to the unknown force X. It remains to find any point P in the line of action of the force X by means of an equilibrium polygon. (35 : 1.) A pole p was chosen at random, rays were drawn, and, beginning at any point in the line of action of A, as k, the equilibrium polygon was drawn. From the force polygon it is seen that the rays a', x', and the force X form a force triangle, hence the three corresponding lines in the space diagram must intersect in a point. (35 : 1 (b).) The point P of the equilibrium polygon in which the sides a and x intersect must, therefore, be a point in the line of action of X; consequently, X may be drawn through this point parallel to X (that is, mn) in the force diagram.

Question. Must the equilibrium polygon be started at some point in the line of action of A, or could it be begun at some point in another line of action?

Comment

If another pole is assumed and a new equilibrium polygon is drawn, another point in the line of action of X may be determined; the line between this point and the first point P should be parallel to mn (Why?) and hence is a check on the inclination of the line of action of X. Check the equilibrium polygon by inspection, to see if the three lines that meet at each apex of the equilibrium polygon correspond to three lines that form a triangle in the force diagram (35 : 1 (a)).

Illustrative Problem in Case 2, — Magnitude of One Force and Magnitude of a Second Force Unknown.

Assume that the six forces A, B, C, E, F, and G, shown in Fig. 57 hold the beam in equilibrium. All of these forces are completely known except forces B and G, the magnitudes of which are required.

Analysis

Body in equilibrium: The beam.
Known: A, B(L), C, E, F, and G(L).
Unknown: B(M) and G(M) (Case 2).
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.

Algebraic Solution.

$$\Sigma H = A_H + B_H + C_H - E_H + F_H - G_H = 0$$
$$= 70.7 + .866 \times B + 56.6 - 232 + 0 - .5 \times G = 0$$
$$\Sigma V = -A_V + B_V + C_V + E_V - F_V + G_V = 0$$
$$= -70.7 + .5 \times B + 56.6 + 0 - 185 + .866 \times G = 0$$

$190^{\#} = B\nearrow$ Magnitude and sense of B
$120^{\#} = G\nwarrow$ Magnitude and sense of G

Comments

1. The analysis is exactly like that for Case B, concurrent forces (page 28), and needs, therefore, no further explanation. In solving the equations the sense of each of the unknown forces B and G must be assumed. (28 : 4.)

2. If the system of non-concurrent forces shown in Fig. 57 is in equilibrium, the conditions $\Sigma H = 0$ and $\Sigma V = 0$ must be fulfilled; but it does not follow that because these conditions are fulfilled, the system is in equilibrium — the condition $\Sigma M = 0$ must also be satisfied.

3. *Exercise.* Determine whether or not $\Sigma M = 0$. Suggestion: Substitute for each of the forces A, B, C and G its H and V components, and assume each pair of these components to act where the line of action of the force that they replace intersects the horizontal line through g.

4. *Second Algebraic Method for Case 2.* In this method two moment equations are used instead of two resolution equations. The two unknowns are M of B and M of G. To find M of B assume the center of moments anywhere in the line of action of G, and to find M of G take the center of moments anywhere in the line of action of B. (34 : 3.) Unless the lengths of the lever arms are easily determined, this second method for Case 2 will usually involve more work than the first.

GRAPHIC SOLUTION, CASE 2

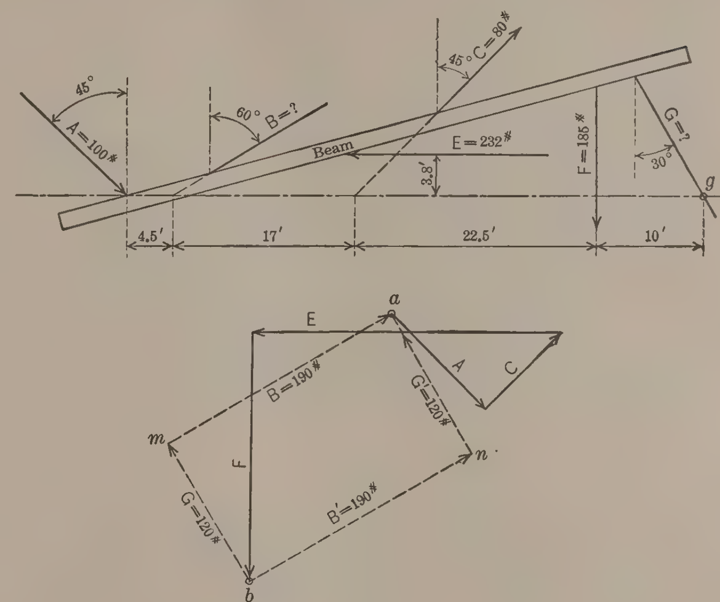
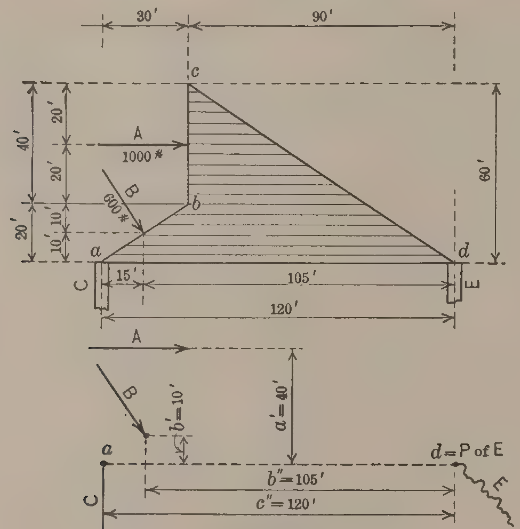


Fig. 57.

Since there are only two unknowns and both of these unknowns are magnitudes, they may be found by means of the force polygon. (14:8.) Of the four common cases in non-concurrent forces, this is the only one in which it is not necessary to use the equilibrium polygon. The graphic solution of the problem follows.

Draw first the space diagram. Draw next as much as possible of the force polygon (from a to b) by laying off the forces of known magnitude, A, C, E, and F. Complete the force polygon by drawing forces B and G until they intersect at m, or B' and G' until they intersect at n; it is immaterial which of these two methods of closing the polygon is used, since the results, both as to magnitude and as to sense, are the same for both methods. The method of drawing the force polygon just explained does not differ from the method of drawing the force polygon in Case B, page 29.

Illustrative Problem in Case 3, — Magnitude of One Force, Magnitude and Direction of a Second Force Unknown.



Space Diagram

Fig. 58.

A rigid body *abcd*, shown in the sketch, stands in a vertical plane; it is fastened to a support E but merely rests without friction on the top of a support C. Two other forces act upon this body, namely, $A = 1000$ lbs. and $B = 600$ lbs., perpendicular respectively to *cb* and *ba*. The line of action of the force exerted by the support C is vertical, (Why?) but the line of action of the force exerted by the support E is unknown. Required: The forces C and E exerted by the supports.

Analysis

Body in equilibrium: Body abcd.
Known: A, B, C(L), and E(P).
Unknown: C(M) and E(M and D) (Case 3).
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.

Algebraic Solution

$$\begin{aligned}
\Sigma M_d &= A \times a' + B_H \times b' - B_V \times b'' + C \times c'' = 0 \\
&= 1000 \times 40 + 333 \times 10 - 500 \times 105 + C \times 120 = 0 \\
76.4^{\#} &= C \circlearrowright \uparrow = \text{Magnitude and sense of } C. \\
\Sigma H &= A_H + B_H + C_H + E_H = 0 \\
&= 1000 + 333 + 0 + E_H = 0 \\
-1333^{\#} &= E_H \leftarrow \text{Magnitude and sense of } E_H \\
\Sigma V &= A_V - B_V + C_V + E_V = 0 \\
&= 0 - 500 + 76.4 + E_V = 0 \\
+424^{\#} &= E_V \uparrow \ \text{Magnitude and sense of } E_V \\
1400^{\#} &= E = \sqrt{(E_H)^2 + (E_V)^2} = \sqrt{(1333)^2 + (424)^2} = \text{Magnitude of } E \\
3.14 &= E_H \div E_V = \text{Slope of } E \nwarrow
\end{aligned}$$

Comments

1. Since there are three unknowns, a moment equation must be used at least once, and if it is used to determine the M of the first force, the M and the D of the second force may be determined from resolution equations.

2. The point d was assumed as the center of moments. (Why?) In the moment equation, the force B was replaced by its components B_H and B_V in order to save the calculation of the length of the lever arm of B. When, as in this case, the components of an inclined force must be determined for use in the resolution equation and when the lever arms of these components are known without calculation, the substitution of the components for the force itself in a moment equation is usually expedient.

3. The plus sign of C from the moment equation means clockwise rotation; hence C must act upward. E_H is minus and E_V is plus; hence E acts toward the left and upward.

4. *Problem.* Check the magnitude of C by using B in the moment equation instead of B_H and B_V. (Calculate the lever arm of B.)

GRAPHIC METHOD, CASE 3

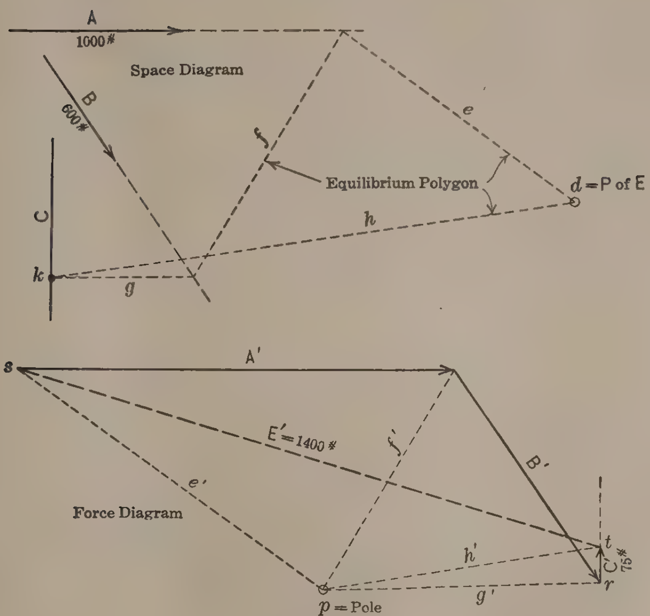


Fig. 59.

The analysis on the preceding page showed that there are three unknown elements, namely, M of C, and M and D of E. When there are three unknowns, an equilibrium polygon must be used as well as a force polygon. (52:2.) The only element of the force E that is known is a point d in its line of action. One apex of the equilibrium polygon must lie in this line of action of E (Why?), and since d is the only point of this line that is known, the point d must be made an apex of the equilibrium polygon. The best way of making d an apex is *to begin the equilibrium polygon at that point.*

The graphic solution of the problem is shown in Fig. 59 and may be carried out as follows:

First: Draw the space diagram by plotting the lines of action of forces A, B, and C and the given point d of the unknown line of action of E.

Second: Draw as much as possible of the force polygon by plotting the magnitudes A' and B' and drawing C' parallel to C but indefinite in length.

Third: Draw the equilibrium polygon, first choosing a pole p and drawing the rays e', f', and g'; the ray h' cannot be drawn until later. Begin the equilibrium polygon by drawing through the point d a side e parallel to e'. Continue this construction of the polygon by the usual method until the point k is reached. (35 : 1.) Make the equilibrium polygon close by drawing the side h from k to d. Draw ray h' parallel to h and intersecting C' in the point t.

Fourth: Make the force polygon close by drawing the side E' from s to t. This side E' represents the magnitude and direction of the unknown force E, and the side C', ending at t and equal to rt, represents the magnitude of the force C.

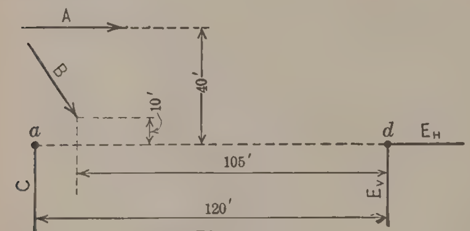
Comments

1. The construction just explained may be checked by applying principle 36 : 11 as follows: The force E and the sides of the equilibrium polygon e and h meet in a point d; hence E', e', and h' should form a triangle in the force diagram. The force C and the sides of the equilibrium polygon g and h meet in a point k; hence C', g', and h' should form a triangle in the force diagram.

2. If the forces A and B are replaced by their resultant R there will be only three forces in equilibrium, namely, C, R, and E. If, therefore, the line of action of R is drawn in the space diagram until it intersects the line of action of C, this point of intersection and the point d will determine the direction of the force E. (34 : 10.) This affords a check on the direction of force E. Note that the resultant of forces A and B will act at the intersection of their lines of action in the space diagram and be parallel to the line joining points s and r in the force diagram. (Why?)

3. Whenever the only known element of a force is a point P in its line of action and an equilibrium polygon is to be drawn, the first thought should be that this polygon must be begun at this known point P. This is the key to the graphic solution of Case 3.

First Illustrative Problem in Case 4, — Three Magnitudes Unknown.



Space Diagram

Fig. 60.

Given the same problem as that just used to illustrate Case 3. Change it to Case 4 by replacing the unknown force E by its two components E_H and E_V. With the exception of this substitution, Fig. 60 is exactly the same as the space diagram in Fig. 58.

Analysis

Body in equilibrium: Body abcd.
Known: A, B, C(L), E_H(L) and E_V(L).
Unknown: C(M), E_H(M), and E_V(M) (Case 4).
Equations: $\Sigma M = 0$, $\Sigma M = 0$ and $\Sigma H = 0$

Algebraic Solution.

$$\Sigma M_d = A \times 40 - B \times 81.9 + C \times 120 = 0$$
$$= 1000 \times 40 - 600 \times 81.9 + C \times 120 = 0$$
$$76.2^{\#} = C \downarrow \uparrow \text{ Magnitude and sense of } C$$
$$\Sigma M_a = A \times 40 + B \times 18.03 + E_V \times 120 = 0$$
$$= 1000 \times 40 + 600 \times 18.03 + E_V \times 120 = 0$$
$$+424^{\#} = E_V \uparrow$$
$$\Sigma H = A_H + B_H + C_H + E_H = 0$$
$$= 1000 + 333 + 0 + E_H = 0$$
$$-1333^{\#} = E_H \leftarrow$$
$$1440^{\#} = E = \sqrt{(E_H)^2 + (E_V)^2} = \sqrt{(1333)^2 + (424)^2} = \text{Magnitude of } E$$
$$3.14 = E_H \div E_V = \text{Slope of } E \nwarrow$$

Comments

1. Before the first moment equation $\Sigma M_d = 0$ can be solved, the unknown lever arm of the force $B = 81.9$ ft. must be determined. This may easily be calculated, but if the diagram has been drawn to scale, it may be scaled. Similarly, the lever arm of $B = 18.03$ must be determined before the equation $\Sigma M_a = 0$ can be solved.

2. If the force B be resolved into its H and V components, *all* of the external forces will then be either vertical or horizontal. Under these conditions, the best combination of equations is: $\Sigma M_d = 0$, $\Sigma H = 0$, and $\Sigma V = 0$ (49:1). The algebraic solution then becomes identical with that for Case 3 on page 58, since the latter solution is for exactly the same combination of equations. It is doubtful in this particular problem whether anything is gained by changing from Case 3 to Case 4.

3. Still another solution is to resolve B into its H and V components, and use the same combination of equations as that used on this page, namely, $\Sigma M_d = 0$, $\Sigma M_a = 0$, $\Sigma H = 0$. (Write out these three equations, using B_H and B_V in place of B.)

GRAPHIC SOLUTION, CASE 4

4. Since there are three unknown elements, an equilibrium polygon must be used as well as a force polygon. By beginning the equilibrium polygon at the intersection of E_V and E_H, namely, at the point d, the graphic solution may be made identical with that shown in Fig. 59. It is equally feasible to begin the equilibrium polygon at the intersection of C and E_H, namely, at the point a; but since it is the resultant of E_H and E_V that is desired and not the resultant of C and E_H, it is better to begin the equilibrium polygon at d.

5. The key to the graphic solution of problems in Case 4 is to begin the equilibrium polygon at the intersection of the lines of action of two forces whose magnitudes are unknown.

6. The graphic solution will not be used in this book for problems in Case 4 as the algebraic solution for such problems is almost always better. Hence no further illustrations of the graphic method for Case 4 will be given.

Second Illustrative Problem in Case 4, — Magnitudes of Three Forces Unknown.

A rigid body *abc* shown in the sketch stands in a vertical plane with one end resting on a support *E* at *a*. The support exerts a vertical pressure upward on the body of 2500 lbs., while two loads *F* and *G*, of 1000 lbs. each, also act on the body as shown. In addition to these three forces *E*, *F*, and *G*, three other forces *A*, *B*, and *C* act on the body to hold it in equilibrium. For the present, it is immaterial what causes these three additional forces. Assume all forces to act in a vertical plane. Find the magnitudes of *A*, *B*, and *C*.

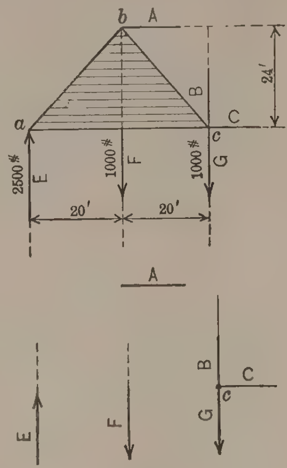


Fig. 61.

Analysis

Body in equilibrium: Body abc.
Known: E, F, G, A(L), B(L), and C(L).
Unknown: A(M), B(M), and C(M) (Case 4).
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.

Algebraic Solution

$$\Sigma M_c = E \times 40 - F \times 20 + A \times 24 = 0$$
$$= 2500 \times 40 - 1000 \times 20 + A \times 24 = 0$$
$$-3330^{\#} = A \circlearrowright \leftarrow \text{Magnitude and sense of A}$$
$$\Sigma H = -A + C = 0$$
$$+3330^{\#} = C \rightarrow \text{Magnitude and sense of C}$$
$$\Sigma V = E - F - G + B = 0$$
$$= 2500 - 1000 - 1000 + B = 0$$
$$-500^{\#} = B \downarrow \quad \text{Magnitude and sense of B}$$

Comments

1. Since all forces are either vertical or horizontal, the first combination for Case 4 is used. (49:1.)

2. From inspection of the unknowns in the written analysis, it is evident that if the magnitude of *A* is to be found from a single moment equation, the unknowns B(M) and C(M) must be eliminated by assuming the center of moments at the intersection of the lines of action of *B* and *C*. Thus the *first step* in choosing a center of moments in Case 4 is to determine from the analysis what two unknown magnitudes must be eliminated from the equation; the *second step* is to find the two corresponding lines of action in the sketch or the space diagram and then follow these lines to their point of intersection. (34 : 5.)

3. The force *A* was found from the moment equation. The force *C* could have been found from a moment equation just as easily, in which case *A* would have been determined from the equation $\Sigma H = 0$. (Where would the center of moments be taken in order to find *C*?) It is immaterial whether *A* or *C* is found from a moment equation; if, however, one attempts to find the magnitude of *B* by means of a moment equation it is evident from inspection of the unknowns in the analysis that the center of moments should be assumed at the intersection of *A* and *C*, and from the sketch or space diagram it is seen that *A* and *C* do not intersect but are parallel. For this reason it is better to find *B* from the equation $\Sigma V = 0$.

4. Attention is again called to the significance of the algebraic signs as indicated by the arrows at the answers. *A* must cause minus rotation about *c*; hence it must act to the left toward *b* (compression). *C* must act to the right or away from *c* (tension) and *B* must act downward or toward *c* (compression). (Why?)

5. The force *C* could be exerted by anything capable of taking tension, as, for example, a chain or steel bar. Force *A* or force *B*, however, could be exerted only by something that is capable of taking compression, as, for example, a strut or post.

Third Illustrative Problem in Case 4, — Magnitudes of Three Forces Unknown.

A rigid body *abcd*, shown in the sketch, stands in a vertical plane with one end resting on a support *E* at *a*. The support exerts a vertical pressure upward on the body of 2500 lbs., while two loads *F* and *G*, of 1000 lbs. each, also act on the body as shown. In addition to these three forces *E*, *F*, and *G*, three other forces *A*, *B*, and *C* act on the body to hold it in equilibrium. The line of action of *B* passes through the point *e*, located as shown. Assume all forces to act in a vertical plane. Required: The magnitudes of *A*, *B*, and *C*.

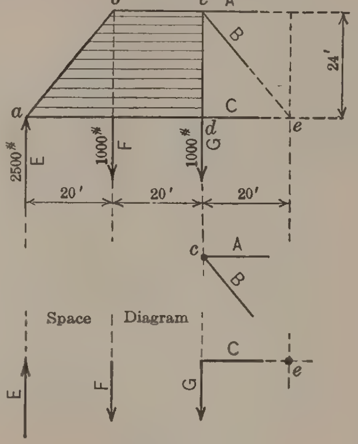


Fig. 62.

Analysis

Body in equilibrium: Body abcd.

Known: E, F, G, A(L), B(L), and C(L).

Unknown: A(M), B(M), and C(M) (Case 4)

Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

Algebraic Solution

$\Sigma M_e = E \times 60 - F \times 40 - G \times 20 + A \times 24 = 0$

$= 2500 \times 60 - 1000 \times 40 - 1000 \times 20 + A \times 24 = 0$

$-3750^{\#} = A$ ↻ ← Magnitude and sense of A

$\Sigma M_c = E \times 40 - F \times 20 + C \times 24 = 0$

$= 2500 \times 40 - 1000 \times 20 + C \times 24 = 0$

$-3330^{\#} = C$ ↻ → Magnitude and sense of C

$\Sigma V = E - F - G + B_v = 0$

$= 2500 - 1000 - 1000 + B_v = 0$

$-500^{\#} = B_v \downarrow$

$-650^{\#} = B = B_v \times (\sqrt{(20)^2 + (24)^2} \div 24)$ Magnitude of B

Comments

1. Since one of the six forces involved is inclined, the second combination for Case 4 is used. (49:4.) In all other respects the problem is similar to the preceding problem.

2. Note that each of the unknown horizontal forces was determined by a moment equation, but that the inclined force was determined by a resolution equation.

3. From inspection of the unknowns in the analysis it is evident that in order to find *A* from a moment equation the center of moments should be taken at the intersection of *B* and *C*, i.e., at the point *e*; likewise, in order to find *C* the center of moments should be taken at the intersection of *A* and *B*, i.e., the point *c*. (Why is the force *B* determined by a resolution equation instead of a moment equation?)

4. The force *B* could have been determined from the equation $\Sigma H = 0$, i.e., $-A + C + B_H = 0$, by inserting the values of *A* and *C* as found from the moment equations; or if values have been determined for the force *B* and either of the horizontal forces *A* or *C* these two values can be inserted in the equation $\Sigma H = 0$ and the other horizontal force can then be determined from that equation. It is better, however, not to use an equation that involves quantities previously found from other equations unless by so doing the work of computation is considerably shortened.

5. Note that the equation $\Sigma M_e = 0$ could have been written in the following form: $\Sigma M_e = 2500 \times 60 - 1000 \times (40 + 20) + A \times 24$. When there are a number of equal parallel forces equally spaced, such a method of writing the equation may be used to advantage.

6. Note that the algebraic signs for *A* and *C* are both minus (counter-clockwise rotation) yet *A* acts to the left (compression) while *C* acts to the right (tension). Why is this? (30 : 7.)

7. Note that force *B* or force *C* could be exerted by anything capable of taking tension, but that force *A* could be exerted only by something that is capable of taking compression.

FOURTH ILLUSTRATIVE PROBLEM IN CASE 4,—MAGNITUDES OF THREE FORCES UNKNOWN.

A rigid body *abcd*, shown in the sketch, stands in a vertical plane with one end resting on a support E at a. The support exerts a vertical pressure upward on the body of 2500 lbs., while two loads F and G, of 1000 lbs. each, also act on the body as shown. In addition to these three forces E, F, and G, three other forces A, B, and C act on the body to hold it in equilibrium. The line of action of A passes through m and the line of action of B passes through e. Assume all forces to act in a vertical plane. Required: The magnitudes of A, B, and C.

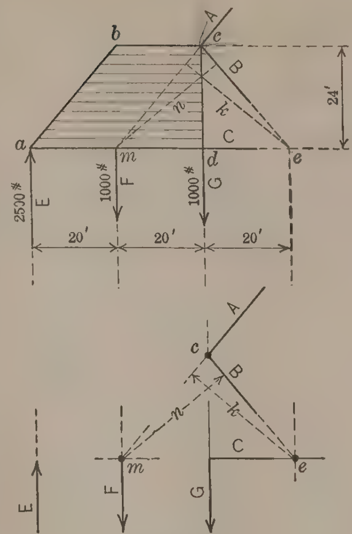


Fig. 63.

Analysis

Body in equilibrium: Body abcd.
Known: E, F, G, A(L), B(L), and C(L).
Unknown: A(M), B(M), and C(M) (Case 4).
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma M = 0$.

Algebraic Solution

$$\Sigma M_c = E \times 40 - F \times 20 + C \times 24 = 0$$
$$= 2500 \times 40 - 1000 \times 20 + C \times 24 = 0$$
$$-3330^{\#} = C \circlearrowright \rightarrow \text{Magnitude and sense of } C$$
$$\Sigma M_e = E \times 60 - F \times 40 - G \times 20 + A \times k = 0$$
$$= 2500 \times 60 - 1000 \times 40 - 1000 \times 20 + A \times 30.8 = 0$$
$$-2930^{\#} = A \circlearrowright \swarrow \text{Magnitude and sense of } A$$
$$\Sigma M_m = E \times 20 + G \times 20 + B \times n = 0$$
$$= 2500 \times 20 + 1000 \times 20 + B \times 30.8 = 0$$
$$-2280^{\#} = B \circlearrowright \nwarrow \text{Magnitude and sense of } B$$

Comments

1. This problem is similar to the preceding one except that, of the three unknown forces, two are inclined instead of one. This problem will illustrate how it is possible to solve certain problems in Case 4 by the use of three moment equations, i.e., the third combination for that case. It does not follow, however, that this is the best combination for this particular problem.

2. In determining the magnitude of C, the center of moments was taken at the point c; for A, the center of moments is at e; for B, the center of moments is at m. In each case the center of moments for one unknown is at the intersection of the two lines of action that correspond to the other two unknowns. (Why?)

3. In this particular problem the points m and e, in which the lines of action of A and B respectively intersect the horizontal line ae, are at known distances from a; hence in the moment equations it may be advantageous to substitute for A its H and V components applied at m, and for B its H and V components applied at e. (A force may be replaced by its H and V components applied at any point in the line of action of the force.) The points m and e were taken in order to make the lever arm of A_H in one case, and of B_H in the other, equal to zero. The second and third moment equations can then be written as follows:

$$\Sigma M_e = E \times 60 - F \times 40 - G \times 20 + A_V \times 40 + A_H \times 0 = 0.$$
$$\Sigma M_m = E \times 20 + G \times 20 + B_V \times 40 + B_H \times 0 = 0.$$

Solve the first of these equations for A_V, and from this value of A_V find A. Solve the second equation for B_V, and from this value of B_V find B. This method avoids the necessity for calculating the lever arms of A and B, but little if any work would be saved if it were necessary to calculate the distances to points m and e.

4. The value of B_V could have been determined from the equation $\Sigma V = 2500 - 1000 - 1000 - A_V + B_V = 0$ by substituting in this equation the value of A_V as found previously from a moment equation; or, similarly, the value of A_V could have been found from $\Sigma V = 0$ by substituting in this equation the value of B_V found previously from a moment equation.

5. The magnitude of the forces A, B, and C can be checked from the resolution equation $\Sigma H = -A_H - B_H + C_H = 0$.

6. The algebraic signs of the moments of C, A, and B are all negative; yet C is a tensile force, while A and B are compressive forces. (Why?)

7. Note that force C could be exerted by anything capable of taking tension, but that force A or force B could be exerted only by something that is capable of taking compression.

ASSIGNMENTS

(1) Solve a problem in Case 4 by means of the graphic method and be prepared to explain it in detail.

(2) Explain how a problem in concurrent forces (Case A or Case B) can be solved by means of moment equations without changing the problem to one in non-concurrent forces, and show that the work involved is practically the same as that involved in the use of two resolution equations.

CHAPTER X

SUMMARY OF THE FUNDAMENTALS OF STATICS

This chapter is a summary of the fundamental principles and methods of statics that have been explained in detail in the preceding chapters. Since these fundamentals of statics are the fundamentals of stresses, it is essential that the student master them thoroughly before proceeding with the study of stresses.

ELEMENTS OF A FORCE

1. The elements of a force are (1) **magnitude** (M), (2) **direction** (D), and (3) **point in the line of action** (P). Magnitude is usually expressed in pounds or tons. Direction includes **inclination** (A) and **sense.** The inclination is measured by the **acute angle** which the line of action makes with the vertical. This angle may be expressed in angular measure (degrees and minutes) or in linear measure (slope in feet or inches). The **sense** is the direction of a force along its line of action — right or left, up or down — and is usually indicated by an **arrow** on or parallel to the line of action. The term *direction* should not be used in place of either *inclination* or *sense,* since it includes both. A **point in a line of action** may be any point in the line and not some particular point, such as that implied by the term **point of application. Line of action** (L) includes the element P and part of the element D, namely, the inclination A; it does not include sense. To determine the line of action of a force completely, not only must the inclination of the force be known, but at least one point in its line of action must also be known. The line of action of a force may be known, yet the direction be partly unknown since the sense may be unknown. The direction of a force may be known, yet the line of action be unknown since no point in that line may be known. (Pages 3 and 4.)

2. To determine a force completely, it is necessary to determine all three of its elements (M, D, and P). In determining D it is necessary to find both inclination (A) and sense, but sense is determined by algebraic signs (or their graphic equivalents), and not by a separate equation. The **unknowns** (M, D, and P) are therefore three and may be determined by three equations or by the graphic equivalents of three equations.

THE H AND V COMPONENTS OF A FORCE

3. In the algebraic or graphic solution of a problem, any force may be replaced by any two **components,** rectangular or oblique, provided these two components are assumed to act at some point in the line of action of the force. The point at which the components act may be theoretically any point in the line of action of the force; practically, in any given problem that point is chosen which will facilitate the solution of the problem. The components most used in stresses are **horizontal** (H) and **vertical** (V) **components.** The relations of the magnitude of a force to the magnitudes of its H and V components may be represented by a right triangle in which the force F is represented by the hypotenuse, the angle of inclination (A) by the angle that the hypotenuse makes with the vertical side, the H component (F_H) by the horizontal side, and the V component (F_V) by the vertical side. A rectangular component of a force may be determined from the force itself by any algebraic or graphic method that can be used for determining the corresponding side of a right triangle from the hypotenuse; conversely, a force may be determined from either or both of its rectangular components by any algebraic or graphic method that can be used for determining the hypotenuse from one or both of the other two sides. (7 : 1.)

TO DETERMINE THE COMPONENTS OF A FORCE

4. *The H and V components of a force may be determined* from the force itself by three methods, namely, (a) graphic, (b) trigonometric, and (c) geometric.

5. *Graphic.* Along the line of action of the force (or on a line parallel

to it) lay off the magnitude of the force to any convenient scale, draw the horizontal and vertical projections of this magnitude, and scale these H and V projections. (7 : 2.)

1. *Trigonometric.* If the inclination A of a force F is expressed in angular measure, then:

$$F_H = F \sin A \text{ and } F_V = F \cos A. \quad (7:5.)$$

2. *Geometric.* If the slope of the line of action of a force is expressed in units of linear measure, the geometric method may be used to advantage. The line of action of the force F is unlimited in length. If any portion of this line is the hypotenuse a of a right triangle with a horizontal side h and a vertical side v, and if the lengths a, h, and v are known, then:

Force : Length : : Force : Length

$$F : a :: F_H : h \quad \text{or } F_H = F\frac{h}{a}$$

$$F : a :: F_V : v \quad \text{or } F_V = F\frac{v}{a} \quad (8:1.)$$

3. *The algebraic signs for H and V components* are:
H components: Sense to the right, *plus;* sense to the left, *minus.*
V components: Sense upward, *plus;* sense downward, *minus.* (8 : 4.)

TO DETERMINE A FORCE FROM ITS H AND V COMPONENTS

4. An unknown force F may be determined from either or both of its H and V components, graphically or algebraically.

I. WHEN THE H AND V COMPONENTS OF F ARE BOTH KNOWN AND A POINT P IN THE LINE OF ACTION OF F IS ALSO KNOWN

5. *Graphic method.* Through P lay off the magnitudes of F_H and F_V to any convenient scale and with the correct sense for each component. F will be the diagonal of the rectangle of which F_H and F_V are the two sides. (Or lay off a right triangle with F_H and F_V as its two sides; then F will be its hypotenuse) (9 : 3.)

6. *Algebraic Method:*

Magnitude of $F = \sqrt{F_H^2 + F_V^2}$.

Inclination of $F = \dfrac{F_H}{F_V}$ expressed either as the tangent of the angle of inclination A or as a slope expressed in linear measure (9 : 5.)

II. WHEN ONE COMPONENT OF F AND THE LINE OF ACTION OF F ARE KNOWN

7. *Graphic method:* From any point P in the line of action of F, lay off to a convenient scale F_H or F_V, whichever is known, and find the corresponding length of F along the line of action. (9 : 7.)

8. *Trigonometric method:* (A = angle of inclination)

$$F = F_H \div \sin A \text{ or } F_H \operatorname{cosec} A.$$
$$F = F_V \div \cos A \text{ or } F_V \sec A. \quad (10:1.)$$

9. *Geometric method:* (Inclination given by horizontal and vertical distances h and v; a is the length of the corresponding hypotenuse, all expressed in any unit of linear measurement.)

Force : Length : : Force : Length

$$F : a :: F_H : h \quad \text{or } F = F_H\frac{a}{h}$$

$$F : a :: F_V : v \quad \text{or } F = F_V\frac{a}{v} \quad (10:2.)$$

TO FIND ONE COMPONENT FROM THE OTHER COMPONENT

10. *Trigonometric method:* $F_H = F_V \tan A$
$F_V = F_H \cot A$ (10 : 4.)

11. *Geometric method:* $F_H : h :: F_V : v$ or $F_H = F_V\dfrac{h}{v}$

$$F_V = F_H\frac{v}{h} \quad (10:4.)$$

12. *Note:* It is sometimes advantageous to use rectangular components parallel respectively to two rectangular axes X and Y, neither of which is horizontal or vertical. The general relations between a force and its components, just given, will hold for such components if " X " is substituted for " H " and " Y " for " V," provided the angle of inclination A is taken as the angle that the line of action of the force makes with the axis Y. (10 : 6.)

13. *Note:* Occasionally it is advantageous to resolve a force into **non-rectangular components.** The general relations between a force and such oblique components are treated in 11 : 1.

MOMENT OF A FORCE

14. The **moment of a force** always involves a center of moments in the plane of the force. The perpendicular distance from the center of moments to the line of action of the force is the **lever arm.** (30 : 3.)

1. The **magnitude of the moment** of a force is a *product*, namely, the product of the magnitude of the force itself by the length of the lever arm. Moment should therefore always suggest a product, namely, force × distance. (30 : 4.)

2. *Note:* The term *moment* is also used quite generally to denote the algebraic sum of two or more moments (products), but in this case what is really meant is the **resultant moment.** (30 : 5.)

3. Since the magnitude of the moment of a force is a product of two quantities and since one of these quantities is measured in units of force and the other in units of length, the magnitude of the moment must be expressed in a compound unit, partly force and partly length. The most common combinations are:

Force		*Lever Arm*		*Moment*
Pounds	×	Feet	=	Pound-Feet
Pounds	×	Inches	=	Pound-Inches
Tons	×	Feet	=	Ton-Feet

4. *Note:* The term **pound-feet** is used in this book in preference to **foot-pounds.** (30 : 6.)

5. The moment of any force has not only magnitude but also direction. If a force tends to produce **clockwise rotation** about a point, the moment with respect to that point is **plus** (in this book); if **counter-clockwise,** the moment is **minus.** The direction of rotation (and the corresponding algebraic sign) depends not only upon the sense of the force but also upon whether the center of moments is on one side or the other of the line of action of that force. (30 : 7.)

6. *Note:* The decision to call clockwise rotation plus and counter-clockwise minus seems to be in accord with the general tendency among engineers. (31 : 1.)

7. Since the moment of a force has both magnitude and direction, it is a vector quantity and may be represented as a vector; the treatment of moments as vectors permits simple solutions of problems involving two or more moments. (31 : 2.)

8. The moment of a force with respect to any center of moments can be determined graphically, but the method is much more laborious than the algebraic method. (31 : 5 and 6.)

9. If a force is replaced by any pair of components at any point P in its line of action, the algebraic sum of the moments of these two components (resultant moment) with respect to any center of moments is equal to the moment of the force itself with respect to the same point. (31 : 4.)

10. Two parallel forces, equal in magnitude but opposite in sense, are called a **couple.** The magnitude of the resultant moment of a couple is always the same, regardless of where the center of moments is taken; it is equal to the product of the magnitude of either force by the perpendicular distance between the two lines of action (**lever arm of the couple**). (32 : 3.)

11. The tendency of a couple is always to produce rotation, and hence if the forces of a couple are the only forces acting on a body that body cannot be in equilibrium. (32 : 5.)

12. For any system of parallel forces, the difference in magnitudes of any two resultant moments is equal to the algebraic sum of the forces multiplied by the lever arm difference for the two corresponding centers of moments. (32 : 7.)

CONDITIONS OF EQUILIBRIUM

13. *A particle* acted on by co-planar forces is said to be in equilibrium if there is no motion of translation, *i.e.*, no motion with respect to each of two rectangular axes in the plane of the forces. (12 : 3.) There will be no motion of translation if each of the following two conditions is fulfilled:

I. The algebraic sum of the horizontal components of the forces must equal zero.

II. The algebraic sum of the vertical components must equal zero.

These **two conditions of equilibrium** may be expressed algebraically thus:

$$\Sigma H = 0 \quad \text{and} \quad \Sigma V = 0. \qquad (13 : 4.)$$

The same conditions are expressed graphically when the forces laid off to scale (units of force) form a **force polygon.** (13 : 6 to 14 : 6.)

14. *A rigid body* acted upon by co-planar forces is in equilibrium if the conditions of equilibrium, just given for a particle, are fulfilled, and if, in addition, there is no rotation about any point whatsoever in the plane of the forces. (32 : 8.) These conditions may be stated as follows:

I. The algebraic sum of the horizontal components of all the forces that act on the body must equal zero.

II. The algebraic sum of the vertical components of all the forces that act on the body must equal zero.

III. The algebraic sum of the moments of all the forces that act on the body about any point of moments whatsoever must equal zero. (32 : 8.)

These **three conditions of equilibrium,** expressed algebraically, are:

$$\Sigma H = 0, \ \Sigma V = 0, \ \text{and} \ \Sigma M = 0.$$

Each of the first two of these equations is known as a **resolution equation** and the third is known as a **moment equation.** (33 : 4.)

The same three conditions may be expressed graphically by two polygons: (1) the **force polygon** (equivalent to $\Sigma H = 0$ and $\Sigma V = 0$), and (2) the **equilibrium polygon** (equivalent to $\Sigma M = 0$). (36 : 8, 9, and 10.)

1. *Note:* The first two conditions of equilibrium may be expressed in more general terms if, in place of H and V axes, any pair of rectangular axes X and Y are taken. Then the two conditions will be $\Sigma X = 0$ and $\Sigma Y = 0$.

2. *Note:* Each of the following two statements of conditions of equilibrium is equivalent to a combination of the three conditions previously given for the equilibrium of a rigid body:

IV. A rigid body is in equilibrium if the algebraic sum of the moments of all the forces that act on the body is zero for each of three centers of moments not in the same straight line. (33 : 2.)

V. A rigid body is in equilibrium if the algebraic sum of the moments of all the forces that act on the body is zero for each of two centers of moments and the algebraic sum of the components parallel to one axis is zero, provided that axis is not perpendicular to the line joining the two centers of moments. (33 : 2.)

USE OF THE EQUILIBRIUM EQUATIONS

3. The H and V components involved in resolution equations may be found by any of the three methods: graphic, trigonometric, or geometric. (Chapter III.) If the inclinations are given in angular measure, the trigonometric method is most convenient; if in linear measure, the geometric method is better. The graphic method may sometimes be used to advantage even for finding components that are to be inserted in a resolution equation.

4. When there is only one unknown component in a resolution equation, that unknown is given a *plus* sign. When the equation has been solved, the algebraic sign of the result indicates *sense;* for $\Sigma H = 0$, *plus* indicates that the unknown component acts to the *right,* — *minus,* to the *left;* for $\Sigma V = 0$, *plus* indicates that the unknown component acts *upward,* — *minus, downward.* (16 : 3, 4, and 5.)

5. Where there are two unknown components in each of two resolution equations, these two equations are solved as simultaneous equations, and the algebraic sign of each unknown component is assumed in accordance with what is known or thought to be the sense of the corresponding force. (16 : 6.)

6. If an unknown is to be determined from a single moment equation, all other unknowns must be eliminated from that equation. If the unknown to be eliminated is a magnitude, assume the center of moments in the corresponding line of action; if the unknowns to be eliminated are M and D of a force for which the element P is the only element known, assume this point P as the center of moments; if the unknowns to be eliminated are two magnitudes, assume the center of moments at the intersection of the two corresponding lines of action. (34 : 1 to 8.)

7. When the magnitude of a force is to be determined from a single moment equation, the moment of the unknown force is given a plus sign in the moment equation. If, upon solving the equation, the algebraic sign of the result is *plus,* the sense of the unknown force must be such as to produce *clockwise* rotation about the center of moments; if *minus,* the sense must be such as to produce *counter-clockwise* rotation. (34 : 9.)

8. A known term in a resolution equation is plus or minus according to the sense of the component corresponding to that term; a known term in a moment equation is plus or minus according to the direction of rotation of the moment corresponding to that term; an unknown term in either a resolution or a moment equation is given the plus sign except in simultaneous equations. When an equilibrium equation has been solved, the algebraic sign of the result indicates *sense* if it is a resolution equation (16 : 2 to 6), or *direction of rotation* if it is a moment equation. (33 : 5.)

9. *Note:* Failure to observe the difference between the significance of algebraic signs in resolution equations and the significance of the same signs in moment equations is a common source of confusion. (33 : 6.)

CONDITIONS OF EQUILIBRIUM EXPRESSED GRAPHICALLY

1. If a force polygon closes, there can be no motion of translation, *i.e.*, a force polygon is equivalent to two resolution equations of equilibrium. (14 : 6 to 9.)

Note: See 14 : 6 for the construction of a force polygon.

2. If the equilibrium polygon closes, there can be no motion of rotation, *i.e.*, the equilibrium polygon is equivalent to a moment equation of equilibrium. (36 : 2.)

Note: See 35 : 1 for the construction of an equilibrium polygon.

3. If a system of concurrent forces is in equilibrium, the corresponding force polygon must close. (14 : 8.) If a system of parallel forces or a system of non-concurrent forces is in equilibrium, both the force polygon and the equilibrium polygon must close. (36 : 3.)

4. *Note:* If three non-parallel forces are in equilibrium, the corresponding force polygon is a triangle. (14 : 3.)

FUNDAMENTAL PRINCIPLES PERTAINING TO FORCE POLYGONS

5. If a system of forces is in equilibrium, the forces will form a force polygon when laid off to scale (units of force) in any order, each parallel to its line of action and each beginning at the arrow end of the preceding force. This principle holds true for non-concurrent as well as for concurrent forces. (14 : 8.)

6. *Note:* If the forces in equilibrium are parallel, all sides of the force polygon will lie in the same straight line.

7. If any number of concurrent forces laid off to scale form a force polygon, these forces must be in equilibrium. (This principle does not hold true for parallel or non-concurrent forces.) (14 : 9.)

8. The arrows representing the senses of the forces all point in the same general direction around the force polygon. (14 : 10.)

9. Any one side of a force polygon represents the equilibrant of all the forces represented by the remaining sides. If the arrow on any one side be reversed so that it points in the opposite direction around the polygon from all the other arrows, that side will then represent the resultant of all the other forces. (14 : 11.)

10. If from any apex of a force polygon a line be drawn across the polygon to any other apex, dividing the polygon into two parts, that line will represent the magnitude of the resultant and of the equilibrant of both sets of forces in the polygon with which it forms a closed figure. (14 : 12.)

PRINCIPLES PERTAINING TO EQUILIBRIUM POLYGONS

11. Two sides of an equilibrium polygon and the line of action in the space diagram at which they intersect are parallel respectively to three sides of a triangle in the force polygon; conversely, the side of a force polygon and the two rays that form a triangle with that side are parallel respectively to the corresponding force in the space diagram and the two sides of the equilibrium polygon that intersect in the line of action of that force. (36 : 11.)

12. The ray that is drawn to the intersection of any two consecutive forces in the force polygon is parallel to the side of the equilibrium polygon that extends between the two corresponding forces in the space diagram; conversely, a side of an equilibrium polygon between two adjacent forces in the space diagram is parallel to the ray that is drawn to the intersection of the two corresponding forces in the force polygon. (37 : 1.)

13. If a system of forces is in equilibrium, each side of an equilibrium polygon is the line of action of the resultant of all of the forces to the left or to the right of that side. (37 : 2.)

THREE CLASSES OF PROBLEMS AND THEIR CHARACTERISTICS

14. *Problems in concurrent forces:*

(*a*) A problem is indeterminate if there are more than two elements unknown. (22 : 3.)

(*b*) An unknown element may be magnitude (M) or direction (D). The element P (point in a line of action) is never unknown and is not involved in the problem. A force is never wholly unknown.

Note: A problem in concurrent forces may be changed to one in non-concurrent forces by replacing any one of the forces by its two components applied at any point in the line of action of the force except the point of concurrence. (42 : 2.)

15. *Problems in parallel forces:*

(*a*) A problem is indeterminate if there are more than two unknown elements. (38 : 4.)

(*b*) An unknown element may be magnitude (M) or point in a line of action (P). The element (D) is never unknown and is not involved in the problem. A force is never wholly unknown.

1. *Problems in non-concurrent forces:*

(*a*) A problem is indeterminate if there are more than three unknown elements. (47 : 3.)

(*b*) An unknown may be any one of the three elements (M, D or P). A force may be wholly unknown.

CHOICE OF EQUILIBRIUM EQUATIONS

2. If there are three unknowns, at least one moment equation must be used.

3. A magnitude (M) may be determined by means of one or both resolution equations or by a moment equation.

4. A moment equation must always be used to determine a point in a line of action (P).

5. The element of direction (D) is best determined by resolution equations though it can be determined by using two moment equations.

6. *A problem in concurrent forces:* There can be only two unknown elements; hence only two equations are necessary, and since an unknown must be either M or D, the two equations used are the resolution equations. (23 : 6.) (P is never an unknown; hence it is not necessary to use a moment equation.)

If the graphic method is used, it will involve only a force polygon.

7. *A problem in parallel forces:* There can be only two elements unknown; hence only two equations are necessary. One of these equations must be a moment equation; the other equation may be another moment equation or it may be a resolution equation. (38 : 6.)

If the graphic method is used, it will involve an equilibrium polygon as well as a force polygon.

8. *A problem in non-concurrent forces:* In all but one of the common cases in non-concurrent forces, three elements are unknown. When three elements are unknown, three equations are necessary and one must be a moment equation; either of the other two may be a moment equation or it may be a resolution equation according to what elements are unknown. (48 : 3 to 8.)

If the graphic method is used and there are three unknown elements, an equilibrium polygon must be drawn as well as a force polygon.

EIGHT COMMON CASES OF PROBLEMS AND THE CORRESPONDING SOLUTIONS

9. *Case A* (Concurrent) M and D of the same force unknown. (22 : 3.)

Algebraic solution: $\Sigma H = 0$ and $\Sigma V = 0$. (Page 26.)

(a) Graphic solution: Force polygon. (Page 27.)

10. *Case B* (Concurrent) M and M of two forces unknown. (22 : 3.)

Algebraic solution: $\Sigma H = 0$ and $\Sigma V = 0$. (Page 28.)

(a) Graphic solution: Force polygon. (Page 29.)

In the algebraic solutions of problems in concurrent forces, it is occasionally advantageous to use two rectangular axes X and Y, neither of which is horizontal or vertical. (28 : 7.)

11. *Case A′* (Parallel) M and P of the same force unknown. (38 : 4.)

Algebraic solution: $\Sigma V = 0$ or $\Sigma H = 0$ and $\Sigma M = 0$. (Page 42.)

Find M from $\Sigma V = 0$ or $\Sigma H = 0$.

Find P from $\Sigma M = 0$. Center of moments anywhere, preferably in the line of action of a known force.

The values of P and M may both be checked by means of a second moment equation with a different center of moments.

(a) Graphic solution: Force polygon and equilibrium polygon. Begin the equilibrium polygon at any point in any known line of action. (Page 43.)

12. *Case B′* (Parallel) M and M of two forces unknown. (38 : 4.)

Algebraic solution: Either of two combinations

$$\Sigma M = 0 \text{ and } \Sigma V = 0 \text{ or } \Sigma H = 0$$
$$\Sigma M = 0 \text{ and } \Sigma M = 0$$

(a) First combination *Case B′:* $\Sigma M = 0$ and $\Sigma V = 0$ or $\Sigma H = 0$. (Page 44.)

Find one M from $\Sigma M = 0$. Center of moments in the line of action of the other unknown M.

Find the other M from $\Sigma V = 0$ or $\Sigma H = 0$.

(b) Second combination *Case B′*: $\Sigma M = 0$ and $\Sigma M = 0$. (44 : 2.)

Find one M from $\Sigma M = 0$.

Find second M from $\Sigma M = 0$.

In each equation assume the center of moments in the line of action corresponding to the unknown M that is to be eliminated from that equation.

(c) Graphic solution: Force polygon and equilibrium polygon. Begin the equilibrium polygon at any point in either of the two lines of action corresponding to the two unknown magnitudes. (Page 45.)

Case B′ corresponds to *Case B* concurrent forces (M and M unknown), but in parallel forces at least one M must be determined by a moment equation. If a resolution equation is used to determine the other M the results may be checked by a second moment equation, and *vice versa*.

1. *Case 1* (Non-concurrent) P, M, and D of the same force unknown. (47 : 4.)

Algebraic solution: $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$. (Page 54.)

Find M and D from $\Sigma H = 0$ and $\Sigma V = 0$. (Exactly as for *Case A* concurrent forces.)

Find P from $\Sigma M = 0$. It is convenient to replace all forces by their H and V components applied at points in the same horizontal line and then to assume the center of moments at any convenient point in this horizontal line. (55 : 2.)

(a) Graphic solution: Force polygon and equilibrium polygon. Begin the equilibrium polygon at any convenient point in the line of action of a known force. (Page 56.)

2. *Case 2* (Non-concurrent) M and M of two forces unknown. (47 : 4.)

Algebraic solution: $\Sigma H = 0$ and $\Sigma V = 0$. (Page 57.)

Find the two unknown magnitudes by means of the two resolution equations. (The problem and its solution are identical with *Case B* concurrent forces.)

Two other combinations of equilibrium equations are possible, namely: (1) $\Sigma M = 0$, and $\Sigma H = 0$ or $\Sigma V = 0$; and (2) $\Sigma M = 0$ and $\Sigma M = 0$. Little is gained, however, by the use of either of these combinations unless the necessary lever arms are known or can easily be found. (57 : 4.)

(a) Graphic solution: Force polygon. (Exactly like the graphic solution for *Case B*.) (Page 57.)

Case 2 is the only common case in non-concurrent forces in which a moment equation is not necessary in the algebraic solution or an equilibrium polygon in the graphic solution.

3. *Case 3* (Non-concurrent) M of one force, M and D of a second force unknown. (47 : 4.)

Algebraic solution: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$. (Page 58.)

Find M of the first force from $\Sigma M = 0$. (Center of moments at the given point P of the second force.)

Find M and D of the second force from $\Sigma H = 0$ and $\Sigma V = 0$. This second step is identical with the first step of *Case 1* and also with the entire solution of *Case A*.

(a) Graphic solution: Force polygon and equilibrium polygon. Begin the equilibrium polygon at the given point P of the force whose M and D are unknown. (Page 59.)

If a problem in *Case 3* is to be solved algebraically, it is often simpler to change it to a problem in *Case 4*. This is done by replacing the force whose M and D are unknown by its H and V components applied at the known point P of the force. (58 : 2.)

4. *Case 4* (Non-concurrent) M, M, and M of three forces unknown. (47 : 4.)

Algebraic solutions: One of three combinations of equilibrium equations may be used.

$$\Sigma M = 0,\quad \Sigma H = 0 \quad\text{and}\quad \Sigma V = 0.$$
$$\Sigma M = 0,\quad \Sigma M = 0 \quad\text{and}\quad \Sigma V = 0 \text{ or } \Sigma H = 0.$$
$$\Sigma M = 0,\quad \Sigma M = 0 \quad\text{and}\quad \Sigma M = 0.$$

(a) First combination *Case 4*: $\Sigma M = 0$, $\Sigma H = 0$ and $\Sigma V = 0$. This combination is used when all forces are parallel to one or the other of the co-ordinate axes H and V, or X and Y. (Page 61.)

Find one unknown M from $\Sigma M = 0$. Center of moments at the intersection of the two lines of action that correspond to the other two unknown magnitudes.

Find the other two unknown M's from $\Sigma H = 0$ and $\Sigma V = 0$. This second step is identical with the solution of *Case 2* and also *Case B*.

(b) Second combination *Case 4*: $\Sigma M = 0$, $\Sigma M = 0$ and $\Sigma V = 0$ or $\Sigma H = 0$. This is the combination most used for *Case 4*. It is a particularly good combination for a problem in which there is only one inclined force in the whole system of forces and the magnitude of that inclined force is one of the three unknown magnitudes. (Page 60.)

Find one unknown M from $\Sigma M = 0$.

Find second M from $\Sigma M = 0$.

Find third M from $\Sigma V = 0$ or $\Sigma H = 0$ whichever can be used to advantage.

In each of the first two equations the center of moments is taken at the intersection of the lines of action corresponding to the two unknown magnitudes that should be eliminated from the equation.

If there is only one inclined force in the system and the magnitude of that inclined force is one of the three unknown magnitudes, the magnitude of a horizontal or vertical force is determined from a moment equation while the magnitude of the inclined force is determined from a resolution equation. (If possible, use a resolution equation that involves only one of the unknown magnitudes.)

(c) Third combination, *Case 4:* $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma M = 0$. This combination can be used to advantage only when the three lines of action corresponding to the three unknown magnitudes intersect each other, *i.e.*, no two are parallel. (Page 63.)

Find the first unknown M from $\Sigma M = 0$. Center of moments at the intersection of the second and third unknown forces.

Find the second unknown M from $\Sigma M = 0$. Center of moments at the intersection of the first and third unknown forces.

Find the third unknown M from $\Sigma M = 0$. Center of moments at the intersection of the first and second unknown forces.

(d) Graphic solution *Case 4:* Force polygon and equilibrium polygon. Begin the equilibrium polygon at the intersection of any two lines of action that correspond to two of the three unknown magnitudes. (60 : 4.)

COMPARISON OF THE FUNDAMENTALS INVOLVED IN THE THREE CLASSES OF PROBLEMS — CONCURRENT, PARALLEL, AND NON-CONCURRENT

The solution of any problem in statics is based on certain fundamentals. These fundamentals differ according to whether the forces involved are concurrent, parallel, or non-concurrent. The fundamental principles and methods for each of these three classes of problems have just been summarized. A comparison of the fundamentals that are characteristic of problems in non-concurrent forces with those that are characteristic of problems in concurrent forces and in parallel forces is made on pages 52 and 53, and also, in somewhat different form, on page 74.

SPECIAL CASES OF FORCES IN EQUILIBRIUM

1. If two forces are in equilibrium, they must have a common line of action, and be equal in magnitude but opposite in sense. (15 : 6.)

2. If three forces are in equilibrium, they may be laid off to scale to form a force triangle. (14 : 3.)

3. If three forces and only three are in equilibrium, all three lines of action must meet in a point or be parallel. (34 : 10.)

4. If there are apparently three lines of action and only three which meet in a point and two are in the same straight line, there can be no force in the third, and the forces in the first two lines must be equal in magnitude and opposite in sense. (15 : 7.)

5. If four concurrent forces in equilibrium act in two and only two straight lines, any two that act in the same line must be equal in magnitude and opposite in sense. (15 : 8.)

6. If four concurrent forces are in equilibrium and two act in the same horizontal line, the V components of the other two must be equal in magnitude and opposite in sense; similarly, if two of the four forces act in the same vertical line, the H components of the other two must be equal in magnitude and opposite in sense. If two of the four forces act in either the same horizontal or the same vertical line and the other two have equal inclinations, the two inclined forces must be equal in magnitude but opposite in sense. (15 : 9.)

7. If four concurrent forces A, B, C, and D are in equilibrium and the lines of action of any two, as A and B, are in the same straight line, the two components normal to that line of the two remaining forces C and D must be equal in magnitude but opposite in sense. If the lines of action of these last two forces C and D are equally inclined to the common line of action of the first two A and B, the last two forces are equal in magnitude. (16 : 1.)

8. The fundamental principle stated in (71 : 4) may be used to demonstrate another principle, not given in the preceding chapters. Let the truss in Fig. 72 be acted upon by external forces *1*, *2*, *3*, *4*, *5* and *6*. Three lines of action meet at joint *i*, but since there are only three and two of these (*gi* and *ij*) are in a straight line, there can be no force in the third (*hi*). The member *hi* may therefore be taken out, leaving only three lines of action at *h*; two of these (*eh* and *hj*) are in a straight line, hence there can be no force in the third (*gh*). Take out the member *gh* and there are only three lines of action at *g*;

since two of these (*fg* and *gi*) are in a straight line, there can be no force in the third (*eg*). Removing the member *eg* in addition to *hi* and *gh* reduces the entire right-hand half of the truss to a single large triangle *efj* acted on by two external forces *4* and *5* and by the forces in the members *ce*, *de* and *df*. (Fig. 72.) Each of these five forces is applied at an apex of the triangle of *efj*, and there are no external forces applied at apices *g*, *h* and *i*. The general principle illustrated by this example may be stated as follows:

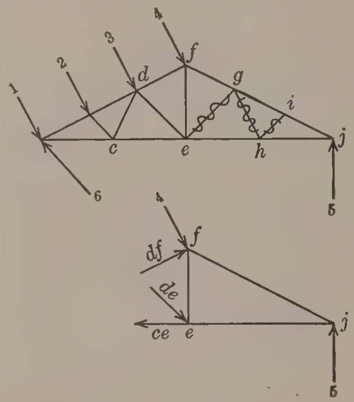


Fig. 72.

1. If the perimeter of a truss or any portion of a truss is a triangle that is subdivided into smaller triangles by truss members within the main triangle, and if there are no external forces acting on the main triangle except those which are applied at its three apices, there can be no stress in any member that is within the main triangle, and the stresses in all members that lie in any side of the main triangle will be equal in magnitude.

STRESS, TENSION AND COMPRESSION, STRAIN

2. Stresses are internal forces caused by external forces. (3 : 1.) Axial stress in any member of a structure is the resultant stress of all internal forces in the member that act in lines parallel to the longitudinal axis of the member; the line of action of the axial stress coincides with the axis of the member (3 : 1 and 2). Unless otherwise stated, the stress in any member is understood to be the axial stress. Axial stresses may be either tension or compression. (5 : 3.)

Other stresses, such as secondary stresses, will not be considered in this book. (3 : 2.)

3. A tension member of a truss always acts away from (exerts a pull on) a joint of a truss, whether that joint is at one end or the other of the member. A compression member acts toward a joint. (6 : 2 and 3.)

4. Any arrow near a joint of a truss diagram indicates the sense of the force that the corresponding member exerts on that joint. The two arrows on any member in a truss diagram, one at either end, are opposite in direction, and point toward each other if the member is in tension, — away from each other if the member is in compression. (6 : 5 and 6.)

5. Structural materials are elastic. External forces tend to lengthen tension members and shorten compression members. (6 : 8.)

6. Strain in a member of a structure is the amount by which the member is lengthened or shortened by the action of external forces. Within the elastic limit, strain is proportional to stress. Strain is measured in units of length, while stress is measured in units of force. (6 : 8.)

GENERAL METHOD OF ATTACK

7. In the general method of attack there are three steps, namely, (1) to determine the knowns; (2) to determine the unknowns; (3) from these unknowns to determine the case under which the problem falls and to use the standard algebraic or graphic method of solution for that case.

8. *To determine the knowns:*

(a) Select the body in equilibrium. The first thought in attacking any problem should be: "What is the body to be considered in equilibrium?"

Suggestions: (1) If the body is larger than a particle it must be a rigid body; if it is part of a structure, that part must be in itself rigid.

(2) The body must be such that at least one of the forces acting upon it is of known magnitude. (Frequently this force is due to gravity acting upon the body itself.)

(3) The body must be such that in the entire system of forces there are not more than two unknown elements if the forces are concurrent or parallel, and not more than three unknown elements if the forces are non-concurrent.

(4) In problems in stresses, a body in equilibrium is usually a joint of a truss (considered as a particle), a rigid portion of the truss, or a complete truss.

(b) To determine the known forces that act on the body in equilibrium and the known elements of partly known forces. This is best done by making a sketch, particularly in problems in parallel and non-concurrent forces.

Suggestions: In making sketches for problems in parallel and non-concurrent forces:

(1) Begin by making a sketch of the free body.

(2) If the weight of the body is to be taken into account, indicate the corresponding force.

(3) Observe carefully all bodies that are in contact with the body in equilibrium and represent as far as possible the lines of action and the magnitudes of the forces exerted by these bodies.

Caution: Do not consider any element of a force as known unless you are certain that it really is known. A common mistake, for example, is to consider that the line of action of a reaction at a support is known, when in reality only one point (P) in that line is known.

(4) Represent other forces, if any, such for example as those due to wind.

(5) Show on the sketch all data needed for solving the problem.

Note: The most common mistakes made in problems in statics or stresses result from overlooking one or more forces that act on the body in equilibrium or from assuming a force to act differently from the way in which it really does act. The surest way of avoiding such mistakes is to make adequate sketches. The battle is half won when the forces in equilibrium have been completely and correctly indicated on a sketch. More complete suggestions for making sketches and representing forces will be found on pages 19 and 50.

1. *To determine the unknowns:* If the knowns have been determined correctly, the forces that are either wholly unknown or partly unknown will be evident.

2. *To solve the problem:* From the unknowns, determine under which of the eight cases the problem falls, and follow the standard method of solution for that case.

Suggestions: (1) If the problem is one in concurrent forces it will fall under one of two cases, either of which may be solved by the resolution equations or by a force polygon. (69 : 9, and 69 : 10.)

(2) If the problem is one in parallel forces it will fall under one of two cases, either of which may be solved by a combination of a moment equation and a resolution equation. If the graphic method is used it will involve both a force polygon and an equilibrium polygon. (69 : 11, and 69 : 12.)

(3) If the problem is one in non-concurrent forces it will fall under one of four cases. A problem in *Case 2*, like a problem in concurrent forces, may be solved by two resolution equations or by a force polygon. A problem in any one of the other three cases involves three unknown elements and hence it will be necessary to use three equations, at least one of which must be a moment equation, or if the graphic method is used it will be necessary to draw both a force polygon and an equilibrium polygon. (70 : 1, 70 : 3, and 70 : 4.)

(4) It will be helpful in choosing equilibrium equations to keep in mind what elements can and cannot be determined by resolution equations, and what elements can and cannot be determined by moment equations. (69 : 2 to 69 : 8.)

(5) In using equilibrium equations keep in mind principles 67 : 3 to 67 : 9.

(6) In drawing force polygons keep in mind principles 68 : 5 to 68 : 10.

(7) In drawing equilibrium polygons keep in mind principles 68 : 11 to 68 : 13.

(8) An algebraic solution is usually preferable to a graphic solution except for *Case B* (concurrent forces). In an algebraic solution, first indicate all equations, making sure that all forces are included that should be included, and that all components or moments are correctly indicated. Do not indicate the method (trigonometric or geometric) of finding components; this work should be done on other paper and only the resulting values inserted in the equation. When you are sure that all equations have been correctly indicated, and not until then, begin the mechanical work of computation.

3. *Analysis:* The form of analysis used throughout this book, in applying the fundamentals of statics in the determination of stresses, is as follows:

Body in equilibrium:
Known:
Unknown:
Equations:

The form just outlined corresponds to the four steps in the general method of attack. The student will find it advantageous at first to write out this analysis. Eventually a corresponding mental analysis should become a fixed habit in attacking any problem in stresses.

AN EPITOME OF STATICS

4. A problem in statics not infrequently falls under one of the special cases summarized in 71 : 1 to 71 : 7. The great majority of such problems, however, must be solved by means of the equations of equilibrium or their graphic equivalents. The fundamental principles and methods that enable one to analyze a problem, to determine under which case it falls, and to apply the best combination of equations, are, after all, relatively few. Once these fundamentals are understood, they can be summarized concisely and in such form as to bring out by contrast the essential differences between the solutions of problems that fall under different cases. Such a summary, arranged in three parallel columns for the sake of contrast, will be found on the next page. This page is an epitome of statics. The student who has mastered all that is involved in this epitome should have little difficulty in the study of stresses.

COMPARISON OF FUNDAMENTAL PRINCIPLES AND METHODS

CONCURRENT

1. *Indeterminate* if there are more than *two* unknown elements.

2. *An unknown* may be M or D; the element P is never unknown.

3. No force is wholly unknown.

4. *Algebraic solution:* Two resolution equations. A moment equation is not used.

5. *Graphic solution:* Force polygon.

6. *Case A:* M and D unknown. (Same force.)
$\Sigma H = 0$ and $\Sigma V = 0$.

Case B: M and M unknown.
$\Sigma H = 0$ and $\Sigma V = 0$.

Notes: Since the element P is never unknown it is not necessary to use a moment equation.

Case B is one of the three cases that occur most frequently in stresses.

PARALLEL

1. *Indeterminate* if there are more than *two* unknown elements.

2. *An unknown* may be M or P; the element D is never unknown.

3. No force is wholly unknown.

4. *Algebraic solution:* At least one moment equation must be used; the other equation may be another moment equation or it may be a resolution equation.

5. *Graphic solution:* Force polygon and equilibrium polygon.

6. *Case A′:* M and P unknown. (Same force.)
$\Sigma V = 0$ (or $\Sigma H = 0$) and $\Sigma M = 0$.

Case B′: M and M unknown.
$\Sigma M = 0$ and $\Sigma V = 0$ (or $\Sigma H = 0$); also, $\Sigma M = 0$ and $\Sigma M = 0$.

Notes: Although P is not an unknown in *Case B′*, it is necessary, nevertheless, to use a moment equation.

Case B′ is one of the three cases that occur most frequently in stresses.

NON-CONCURRENT

1. *Indeterminate* if there are more than *three* unknown elements.

2. *An unknown* may be any one of the three elements M, D or P.

3. A force may be wholly unknown.

4. *Algebraic solution:* At least one moment equation must be used except in *Case 2;* either of the other two equations may be a moment equation or a resolution equation, depending upon the case.

5. *Graphic solution:* Force polygon and equilibrium polygon except in *Case 2.*

6. *Case 1:* M, D, and P unknown. (Same force.)
$\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$.

Case 2: M and M unknown.
$\Sigma H = 0$ and $\Sigma V = 0$.

Case 3: M, M and D unknown.
$\Sigma M = 0$, $\Sigma H = 0$ and $\Sigma V = 0$.

Case 4: M, M, and M unknown.
$\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

Notes: In *Cases 1, 3,* and *4* there are three unknowns, hence at least one moment equation must be used in each of these cases.

The unknowns in *Case 2* are the same as those in *Cases B* and *B′* and the solution is identical with that for *Case B*, namely, $\Sigma H = 0$ and $\Sigma V = 0$.

Case 4 is one of the three cases that occur most frequently in stresses. *Case 3* also occurs frequently but it may be changed to *Case 4.*

When all of the forces involved in *Case 4* are either horizontal or vertical, the best combination of equations is $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$.

1. **Illustrative Problems.** The aim in this book thus far has been to give to the student the working knowledge of statics that he will need in the study of stresses. This working knowledge consists mainly of: (1) A knowledge of the fundamentals pertaining to a single force, including the elements of a force, the methods of resolving a force into any two components, and the methods of determining a force from either or both of two components. (Chapters II and III.) (2) A knowledge of the conditions of equilibrium (Chapters IV and VII), including special cases. (71 : 1 to 8.) (3) A knowledge of the eight common cases under one of which a problem in statics usually falls, particularly a knowledge of the unknown elements that determine each case and the combination of equilibrium equations used in the standard solution for each case. (Page 74.) (4) A knowledge of a method of attack that enables one to see clearly what forces or elements of forces are known and what are unknown in a problem, to recognize under which case the problem falls, and finally to apply with ease and certainty the standard method of solution for that case. (Chapter V.)

2. The student is advised to test for himself his working knowledge of statics by a study of the illustrative problems that comprise the remainder of this chapter. The statement of each problem is followed by an analysis of the problem, but the actual equations used in the solution are not given in complete form. By rapidly scanning the statement and the corresponding analysis, the student should be able to recognize at once under which case each problem falls and to see how the standard solution for that case is applied.

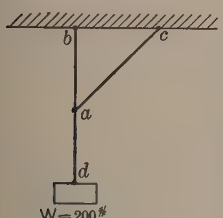


Fig. 75 (*a*).

3. *Notes:* (*a*) In the majority of the problems, only *sketches* corresponding to the given conditions are shown; it is expected that the student will draw for himself any *space diagram* that may be needed. (*b*) Some of the more simple problems fall under special cases, and some of the more complex involve two or more of the common cases. (*c*) In certain problems, particularly in those in which a moment equation of equilibrium is used, it is expedient to replace a force by its two components, and to consider these components applied at such a point in the line of action of the force that the lever arm of one of them is zero with respect to the center of moments selected for the moment equation. (*d*) Having first checked the analysis for *all* of the problems, the student will find it good practice to go through the problems a second time and actually solve each problem.

4. *Problem 1.* (Fig. 75 (*a*).) Given: A block W of 200 lbs. weight suspended by three ropes ad, ab, and ac. Required: The stresses in the ropes.

Since there are only three forces at a and since ab and ad have a common line of action, there can be no force in ac, and $ab = -ad = 200$ lbs. (71 : 4.)

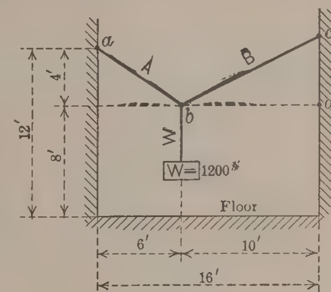


Fig. 75 (*b*).

5. *Problem 2.* (Fig. 75 (*b*).) A weight W of 1200 lbs. is suspended by a rope that passes over a pulley at b and is fastened to a wall at a. Assuming that the pulley is held in equilibrium by a rope bc, what is the stress in this rope and the distance of the point c above the floor? The dimensions of the pulley and friction are negligible.

Analysis

Body in equilibrium: The pulley considered as a particle.
Known: W, and A. (Each equal to 1200 lbs.)
Unknown: B(M and D). (Case A.) (69 : 9.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.
From $\Sigma V = 0$ determine B_V.
From $\Sigma H = 0$ determine B_H.
From B_H and B_V determine B. (65 : 6.)
From the direction of B determine cd.
Answers: $B = 1130$ lbs. tension; c is 13.3 ft. above the floor.

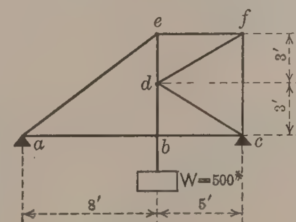


Fig. 75 (*c*).

6. *Problem 3.* (Fig. 75 (*c*).) Given: The framework shown in the figure, supported at a and c, and carrying a load W of 500 lbs. suspended from the joint b. Required: To determine the forces due to the load W in all four members that meet at joint d by considering only the forces at joints b and d.

Analysis

At joint b the force in bd is 500 lbs. tension from fundamental principle. (71 : 5.)

At joint d there are three unknown elements, namely, the magnitudes of de, df, and dc, hence the problem is indeterminate. (68 : 14 (*a*).) From fundamental principle (71 : 7), the force in df must equal in magnitude the force in dc, and this fact, at first thought, may appear to furnish an equation which is in addition to the two resolution equations of equilibrium, thus making the solution possible, but this fundamental principle is based on the equation $\Sigma H = 0$, and does not, therefore, furnish a third equation. In order to solve the problem, it is necessary to consider forces other than those at joints b and d.

1. *Problem 4.* (Fig. 76 (*a*).) Given: A beam of weight $W = 3200$ lbs. that supports three columns A, B, and C, with column loads of $A = 4000$ lbs., $B = 10{,}000$ lbs., and $C = 12{,}000$ lbs. On the right hand end of the beam is a counterweight E of 20,000 lbs. The beam is supported by a single column F, the center of which is an unknown distance x from the point e. Required: To determine the force F.

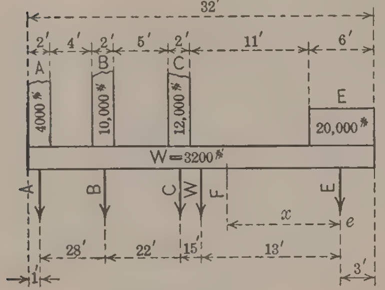


Fig. 76 (*a*).

Analysis

Body in equilibrium: The beam.
Known: A, B, C, W, and E.
Unknown: F. (Case A′.) (69 : 11.)
Equations: $\Sigma V = 0$ and $\Sigma M = 0$.
From $\Sigma V = 0$ determine M of F.
From $\Sigma M_e = 0$ determine x, i.e., P of F.

Answers: $F = 49{,}200$ lbs. upward; $x = 11.25$ ft.

2. *Problem 5.* (Fig. 76 (*b*).) Given: A framework *abcd* of negligible weight. A force A of 100 lbs. applied at a acts toward the framework. The only other force that acts on the framework is a force C applied at c. Required: The direction of the force A and the magnitude and direction of the force C.

Since A and C are the only forces acting on the framework, they must have a common line of action, be equal in magnitude but opposite in sense. (71 : 1.) The common line of action is a line through both a and c. The force C must be equal to 100 lbs. and act toward the framework at c.

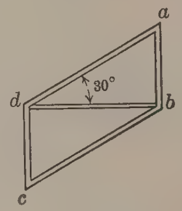


Fig. 76 (*b*).

3. *Problem 6.* (Fig. 76 (*c*).) Given: A block A weighing 100 lbs. supported on the inclined surface of a body B. The block is kept from sliding by friction. Required: (a) The force R exerted on the block by the body B. (b) The frictional force F on the block A (parallel to the inclined surface), and the force N that the body B exerts on the block A normal to the inclined surface.

The force $W = 100$ lbs. acts vertically through the center of gravity of the block, and its line of action intersects the inclined surface at a. Considering the block A as the body in equilibrium, the force R that the body B exerts on the block A must have a common line of action with W, and be equal in magnitude but opposite in sense. (71 : 1.)

To find the forces F and N, consider R resolved into components R_X and R_Y, parallel and perpendicular respectively to the inclined surface. Then $F = R_X$ and $N = R_Y$. This is equivalent to considering the point a in equilibrium.

Analysis

Body in equilibrium: Particle a.
Known: W, N(L), and F(L).
Unknown: N(M) and F(M). (Case B.) (69 : 10.)
Equations: $\Sigma X = 0$ and $\Sigma Y = 0$.
From $\Sigma X = 0$ determine F.
From $\Sigma Y = 0$ determine N.

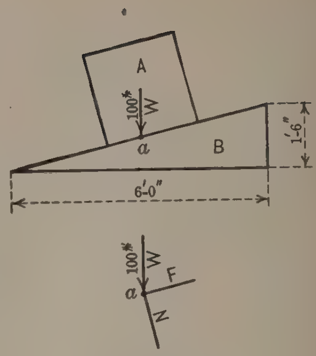


Fig. 76 (*c*).

(This is equivalent to determining the X and Y components of R.)

The solution is simplified by using the X and Y rectangular components instead of the H and V components. (10 : 6.) (How are R_X and R_Y calculated by the geometric method?)

Graphic method: W, F and N must form a force triangle. (14 : 3.)

Answers: $F = 24.3$ lbs.; $N = 970$ lbs. (both upward).

4. *Problem 7.* (Fig. 76 (*d*).) A beam *ab* 15 ft. long, is suspended from a point c by two chains, *ca* and *cb* 12 and 9 feet long respectively. Will the beam remain horizontal as shown, — if not in what position will it come to rest. Let d be the center of the beam. The weight W of the beam may be considered as applied at d. The forces that act on

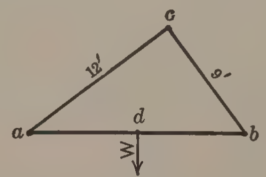


Fig. 76 (*d*).

the beam are three, namely, *ac*, *bc*, and W. The beam cannot be in equilibrium since the lines of action of these three forces do not meet in a point. (71 : 3.) The beam will therefore revolve about c, a moving with a radius of 12 ft. and b with a radius of 9 ft., until the vertical line of action of W passes through c.

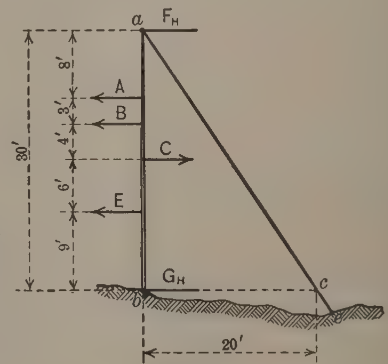


Fig. 76 (*e*).

5. *Problem 8.* (Fig. 76 (*e*).) Given: A pole *ab* of weight $W = 900$ lbs. Four horizontal wires are attached to the pole as shown, and a guy wire extends from a to e. The pull exerted by each of the wires A, B, C, and E is 300 lbs. Required: The horizontal components of the forces F and G exerted on the pole at points a and b respectively.

Analysis

Body in equilibrium: The pole *ab*.
Known: A, B, C, E, F_H(L), and G_H(L).
Unknown: F_H(M) and G_H(M). (Case B′.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma H = 0$.
From $\Sigma M_b = 0$ determine F_H.
From $\Sigma H = 0$ determine G_H.
Answers: $F_H = 350$ lbs.; $G_H = 250$ lbs. (both toward the right).
Question: What is the best method of determining the vertical force exerted on the pole at b, having determined F_H? (Vertical force at b = 1425 lbs. (upward).)

1. *Problem 9.* Given: The simple derrick shown in Fig. 77 (*a*) with a weight W of 1000 lbs. suspended at a. Required: The forces that the ground exerts at points e and c due to the weight W. (Weight of derrick neglected.)

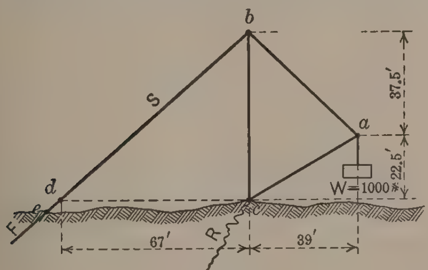


Fig. 77 (*a*).

Considering the point e as a particle in equilibrium, the force F that the ground exerts on e must have a common line of action with the stress S in the guy *eb*, and must be equal in magnitude to that force but opposite in sense. (71 : 1.) The stress S in *eb* will therefore be determined first, and as part of the same problem the force R that the ground exerts at c will also be found.

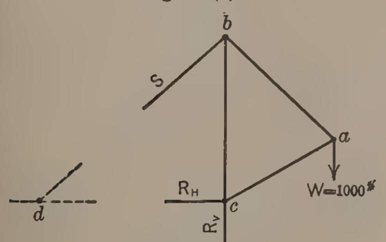


Fig. 77 (*b*).

Analysis

Body in equilibrium: Triangle *abc*. (Fig. 77 (*b*).)
Known: W, S(L), and R(P).
Unknown: S(M) and R(M and D). (Case 3.) (70 : 3.)

Replace R by R_H and R_V thus changing to Case 4. (47 : 5.)
Unknown: S(M), R_H(M), and R_V(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$ or $\Sigma H = 0$. (70 : 4.)
From $\Sigma M_d = 0$ determine R_V.
From $\Sigma M_b = 0$ determine R_H.
From $\Sigma V = 0$ determine S_V.
From R_V and R_H determine R. (65 : 6.)
From S_V determine S. (65 : 9.)
From S determine F.

Answers: R = 1710.4 lbs. upward; $4\frac{15}{16}$ horizontal to 12 vertical; F = 872.5 lbs. downward, line of action *db*.

2. *Problem 10.* (Fig. 77 (*c*).) Given: The weight W of the beam *bd* is 360 lbs. The beam is held in a horizontal position by two chains, *ba* and *de*, and a strut *bc*. Two loads are suspended by chains *fg* and *hi*. Required: The stresses in the two inclined chains and the strut.

Analysis

Body in equilibrium: The beam.
Known: W, *fg*, *hi*, *ba*(L), *bc*(L), and *de*(L).
Unknown: *ba*(M), *bc*(M), and *de*(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.
Replace *de* by de_H and de_V applied at d.
From $\Sigma M_b = 0$ determine de_V.
From de_V determine de_H. (65 : 11).

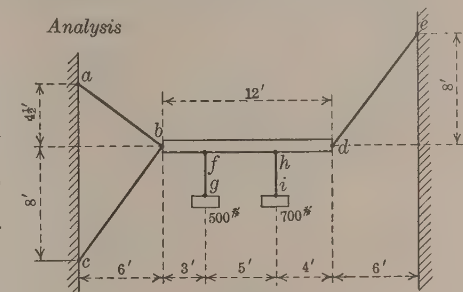


Fig. 77 (*c*).

Replace *bc* by bc_H and bc_V applied at c, and *de* by de_H and de_V applied at d.
From $\Sigma M_a = 0$ determine bc_H.
From $\Sigma H = 0$ determine ba_H.
From de_H, bc_H, and ba_H determine respectively *de*, *bc*, and *ba*. (65 : 9.)

Answers: *de* = 964.6 lbs. tension; *bc* = 283.3 lbs. compression; *ba* = 936.0 lbs. tension.

Question: Could this problem be solved by means of three moment equations and without using the H or V component of any one of the unknown forces?

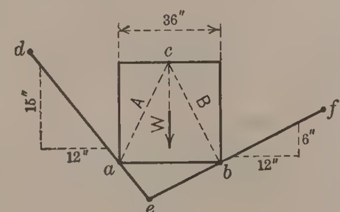


Fig. 77 (*d*).

3. *Problem 11.* (Fig. 77 (*d*).) Given: A 36 inch cube whose weight W is 4000 lbs. Two edges of the cube rest on two inclined planes *ed* and *ef* at points a and b as shown in the figure. Two faces of the cube are horizontal. Required: The forces exerted by the planes on the edges of the cube at a and b. Let A represent the force at a, and B the force at b. The unknown elements are A(M and D) and B(M and D). Since there are four unknown elements, the problem is indeterminate (69 : 1 (a)).

4. *Problem 11-A.* Required: To solve the problem when the following conditions are added: The cube is fastened to plane *ed* at corner a by a hinged joint, and the other corner at b is free to move along the plane *ef*, i.e., the force B is normal to the plane *ef* at b.

Analysis

Since there are three and only three forces acting on the cube, namely W, A, and B, the lines of action must meet in a point. (71 : 3.) Since the line of action of B is normal to the plane at b, the point c in which the lines of action of B and W intersect must be the point of concurrence. The line of action of A is, therefore, ac, and since ac and bc have equal inclinations, $A_V = B_V$. From $\Sigma V = 0$, $A_V = B_V = \frac{1}{2}W$. Either A or B may be determined from its V component and $A = B$.

Answers: $A = B = 2236.1$ lbs. (both upward).

Note that although the forces A and B are equal in magnitude and their lines of action have equal inclinations, the force A has a component parallel to the plane de, whereas, by assumption, the force B has no component parallel to the plane ef.

1. *Problem 12.* (Fig. 78 (a).) Given: A load of 500 lbs. at d suspended by a rope that passes over a sheave 4 ft. in diameter to a point e. The sheave turns on an axle at a, and this axle is supported by a framework bac. The distance $bc = 6$ ft., and the point a is 16 inches above the center of bc. Friction and weights of sheave and framework may be neglected. Required: The force exerted on the axle by the sheave and by the members ba and ca.

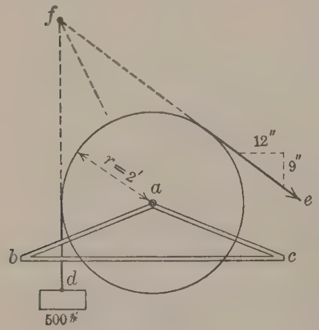


Fig. 78 (a).

Let F represent the force exerted on the axle by the sheave and F' the equal but opposite force exerted on the sheave by the axle. The forces acting on the sheave are three, and the three corresponding lines of action must meet in a point (71 : 3), hence these lines are fd, fa and fe. The forces fd and fe are equal, and F' is the equilibrant of these two forces. This equilibrant may be found directly from fd and fe algebraically or graphically. (14 : 5.)

To find ba and ca:

Analysis

Body in equilibrium: The axle (particle a).
Known: F (from F'), ba(L), and ca(L).
Unknown: ba(M) and ca(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine ba and ca.
Corresponding graphic method: Force triangle.

Answers: $ba = 766.1$ lbs.; $ca = 1203.8$ lbs. (both compression).

2. *Problem 13.* (Fig. 78 (b).) Given: A building temporarily supported on girders, one of which ab is shown. The weight W of that portion of the building that is supported by the girder ab is 80 tons, symmetrically distributed with respect to the center of the girder. On the floor of the building are three uniformly distributed loads, namely, $A = 15$ tons, $B = 30$ tons, and $C = 10$ tons. The weight G of the girder itself is 4 tons. The girder is supported at two points e and f, 28 ft. apart, so located that the force E that the support at e exerts, is equal to the force F that the support at f exerts. Required: To determine the forces E and F.

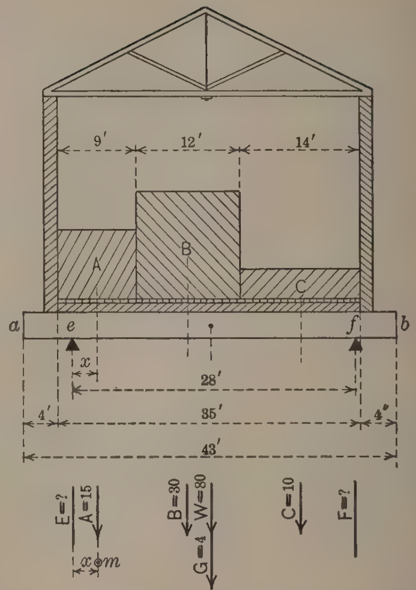


Fig. 78 (b).

Let x represent the unknown distance from e to the line of action of A.

Analysis

Body in equilibrium: The beam ab.
Known: A, B, C, W, and G.
Unknown: $E = F$. (Case A′.) (69 : 11.)
Equations: $\Sigma V = 0$ and $\Sigma M = 0$.
From $\Sigma V = 0$ determine M of E = M of F.
From $\Sigma M_m = 0$ determine x, i.e., P of E.

Answers: $E = F = 139{,}000$ lbs. upward; $x = 2.2$ ft.

3. *Problem 14.* (Fig. 78 (c).) Given: A rigid body abc of weight $W = 100$ lbs Three forces $A = 300$ lbs., $B = 200$ lbs., and $C = 150$ lbs. act on the body as shown. Required: A fifth force F which, if applied on the edge ac somewhere between a and c, will hold the body in equilibrium.

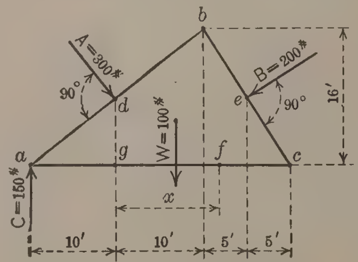


Fig. 78 (c).

Analysis

Replace A by A_H and A_V applied at d, and B by B_H and B_V applied at e.
Body in equilibrium: Body abc.
Known: A_H, A_V, B_H, B_V, C, and W.
Unknown: F. (Case 1.) (70 : 1.)
Equations: $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$.
From $\Sigma H = 0$ determine F_H.
From $\Sigma V = 0$ determine F_V.

Assume F_H and F_V to be applied at f an unknown horizontal distance x from g a point vertically below d.

From $\Sigma M_d = 0$ determine x, i.e., P of F.
From F_H and F_V determine M and D of F.

Answers: $x = 13.4$ ft.; $F = 291.0$ lbs., $\frac{3}{4}$ horizontal to 12 vertical, upward and to the left.

1. *Note:* Each of the forces A and B was replaced by its H and V components in order to simplify the calculation of lever arms, and the point d was chosen as a center of moments in order to reduce the number of forces in the moment equation by eliminating A_H, A_V, and B_H from that equation. This is an excellent illustration of how the application of a moment equation of equilibrium may be simplified by replacing forces by their components, and by choosing the center of moments in the line of action of one or more of the forces involved in the equation.

2. There are a number of other ways in which the moment equation of equilibrium could be applied. For example, the point a could be selected as a center of moments; the forces C and F_H would then be eliminated from the equation, and x, the lever arm of F_V, would extend from a to f.

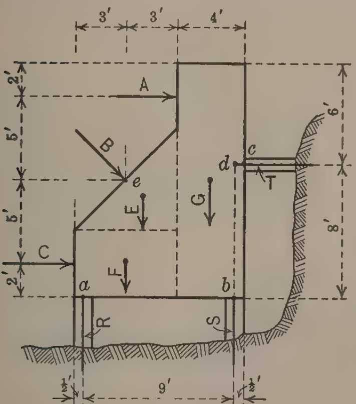


Fig. 79 (*a*).

3. *Problem 15.* (Fig. 79 (*a*).) Given: A body supported at a and b and braced horizontally at c. The weight of the body is 130 lbs. per square foot of its cross-section. (This weight may be divided into three forces E, F, and G, each acting at the center of gravity of a portion of the cross-section as shown.) Three external forces $A = 500$ lbs., $B = 3500$ lbs., and $C = 3000$ lbs. act on the body. Each of these three forces acts at the center of the surface to which it is applied. The supports at a and b are designed to take vertical forces only, and the horizontal brace at c exerts only a horizontal force. Required: The forces exerted on the body by the two supports and the brace, i.e., the forces R, S, and T.

Analysis

Replace B by B_H and B_V applied at e.
Body in equilibrium: The given body.
Known: A, B_H, B_V, C, E, F, G, R(L), S(L), and T(L).
Unknown: R(M), S(M), and T(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$ (since all forces are vertical or horizontal). (70 : 4 (a).)
From $\Sigma M_d = 0$ determine R.
From $\Sigma H = 0$ determine T.
From $\Sigma V = 0$ determine S.
Answers: $R = 8737$ lbs. (up); $T = 5975$ lbs. (to the left); $S = 6478$ lbs. (up).

4. *Problem 16.* (Fig. 79 (*b*).) Given: A block W weighing 2000 lbs. supported by two chains ac and bc that are fastened to a ring at c. A wire rope fastened to the same ring runs from c over a pulley at d to a point j where it is attached to the post df. The stress in this rope may be assumed to be the same from d to j as it is from d to c. A second wire rope in a similar manner runs over a pulley at e to a point k on the post eg. A wire guy runs from d to h, and another from e to i. Required: The forces in the guy ropes dh and ei, in the posts df and eg, and in the chains ac and bc.

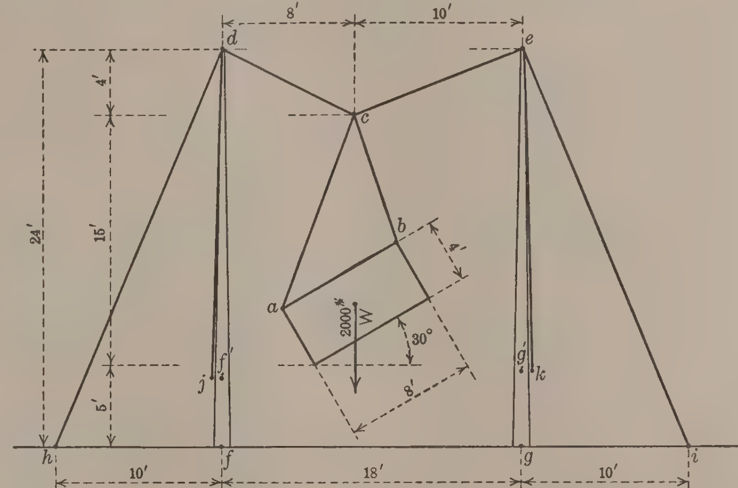


Fig. 79 (*b*).

This problem may be broken up into four problems, namely, (1) to obtain the stress in chains ac and bc, (2) to obtain the stresses in the wire ropes cdj and cek, (3) to obtain the forces in the guy dh and the post df, and finally (4) to obtain the forces in the guy ei and the post eg. It will be seen that each of these problems falls under Case B. (69 : 10.)

Analysis for (1)

Since there are only three forces acting on the block, W, ac, and bc, these forces must be concurrent (71 : 3), i.e., the line of action of W must pass through c.
Body in equilibrium: The block W.
Known: W, ac(L), and bc(L).
Unknown: ac(M) and bc(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine ac and bc.
Answers: $ac = 981.6$ lbs.; $bc = 1140.8$ lbs. (both tension).

Analysis for (2)

Body in equilibrium: Particle c.
Known: ca, cb, cd(L), and ce(L).
Unknown: cd(M) and ce(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine cd and ce.
Answers: $cd = 2484.4$ lbs.; $ce = 2393.6$ lbs. (both tension).

Analysis for (3)

Body in equilibrium: Particle d.
Known: dc, dj (rope), dh(L), and df'(L).
Unknown: dh(M) and df'(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine dh and df'.
Answer: $dh = 5778$ lbs. (tension); $df' = 8929$ lbs. (compression).

1. *Question:* The stress df' just determined is the stress in the post from d to f'. What is the magnitude of the stress in the post from f' to f?

Analysis for (4)

The analysis for the problem of determining the stresses in ei and eg' is similar to that just given for obtaining stresses in dh and df'.

Answers: $ei = 5778$ lbs. (tension); $eg' = 8616$ lbs. (compression); $g'g = ?$

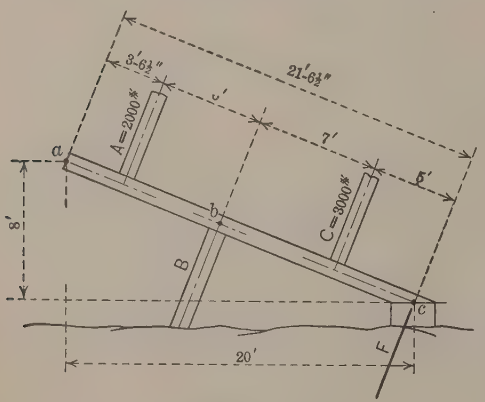


Fig. 80 (*a*).

2. *Note:* The graphic method for Case B (24 : 4 and page 29) could be used to advantage throughout this entire problem in place of the algebraic method. In this case, the first step is to draw the entire figure carefully to scale, replacing, however, the posts by single lines representing their axes.

3. *Problem 17.* (Fig. 80 (*a*).) Given: A beam ac of negligible weight, $21'$-$6\frac{1}{2}''$ long, supported by a brace B at b and a post at c. This beam supports two braces A and C on which there are axial loads of 2000 and 3000 lbs. respectively. The forces exerted on the beam by the three braces A, B, and C are all normal to the axis of the beam. Required: (a) The forces B and F exerted on the beam respectively by the brace at b and the post at c. (b) Assuming that the beam is spiked to the post at c and disregarding friction, what is the force that the spikes must withstand?

Analysis for (a)

Assume one coördinate axis (X) parallel to ac and the other (Y) perpendicular to ac (10 : 6). Since each of the three forces A, B, and C are perpendicular to the X axis, the fourth force F must also be perpendicular to this axis. (Why?)

Body in equilibrium: Beam.
Known: A, C, B(L), and F(L).
Unknown: B(M) and F(M). (Case B'.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma Y = 0$.
From $\Sigma M_c = 0$ determine B.
From $\Sigma Y = 0$ determine F.
Answer: $B = 4250$ lbs.; $F = 750$ lbs. (both upward).

Analysis for (b)

The force that the spikes must withstand is that component of F that is parallel to the plane of contact between the beam and the post, i.e., the H component of F. This component may be found by the geometric method (8 : 1).

Answer: $F_H = 278.6$ lbs.

4. *Problem 18.* (Fig. 80 (*b*).) Given: A framework abc attached to a wall by a hinged joint at b and by a tie cd. A rope supporting a weight W of 1000 lbs. passes over a pulley at a and is fastened to the wall at e. The dimensions of the pulley, friction, and all weights except W may be neglected. Required: The force F that the tie cd exerts at c, the force B that the wall exerts at b, and the forces in the members ab and ac.

First find the forces in ab and ac.

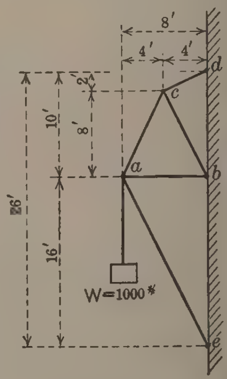


Fig. 80 (*b*).

Analysis

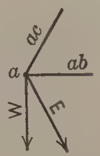


Fig. 80 (*c*).

Body in equilibrium: Joint a. (Fig. 80 (*c*).)
Known: W, E, ab(L), and ac(L).
Unknown: ab(M) and ac(M). (Case B.) (69 : 10.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.
From $\Sigma V = 0$ determine ac_V.
From ac_V determine ac and ac_H. (65 : 9 and 65 : 11.)
From $\Sigma H = 0$ determine ab.

Answers: $ab = 1394$ lbs. (compression); $ac = 2118$ lbs. (tension).

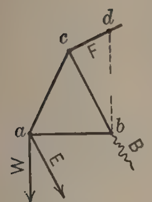


Fig. 81 (*a*).

Analysis

Body in equilibrium: Framework *abc*. (Fig. 81 (*a*).)
Known: W, E, F(L), and B(P).
Unknown: F(M), and B(M and D). (Case 3.) (70 : 3.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.
From $\Sigma M_b = 0$ determine F.
From $\Sigma H = 0$ determine B_H.
From $\Sigma V = 0$ determine B_V.
From B_H and B_V determine B. (65 : 6.)

Answers: $F = 758$ lbs. (upward); $B = 2268$ lbs. 20$\frac{3}{32}$ horizontal to 12 vertical, upward toward the left.

1. *Note:* Instead of determining the magnitude of F directly from $\Sigma M_b = 0$, F may be replaced by F_H and F_V applied at d (48 : 2); F_H may then be determined from $\Sigma M_b = 0$, and F calculated from F_H. Likewise the force E may be replaced by E_H and E_V applied at a, and these two components can then be used in place of E in the equation $\Sigma M_b = 0$. The replacement of F and E by their H and V components renders unnecessary the calculation of lever arms, but little would be gained were it not for the fact that these components must be determined in any case for use in $\Sigma H = 0$ and $\Sigma V = 0$.

2. *Problem 19.* (Fig. 81 (*b*).) Given: A wheel 48″ in diameter. The load W on the wheel, including its own weight, is 300 lbs. A force ab exerted on the axle at a is just sufficient to start the wheel over an obstacle at c. Required: The force ab, and the force C exerted on the wheel by the obstacle.

Under the given conditions the wheel is just leaving the ground at d, hence the force which the ground exerts on the wheel at d has become zero. This leaves only three forces acting on the wheel, namely W, ab, and C, and the lines of action of these three must meet in a point (71 : 3). The line of action of C must therefore pass through a as well as c.

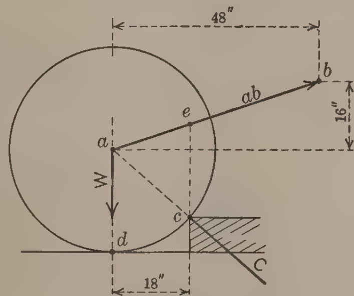


Fig. 81 (*b*).

Analysis

Body in equilibrium: *The axle* assumed as a particle *a*.
Known: W, ab(L), C(L).
Unknown: ab(M) and C(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine ab and C. (16 : 6.)
Graphic method: Force triangle. (68 : 3 and 68 : 4.)

Answers: $ab = 260.1$ lbs. (upward); $C = 329.0$ lbs. 13$\frac{5}{8}$ horizontal to 12 vertical (upward).

A more general solution of the problem, one that can be used when there are more than three external forces on the wheel, is the following:

Analysis

Body in equilibrium: The wheel.
Known: W, ab(L), C(P).
Unknown: ab(M) and C(M and D). (Case 3.) (70 : 3).
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.
Replace ab by ab_H and ab_V applied at e, the intersection of ab and a vertical line through c.
From $\Sigma M_c = 0$ determine ab_H.
From ab_H determine ab_V and ab. (65 : 11 and 65 : 9.)
From $\Sigma H = 0$ determine C_H.
From $\Sigma V = 0$ determine C_V.
From C_H and C_V determine C (65 : 6).

3. *Problem 20.* (Fig. 81 (*c*).) Given: A block K weighing 1000 lbs. suspended by a chain fg. The boom cf is free to revolve in a vertical plane about the joint c; its weight W is 300 lbs. The point d is in a horizontal line through b. Required: The axial forces in the rope de and in the braces ca and cb.

Analysis

Body in equilibrium: Boom *cf*.
Known: W, fg, de(L), ca(L), and cb(L).
Unknown: de(M), ca(M), and cb(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.
Replace de by de_H and de_V applied at a point n in a vertical line through c.
From $\Sigma M_c = 0$ determine de_H.
From de_H determine de_V and de. (65 : 11 and 65 : 9.)
From $\Sigma H = 0$ and $\Sigma V = 0$ determine cb and ca. (16 : 6.)

Answers: $de = 1479.4$ lbs. (tension); $cb = 646.0$ lbs. and $ca = 922.0$ lbs. (both compression).

Moment equations can be used in place of $\Sigma H = 0$ and $\Sigma V = 0$ for determining cb and ca. Proceed as follows:

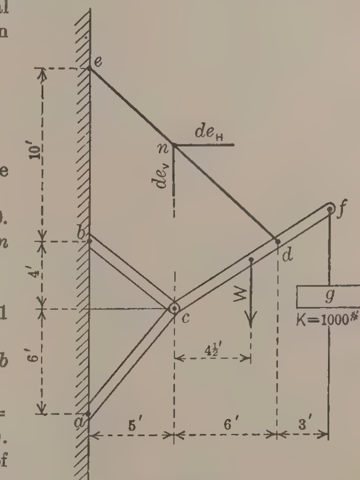


Fig. 81 (*c*).

Replace de by de_H and de_V at e; cb by cb_H and cb_V at b; ca by ca_H and ca_V at a.
From $\Sigma M_b = 0$ determine ca_H.
From $\Sigma M_a = 0$ determine cb_H.
From ca_H and cb_H determine, respectively, ca and cb. (65 : 9.)

1. *Problem 21.* (Fig. 82 (*a*).) Given: A block $6' \times 2' \times 3'$ that weighs 130 lbs. per cu. ft. The block rests on an inclined plane *ab*, and is fastened to this plane by a hinged joint at *a* and by a connection at *b* capable of exerting a pull on the block of 2000 lbs. normal to *ab*. The connection at *b* cannot withstand any force parallel to *ab*. A chain *cd* exerts a force *C* just great enough to break the connection at *b* and start the block revolving about the hinged joint at *a*. Required: The forces *A* and *C* exerted on the block respectively at *a* and *c*.

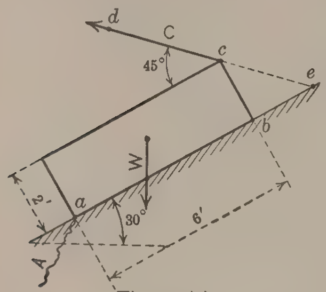


Fig. 82 (*a*).

Let *W* represent the weight of the block and *B* the force of 2000 lbs. exerted by the connection at *b*.

Analysis

Body in equilibrium. The block.
Known: *W*, *B*, *C*(L), and *A*(P).
Unknown: *C*(M) and *A*(M and D). (Case 3.) (70 : 3.)
Equations: $\Sigma M = 0$, $\Sigma X = 0$, and $\Sigma Y = 0$. (Assume axis X parallel to *ab*.)

From $\Sigma M_a = 0$ determine *C*.
From $\Sigma X = 0$ determine A_X.
From $\Sigma Y = 0$ determine A_Y.
From A_X and A_Y determine *A*. (65 : 6.)

Answers: $C = 3856.4$ lbs. (upward); $A = 6061.1$ lbs. $6\frac{1}{32}$ horizontal to 12 vertical, upward to the right.

2. *Exercise:* If the force *C* is replaced by C_X and C_Y, what modifications or additions will be necessary in the analysis just given?

3. *Note:* An alternative method is to replace *A* by A_X and A_Y, applied at *a*. The problem then becomes one in Case 4 (70 : 4), and may be solved as follows:

From $\Sigma M_a = 0$ determine *C*.
From $\Sigma M_e = 0$ determine A_Y.
From $\Sigma M_c = 0$ determine A_X.
From A_X and A_Y determine *A*. (65 : 6.)

4. *Problem 22.* (Fig. 82 (*b*).) Given: A bracket *abc* attached by hinged joints to a wall at *b* and *c*. The member *ac* also acts as a beam to support a platform *de* that rests on supports at *d* and *e*. The loads that the platform brings to the supports at *d* and *e* are, respectively, $E = 300$ lbs. and $F = 600$ lbs. Required: The axial stresses in the members *ac*, *ab*, and *cb*. (Since the member *ac* acts as a beam to support *E* and *F*, there will be in this member certain stresses that are not axial and cannot be determined by methods thus far given.) In order to determine the required stresses, it will be necessary first to determine the external forces *A* at *a* and *C* at *c* due to loads *E* and *F*. Let *A′* and *C′* represent the reactions at *a* and *c* due to *E* and *F*, i.e., $A' = -A$ and $C' = -C$.

Analysis

Body in equilibrium: The beam *ac*.
Known: *E*, *F*, *A′*(L), and *C′*(L).
Unknown: *A′*(M) and *C′*(M). (Case B′.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma V = 0$.

From $\Sigma M_c = 0$ determine *A′*.
From $\Sigma V = 0$ determine *C′*.

Answers: $A = 300$ lbs., and $C = 600$ lbs. (both down).

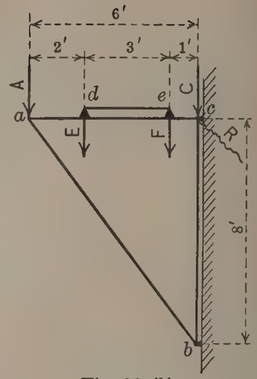


Fig. 82 (*b*).

Analysis

Body in equilibrium: Joint *a*.
Known: *A*, *ac*(L), and *ab*(L).
Unknown: *ac*(M) and *ab*(M). (Case B.) (69 : 10.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.

From $\Sigma V = 0$ determine *ab*.
From $\Sigma H = 0$ determine *ac*.

Answers: $ab = 375$ lbs. (compression); $ac = 225$ lbs. (tension).

To determine *cb* select *c* as the joint in equilibrium. The force *R* that the wall exerts on *c* is unknown both in magnitude and direction, hence the unknowns are *cb*(M), and *R*(M and D). The problem in concurrent forces at *c* is, therefore, indeterminate. (Three unknown elements, 68 : 14 (a).) If the line of action of *R* is assumed as horizontal, then from 71 : 5 the stress in the member *cb* is equal to *C* and *R* is equal to *ac*.

Exercise: Assume *R* to be horizontal. Write out the analysis for determining *R* and the force *B* that the wall exerts on the joint at *b*, taking the triangle *abc* as the body in equilibrium.

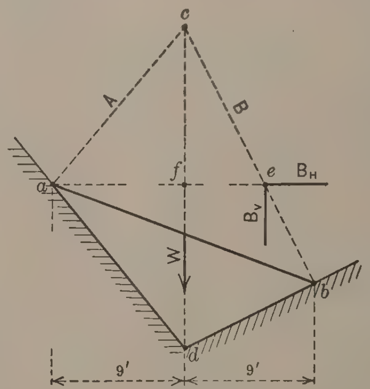


Fig. 82 (*c*).

5. *Problem 23.* (Fig. 82 (*c*).) Given: A beam *ab* whose weight *W* is 2000 lbs. The beam is supported at a point *a* on a plane *da* and at a point *b* on a plane *db*. The inclination of *da* is $1\frac{1}{4}$ vertical to 1 horizontal; of *db*, $\frac{1}{2}$ vertical to 1 horizontal. The beam is fastened to the plane at *a* by a hinged joint but is free to move along the plane *db*, i.e., the force *B* that the plane *db* exerts on the beam at *b* is normal to that plane. Required: The force *A* that the plane *da* exerts on the beam at *a*, and the force *B*.

Analysis

The lines of action of A, B, and W must meet in a point (71 : 3), and since B is normal to the plane db, the point of concurrence must be c, the intersection of the lines of B and W. The line of action of A will then be ac.

Analysis

Body in equilibrium: The beam ab.
Known: W, A(L), and B(L).
Unknown: A(M), and B(M). (Case B.) (69 ; 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine A and B. (16 : 6.)
Answers: $A = 985.1$ lbs., and $B = 1376.0$ lbs. (both upward).

The following is a more general method of analysis: Let e be the point in which a horizontal line through a intersects the line of action of B.

Analysis

Body in equilibrium: The beam ab.
Known: W, B(L) and A(P).
Unknown: B(M) and A(M and D). (Case 3.) (70 : 3.)
Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$.
Replace B by B_H and B_V applied at e.
From $\Sigma M_a = 0$ determine B_V.
From B_V determine B_H and B. (65 : 11 and 65 : 9.)
From $\Sigma V = 0$ determine A_V.
From $\Sigma H = 0$ determine A_H.
From A_H and A_V determine A. (65 : 6.)

This second method is particularly advantageous when, in addition to its own weight, the beam carries other loads whose lines of action do not pass through the center of gravity of the beam.

1. **Comments on the Illustrative Problems.** A study of the twenty-three problems just analyzed leads to certain conclusions that are in accord with statements previously made. Among the most important of these conclusions are the following:

2. The problem in concurrent forces in which the two unknowns are two magnitudes (*Case B*) occurs most frequently. Although an attempt was made to formulate a set of problems that would illustrate all eight cases, ten of the twenty-three problems fall under *Case B*. This case is also of frequent occurrence in stresses and is the basis of two important methods of determining stresses, namely, the *algebraic method of successive joints* and the *graphic method of successive joints*. This means that many of the problems in stresses may be solved solely by the use of the two resolution equations of equilibrium or their graphic equivalent — the solution for *Case B*.

3. The problem in non-concurrent forces in which the three unknowns are the magnitude of one force, and the magnitude and direction of a second force (*Case 3*) is also of frequent occurrence. Five of the twenty-three problems fall under this case, but in three of the five the problem is changed to *Case 4*. In stresses, *Case 3* occurs most frequently in determining reactions.

4. The problem in non-concurrent forces in which the three unknowns are three magnitudes (*Case 4*) is another common case. In five of the problems the solution for this case is used. In stresses, *Case 4* occurs almost, if not quite, as often as *Case B*, and is the basis of a most important method of determining stresses called the *method of sections*.

5. The problem in parallel forces in which the two unknowns are two magnitudes (*Case B′*) is a fourth case of frequent occurrence. Although it occurs in only three of the twenty-three problems, it occurs in stresses over and over. In the great majority of problems, all of the external forces on a structure (loads and reactions) are parallel (usually vertical), and therefore the problem of determining the magnitudes of the two reactions falls under *Case B′*.

6. The four remaining cases (*Case A*, *Case A′*, *Case 1*, and *Case 2*) are of rather infrequent occurrence in statics and in stresses. In the twenty-three problems, *Case A* occurs once, *Case A′* twice, *Case 1* once, and *Case 2* not at all.

7. It is to be noted that in the solutions for the four cases of frequent occurrence, the moment equation of equilibrium must be used at least once in all except the solution for *Case B*. (69 : 9 to 70 : 4.)

8. The first and most difficult step in the solution of any problem is to indicate clearly and correctly *all* of the forces that are completely known and *all* of the unknown elements of the forces that are partially known (or, occasionally, the single force that is completely unknown). When this has been done, the remainder of the solution, as the twenty-three problems clearly show, is more or less of a routine character. Most of this routine work consists in calculating H and V components, and the

lengths of lever arms — simple problems in geometry and trigonometry. In the first step, it is easy to overlook a force that is completely known or an element of a force that is partially known, or to assume an element of a force different from what it really is.

1. Even a casual study of the twenty-three problems must impress one with the importance of a good sketch in determining what forces are acting on the body in equilibrium. (18 : 2 and 3.) From a study of the sketches, it is evident that practically every force that acts on a body in equilibrium, except the force of gravity, is exerted by some other body that is in contact with the body in equilibrium, but that the line of action of the force exerted by this other body depends upon the conditions of contact, such as the inclination of the surface of contact and whether or not the bodies are free to move along this surface.

2. Many problems occur in statics and in stresses that are most easily solved by means of one or more of the general principles given under special cases on page 71. *Problems 1, 5, 7,* and *12* are of this character.

3. A problem must frequently be broken up into several problems (21 : 2), just as *Problems 16, 17, 18,* and *22* were each broken up into two or more problems. This is particularly true in stresses. To determine the stresses in a truss it is necessary first to determine the reactions, which is one kind of a problem (usually under *Case B′*), and then to determine the stresses, which is a different kind of a problem. The stresses in some members may be most easily determined by the use of the resolution equations of equilibrium (*Case B*), whereas the stresses in others may be best determined by moment equations and resolution equations (*Case 4*). Still other stresses may be determined by general principles for special cases. It will be the aim in **PART II** to explain how best to choose and combine these different methods.

4. Finally, in the study of the illustrative problems, the value of a systematic method of analysis must have become apparent. Such a method of attack, that reduces a problem to its simplest terms and thereby suggests a standard solution, has been illustrated by nearly every problem. This same method of analysis will be used throughout the remainder of this book.

PART II—STRESSES DUE TO DEAD LOAD

INTRODUCTION TO PART II

1. The aim in **PART II** is to show how the fundamentals of statics, explained in **PART I** and summarized in CHAPTER X, are applied in determining stresses in simple beams and trusses. There is little in **PART II** that is new in theory; in fact, most of the typical problems in **PART II** are identical in principle and method with the typical problems explained in **PART I**. The student who has mastered **PART I** can, therefore, begin the study of **PART II** with the assurance that he has merely to learn to apply what he already knows.

2. It is important, first of all, to understand how loads on structures are determined and at what points of the structure they are applied. For this purpose a general knowledge of structures and parts of structures is necessary, and hence the first chapter of **PART II** (CHAPTER XI) is devoted to a brief description of bridges and roofs with particular reference to the way in which loads on all parts of the structure are brought to the trusses.

3. Since stresses are caused by external forces, namely, loads and reactions, these forces must be known before the stresses can be determined. The calculation of loads frequently involves a knowledge of structural design that the student beginning a course in stresses does not ordinarily have; such knowledge is best acquired later in a course in design, and for this reason comparatively little concerning the calculation of loads is given in this book. As a matter of fact, in studying the theory of stresses it is immaterial whether the loads are computed or are merely assumed. In many illustrative problems it is advantageous to assume fictitious loads which are quite different from the loads that the structure would actually carry. This assumption permits the use of small loads expressed in round numbers, such, for example, as 10 tons or 1000 lbs.; the amount of numerical work involved is thus reduced to a minimum without sacrificing any of the desired practice in solving problems.

4. *Note:* Throughout the illustrative problems in **PART II** the external forces and the stresses are expressed, for the sake of clearness, in terms of *pounds*. Much time may be saved in actual computation by expressing forces in *thousands of pounds*, or "kips." The unnecessary work of writing large numbers of zeros is thus avoided. For example, "8.5" is used in place of "8500 lbs." or "101.8" in place of "101800 lbs." Later on, in **PART III**, after the fundamental methods of determining stresses have been covered, this shorter method of expressing forces will be followed.

5. When the loads on a beam or truss have been assumed or calculated, the corresponding reactions must be determined. Problems in determining reactions usually fall under *Case B′* of parallel forces, though occasionally such a problem falls under *Case 3* or *Case 4* of non-concurrent forces. Typical problems in reactions are treated in the latter part of CHAPTER XII.

6. Since stresses are frequently determined from shears or bending moments, it is desirable to explain early in the course the methods of calculating shears and bending moments, and this is done in CHAPTER XIII. The remaining chapters of **PART II** are devoted to the algebraic and graphic methods of determining stresses.

7. There are two general methods of determining stresses: (1) one in which the body in equilibrium is a *rigid portion* of the truss (**method of sections**) and (2) one in which the body in equilibrium is a *joint* of the truss considered as a particle (**method of successive joints**). It is well to remember that since stresses are forces within a truss they cannot be determined by assuming the *whole* truss as the body in equilibrium. The forces that hold a whole truss in equilibrium are external forces, hence it is only when an external force, such as a reaction, is to be determined that the whole truss is taken as the body in equilibrium. The method of sections and the method of successive joints may each be subdivided into an algebraic method and a graphic method. The graphic method of sections is

seldom used and hence it has been omitted in this book. This leaves three methods of determining stresses, namely:

Algebraic Method of Sections (CHAPTER XIV)
based on *Case 4* non-concurrent forces.

Algebraic Method of Successive Joints (CHAPTER XV)
based on *Case B* concurrent forces.

Graphic Method of Successive Joints (CHAPTER XVI)
based on *Case B* concurrent forces.

1. It is evident from the statements just made that a problem in stresses usually falls under one of two cases, namely, *Case B* or *Case 4*. Since a problem in reactions also falls under one of two cases, *Case B′* or *Case 3*, it follows that of the eight cases explained in **PART I**, only four are involved, as a general rule, in **PART II**. Occasionally a problem will fall under one of the remaining four cases.

2. Only static loads are considered in **PART II**. This gives the student an opportunity to become familiar with the methods of determining stresses due to static loads, before taking up the subject of moving loads. It permits him to realize how simple these methods are when free from complications caused by moving loads. Later on, in **PART III**, he will learn where to place a moving load on a structure in order to obtain maximum live-load stresses; but once a moving load has been placed in the correct position, it is assumed to be *static* in that position. Hence the methods for determining live-load stresses do not differ materially from those used for determining dead-load stresses. This means that the student who has learned these methods in **PART II** will have little more to learn in **PART III** except how to determine, in any given case, the correct position of the moving load.

3. *Note:* If it is desired to study simultaneously the subject of dead-load stresses and the subject of live-load stresses, this may be done by combining certain portions of **PART II** with certain portions of **PART III**. Such combinations are suggested in the preface.

4. Typical parallel-chord trusses have been used almost exclusively in the illustrative problems in **PART II** except in the chapter on *Roof Trusses*. Bridge trusses with inclined chords are treated in **PART III**.

5. Since the principal aim in studying **PART II** should be to learn the basic *general* methods of determining stresses, it is felt that too much time should not be spent on details which can better be taken up in **PART III**. Hence the following suggestions:

(*a*) Solve only enough problems in connection with each method to fix the method in mind. The more practical considerations are best left until **PART III** is reached.

(*b*) In applying an algebraic method of determining stresses, instead of calculating the stresses in all of the members, calculate merely the stresses in three or four typical members, such, for example, as an upper-chord member, a lower-chord member, a vertical web member and an inclined member. To calculate the stresses in all members involves much duplication of effort and by avoiding this duplication the student will have time for problems in a greater variety of trusses.

(*c*) In **PART III** the student will be required to determine both dead- and live-load stresses and thus will gain additional practice in the application of the various methods of **PART II**. This should be taken into account in determining the amount of time to be spent on **PART II**.

6. *Note:* Some of the suggestions just given do not apply to the study of stresses in roof trusses. Roof trusses are not ordinarily required to carry live loads. Wind pressure is in a sense a live load, but it is applied to the entire exposed surface on one side or the other, consequently there is no question concerning the position of the load. None of the problems peculiar to live loads arise in connection with roof trusses, and it has seemed best, therefore, to include in **PART II** all that will be given in this book pertaining to stresses in roof trusses. For this reason it is suggested that the study of stresses in roof trusses be completed before proceeding to **PART III**.

7. **ASSUMPTIONS.** In determining stresses in ordinary trusses certain assumptions are made, some of which are not strictly in accord with actual conditions. The differences between the stresses thus obtained and the actual stresses are, however, negligible except for extraordinarily large and heavy trusses. The assumptions are as follows:

(*a*) That the stress in each member of the truss is a resultant stress acting along the axis of the member, i.e., an axial stress. (3 : 1 and 2.)

(*b*) That all axial stresses in the truss and all external forces acting on the truss to cause these stresses are co-planar. (12 : 2.)

(*c*) That the axes of all members meeting at a joint of a truss intersect in a common point.

(*d*) That joints of a truss are frictionless hinges. This means that any effect on the stresses due to friction on pins at joints, or due to joints' being riveted, is neglected.

(*e*) That loads are applied only at joints. Framed structures are generally so designed that external loads take effect only at joints of trusses. The weight of any member of a truss is really distributed throughout its length and, except in the case of vertical members, causes shearing and bending stresses, but in ordinary trusses these stresses are so small compared with axial stresses that they may be neglected; the weight of the member may then be assumed to be applied at two joints of the truss, one at either end of the member.

(*f*) That the truss is perfectly rigid. A truss cannot be perfectly rigid because the materials of which it is composed are elastic. Under the action of external forces on a truss, tension members are lengthened and compression members are shortened. (6 : 8.) These strains permit a movement in space of every part of the truss, including its supports, and the truss assumes a form slightly different from its original geometric form (truss diagram). The effect, however, of this deformation on stresses and reactions may be neglected in ordinary trusses that are otherwise statically determinate.

1. *Note:* Although the effect of deformation may be neglected in determining stresses, the deformation itself, particularly vertical deflection, frequently affects the appearance of a structure. When this is the case, allowance is made for deflection by so changing the lengths of certain members of the truss from their geometrical lengths, that the principal horizontal line, usually the bottom chord, is made to rise slightly in the center portion of the span. The amount of camber thus introduced in a bridge truss is such that, under the action of live load, the principal horizontal line will not deflect below the horizontal.

The appearance of a structure is often improved by making the camber still greater, so that under live load there will still be a slight curve upward; this is because a strictly horizontal line may appear to sag a little, suggesting weakness. In many long-span trusses the camber is intentionally made such that the upward curve is quite noticeable, even under the greatest deflection, such a curve suggesting strength.

2. **The Difference Between Theory and Practice.** Stresses may be determined with considerable mathematical exactness, and hence engineers in practice adhere more closely to theory in determining stresses than it is possible to do in work in which data and methods are less exact. Nevertheless, it is often possible in practice to eliminate worthless refinements in determining stresses, by the use of methods which are not strictly correct theoretically, but which give results that are accurate enough for all practical purposes. In the study of stresses, however, it is necessary for both teacher and students to adhere, for the most part, strictly to theory. This results in doing many things that are not always done in practice, and sometimes in doing things which, in practice, would not be in accord with either facts or common sense.

3. One example of the truth of the last statement may be found frequently in illustrative problems in this book. When relatively small forces are involved, as, for instance, a fictitious load of 1000 lbs., it is often necessary, for the sake of clearness, or for the purpose of checking, or for other reasons, to give results of calculations to the nearest 10 lbs. and sometimes to the nearest pound — results that would be absurd in practice. A similar statement may be made concerning linear measurements such as lengths of lever arms and lengths of truss members. It is unfortunate that this is so, for too many engineers are prone to use worthless refinements in calculations. In order to counteract this tendency, particularly among students, the next article is printed almost verbatim from portions of an article on the same subject in the author's book on *Plane Surveying*.* This article is placed here, at the very beginning of the study of stresses, in order that the student may not be misled by the apparent accuracy with which quantities and results are given in some of the illustrative problems, and also that he may be guided in his own computations by the general rules for rejecting worthless figures.

4. **The Elimination of Worthless Refinements in Computations** consists chiefly in avoiding the use of unnecessary figures. Data may be given to more places of figures than there is any need of, or time may be wasted in trying to attain a higher degree of precision than the data will warrant. The aim in this article is to show that in all calculations it is desirable to make a critical inspection of the numbers which enter into every computation for the purpose of rejecting figures that cannot affect the accuracy of the final results. Not only is it a waste of time to use an unnecessary number of figures in computations, but the results thus obtained are misleading, for they appear to be more accurate than they really are. Worthless figures may be avoided by the intelligent use of the following facts: (1) Numbers expressing measurements cannot be exact, for the true value of a measurement is never known. (2) A result cannot be more accurate than the data from which it is obtained, no matter how many places of figures are used in each step of the calculation. (3) In dealing with inexact numbers, *consistent* accuracy should be the aim instead of *absolute* accuracy, and consistent accuracy is not gained by carrying all numbers to the same number of decimal places regardless of how many figures each number contains.

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1. *Illustration.* A distance measured to the nearest *foot* is 3216 ft.; a second distance measured to the nearest *tenth* is 321.6 ft.; a third distance measured to the nearest *hundredth* is 32.16 ft.; a fourth distance measured to the nearest *thousandth* is 3.216 ft. The last figure in each number is *uncertain*, whereas the first three are *certain*, and the *relative precision* of each measurement is the same, namely, 5 to 32160 or 1 to 6432. The position of the decimal point has nothing to do with the *relative precision* of a measurement, and the mistaken idea that the number of decimal places to which a result is carried indicates the accuracy of that result is responsible for a great deal of stupid and unnecessary labor.

2. *The accuracy to be attained* in any computation is determined by the *number* of certain figures in the data. In order to determine the number of certain figures in any measurement, some approximate expression of its accuracy must be obtained. For example, 5600 lbs. may contain one, two, or three certain figures, according to whether the expression is to the nearest 100 lbs., to the nearest 10 lbs., or to the nearest pound.

3. *Remark.* The number of significant figures in any number representing a measurement is always one greater than the number of certain figures; the last significant figure is the first uncertain figure. It must not be assumed that because a figure is uncertain it has no significance. Its uncertainty may be very small, hence it has significance, but there can be no advantage gained in writing a number beyond the *second* doubtful figure. It is seldom necessary to give a *final* result beyond the *first* uncertain figure, except when it is desired to show that the uncertainty is small. Not only is it misleading to retain more than two uncertain figures, but it is a reflection on the intelligence of the computer.

4. *General principles that are useful in computing are:* (1) In addition or subtraction, the result cannot be accurate beyond the first doubtful place in any of the numbers added or subtracted. (2) In multiplication the number of figures in a product can never exceed by more than one the number of certain figures in the multiplier or the multiplicand, whichever has the smaller number, and it frequently is the same as this smaller number. (3) In division, the number of certain figures in a quotient can never exceed the number of certain figures in the divisor or dividend, whichever has the smaller number, and it frequently is one less than this smaller number.

5. *Illustration.* A force, given to the nearest thousand pounds, is $\mathbf{4}\mathit{5}000$ lbs., and a lever arm, given to the nearest hundredth of a foot, is $\mathbf{30.2}\mathit{5}$ ft. To how many figures should the product (moment) be carried? The certain figures are printed in bold-face, the first uncertain figure in italics, and the remaining uncertain figures in Roman. The " $\mathbf{4}\mathit{5}000$ " has the smaller number of certain figures, and since this number is *one*, there cannot be more than *two* certain figures in the product. (1) If the measurements were exact, the product would be $\mathbf{45000} \times \mathbf{30.25} = \mathbf{1361250}$; (2) if there are deviations of only 1 lb. and .001 ft. the product is $\mathbf{4}\mathit{4}999 \times \mathbf{30.2}\mathit{4}9 = \mathbf{13}\mathit{6}1174$; (3) if there are deviations of 250 lbs. and 0.0025 ft. (the averages of all possible deviations), the product is $\mathbf{4}\mathit{4}750 \times \mathbf{30.2}\mathit{4}75 = \mathbf{13}\mathit{5}2675$; (4) if there are deviations of 500 lbs. and 0.005 ft. (the greatest possible deviations), the product is $\mathbf{4}\mathit{4}500 \times \mathbf{30.2}\mathit{4}5 = \mathbf{13}\mathit{4}5902$. The first combination of exact measurements is impossible; the second combination of extremely accurate measurements is highly improbable; either of the remaining two combinations is quite probable. In the products obtained from the second, third, and fourth combinations, the first two figures are the same, namely, "**13**," but the third figure differs, i.e., it is *6* in the second product, *5* in the third, and *4* in the fourth. Obviously, this third figure is uncertain, and the result should be given as $\mathbf{13}\mathit{6}0000$ lb.-ft.

6. In applying the general principles, it is well to retain through the different steps of a calculation enough figures to correspond to two uncertain figures in the result, in order that the first uncertain figure may be free from accumulated rejection errors. In dropping figures, add 1 to the last figure retained when the first left-hand rejected figure is greater than 5; when it is less than 5, leave the last figure retained unchanged; if the rejected figure is 5, add 1 to the last figure retained if this will make it even, otherwise leave it unchanged.

7. It is generally known before beginning a calculation, what degree of precision is required in a result, or else it is desired to compute a result as accurately as the data will permit. In either case, one may be guided by the general principles for consistent accuracy. It is unwise to adopt hard and fast rules for shortening computations, but by an intelligent observance of the rules for certain and uncertain figures, much time may be saved without sacrificing accuracy in results.

CHAPTER XI

DESCRIPTION OF BRIDGES AND ROOFS

The aim in this chapter is to describe the more important parts of bridges and roofs, and to explain how external loads are carried from one part to another until they finally take effect on the trusses.

1. **Beams and Girders.** The most common conception of a beam is that of a relatively long, horizontal piece of wood, steel, or concrete which supports one or more loads. Strictly speaking, however, any structure or any member of a structure that resists the direct application of forces which are perpendicular or oblique to its longitudinal axis acts as a beam, i.e., it acts to resist bending caused by the forces.

2. A **steel beam** is generally understood to be a single piece of rolled steel. A **steel girder** is composed of several pieces, such as angles and plates, riveted together to act as a solid beam. Engineers, however, do not always make this distinction. For example, in the floor of a building a beam which supports other beams is called a *girder* regardless of whether it is a single rolled beam or a " built beam," whereas in the floor of a bridge such a beam is called a *floor beam* even though it is a girder " built up " of plates and angles.

3. **Plate Girders.** Rolled steel beams are limited in size to relatively small depths and short lengths. When deeper, longer, or stronger beams are required, it becomes necessary to use some form of " built beam " or steel girder. The most common form is the **plate girder** composed of plates and angles. The principal parts of a plate girder are the web, the flanges, and the stiffeners.

4. The **web** is a rolled plate, usually from $\frac{5}{16}$ to $\frac{5}{8}$ inch thick. This web is equal, approximately, in depth and length to the girder itself. It is often necessary to splice two or more web plates end to end in order to obtain the required length of web.

5. In the simplest for of plate girder, the **top flange** is usually composed of two **flange angles** which are riveted along the upper edge of the web, one on either side. The **bottom flange** extends along the lower edge of the web and is similar to the top flange. A cross section of a plate girder of the simplest type is shown in Fig. 89 (*a*). Another common form of plate girder shown in Fig. 89 (*b*) is similar to that shown in Fig. 89 (*a*) except that in each flange a **cover plate** is riveted to the outstanding legs of the flange angles. The cover plate on the top flange of a bridge girder should ordinarily extend the entire length of the girder as a protection from moisture. In a large girder, additional shorter cover plates often extend for varying distances on either side of the center of the girder in order to provide for the different flange stress, which is greatest near the center and decreases toward the ends of the girder.

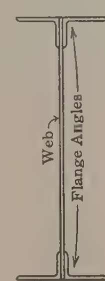


Fig. 89 (*a*).

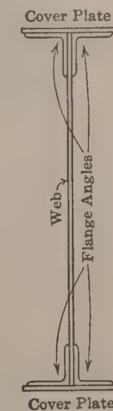


Fig. 89 (*b*).

6. A **stiffening angle** is one that extends vertically from the under side of the outstanding leg of a top flange angle to the upper side of the outstanding leg of a bottom flange angle. Stiffening angles are usually riveted to the girder in pairs, one on either side of the web. A **stiffener,** as the name implies, stiffens the web to prevent buckling; it may be composed of a single stiffening angle or a pair of such angles. Stiffeners should be placed at the ends of the girder, at points of bearing and also at points of concentrated loads. Additional intermediate stiffeners are used in

Fig. 90. One Through Plate-girder Span and Eight Deck Plate-girder Spans.

the deeper girders in order to meet the usual specification that the distance between stiffeners shall not exceed the depth of the girder, or at the most 6 feet. The specification just quoted merely limits the maximum distance between successive stiffeners; it may be necessary to space stiffeners closer than this maximum. Spaces between successive stiffeners are not necessarily equal; on the contrary, stiffeners are frequently spaced nearer and nearer together from the center of the girder toward either end. (Fig. 90.)

1. *Dimensions of girders.* The **effective length** of a plate girder resting on two supports is the distance from the center of bearing at one support to the center of bearing at the other support; if the girder is supported by other girders or trusses, as in the case of a floor beam, the effective length is the length, center to center, of the supporting girders or trusses. The **effective depth** of a plate girder is usually assumed as equal to the distance between the centers of gravity of the top and bottom flanges; if, however, this distance is greater than the depth, back to back, of the flange angles, the latter depth is considered as the effective depth. The depth of a plate girder preferably should be not less than one twelfth of the span.

2. **Plate-Girder Bridges.** A plate-girder bridge is one in which the track or the floor system is supported by plate girders; as a rule, there are two such main girders. The simplest type of plate-girder bridge is a single-track *deck* railway bridge in which the ties rest directly on top of two plate girders. The floor systems for *through* plate-girder bridges are similar to the floor systems which will be described later in connection with truss bridges.

3. *Dimensions of plate-girder bridges. The lengths* of span for plate-girder bridges vary from 30 feet to 100 feet, and this upper limit has often been exceeded by from 25 to 30 per cent. When the span is less than 30 feet, rolled beams may often be used; when the span is too long for plate girders, trusses are used. The difficulty in shipping very long or deep plate girders from the bridge shop to the site often determines the maximum length of span.

1. *The depth of the main girders* in a plate-girder bridge is to a great extent a function of the length — the longer the span the greater the depth. As a rough guide, the following lengths and corresponding depths are given.

Length in feet	30	40	50	60	70	80	90	100	110	120
Depth in feet	4	5	$5\frac{1}{2}$	6	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$	9	$9\frac{1}{2}$

The best depth for a given length of girder may be several inches greater or less than that in the table, depending upon such conditions as the character of the bridge and the load carried.

2. *The minimum width* between the two girders of a single-track deck plate-girder bridge is usually specified as $6\frac{1}{2}$ feet; it may be necessary to make the width greater to resist overturning. In a single-track through plate-girder bridge, the distance in the clear between girders is usually specified as 16 feet in order to provide adequate clearance for a train passing between the girders. To obtain this clearance it is necessary to make the distance center to center of girders at least 16 feet plus the extreme width of one girder. In a highway bridge the distance between girders is determined by the width of roadway.

3. **Trusses.** It was stated in paragraphs 5 : 7 and 8 that a **truss** is a rigid framework composed of straight members which extend between **apices** or **joints,** that the triangle is the truss element, and that each of the different types of standard trusses is an assemblage of triangles. It was pointed out in 3 : 1 that the innumerable forces in any member that act parallel to its axis may be replaced by one resultant force and that the axis of the member is the line of action of this force. A **truss diagram** is composed of straight lines each of which represents the axis of a member of the truss which in turn is the line of action of the resultant stress in the member. A truss should be so designed that the axes of all members which meet at any joint will intersect as nearly as practicable in a point; the corresponding lines in the truss diagram will then intersect in a point and this point will represent a joint of the truss.

4. Trusses, particularly bridge trusses, are usually divided lengthwise into equal **panels,** and the length of one of these divisions or panels is called the **panel length.**

5. *Note:* A member of a truss is ordinarily assumed to be subject to stress only in the direction of the longitudinal axis. Strictly speaking, it also acts as a beam to support its own weight; but except in large bridges, in which the weight of a member is great, the stresses due to this cause are negligible. Other secondary stresses in ordinary small bridges are also usually negligible. (3 : 2.)

The truss as a whole acts as a beam since it resists the application of forces which are perpendicular or oblique to its longitudinal axis. A truss, therefore, may be considered as a special form of beam.

6. *The correct number of members in a truss.* It is possible to draw a truss diagram in which there are too few members for rigidity. Strictly speaking, a structure represented by such a diagram could hardly be considered a truss because it is not a rigid structure. It is also possible to draw a truss diagram in which there are too many members, that is, there are unnecessary or **redundant members.** The following criterion may be used to determine whether or not a truss has the correct number of members:

Number of members = twice the number of apices minus three.

$$n = 2a - 3$$

7. *Application of the criterion.* Apply the above criterion to the incomplete truss Fig. 91 (*a*), to the truss with a redundant member Fig. 91 (*b*), and then to any of the typical forms of trusses shown in the next article. The two diagrams shown here are examples of badly designed trusses that were actually constructed.

Fig. 91 (*a*). Example of a Truss with Too Few Members.

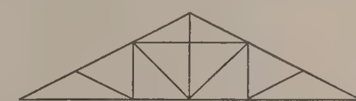


Fig. 91 (*b*). Example of a Truss with Too Many Members.

8. *Note:* The criterion just given is not absolutely conclusive unless it is applied to every part of a truss as well as to the truss as a whole. This is illustrated by the truss in Fig. 91 (*c*). Since there is no diagonal in the second panel, this truss cannot be rigid, yet the omission of the diagonal is not revealed by the application of the criterion to the truss as a whole. There are 13 apices and 23 members; the number of members therefore satisfies the criterion. If, however, the criterion is applied either to the portion of the truss that includes the first and second panels or to the portion that includes the second panel only, the lack of one member will be revealed. The number of members in the portion of the truss that includes the first two panels is six, whereas the criterion requires seven. $(2 \times 5 - 3 = 7.)$ In the portion of the truss that in-

Fig. 91 (*c*).

Fig. 92. Terms Used for Members of a Pratt-truss Single-track Railroad Bridge.

cludes the second panel only, the number of members is four, whereas the criterion requires five. $(2 \times 4 - 3 = 5.)$ It should be noted that in the truss in Fig. 91 (*c*), the number of members missing in one portion of the truss is equal to the number of redundant members in the other portion, thus making the total number of members correct. When this is the case the defects in the truss are usually obvious from inspection without the use of the criterion. The criterion is at least conclusive to the extent that when the total number of members in a truss is incorrect, i.e., when there are too few or too many members, the correct use of the criterion will reveal the discrepancy.

1. *Chord members.* The upper portion of the perimeter of a truss is called the **upper chord** or **top chord,** and the corresponding members of a truss are called **upper-chord members** or **top-chord members**; the lower portion of the perimeter is called the **lower chord** or **bottom chord,** and the corresponding members are called **lower-chord members** or **bottom-chord members.** In the majority of trusses the lower chord is horizontal, and in bridge trusses of comparatively short spans the top chord is also horizontal. Such trusses are called **parallel-chord trusses.** In most roof trusses and in many bridge trusses the upper chord is not horizontal though some of the chord members in it may be horizontal.

2. *Web members* of a truss are those that extend between top and bottom chords. They may be either **vertical** or **inclined**. A vertical web member in compression is commonly called a **post**; when a vertical web member is in tension and the tension is due entirely to the external forces applied at its lower end, the member acts as a **hanger.** In many trusses, all vertical web members in tension are of this character. The joint at which an inclined **end post** (sometimes called a **batter post**) intersects the top chord is called the **hip,** and a vertical hanger at the hip is called a **hip vertical.** Inclined web members are called **diagonals.** A **tie** is a diagonal in tension, and a **strut** is a diagonal in compression. The term " strut " is also used for certain other compression members as explained later. A **sub-diagonal** or **half-diagonal** is a secondary member connecting the midpoint of a main diagonal with an adjacent panel point. A **sub-strut** is a sub-diagonal carrying compression, and a **sub-tie** is a sub-diagonal carrying tension. A **sub-vertical** is a vertical member extending from the intersection of a sub-diagonal and a main diagonal to one of the chords. A sub-vertical in tension is a **sub-hanger**; a sub-vertical in compression is a **sub-post**—also called a **sub-strut.**

3. *Supplementary members.* The simplest form of any type of truss, i.e., the form in which there is the smallest possible number of members, may be modified by the addition of supplementary members. These supplementary members may be divided into two classes, namely, those that act as **braces** and those that act as **counters.**

4. *A brace* is a member that acts at some point of another member to fix that point in position; it may act merely to support the other member, it may act to prevent the other member from buckling, or it may serve both of these purposes. Supplementary members of this character are usually either horizontal or vertical. An example of a *horizontal* supplementary member is that of a member which extends from the midpoint of a post to the midpoint of a diagonal. Such a member reduces the unsupported length of the post and thus acts as a brace to prevent buckling. An example of a *vertical* supplementary member is that of a vertical which intersects a horizontal chord at a joint where there is no other web member. Such a member supports the horizontal chord at the joint, and if the chord is in compression the supplementary member also acts as a brace to prevent buckling.

5. A **knee brace** is a special form of brace used to stiffen the connection of one member to another. It is usually a short piece that forms the hypotenuse of a triangle, the other two sides of which are short portions of the members connected. Sometimes a knee brace forms a solid triangle, in which case it is often called a **bracket.**

6. *A counter* is a diagonal or half-diagonal that slopes in a direction opposite to that of the main diagonal in the same panel. (Fig. 92.) The purpose of a counter is to prevent compressive stress from occurring in a main diagonal that is designed to take tension only. This compressive stress would only occur for certain positions of the live load, consequently the counter would act only when the live load was in any of these positions. *A counter carries no dead-load stress.*

7. *Pin-connected trusses and riveted trusses.* A **pin-connected truss** is one in which members are held together at each joint by a pin, i.e., a short steel cylinder threaded at the ends for pin nuts which keep it in place. The ordinary diameter of a pin is from 3 to 8 inches, though occasionally pins are smaller than 3 inches and in long bridges they may be much larger than 8 inches. In the largest bridges, pins 20 inches in diameter or larger have been used. A **riveted truss** is one in which the members at a joint are held together by a rigid riveted connection. (Fig. 92.)

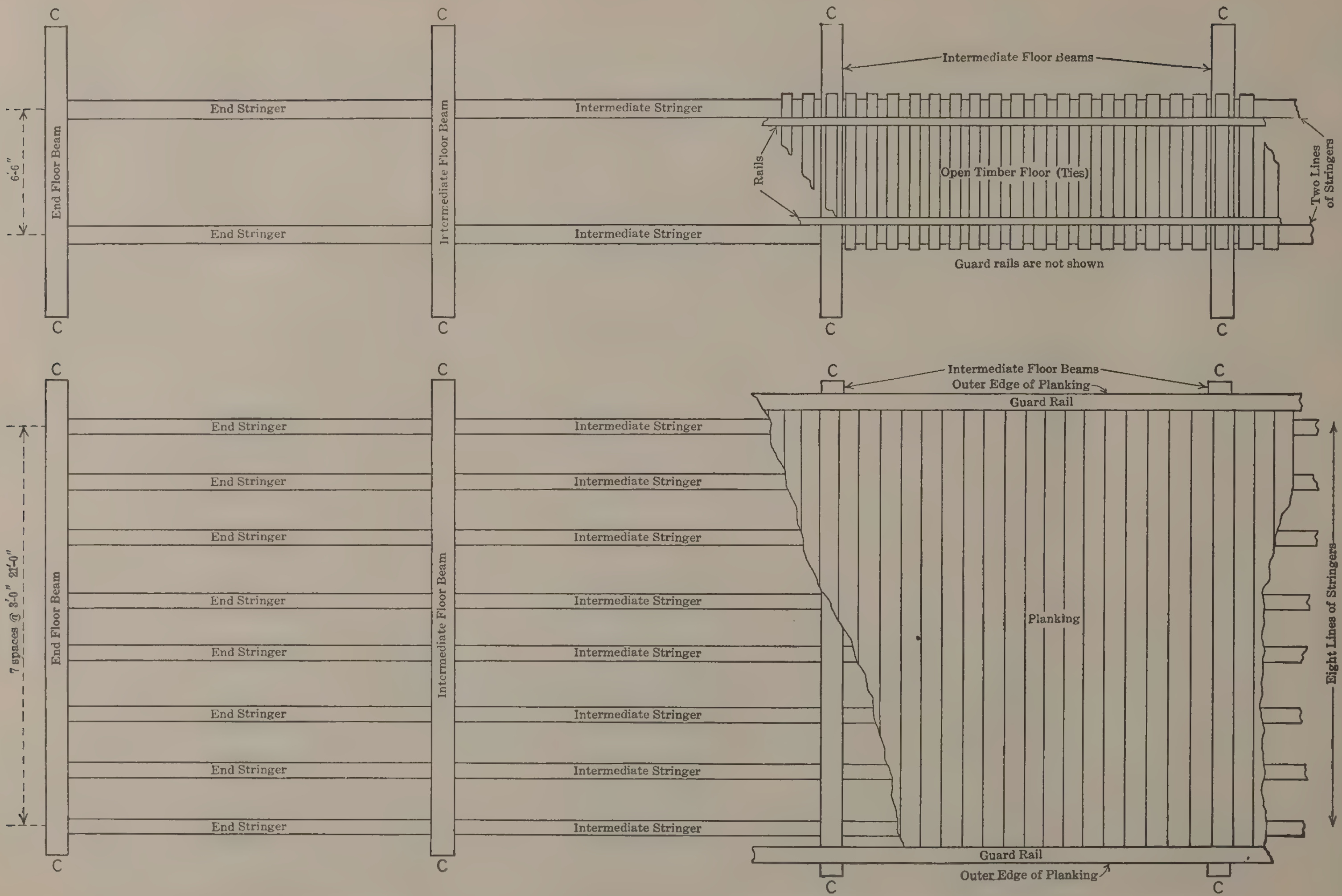


Fig. 94. Plans of Floor Systems for a Railroad and a Highway Bridge. Floor beams are connected to the trusses at the points marked "C."

1. *A pony truss* is a term sometimes used for the truss of a short-span bridge in which the height of the truss is so small that there can be no bracing of any kind above the roadway from one truss to the other truss (Fig. 95.)

Fig. 95. Five-panel, Pony-truss Highway Bridge.

2. *Note:* The term "girder" is sometimes used instead of "truss." For example, a riveted truss with parallel chords is frequently called a **riveted girder** or a **lattice girder**; the term "lattice girder," strictly speaking, should be reserved for a truss that has two or more web systems, but this distinction is not now generally observed.

3. **Truss Bridges.** The ordinary single-span bridge rests on two supports only — one at each end. These supports may be **abutments** or **piers** constructed of stone or concrete, or they may be **bents** or **towers** constructed of steel. A truss bridge may be divided into three parts, namely, the **floor system,** the **trusses,** and the **bracing** between trusses. **A through truss bridge** is one in which the floor system is near the bottom chords of the trusses and the traffic is through the bridge between the trusses. A **deck truss bridge** is one in which the floor system is near the top chords of the trusses and the traffic is over the top of the bridge. A **half-through truss bridge** is one in which the floor system is approximately half way between the top and bottom chords of the trusses.

4. *Note:* The term "bridge" may mean a single span between two supports or it may mean a structure composed of two or more successive spans, each a complete unit in itself. Unless otherwise stated, the term will be used in this book to denote a single-span structure.

5. *Floor systems.* (Fig. 94.) The floor system of a truss bridge usually consists of **floor beams, stringers,** and **flooring.** The entire system is supported by the trusses and is usually connected to these trusses at joints called **panel points.** A **floor beam** is a beam or girder which extends crosswise from a truss on one side of the bridge to the truss on the opposite side — usually at right angles to the axis of the bridge. Since each floor beam is connected to each truss at a panel point, the distance between the centers of two successive floor beams is equal to the panel length. The floor beams support the stringers. A **stringer** is a beam or girder parallel to the axis of the bridge and extending between two adjacent floor beams; its depth is considerably less than that of the floor beam, and its length is approximately equal to the panel length. Stringers support the flooring.

6. *Note:* The floor beam at the extreme end of a truss over a support, i.e., the **end floor beam,** carries approximately half as great a load as an **intermediate floor beam** and consequently may be of lighter design. In some bridges the end floor beams are omitted and the stringers in each end panel are allowed to rest at one end on the abutment, pier, or other support at that end.

7. *Floor systems for railroad bridges.* (Figs. 94 and 96.) In a single-track railroad bridge there are usually two parallel lines of steel stringers $6\frac{1}{2}$ feet apart; this brings each line about 10 inches outside of the corresponding rail. The rails rest on the ties, and the ties rest on the tops of the stringers. The stringers are generally set down between the floor beams with their tops in a horizontal plane a little less than the thickness of the ties below the tops of the floor beams, so that the base of the rail will be about 1 inch above the floor beams; the ends of the stringers are rigidly connected to the supporting floor beams. In a double-track bridge with two supporting trusses there will be four lines of stringers instead of two. In the better class of railroad bridges some "solid" type of floor system is often used. In addition to giving solidity to the road bed, such a floor system, if water proof, protects the steel work below from corrosion, which would otherwise be caused by rain water or by the drippings from refrigerator cars.

8. *Floor systems for highway bridges.* (Fig. 94.) In a highway bridge there is a floor beam at each panel point, just as in a railroad bridge, and there are also longitudinal lines of stringers upon which the flooring rests, but these lines are spaced nearer together than they are in railroad bridges (usually from 2 to 4 feet). The number of lines will depend largely upon the width of the roadway. For example, if the roadway is 24 feet and the

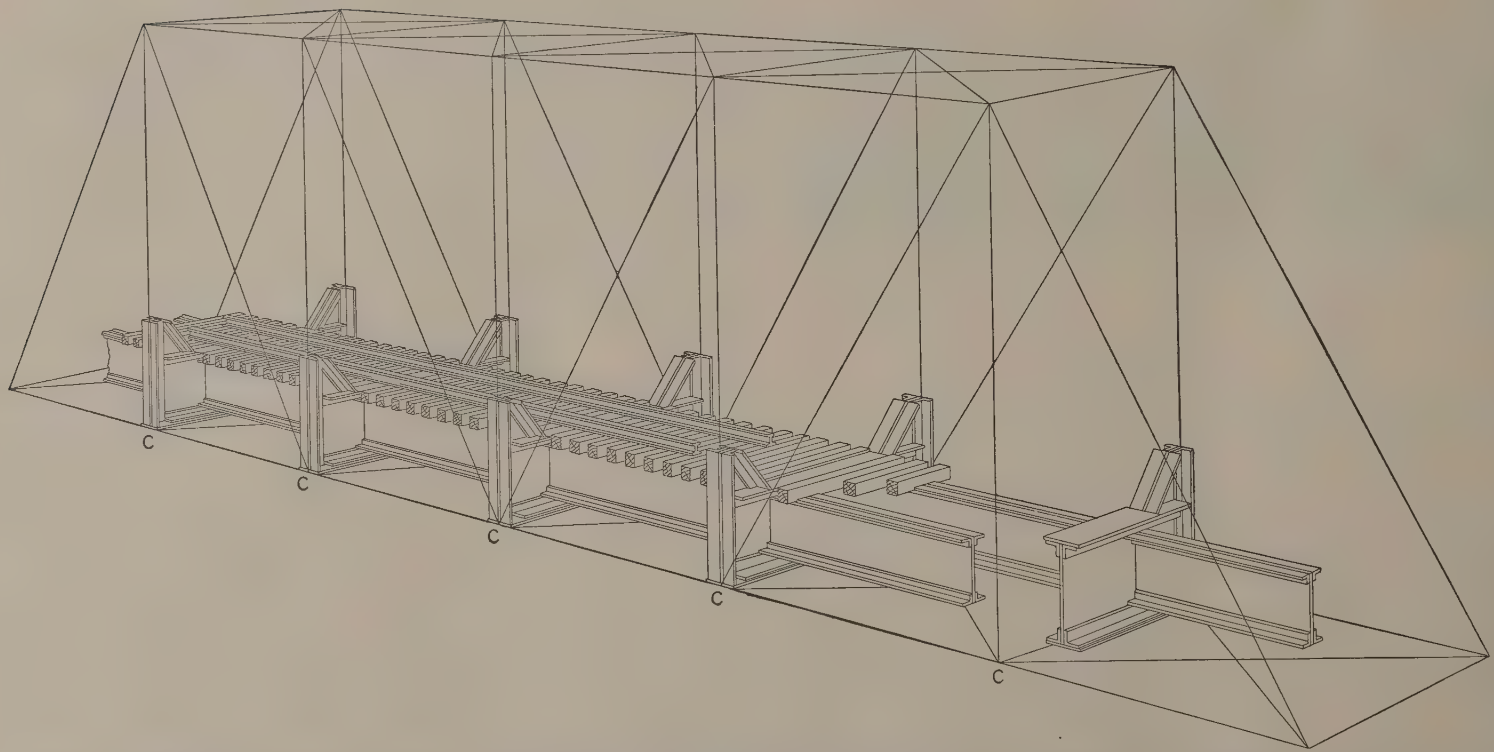


Fig. 96. Diagram of a Single-track Through Pratt-truss Railroad Bridge Showing How the Floor System is Connected to the Trusses at Panel Points Only. (Compare with the photograph on page 92.)

lines of stringers are spaced approximately 3 feet apart, there will be eight or nine lines. In the cheaper type of highway bridge, lines of wooden joists, spaced about 2 feet apart, are used instead of steel stringers. Wooden joists frequently rest on the tops of the floor beams and the same construction is sometimes used for light steel stringers. The simplest form of flooring is planking, usually laid at right angles or inclined to the axis of the bridge; the flooring rests either directly on the stringers or upon wooden **spiking pieces** which are fastened lengthwise along the tops of steel stringers to take the spikes that hold the planking. In the better class of highway bridges some form of pavement is laid on top of the planking or some "solid" type of floor system is used. If there are sidewalks, each sidewalk may be either just inside or just outside of a truss. In the latter case, sidewalk stringers are usually supported by **sidewalk brackets;** each bracket is practically an extension of the corresponding end of the floor beam. If a highway bridge supports tracks of a street railway, the stringers under the tracks are spaced approximately under the rails, and may be heavier than the stringers which support the other portions of the roadway.

1. *Connection of floor systems to trusses.* It is important to note that in nearly all bridges, both railroad and highway, the floor system is connected to the trusses *at panel joints only*, and hence it is only at these joints that the weight of the floor system takes effect. Any moving load which comes upon the floor system also takes effect on the trusses at panel joints only, since it is distributed by the flooring to the stringers, carried by the stringers to the floor beams, and carried by the floor beams to the panel joints of the trusses. (Fig. 96.)

2. When the floor system is connected to the trusses below the joints of the bottom chord it is said to be "below chord"; when above the joints, it is "above chord." The latter arrangement may require a greater height of truss, at least in a short-span bridge. The chord to which the floor system is connected is called the **loaded chord,** i.e., the lower chord of a through bridge and the upper chord of a deck bridge.

3. *Effective lengths.* The effective lengths used in the design of stringers and floor beams are, for stringers, the length, center to center, of floor beams (panel length); and for floor beams, the distance, center to center, of trusses (or plate girders if it is a plate-girder bridge).

4. **TRUSS DIAGRAMS.** Since a truss diagram is composed of straight lines, each of which represents the axis of a truss member, the length of the truss represented by the truss diagram is a theoretical length called the **effective length.** This effective length is shorter than the actual length "over all" of the truss and longer than the "length in the clear" of the span. In a like manner, the length of any member as represented in the truss diagram is a theoretical length and not necessarily the actual length "over all" of the member itself. Compression members are frequently represented in a truss diagram by heavy lines, and tension members by fine lines. In some types of trusses there are supplementary web members near the center of the truss called **counters.** Counters are often shown by broken lines in a truss diagram.

5. When the top and bottom chords of a truss are parallel (**parallel-chord truss**), the vertical distance from the axis of one chord to the axis of the other is the **effective depth of truss** or **height of truss.** When the two chords are not parallel, the vertical distance between their axes measured at any point is the effective depth at that point. In any case, the effective depth is less than the depth of the truss "over all."

6. The distances used in determining stresses are the theoretical distances represented by the truss diagram, such, for example, as the effective length and the effective depth of truss.

7. In a parallel-chord truss the angle between the diagonal and the vertical is the same for all diagonal web members. This angle is called the **truss angle.**

8. *Note:* The last statement is true only when the panels are equal in length. This is typical of a number of statements which are true for common forms of trusses but not true for exceptional forms. Unless otherwise specified, it is understood that all trusses referred to in this chapter are common forms.

9. *Systems of lettering truss diagrams.* Two common systems of lettering truss diagrams will be used in this book.

(*a*) In the first system (Fig. 99 (*a*)) the joints in the lower chord are lettered consecutively *a*, *b*, *c*, etc., beginning at the first left-hand joint; the upper joints are lettered *B*, *C*, *D*, etc., beginning at the first left-hand joint. When a joint in the upper chord is directly over a joint in the lower chord, the capital letter used for the upper joint should correspond to the lower-case letter used for the lower joint.

Fig. 98. Warren Trusses with Verticals, Single-track Railroad Bridge. (See Truss diagram, Fig. 99 (d).)

(*b*) In the second system (Fig. 99 (*b*)) the joints in the lower chord are lettered L_0, L_1, L_2, etc., beginning at the first left-hand joint; the upper joints are lettered U_1, U_2, U_3, etc., beginning at the first left-hand joint. When an upper joint is directly above a lower joint the "U" for that joint has the same subscript as that used for the "L" of the lower joint.

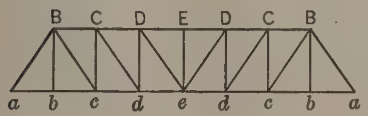


Fig. 99 (*a*).

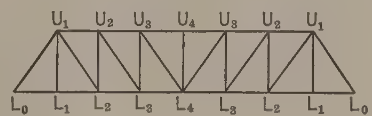


Fig. 99 (*b*).

1. *Note:* The two systems of lettering just described are for trusses with inclined end posts. When the end post is vertical, as in a deck bridge, the first left-hand upper joint of the truss is lettered "A" or "U_0", according to which of the two systems is used.

2. *Note:* A third system of notation, to be explained later, is Bow's notation, a system of lettering which is used in connection with graphic methods of determining stresses.

3. **Standard Types of Bridge Trusses.** Bridge trusses in general may be divided into three classes, namely, (1) **parallel-chord trusses,** i.e., those in which both the top and bottom chords are straight and parallel to each other (usually horizontal); (2) trusses in which one chord is straight and the other curved; (3) those in which both the top and bottom chords are curved. A **curved chord** of a truss is generally understood to mean one in which the joints of the chord are on a curve but in which each chord member between joints is straight. A more precise term for such a chord is **polygonal chord.**

4. In each of the groups just mentioned there are several standard types of trusses. The **Warren truss** and the **Pratt truss** are the types of parallel-chord trusses most used. The **Parker truss** is a common type for trusses with one polygonal chord; it is in reality a Pratt truss with one of its chords curved instead of straight. In some cases there are several names for the same type of truss.

5. A simple type of truss may be subdivided by introducing additional members, as explained later, and the truss thus formed is called a **subdivided truss.** Special names are sometimes given to such subdivided trusses; for example, a subdivided Pratt truss, or a **Sub-Pratt,** is also called a **Baltimore truss.**

6. *Illustrations of different types of trusses.* Many types of trusses formerly used may now be considered obsolete. There are, however, several forms of simple bridge trusses that, having survived the test of time, may be considered standard, and these types will now be illustrated by means of truss diagrams and photographs.

7. *Note:* In each truss diagram the corresponding type of truss will be shown *in its simplest form* by full lines. Broken lines will be used to indicate additional or supplementary members. (97 : 4.)

PARALLEL-CHORD TRUSSES FOR THROUGH BRIDGES

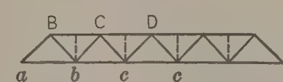


Fig. 99 (*c*). Warren Truss. (See photograph, page 95.)

8. *The Warren truss* (*or triangular truss*) in its simplest form is used principally for short spans, particularly for bridges with pony trusses. The joints are usually riveted. (Fig. 99 (*c*)), and photograph, Fig. 95).

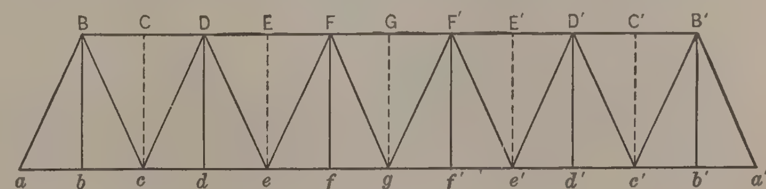


Fig. 99 (*d*). Subdivided Warren Truss or Warren Truss with Verticals. (See photograph, page 98.)

Fig. 100. Baltimore or Sub-Pratt Trusses, Double-track Railroad Bridge. (See Truss diagram, Fig. 101 (c).)

1. *The subdivided Warren truss* (often called the "Warren truss") is merely a modification of the simple Warren truss, formed by adding at each upper joint a vertical member (hanger) which extends to the lower chord and thus subdivides each panel into two panels. In Fig. 99 (*d*) these verticals are the six members *Bb*, *Dd*, *Ff*, *F′f′*, *D′d′*, and *B′b′*. The truss may be modified still further by adding the five vertical supplementary members indicated by the five broken lines, *Cc*, *Ee*, *Gg*, *E′e′*, and *C′c′*. The subdivided Warren truss is usually a riveted truss. Because of its simplicity and rigidity it is extensively used for short-span railroad bridges and for the better class of short-span highway bridges. (See photograph, Fig. 98.)

2. *Note:* Another form of the subdivided Warren is that in which each main panel is made unusually long and then is subdivided into four panels by two sub-verticals, one from the center of each main diagonal, and two subdiagonals from the same two points. This form of truss is not as common as the other form of subdivided Warren, but is sometimes used in long-span bridges.

3. In order to avoid confusion between the different types of the Warren truss, some engineers prefer to call the truss shown in Fig. 99 (*c*) the "Warren truss without verticals," and the truss shown in Fig. 99 (*d*) the "Warren truss with verticals."

Fig. 101 (*a*). Pratt-truss Highway Bridge. (Without counters.)

4. The *Pratt truss* is extensively used for both railway and highway bridges for spans under 200 feet in length, and occasionally for spans of considerably greater length. It is a particularly good type for pin-connected trusses. Pratt trusses may be designed either with or without counters. If no counters are used, the main diagonals must be designed

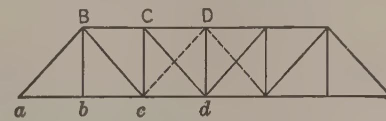


Fig. 101 (*b*). Pratt Truss.

to take any compression (usually small in amount) that may be caused by the live load. The bridge in Fig. 101 (*a*) is a Pratt truss *without* counters; if counters were used, they would correspond to the members represented by broken lines in Fig. 101 (*b*). The bridge in the photograph, page 92, is a Pratt truss *with* counters. In a Pratt truss with an odd number of panels, there are *two* diagonals in the center panel.

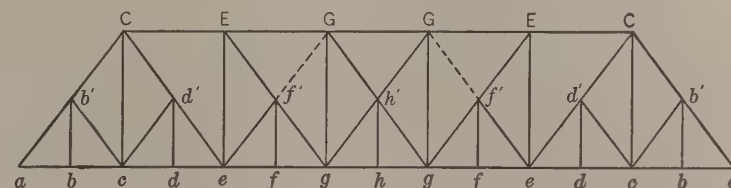


Fig. 101 (*c*). Subdivided Pratt Truss, or Baltimore Truss. (See photograph, page 100.)

5. The *subdivided Pratt truss* (*Sub-Pratt* or *Baltimore truss*) is a simple Pratt truss in which each panel has been subdivided by the addition of two members, a **sub-vertical** and a **sub-diagonal.** (93 : 2.) The sub-vertical extends from the center of the main tie to the lower chord or upper chord; the sub-diagonal or half-diagonal extends from the same point to the lower chord if it is a **sub-strut,** or to the upper chord if it is a **sub-tie.** In Fig. 101 (*c*) the sub-verticals are *bb′*, *dd′*, *ff′*, and *hh′*, and the sub-struts (half-diagonals) are *b′c*, *cd′*, *ef′*, and *gh′*. The counters are the two members indicated by broken lines marked *Gf′*. The two members marked *Gh′* also act as counters.

Fig. 102. Parker Trusses, Double-track, Railroad Bridge. (See Truss diagram, Fig. 103 (*b*).)

1. Another form of Baltimore truss would be that in which sub-struts cd', ef', and gh' are omitted (or act as counters) and in their places sub-ties $d'E$, $f'G$, and $h'G$ are used.

2. *The Warren truss and Pratt truss compared.* In a Warren truss alternate diagonals are primarily compression members. In a Pratt truss there are no inclined compression members except the end posts. In a Warren truss inclined members are "stiff" or "semi-stiff" members capable of taking either tension or compression should reversal of stress occur. In a Pratt truss the main diagonals (normally in tension) are frequently composed of bars which would buckle if subjected to compression; should the live load in any position tend to cause compression in these diagonals, the reversal of stress is prevented by means of additional or supplementary diagonals (counters) which slope in the opposite direction from the main diagonals. As already stated, however, the Pratt truss may be designed without counters.

3. *The Howe truss.* The main diagonals in either half of a Howe truss are parallel to the corresponding inclined end post, instead of sloping in a direction opposite to that of the end post as in a Pratt truss. In other respects the two types of trusses are similar. In a Howe truss the shorter web members (verticals) are normally in tension and the longer members (diagonals) are in compression, whereas in a Pratt truss the shorter web members, with the exception of the hip vertical, are in compression and the longer web members, with the exception of the end posts, are in tension. The Howe bridge truss is used mainly for wooden bridges, and is obsolete as a type for steel bridges.

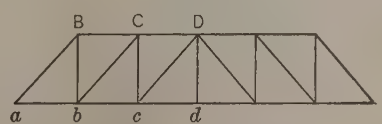


Fig. 103 (*a*). Howe Truss.

4. *Note:* In all of the simple types of parallel-chord trusses the end posts of through bridges are occasionally made vertical instead of inclined; end posts of deck bridges are usually vertical.

BRIDGE TRUSSES WITH ONE POLYGONAL CHORD

5. The type of truss in which one chord is straight and the other is polygonal is quite generally used for spans that exceed 250 feet and often for spans that exceed 175 feet. For exceptionally long spans both chords of a truss may be polygonal. The two most common types of trusses in which only one chord is polygonal are the **Parker truss** and the **Petit truss.**

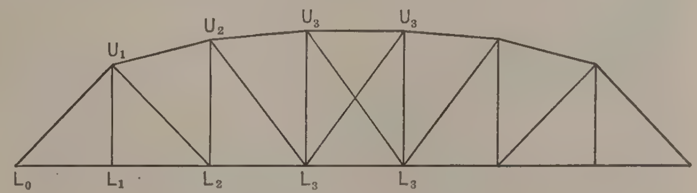


Fig. 103 (*b*). Parker Truss.

6. *The Parker truss* is a Pratt truss with one chord polygonal. (See photograph, Fig. 102, also photograph, Fig. 108.)

7. *The Petit truss* is similar to the Baltimore truss except that one chord is polygonal. It is also known as the **Pennsylvania truss.** (See photograph, Fig. 104, for a long-span Petit truss with twenty panels instead of sixteen as indicated by the diagram of half the truss in Fig. 103 (*c*).)

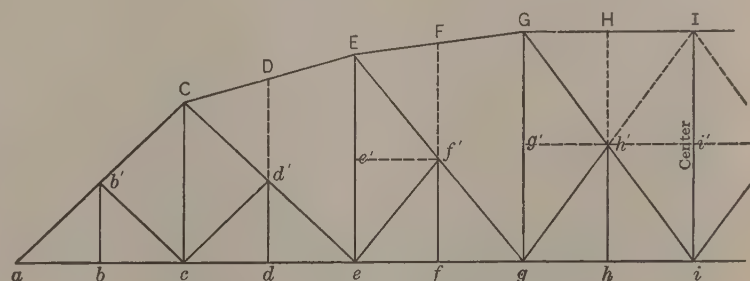


Fig. 103 (*c*). Petit Truss or Pennsylvania Truss.

Fig. 104. Railroad Bridge. Six Spans with Petit Trusses and a Portion of a Deck Pratt-truss Span. (See Truss diagrams, Fig. 103 (*c*) and Fig. 107 (*c*).)

Fig. 105. Deck Warren-truss Railroad Bridge. (See Truss diagram, Fig. 107 (*b*).)

Fig. 106. Deck Baltimore and Through Baltimore Spans. (See Truss diagram, Fig. 107 (*d*).)

1. *The K-truss* is not as common as either of the other two types of trusses just illustrated. It derives its name from the " K's " formed by

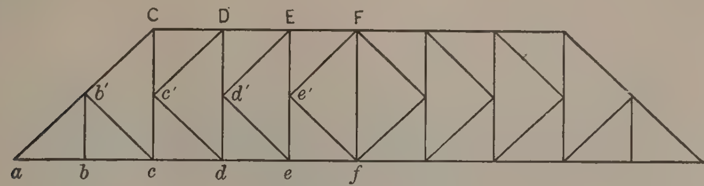


Fig. 107 (*a*). K-Truss.

the web members. It may be designed with parallel chords as shown in Fig. 107 (*a*), or with one chord polygonal, or with both chords polygonal.

2. **Trusses for Deck Bridges.** In a deck bridge the track or floor system is at the top of the truss and hence the top chord of a truss for such a bridge is straight. The principal differences between a truss for a deck bridge and a truss of the same type for a through bridge are in the design of the vertical members, and these differences are due to the fact that the floor loads and the live loads are applied at the upper joints of the truss instead of at the lower joints. Some of the differences will be explained in connection with the truss diagrams which follow.

3. In the *deck Warren truss*, Fig. 107 (*b*), the verticals which carry floor-beam loads (live and dead) are the five members *Aa*, *Cc*, *Ee*, *Cc*, and *Aa*.

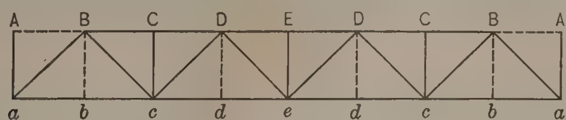


Fig. 107 (*b*). Deck Warren Truss (With Verticals). (See photograph, page 105.)

These verticals are all in compression; in a through truss the verticals carrying floor-beam loads would be the four members *Bb*, *Dd*, *Dd*, and *Bb*, and these verticals would be hangers in tension. In a deck bridge the four members just mentioned are supplementary members. (See photograph, Fig. 105.) (See photograph, Fig. 108, for another illustration of a deck Warren truss.)

4. In the *deck Pratt truss*, Fig. 107 (*c*), the hip verticals *Bb* and *Bb* do not carry floor-beam loads as in a through truss. The verticals *Aa* and *Aa* and the top-chord members *AB* and *BA* are auxiliary members that carry the floor or track in the end panels from the hips to the abutments. Some-

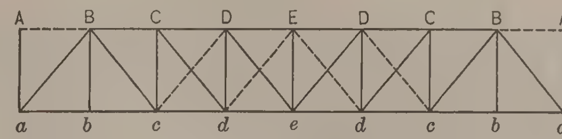


Fig. 107 (*c*). Deck Pratt Truss. (See photograph, page 104.)

times the abutment or pier replaces, wholly or partly, the vertical end posts, as in the case of the deck Pratt truss partly shown in the photograph, Fig. 104.

5. In the *deck Baltimore truss*, Fig. 107 (*d*), the sub-verticals are at the top of the truss and are in compression, whereas in a through bridge they

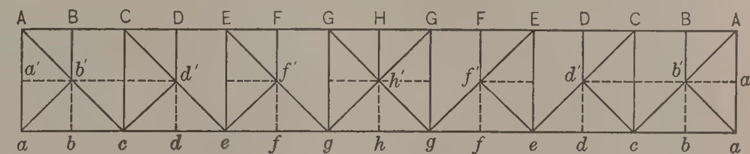


Fig. 107 (*d*). Deck Baltimore Truss. (See photograph, page 106.)

are at the bottom of the truss and are in tension. The diagram is for the deck trusses shown in the photograph, Fig. 106. In this bridge the main diagonals are capable of taking any compressive stresses that may be caused by the live load, consequently no counters are necessary. The supplementary members are indicated by broken lines in the truss diagram. The photograph (Fig. 106) affords an excellent opportunity to compare a deck Baltimore truss with a through Baltimore truss.

6. *Note:* A deck truss of any one of the three types previously described can be so designed that the diagonal web member at each end is a tension member instead of a compression member. The deck Warren truss in the photograph, Fig. 108, is a truss of this character. Note the sub-diagonal and horizontal brace at each end to transfer to the support longitudinal forces due to such causes as traction.

Fig. 108. Portion of a Through Parker Truss. Partly Through, Partly Deck Warren Truss. Deck Plate Girder.

The floor system in the Warren truss span is considerably below the top chord but near enough to the top to make the trusses act very nearly as deck trusses. Note the tension diagonal at each end of this span, and the half-diagonal and horizontal brace. (107 : 6.) Note that there are no counters in the Parker truss, the diagonals being designed to take counter stress.

1. **Principal Dimensions of Truss Bridges.** In designing a bridge an engineer must decide such questions as the number of spans, the length of each span, the type of truss for each span, the height of each truss, and the number of panels for each truss. The total cost of the bridge, including the cost of piers and abutments, will depend to a large extent upon these decisions, but it is not within the scope of this book to enter upon such questions of design except to give some of the more general facts concerning the dimensions of a single-span bridge.

2. The two principal dimensions to be determined for a truss of given span are the **height of truss** and the **length of panel.** (The length of panel determines the number of panels or *vice-versa.*) These two principal dimensions determine in a parallel-chord truss the slope of diagonal web members; the varying heights at panel points of a truss with one polygonal chord determine the varying slopes of different diagonal web members. The slope of any diagonal web member should not vary greatly from 45°, and this requirement may determine to some extent both the height of truss and the length of panel. In general, as spans become longer, the height of truss and the length of panel tend to become greater; when panels would otherwise be too long, they may be subdivided by some form of subdivided truss. More definite information concerning each of the principal dimensions of through bridges of varying spans will now be given.

3. *The height of truss* at any joint of a truss diagram is the vertical distance between the axes of the top and bottom chords. For parallel-chord bridges this height varies, generally speaking, from approximately one-fifth of the span for a 100-foot span to approximately one-seventh of the span for a 200-foot span. The vertical clearance required for the passage of vehicles or trains frequently fixes the height of truss in a through bridge. To provide for the passage of vehicles through a highway bridge, there should be a clearance between the top of the roadway floor and the under side of the overhead bracing of at least 14 feet. This will require a height of truss of at least from 16 to 18 feet. To provide for the passage of trains through a railway bridge there should be a similar clearance between the top of the rail and the overhead bracing of 22 feet. This will require a height of truss of at least from 27 to 30 feet. Thus the height of truss in a through bridge may be almost wholly determined by the requirement for head-room clearance, particularly in short spans. Truss depths for deck bridges may be less than for through bridges for spans up to 150 feet since no overhead clearance must be provided for the passage of vehicles or trains. The height of truss in a deck bridge may, however, be fixed to some extent by the clearance required underneath the bridge, as, for example, when such a bridge passes over a highway. The height of truss for long spans for either through or deck bridges is not ordinarily determined by required clearances but by its effect on the cost of construction; that is, for a given length of span and type of truss, there is an **economic height of truss.**

4. In a truss with one polygonal chord the height of truss at the center is approximately from one-fifth to one-seventh of the span, and the height at the hips must be sufficient to provide headroom clearance. The other joints of the polygonal chord should be approximately on a parabola passing through the two hips and the point determined by the height of the truss at the center of the span.

5. *The panel length.* In certain types of parallel-chord trusses the best panel length will be somewhat less than the height of the truss so that the angle which an inclined web member makes with the vertical (**truss angle**) will be approximately 40°. In any case the slope of an inclined web member will not vary greatly from 45°. This approximate limit to the slope of web members is in itself a limit to the panel length. The panel length is further determined by the number of panels; that is, the number of panels for a truss for a given length will be the same as that for a truss a few feet longer or shorter. The number of panels determines the number of floor beams and the length of stringers, and thus affects the cost of the floor system as well as the cost of the trusses. There is, therefore, for a given length of span, an **economic panel length,** though often it is not practical to use this length.

6. The panel length for parallel-chord trusses for highway bridges varies in general from 16 feet to 21 feet. For ordinary lengths of span, the panel length seldom exceeds 25 feet, even in trusses with polygonal chords. The panel length for parallel-chord trusses for railway bridges varies in general from 22 feet to 28 feet, with 25 feet as an average. For trusses with polygonal chords, the panel lengths may be somewhat greater, but they will rarely exceed 35 feet and this length of panel should be used only for spans of 350 feet or more. In some of the specifications for railway

bridges it is stated that panel lengths shall not exceed one and one-half times the width, center to center, of trusses.

1. *Typical panel lengths and heights of trusses.* The following tables of panel lengths and heights of trusses are intended merely as a rough guide.

HIGHWAY BRIDGES WITH PARALLEL CHORDS

Length	Panels	Height of Truss
90 to 100 ft.	5	18 to 20 ft.
100 to 120 ft.	6	20 ft.
120 to 140 ft.	7	20 ft.
140 to 160 ft.	8	21 ft.

HIGHWAY BRIDGES WITH INCLINED CHORDS*

Length	Panels	Ratios of Heights to Panel Lengths
162 to 180 ft.	9	1.0, 1.16, and 1.25
190 to 220 ft.	9	1.0, 1.24, 1.28, and 1.43
240 to 276 ft.	12	1.0, 1.4, 1.6, and 1.7
294 to 322 ft.	14	1.0, 1.36, 1.6, 1.7, and 2.0

The numbers of panels and the ratios in the last two lines (spans from 240 to 322 ft.) are for Petit trusses.

RAILWAY BRIDGES WITH PARALLEL CHORDS

Length	Panels	Height (Single track)	Height (Double track)
100 ft.	4	27 ft.	31 ft.
125 ft.	5	27 ft.	31 ft.
150 ft.	6	28 ft.	33 ft.

RAILWAY BRIDGES WITH INCLINED CHORDS

Length	Panels	Heights	Heights
175 ft.	7	28 ft., $31\frac{1}{3}$ ft., and 33 ft.	33 ft., $36\frac{3}{4}$ ft., and 38 ft.
200 ft.	8	28 ft., 33 ft., and 36 ft.	33 ft., 39 ft., and 41 ft.

* See Ketchum's "Structural Engineer's Handbook."

2. *Width of bridge.* The **width in the clear** between trusses is the clear space between trusses, measured where this space is the minimum; it is usually equal to the width, center to center, of the trusses minus the width of the widest truss member. For a single-track railway bridge the width in the clear should not be less than 16 feet; for a double-track bridge the distance from the center of either track to the nearer truss should be 8 feet or a distance in the clear between trusses of 29 feet if the tracks are spaced 13 feet centers. For highway bridges the minimum clear width of roadway should be 9 feet for each line of traffic, with a minimum of 12 feet for one line of traffic. Bridges designed for two lines of traffic preferably should have a clear width of roadway of not less than 20 feet. The width in the clear is often much greater than this, particularly if street cars pass over the bridge.

3. *The width, center to center,* of trusses for through bridges, either railway or highway, is usually determined by the required width in the clear between trusses unless the width so determined is less than one-twentieth of the effective length of span. The width, center to center, of trusses for a deck railway bridge should not be less than 10 feet. In some specifications the width is specified as 10 feet for spans from 100 to 110 feet, 12 feet for spans from 110 to 130 feet, and 14 feet for spans from 130 to 150 feet. An additional statement commonly found in specifications is as follows: "The width between centers of trusses shall be sufficient to give lateral stiffness and to prevent overturning by the specified lateral forces, and in no case shall be less than one-twentieth of the span." This applies to both railway and highway bridges.

4. **Bracing for Bridges.** If the two trusses of a bridge were connected to each other merely by floor beams, the bridge would not be sufficiently rigid to resist the action of *horizontal* forces, such as wind pressure against the side of the bridge and against the side of a train on the bridge. It is necessary, therefore, to use **bracing** between trusses in practically all bridges. The primary object of this bracing is to act with the main (vertical) trusses in transmitting to the bridge supports such horizontal forces as wind pressure, centrifugal forces, and tractive forces; but the bracing also provides for lateral forces and vibration due to rapidly moving loads and to the lurching movements of locomotives, assists in distributing unequal train loads between trusses, and braces laterally the two compression chords of the main trusses, each of which would otherwise be a very long column with no bracing to prevent this column from buckling laterally. Frequently there is also special need of bracing during erection.

1. **Lateral systems.** The bracing between the trusses in the plane of the two upper chords or in the plane of the two lower chords is called **lateral bracing.** (Figs. 111 (*a*) and 111 (*b*).) A **lateral strut** is a compression member which extends from a joint of one truss to the corresponding joint of the opposite truss, usually at right angles to the axis of the bridge. (Fig. 111 (*a*).) A **lateral** is a diagonal member of the lateral bracing; usually in each panel of bracing there are two such diagonals crossing each

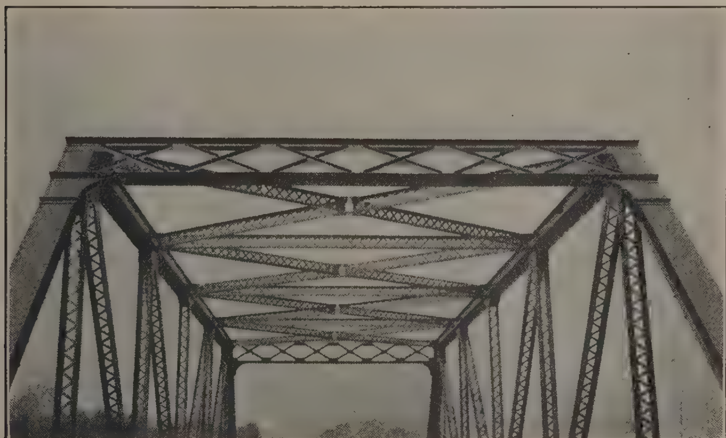


Fig. 111 (*a*). Upper Lateral System and Portal of the Pratt-truss Highway Bridge Shown in the Photograph, Fig. 101.

other. (Fig. 111 (*a*).) In a through bridge the top lateral struts, the top laterals, and the two top chords of the main trusses form a horizontal truss (**top lateral system**) (Fig. 111 (*a*)); similarly, the bottom laterals, the two lower chords of the trusses, and the floor beams form a horizontal truss underneath the floor, the floor beams acting in place of struts (**bottom lateral system**) (Fig. 111 (*b*)). In a deck bridge the floor beams act as struts in the top lateral system and the regular lateral struts are in the bottom lateral system (Fig. 104). In a bridge with polygonal top chords, portions of the two chords that have the same inclination may form with the corresponding laterals and struts a separate lateral truss; the entire upper lateral system will then be composed of several successive trusses, the planes of which make different angles with the horizontal (Fig. 102).

Fig. 111 (*b*). Bottom Lateral System in the First Two Panels of a Baltimore-truss Highway Bridge.

1. Laterals in ordinary country highway bridges are often rods which can be tightened by means of turnbuckles or by nuts on threaded ends. Frequently a lateral is merely a single angle or a pair of angles back to back. In railroad bridges and in the better class of highway bridges more rigid laterals are generally used.

2. **Intermediate lateral struts** (struts at intermediate joints) are comparatively light compression members, usually equal in depth to the depth of the chord members to which they are connected. The **portal struts** or **portals** (struts at the ends of the top chords of through bridges) usually lie in the inclined plane of the end posts, and are deeper and heavier than intermediate struts; they are often little trusses in themselves. A very rigid form of portal is that in which the web is solid or semi-solid. The depths of portal and intermediate struts are limited by the clear headroom required.

3. *Note:* Upper lateral systems are shown more or less clearly in the following photographs: Figs. 92, 102, 111 (*a*), and 112. Portions of bottom lateral systems are shown in the following photographs: Figs. 104 and 111 (*b*).

4. **Transverse bracing** is bracing that lies in a vertical or an inclined plane between trusses; it is deeper than a lateral strut. If it is in the inclined plane of the end posts it is called **portal bracing** (see portal bracing in the photograph, Fig. 111 (*a*)); at intermediate points it is called **sway bracing** (see sway bracing in a vertical plane between each pair of opposite vertical posts in the photograph, Fig. 112). Although all lateral forces can be transmitted to supports by the lateral trusses or systems already described, sway bracing should always be used when the height of truss permits, as it adds greatly to the rigidity of a bridge. Even when the clear headroom required and the height of truss are such that only simple lateral struts can be used, **knee braces** should be placed at the connections of the struts to the trusses. The simplest form of sway bracing for through bridges is the double-diagonal type consisting of an upper and lower strut and two diagonals; if the main trusses are deep, two or more of such panels in the same vertical plane may be used, one above the other. (Such double-panel sway bracing may be seen in the Petit-truss bridge in the photograph, Fig. 104, particularly between the pair of vertical posts nearest the left-hand end.) In a deck bridge, sway bracing extends the full depth of

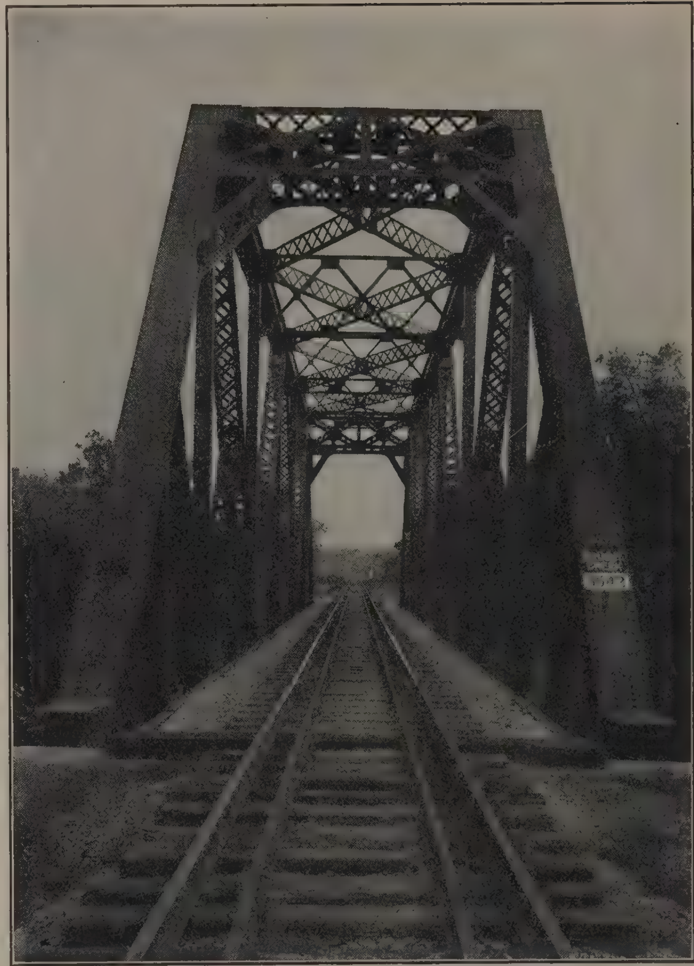


Fig. 112. Upper Lateral and Transverse Bracing.

Note the latticed laterals, the relatively deep sway bracing with lattice web, the double-plane portal bracing and portal knee braces.

the bridge, and the double-diagonal form shown between opposite posts in the photograph of a deck Pratt-truss bridge, Fig. 104, is commonly used for ordinary spans. (See also the sway bracing in the deck spans shown in Fig. 106.) The *portal bracing* may be similar to the sway bracing but heavier in construction, or it may be of the solid web type. In small bridges simple knee braces are used at the portal as well as at the intermediate struts. (See portal knee brace in the photograph, Fig. 112.) A common form of portal is that which has some form of lattice web (Figs. 92 and 111 (*a*)), and the same form of lighter construction may also be used for the sway bracing (Fig. 112). In light bridges the portal may be approximately in the plane of the upper surfaces of the inclined end posts as in Fig. 111 (*a*); it is sometimes set in a plane through the axes of the end posts as in Fig. 98. In heavy construction it is best to use a double-plane portal as in Fig. 112 and Fig. 114.

1. *Note:* Various forms of sway bracing are shown in the following photographs: Figs. 98, 100, 102, 104, 106, 112, and 114. Various forms of portals are shown in the following photographs: Figs. 92, 98, 100, 102, 104, 111 (*a*), 112, and 114.

2. **Bridges of a More Complex Type.** The bridges thus far described are of a comparatively simple type, with trusses in which the stresses are statically determinate. Bridges of a more complex type, particularly those with trusses in which the stresses are statically indeterminate, do not come within the scope of this book. Nevertheless, no description of bridges would be complete without at least brief mention of these more complex types.

3. *Continuous-truss bridges.* A continuous truss or girder is one that extends over three or more supports. The reactions at the supports are statically indeterminate (40 : 5), consequently the stresses in the truss or girder are statically indeterminate. The use of the continuous truss or girder reduces the amount of metal required, particularly in long spans, and often permits the cantilever method of erection. Continuous-truss bridges are not as common in this country as they are in Europe, where even the stringers in floor systems are often designed as continuous beams.

4. *Cantilever bridges.* A cantilever bridge, in its usual form, consists of *two* cantilever spans with a **suspended span** between. (See Fig. 21 (*b*) for a simple illustration.) The portion of a cantilever between its two supports is frequently called the **anchor arm** or the **shore arm,** and the overhanging portion the **cantilever arm** or the **river arm.** The stresses in the usual form of cantilever bridge are statically determinate.

5. *Suspension bridges.* A suspension bridge is one supported by cables that extend over towers to anchorages at either end. Each cable may be composed of many smaller wire cables compressed and bound together or it may be formed of eye bars with pin connections. Towers may be of masonry, or of steel, or of steel encased in masonry.

6. *Steel arches.* A steel arch bridge is one in which the supporting trusses or girders are in the form of an arch. A three-hinged arch is one in which there is a hinged joint at each support and one at the center connection between the two halves of the arch; the stresses in such an arch are statically determinate. A two-hinged arch is one with hinged joints at the supports but none at the center. A third form of arch is one without hinged joints.

7. *Movable bridges.* The **draw bridge** which is opened by drawing back or to one side the entire bridge, or a portion at each end, is one of the oldest types of movable bridges. The **swing bridge** acts as a double-cantilever span when open and as two separate spans when closed, unless it is designed to act as a continuous structure over three supports. The **bascule** bridge or **revolving lift** bridge is one which revolves vertically. In one type, the whole span moves as a unit, one end revolving upward while the other remains on its support. In another type, the span is divided into two segments, each segment revolving upward with one end remaining on its support. The **vertical lift** bridge is one in which the entire span remains horizontal while it is lifted vertically by means of cables and towers at each end.

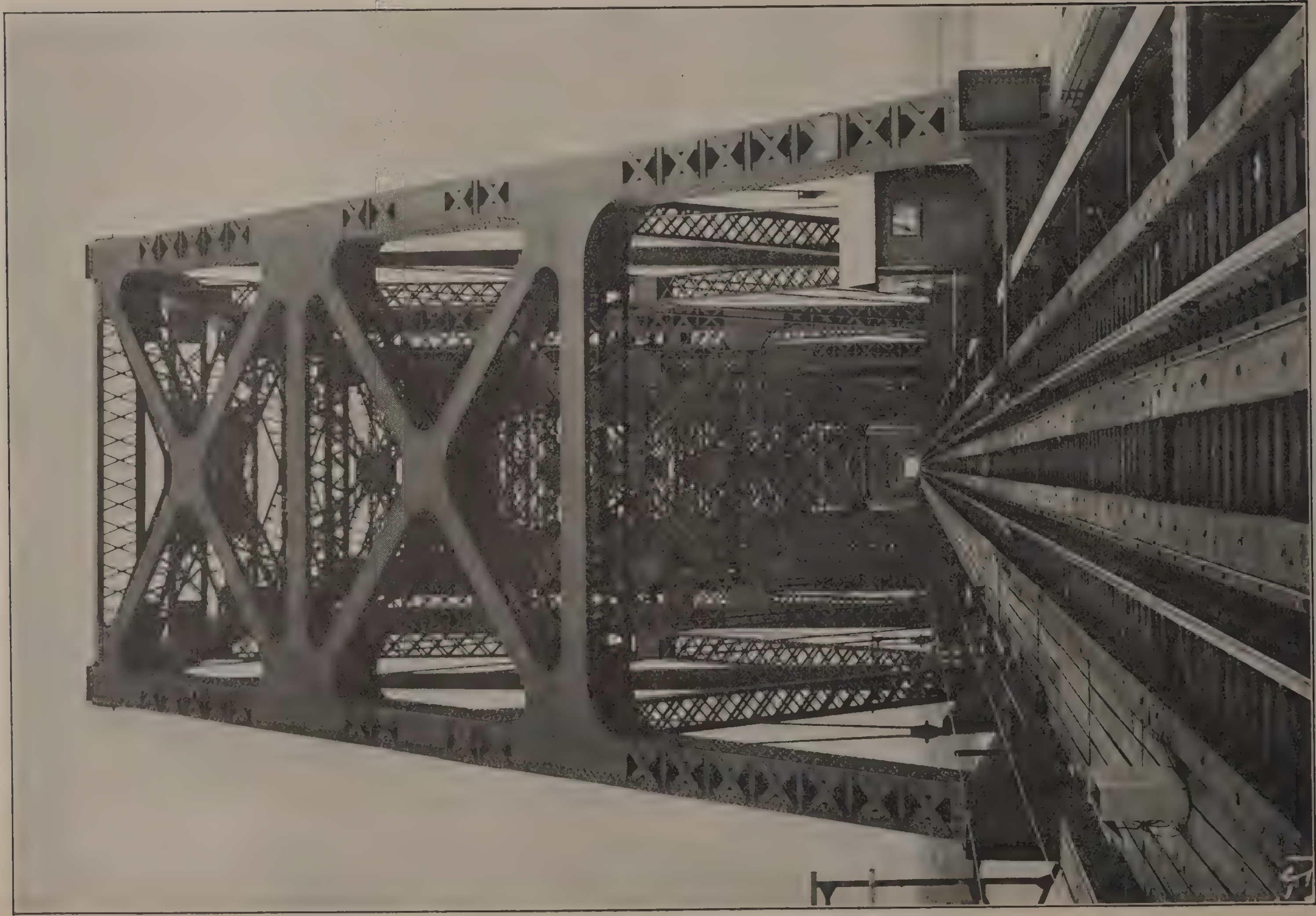


Fig. 114. Transverse Bracing in the Petit-truss Spans Shown in the Photograph, Fig. 104.

Note the latticed members of the sway bracing and the two-panel, double-plane portal.

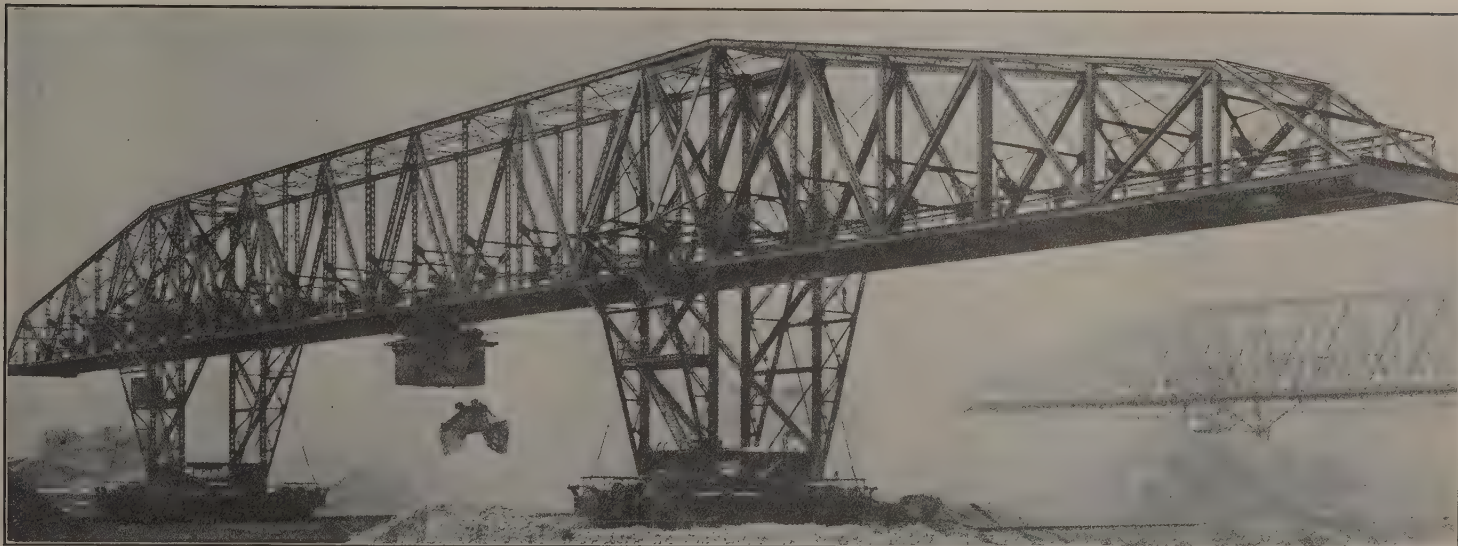


Fig. 115. An Ore Bridge.

There are many special forms of framed structures in addition to those used in bridges and buildings. Steel framework, more or less complex, is used, for example, in derricks, conveyors, cranes, hoists, cableways, head frames for mines, gasometers, and towers for all sorts of purposes. Special forms of framed structures are to be seen everywhere, as, for example, in elevated railroads, subways, tunnels, canals, ships, dams, railroad terminals, ship terminals, mines, and manufacturing plants. The ore bridge shown in the photograph is a typical example of the use of a framed structure for a special purpose.

Special forms of framed structures will not be treated in this book, but the fundamental methods used for determining stresses in bridges and buildings hold good for all other framed structures that are statically determinate.

1. **Roof Structures.** A complete roof structure consists of the **roof,** the **roof trusses** which support the roof, and the **bracing** between trusses. The roof trusses extend across the building between the two sides; they may rest directly on the side walls (Fig. 116) or they may be supported by columns (Fig. 119). The trusses are usually spaced equal distances apart from one end to the other of the building, and the spaces between trusses are called **bays.**

2. **Purlins** are horizontal beams which extend lengthwise along the roof from one truss to another and support the roof covering. On each side of a roof there are several lines of purlins corresponding to lines of stringers in a bridge floor. (See photograph, page 117.) These lines should be spaced, if possible, to cross the trusses at apices of the upper chords. The most common form of purlin is a channel with flanges extending up the slope; when **spiking pieces** are used flanges may be reversed. In order to stiffen the structure purlins may be made to extend over two bays with "broken joints" in adjacent lines. When bays are exceptionally long, trussed purlins are sometimes used.

3. **Roof covering** may be sheathing covered with shingles, tin, slate, or other roofing material; it may be formed of some material that does not require sheathing, as, for example, tiles or corrugated iron; or it may be a more solid type, such as gypsum block or concrete with tile on top. The covering usually rests directly on the purlins or on spiking pieces attached to the purlins. (Fig. 117.) In large roofs the covering may rest on intermediate rafters (**jack rafters**) which are parallel to the main rafters of the trusses; these jack rafters rest on the purlins.

4. **Monitors.** In some buildings, particularly mill buildings, there is often a supplementary roof structure called a **monitor** (or **monitor ventilator**) which runs lengthwise along the ridge of the main roof. The clear opening in a vertical side of a monitor is called a **clerestory.** When only ventilation is desired, a series of fixed horizontal slats, called **louvres,** are placed in the clerestory, so arranged as to provide ventilation and at the same time to exclude rain or snow. If light as well as ventilation is desired, windows may be used in place of louvres. (See photograph, page 121.)

5. **Roof Trusses.** The upper chords or **main rafters** of an ordinary roof truss are inclined to conform to the slope of the roof; the lower chord

Fig. 116. Roof Trusses for a Garage Roof.

Note that the trusses rest directly on the side walls and not on columns. The ends of nine lines of purlins, including that at the peak, are shown resting on the nearest truss, and each of these lines may be seen extending from the front to the rear of the building. The details of the nearest truss and of the purlins are shown more clearly in the next photograph, Fig. 117.

is usually horizontal although it is sometimes inclined upward from the ends toward the center (**cambered**) in order to increase the clear headroom underneath.

6. The top joint at the intersection of the two main rafters is the **peak** of the truss; the joint at either support is a **heel.** The **rise** of a truss is its height measured from the peak to the lower chord, if it is horizontal, or to a horizontal line between apices at the supports if the lower chord is cambered. The **length** represented in a truss diagram is a theoretical or **effective length** as in a bridge truss. The **pitch** is the ratio of the rise to the length; and, since in the ordinary symmetrical truss the **slope** is the ratio of the rise to *one-half* the length, the pitch is *half* as great as the slope of the roof. A **center hanger,** a light auxiliary member which extends from the peak to the bottom chord of a truss, is often used, even though theoretically there may be no stress in such a member. The purpose of the hanger is to stiffen the truss during erection and to support the center panel of the bottom chord if there is any possibility of a load being suspended from that panel. Other supplementary hangers may be introduced

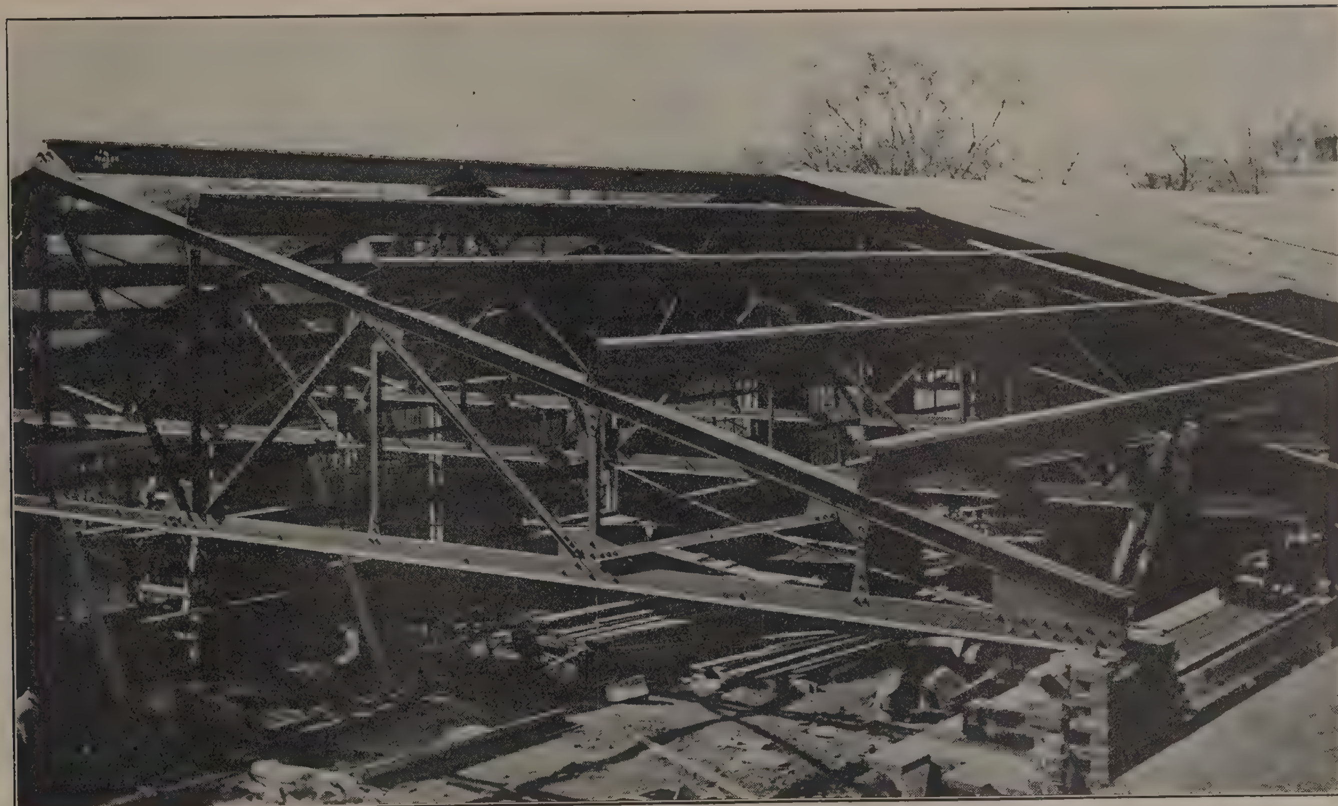


Fig. 117. Details of a Garage Roof.

Note that each line of purlins is connected to the truss at an apex of the upper chord; that along the top of each purlin is bolted a wooden spiking piece, and that the wooden sheathing rests directly on these spiking pieces, being nailed to them. The portion not yet covered with sheathing extends over the first two bays, i.e., the portion between the first and second trusses, and between the second and third trusses. Note the center hanger and the supplementary members (first and third verticals from the heel).

Note that the axes of the top and bottom chords do not intersect at the heel in a *point* as they should. Such eccentric joints have been the cause of failures; they either should be designed to stand the stresses in the connection due to the eccentricity or, better still, they should be avoided whenever possible.

when needed. (See photograph, Fig. 117.) For roofs with monitors a supplementary framework or **monitor truss** is connected to each main truss near its peak.

Howe truss and Pratt truss. In the Howe truss and in the Pratt truss there is a *vertical* member at each intermediate panel point. In the Howe truss these verticals are in tension and the inclined web members are in compression, whereas in the Pratt truss the reverse is true. Notice that

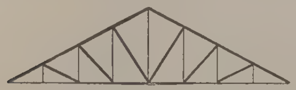


Fig. 118 (*a*). Howe Truss.

Fig. 118 (*b*). Pratt Truss.

the inclined web members of a Howe roof truss slope in the opposite direction from that in which the web members of a Howe bridge truss slope, and a similar statement is true of a Pratt roof truss and a Pratt bridge truss. (Compare the Howe and Pratt roof trusses (Figs. 118 (*a*) and 118 (*b*)) with the Howe and Pratt bridge trusses (Figs. 103 (*a*) and 101 (*b*)).) The Howe roof truss is used mainly for wooden structures.

1. *Fink truss.* In the Fink truss the struts in the web system on either side of the center are normal to the corresponding main rafter. The normal at the center of a main rafter is extended until it intersects the bottom chord, wherever that point may be, and hence any desired rise of

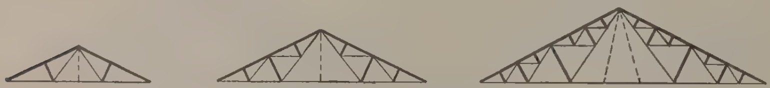


Fig. 118 (*c*). Fink Trusses.

truss may be used. Each side of the Fink roof truss is in reality a separate Fink truss in itself, the two trusses being riveted together at the peak and joined by a tie that forms the central portion of the lower chord. The number of intermediate apices in each main rafter may be one, three, or seven, as shown in the figures. In the simplest form of Fink truss the main rafter is divided into two equal panels by a single normal, as in the first figure. (Fig. 118 (*c*).) If each of these two panels is subdivided by a normal and the corresponding diagonals are drawn, the second figure of the Fink truss, with *four* panels in each main rafter, will be formed. If, in a similar manner, each of these four panels is subdivided, there will be eight panels in a rafter, as in the third figure. This subdividing or doubling process always results in an *even* number of panels in the main rafter in either half of the truss. It is one of the disadvantages of the Fink truss in its original form that this doubling process is the only method of increasing the number of apices in a top chord. This disadvantage may be overcome, however, by slightly modifying the truss to form a " fan truss " as explained later. The Fink roof truss is probably used more than any other type for spans of ordinary length. The web system is economical because the compression members are relatively short in comparison with the tension members. Center hangers may be used as indicated by the dotted lines. A modified form of the Fink truss is that in which the short web members, instead of being normal to the rafters, are *vertical*.

2. *Fan trusses.* When it is desired to increase the number of panels in a top chord without doubling the number, or when an odd number of panels

Fig. 118 (*d*). Fan Trusses.

is desired, the Fink truss may be modified by replacing one or more of the normal struts by two members which are not normal to the top chord. The truss then becomes a fan truss. The two fan trusses shown in Fig. 118 (*d*) are modifications of the second form of Fink truss.

3. *Baltimore truss.* In the Baltimore truss the struts in the web system on either side of the center are normal to the corresponding main rafter as in the Fink truss, but with the difference that each normal extends to the bottom chord. Moreover, since the two normals adjacent to the center of the truss meet at the center of the bottom chord, the rise of the truss cannot be assumed, but is determined by the length of span and the number of panels. If the main rafter in one half of the truss is divided into four

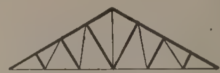


Fig. 118 (*e*). Baltimore Truss.

Fig. 119. Steel Framework for a Mill Building.

The roof is supported by nine Fink trusses and each truss is supported by columns. Knee braces stiffen the connection of trusses to columns. Lines of purlins may be seen extending the entire length of the roof. The framework for a monitor may also be seen. Rods are used for bracing. Horizontal girts to which the vertical sides of the building will be fastened extend between columns. A horizontal **crane girder**, attached to the columns on each side of the building and extending the entire length, supports the track for a traveling crane. Knee braces extend from the columns to the bottom of the crane girder.

equal panels as shown in Fig. 118 (*e*), the height of truss or rise will be the length of span multiplied by $\sqrt{\frac{1}{12}}$; i.e., if L is the length of span between supports and r the rise, $r = 0.2886 \times L$. As this rise is approximately one-fourth of the span, the truss is well proportioned. (Fig. 118 (*e*).)

1. When the required pitch is such that a Baltimore truss cannot be used, a truss of the same general form may be substituted if desired. The diagram for such a truss may be drawn as follows: Draw a vertical from the peak to the center of the bottom chord. Divide each half of the bottom chord into equal parts, one less in number than the number of panels in the corresponding half of the top chord; make each of these parts in the bottom chord the base of a triangle, the upper vertex of which will be the top-chord panel point that lies vertically over some point of the base of the triangle. The diagram thus drawn will be similar to that of a Baltimore truss except that none of the web members will be normal to a rafter. (Fig. 120 (*a*).)

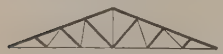


Fig. 120 (*a*).

2. *Warren trusses.* When the slope of a roof is slight, some form of the Warren truss may be used with the top chord inclined slightly to conform to the slope of the roof. If the span is short the simple Warren web system without verticals is generally used. For longer spans, the subdivided system with verticals may be better. This form of truss is often called the **flat Warren.** (Fig. 120 (*b*).)

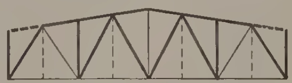


Fig. 120 (*b*). Warren Truss.

3. *Saw-tooth roofs.* A saw-tooth roof is one supported by a series of unsymmetrical trusses, one main rafter of each truss being vertical or nearly vertical, as shown in Fig. 120 (*c*). Saw-tooth roofs are used to provide satisfactory lighting for the space below. Windows or glazed surfaces are placed in the vertical or nearly vertical sides of the roof, which usually face the north (in the northern hemisphere) in order to admit northern light and exclude the direct sunlight. When it is desired to have clear floor space with no columns or as few columns as possible, two or more saw-tooth trusses may be made integral parts of one main truss by making the top chord of this main truss extend from peak to peak of the saw-tooth trusses. For example, the second and fourth columns in Fig. 120 (*c*) could be eliminated by making the first and second trusses into one truss, and the third and fourth into another. The top chord extending from the first to the second peak and that extending from the third to the fourth peak would be entirely above and outside of the roof. (See photograph, Fig. 120 (*d*), of a building in which saw-tooth trusses are made into one truss in groups of trusses.)

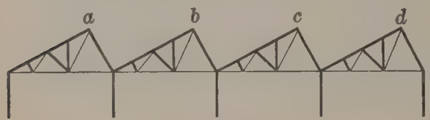


Fig. 120 (*c*).

Fig. 120 (*d*). Building with Saw-tooth Roof. Three Spans Carried as Integral Parts of One Truss.

Note the horizontal top chord of the main truss composed of three saw-tooth trusses.

4. *Fink truss with monitor and lean-to on each side.* In Fig. 120 (*e*) is shown a Fink truss supported by columns, with framework for a monitor, and with a supplementary Fink truss on each side for a lean-to. (**A lean-to** is a part of a building with a roof that slopes in one direction only and " leans " against the side wall of the main portion of the building.) Notice

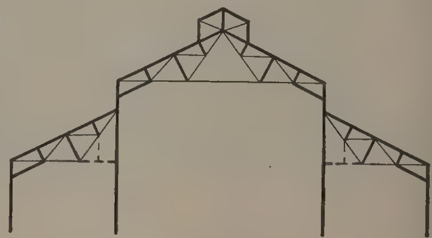


Fig. 120 (*e*).

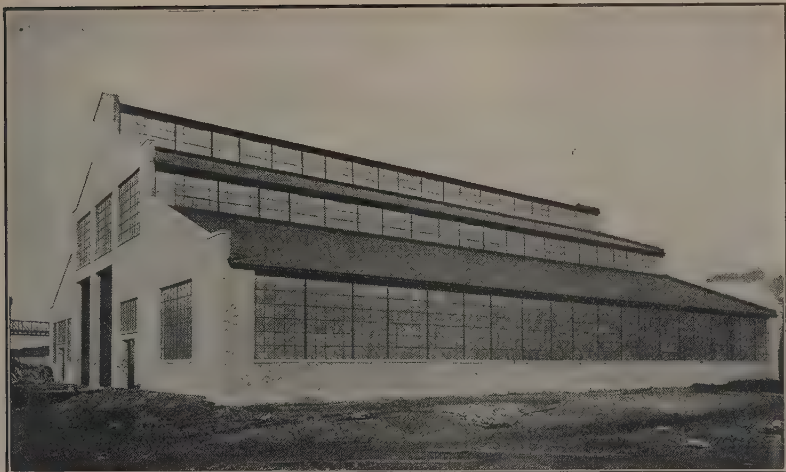


Fig. 121 (*a*). Building with Monitor and Lean-to Trusses.

Note the continuous steel sash construction including sash in the monitor and in the vertical side just above each lean-to.

the knee braces which stiffen the connection of the trusses to the columns. If desired, louvres or windows may be placed in the vertical side of the building just above the roof of each lean-to as well as in the vertical sides of the monitors. (See photograph, Fig. 121 (*a*).)

1. **Standard Types of Roof Trusses Modified.** Practically all of the standard types of roof trusses may be modified to meet special requirements. The most common modified forms are the *unsymmetrical truss* and the truss with *one or both chords curved.*

2. *An unsymmetrical truss* may be used when the slope of the roof on one side is different from that on the other, as in the case of the **saw-tooth truss** (Fig. 120 (*c*)), or when the roof has only one slope, as in the case of a lean-to. It is sometimes necessary to design one portion of a truss to carry loads which the corresponding portion on the other side does not carry. In this case the truss diagram may be symmetrical, but the members in one portion may be heavier than the corresponding members of the other portion; or it may be necessary to introduce in one portion supplementary members that are not in the other corresponding portion, thus making the truss diagram unsymmetrical.

3. *A roof truss with curved top chord* is generally understood to mean one in which the joints of the top chord lie in a curve, but in which each top-chord member between two successive joints is straight. The simplest top chord of this character is that in which there are only two slopes to the roof between the peak and either heel of the truss. (See photograph, Fig. 122.)

4. *A curved bottom chord* in its most common form is a **cambered chord.** Moderate camber may be obtained by making a portion of the bottom chord at the center lie in a horizontal line slightly above the horizontal line between the heels of the truss; the remainder of the bottom chord will then lie in inclined lines from the heels to the horizontal portion. Camber may be desired either to provide greater headroom between the floor and the bottom of the truss, or for the sake of appearance. All of the standard types of roof trusses can be designed with cambered lower chord. A Fink truss thus modified is shown in Fig. 121 (*b*). A truss in which both the top and bottom chords are curved is shown in Fig. 121 (*c*). Roof trusses may also be designed to form arches, as will be explained later.

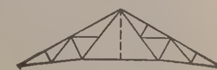


Fig. 121 (*b*).

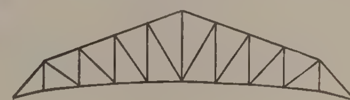


Fig. 121 (*c*).

5. **Architectural Forms of Roof Trusses.** Trusses of an entirely different character from those thus far described are often used for architectural effect. Two of the simplest as well as the most common forms of such trusses are the **scissors truss** (Fig. 121 (*d*) and the **hammer-beam truss** (Fig. 121 (*e*)). The diagrams show these trusses in their simplest forms. Each type may be modified in many ways by the introduction of additional members. Neither, strictly speaking, is a true type of truss since, as in most architectural forms of trusses, provision must be made for horizontal thrust at the supports· i.e., the truss is not of a form that is rigid.

Fig. 121 (*d*). Scissors Truss.

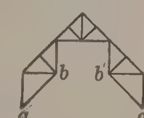


Fig. 121 (*e*). Hammer-beam Truss.

Fig. 122. Trusses with Curved Top Chords.

Note the unusual web system, part of which is similar to a Fink truss with verticals. The purlins are of wood and carry the roof sheathing. Note that most of the lines of purlins do not cross the rafters of the trusses at apices of the truss. This causes bending stresses in the top-chord members in addition to the axial stresses. Note the knee braces between columns and trusses. The unusually light roof construction was possible because of the mild climate and correspondingly light loads.

1. **Principal Dimensions of Roofs.** The rise or height of truss may be determined by the desired proportions of the building, but frequently it is determined by practical requirements pertaining to the slope of the roof. The minimum slope allowable depends to a considerable extent upon the character of the roof covering. If, for example, slate or tile are laid on too flat a slope, moisture will penetrate between the pieces; whereas tin with properly soldered joints may be used on a roof of any slope. For a shingle, slate, or tile roof covering, the pitch should be at least one-quarter; whereas for gravel and tar or asphalt covering, the pitch should not exceed one-fifth and preferably should be considerably less. The slope of a roof should be greater in a place where the snow fall is heavy than in a climate where there is no snow. There is an *economic rise* for each length of span, corresponding to the economic height of a bridge truss. Economy of design is probably best attained and, at the same time, practical requirements of roof covering are most often met, by a rise of one-fourth of the span (pitch $\frac{1}{4}$, slope $\frac{1}{2}$ or 6 inches vertical to 12 inches horizontal); hence this is the most common rise for ordinary roof trusses. Other common pitches are $\frac{1}{5}$ and $\frac{1}{3}$, and a common slope is 30°. In deciding upon the rise of a truss, it is well to bear in mind that $10\frac{1}{2}$ feet is about the maximum height that can be shipped by rail. If the rise does not exceed this, it is usually possible to ship a roof truss completely assembled; otherwise, it must be shipped in two or more sections.

2. *Purlin spacing.* The distance between adjacent lines of purlins depends partly on the nature of the roof covering and partly on the number of panels in the roof trusses. For any form of covering in which sheathing is used, not only must the lines of the purlins be close enough together to support adequately the sheathing and roofing, but the spacing should be such that commercial lengths of lumber (multiples of 2 feet) may be used without excessive waste. If tiles or corrugated steel sheets rest directly on the purlins, the spacing must conform to the length of tiles or to the length of the steel sheets. (The latter may also extend over more than one line of purlins.) Each line of purlins should preferably intersect the main rafter of a truss at an apex of the truss, and, if it is practicable, some adjustment in design should be made to meet this requirement. For this reason, in choosing the type of truss and in determining the number of apices in the upper chord, the desired purlin spacing should be considered. This may lead to some slight modification of a truss, such, for example, as the modification of a Fink truss to form a fan truss. On the other hand, the desired spacing of purlins can often be changed slightly to conform to the spacing of apices in the top chord of a truss. It is sometimes best to place lines of purlins between those which pass through truss apices even though this causes flexure in the main rafters in addition to the usual axial stresses.

3. The *economic length of bay* depends on many factors, such as length of trusses, spacing of purlins, type of truss, and other questions of design and erection which are not within the scope of this book. For ordinary roofs, bays are seldom less than 14 feet or more than 20 feet in length.

4. **Roof Bracing.** The most common types of roof bracing are (1) **diagonal bracing** in the plane of the main rafters or upper chords and in the plane of the bottom chords; (2) **struts** running lengthwise of the building in the plane of the upper chords and in the plane of the lower chords; and (3) **sway bracing** in a vertical plane between successive trusses. Various combinations of these three types of bracing may be used, depending upon the type of building and the purpose for which it is used. Diagonal bracing may be placed in every bay, or trusses may be **braced in pairs** by placing diagonal bracing in the two end bays and in alternate bays, or in the end bays and in every third or fourth bay. When a roof covering is on sheathing rigidly fastened to purlins, diagonal bracing in the planes of the upper chords becomes less necessary, except for purposes of erection, and may often be omitted; whereas such bracing is essential when the roof covering is not rigid, as, for example, when it consists of tile or of corrugated steel laid on purlins. When trusses are braced in pairs, struts are usually placed in the bays in which there is no diagonal bracing. Transverse bracing or sway bracing may consist of a single line located in a vertical plane through the centers of the trusses, or of more than one line, as, for example, of lines located approximately at the third points or the quarter points of the lower chord.

5. The purpose of roof bracing is to hold the trusses in place during erection, to resist lateral pressures on all parts of the finished building, and to make the whole structure as rigid and free from vibration as possible under the action of machinery or of moving loads such as cranes. Less bracing is required when a roof is supported on masonry walls than when

it is supported on columns, since, in the latter case, the rigidity of the whole structure is dependent almost entirely on the bracing.

1. Stresses are indeterminate in much of the roof bracing commonly used. The bracing for the roofs of ordinary structures, however, has become more or less standardized as the result of past experience; and although it may be possible to determine approximately the stresses in such bracing, this is rarely done except when the structure is extraordinary in size or type.

2. **Transverse bents.** When a roof truss is supported by two columns, a brace called a **knee brace** is generally used at each end of the truss. This knee brace extends from a point on the column a short distance below the top of the column to a joint of the lower chord of the truss, usually the joint nearest the end. The truss, the two columns, and the two knee braces form a structure called a **transverse bent,** or merely a **bent.** The stresses due to lateral forces may be determined approximately for all members of such a bent.

ASSIGNMENTS

(1) Visit a bridge with standard trusses, and submit a report which shall cover as far as possible the following: (*a*) Railway or highway, deck or through, riveted or pin-connected. (*b*) Brief description of the floor system, including method of fastening floor to stringers, and of connections of stringers to floor beams and floor beams to trusses. (*c*) Type of trusses, number of panels, height of truss, and panel length. (*d*) Description and diagram of upper and lower lateral systems, portal bracing and sway bracing; method of adjusting lengths of laterals if laterals are adjustable. (*e*) Description of shoes and bed plates; method of providing for expansion or contraction and of anchoring bridge to supports.

(2) Visit a building with a standard type of roof truss, and submit a report which shall cover as far as possible the following: (*a*) Type of truss, length and height, number of trusses, and lengths of bays. (*b*) Method of supporting trusses, of fastening trusses to supports, and provision, if any, for expansion. (*c*) Spacing of lines of purlins (note if all of these lines cross trusses at apices); description of purlins and connections to trusses. (*d*) Description of roof covering and method of fastening covering to purlins. (*e*) Description of roof bracing with diagrams. (*f*) Description of any additional feature, such, for example, as a monitor.

(3) Report on various types of double or multiple intersection trusses and the reasons why such trusses are not more generally used.

(4) Report on the longest spans in existence of the following types: plate girder, parallel-chord truss, Parker truss, and Petit truss. (Brief descriptions and diagrams.)

(5) Report on the essential differences between the arch, cantilever, and suspension bridges (the three types used for extremely long spans) and give a brief description of one or more of the longest spans of each type.

(6) Submit a collection of diagrams of types of bridge trusses not illustrated in this chapter. (Omit movable bridges, arches, cantilevers, and suspension bridges.)

(7) Submit a collection of diagrams of types of roof trusses not illustrated in this chapter.

(8) Report on several exceptionally long-span roof trusses. (Brief descriptions and diagrams.)

(9) Report on the different kinds of movable bridges.

(10) Report on different types of abutments, piers, bents, and towers.

(11) Submit a brief, historical review of the development in the design and construction of wooden, iron, and steel bridges. (Evolution of different types, early contributions to analysis of trusses, and mention of some of the prominent bridge engineers and their work.)

(12) Report on different types of connections of trusses to supports, including different provisions for expansion.

(13) Report on riveted *versus* pin-connected trusses — advantages and disadvantages of each type and whether or not the trend is toward the more extensive use of the riveted truss.

(14) Report on the advantages of plate girders for short spans as compared with trusses.

(15) Report on the advantages and disadvantages of various forms of trusses. Compare particularly the Warren truss and the Pratt truss.

(16) Report on the disadvantages of a subdivided truss such as the "Baltimore truss."

(17) Report on the advantages claimed for the K-truss.

(18) Report on the types of trusses most common for different lengths of span — limits between which any given type is used.

(19) Report on the subject of "Camber for Bridges," including the extent to which trusses change form because of the lengthening of tension members and the shortening of compression members.

(20) Report on the changes in bridge trusses due to the changes in the material used — wood, wrought iron, steel, and various steel alloys.

(21) Report on the advantages and disadvantages of steel bridges as compared with reinforced-concrete bridges.

(22) Report on developments in architecture due to the use of the steel framework in buildings.

(23) Report on the æsthetic element in the design of steel bridges.

(24) Report on the limit to the length of span of steel bridges, including descriptions of bridges of the longest span thus far proposed.

CHAPTER XII

EXTERNAL FORCES

In determining stresses in a structure it is necessary, first of all, to know the external forces that cause those stresses. In this chapter are explained the methods of determining external forces, both loads and reactions. Different kinds of static and moving loads are described and methods of calculating the magnitudes of these loads are given. The latter part of the chapter is devoted to the determination of reactions by the methods of statics already explained in **PART I.**

It is not the object in this chapter to give a comprehensive statement of all the values usually specified for various loads, but rather to give the fundamental facts concerning different kinds of loads, to explain how these loads are carried by one part of the structure to another until they are distributed to the trusses, to show how the line of action of a reaction depends not only on the lines of action and magnitudes of the loads but also upon the conditions at the support, to explain how a problem in the determination of reactions always falls under one of the eight cases treated in **PART I** (usually under *Case B′* or *Case 4*), and, finally, to show how the solution of a problem in reactions is identical with the standard solution given for the corresponding case in **PART I.**

1. **External Forces.** Forces due to gravity, forces due to wind, and reactions at supports are the principal *external forces* which, acting on a structure, cause *internal forces* or *stresses.* The weight of a structure and any vertical load which it supports tend to move it downward, wind pressure tends to move it horizontally or overturn it, and the supports react on the structure to hold it in equilibrium. Other external forces which it may be necessary to take into account are impact, tractive forces, and centrifugal force — all of which are due partly to gravity and partly to the movement of a load, such as a train, that comes upon the structure. A structure rigidly fastened at both ends may be subject to internal forces due to temperature (temperature stresses), but in ordinary bridges and roofs such stresses are avoided as far as possible by providing in the design for contraction or expansion.

2. **Dead load** is the weight of the structure or some part of the structure, as, for example, the dead load due to the weight of a floor system or the dead load due to the weight of a truss. **Static load** is sometimes used in place of "dead load," but it is a more comprehensive term. **Snow load,** for example, is a static load. **Live load** and **moving load** are practically synonymous terms for a load that is not static.

3. A **single concentrated load** is a theoretical load which has at any given instant a single line of action. Strictly speaking, there is no such load any more than there is a single line of stress. The vertical force from a wheel of a locomotive, for example, though usually considered as a single concentrated load, is really distributed over an appreciable length of rail, and by the rail and ties over a considerable length of stringer. A **wheel load** is the most common form of moving concentrated load and is usually one-half of the total **axle load.** Two or more wheel loads or concentrated loads at fixed distances apart constitute a **system of concentrated loads,** such, for example, as a **concentrated-load system** or **wheel-load system** which corresponds to the weights on the wheels of a locomotive.

1. A **distributed load** is one that is distributed over the whole or a portion of a structure. Such loading is usually **uniformly distributed.** The weight of a floor system, for example, is usually considered as a uniformly distributed dead load throughout the entire area of the floor, and the weight of a compact crowd of people is considered as a uniformly distributed live load which may cover the entire floor or only a portion of the floor. A uniformly distributed load may be considered as a **uniform load per square foot,** as, for example, 100 lbs. per square foot of floor surface, but usually for purposes of calculation it must be reduced to a **uniform load per linear foot,** as, for example, 500 lbs. per linear foot of stringer, or 2000 lbs. per linear foot of truss. **Uniform load per linear foot of rail** is one-half the **uniform load per linear foot of track,** and for bridges the **uniform load per linear foot of truss** is usually one-half the **uniform load per linear foot of bridge.**

2. A **panel load** (either dead or live) is the load carried by one panel. For uniform load, the **panel load per bridge** is equal to the load per linear foot of bridge multiplied by the panel length; the **panel load per truss** is usually one-half the panel load per bridge.

3. **Counterbalanced loading** is a term sometimes used when every load on one side of the center of a truss is counterbalanced by an equal load at the same distance from the center on the other side; **symmetrical loading** is a synonymous term. If, for example, all of the panel points of a symmetrical truss are loaded with equal loads, the loading is counterbalanced or symmetrical.

4. Different Kinds of Dead Load in Bridges and the Distribution of Dead Load. The dead load which a bridge truss must carry ordinarily includes (1) one-half the weight of the floor system, (2) one-half the weight of the bracing between trusses, and (3) the weight of the truss itself. The floor system is designed first, and therefore its weight can be accurately determined. The weight of the bracing is comparatively small, and is often negligible in short-span bridges. The weight of the truss must be assumed, and if, after the truss is designed, its actual weight is materially different from the assumed weight, the stresses and design may be revised in accordance with a new assumption of weight more in accord with the actual weight. It will be shown, however, that this "cut and try" process is not a mere guessing of weights. It is important to note that in any bridge the dead load is partly a function of the live load — the greater the live load the greater the dead load.

In the determination of stresses it is necessary to know not only the amount of dead load which each truss must support, but also how this load is carried from part to part and finally distributed to the apices of the truss. This will be best understood by considering what load each principal part of the bridge carries.

5. **Stringer load.** A stringer must sustain, first of all, its own weight, and in designing the stringer this weight must be assumed. A railroad-bridge stringer must carry, in addition, one-half of the weight of flooring, if there are only two stringers; this flooring may be of a simple type (rails and ties), or it may be of some solid type (ballasted flooring). A highway-bridge stringer must carry, in addition to its own weight, its share of the weight of the flooring, whether it be simply planking or a more elaborate and heavier type of flooring. The dead load for a stringer, including its own weight, is usually reduced to a uniformly distributed load per linear foot of stringer. This load per linear foot multiplied by the length of stringer (usually the panel length) *divided by two* is the total dead load brought to a floor beam by a stringer. Two successive stringers *in the same line* are connected to an intermediate floor beam at the same point (Fig. 94), hence the total dead load brought to a floor beam by such a pair of stringers is equal to the total dead load for one stringer, and there will be as many such "stringer loads" on a floor beam as there are lines of stringers.

6. **Floor-beam load.** The dead load for a floor beam is its own weight plus the "stringer loads." The weight of the floor beam is usually uniformly distributed. In a single-track railroad bridge there are generally two lines of stringers, hence two "stringer loads" are concentrated at points about $6\frac{1}{2}$ feet apart, equidistant from the center of the floor beam. In a highway bridge there are usually at least six or eight concentrated stringer loads on a floor beam (95 : 8), and in designing the beam these loads, if they are equal and equally spaced, may usually be reduced to an equivalent uniformly distributed load without serious error. In most cases, the total dead load brought to the panel point of a truss by the floor beam is equivalent to one-half the weight of floor beam plus one-half the "stringer loads."

1. *Note:* The weight of a floor system per linear foot of bridge is quite generally known for a bridge of standard type, given panel length, given width, and given live load. In this event, the weight per linear foot of floor system may be assumed in accordance with this general knowledge, and this weight multiplied by the panel length divided by two (since there are two trusses) is the total dead load brought to a truss panel point by a floor beam. Ordinarily, however, the floor system is designed before the trusses, and its weight determined.

2. **Weight of truss.** Each member of a truss is supported at two truss apices, one at each end of the member. Hence the portion of the weight of a truss concentrated at any apex can be found by adding one-half the weights of all members that meet at that apex, and in a very large bridge this is done. For the purpose of determining stresses in an ordinary bridge, the weight of truss is reduced to a uniformly distributed dead load per linear foot of truss. This weight per linear foot must be assumed. Empirical formulas for such weights are often unreliable. In assuming the weight of truss it is better, if possible, to be guided by actual weights of trusses in similar bridges previously designed. Most structural companies have data that make it comparatively easy to assume correct weights of ordinary trusses. In short-span bridges the weight of the trusses is small as compared with the live load, and hence a considerable error in the assumption of the weight of truss does not affect materially the total stress in any member of the truss. In long-span bridges, on the other hand, the weight of trusses may be great as compared with the live load, and hence not only should the dead load be determined with more care but the distribution of the weight of truss to different apices should be considered.

3. **Panel dead load.** It was explained in 97 : 1 that the weight of a floor system is brought to the panel points of trusses by the floor beams; the total amount thus brought by a single floor beam to a panel point of a truss has been shown to be one-half the weight of the floor beam plus one-half the stringer loads on the beam. If to this amount is added the weight of truss per linear foot multiplied by the panel length, the total is the *panel dead load.* (If the weight per linear foot of floor system is used, this weight multiplied by the panel length divided by two is the amount brought to a panel point by the floor beam.) When the weight of the lateral systems and transverse bracing is great enough to be taken into account, the weight per panel of such bracing is included in a panel dead load. In a through bridge the loaded chord is the lower chord, since the floor beams bring their loads to lower apices; in a deck bridge the loaded chord is the upper chord. (95 : 3.)

4. *The distribution of a panel dead load* between a lower apex and the corresponding upper apex is usually made in accordance with one of the three following assumptions:

(*a*) That the total dead panel load is concentrated at the apex of the *loaded* chord whether it be a lower or an upper apex.

(*b*) That two-thirds of the total dead panel load is concentrated at the apex of the *loaded* chord, and the other third at the corresponding apex of the *unloaded* chord. If the floor system is unusually heavy, the proportion of the dead load that is assumed to act at the loaded chord may be somewhat greater.

(*c*) That the dead load brought by the floor beam to an apex of a truss is concentrated at that apex, and that the weight of the truss per panel is equally divided between an apex of the lower chord and the corresponding apex of the upper chord.

The first assumption is commonly made in determining the stresses in light bridge trusses; the third assumption is more nearly in accord with the actual conditions.

5. *Note:* The dead load at the end apex of a loaded chord is usually one-half of a panel load. This half panel load goes directly to the support, and consequently it cannot cause stress in any member of the truss. Such a load, therefore, may be ignored when determining stresses in the truss. If it were included in the external forces, it would increase the reaction at the support by an amount equal to itself, namely, by half a panel load, but the resultant of this end-apex load and this larger reaction would be equal in magnitude to the smaller reaction found by ignoring the end-apex load. For example, assume that an end-apex load is 20,000 lbs., and that the reaction at the corresponding support when this end-apex load is ignored is 100,000 lbs. If, now, the end-apex load is included in the external forces, the reaction will be increased by 20,000 lbs., and will become 120,000 lbs., but the resultant of the end-apex load and this larger reaction is 120,000 − 20,000, or 100,000 lbs., and this resultant will have the same effect on the stresses in the truss as the smaller reaction of 100,000 lbs. Note that in designing the support at the end of a truss, the half panel load should be included in the total load on the support.

6. **Values Used in Calculating or Assuming Dead Loads for Bridges.** Certain parts of bridges are more or less standardized, and the corresponding weights are known. The weight of a part to be designed

must be assumed. In calculating or estimating weights, the following unit weights in pounds per cubic foot are used:

Steel, 490; concrete, 150; sand, gravel, and ballast, 120; asphalt-mastic and bituminous macadam, 150; granite, 170; paving bricks, 150.

The weight of timber is taken as 4 or 5 lbs. per foot board measure, i.e., 48 or 60 lbs. per cubic foot.

1. *The dead load for a railroad-bridge stringer* consists of a portion of the weight of the flooring plus the weight of the stringer itself. This dead load is usually reduced to the load *per linear foot* of one stringer. The simplest type of flooring (rails, ties, guard rails, and fastenings) weighs from 400 to 600 lbs. per linear foot of track. If there are only two stringers per track, the load per linear foot of stringer is from 200 to 300 lbs. Ballasted flooring will weigh from three to four times as much, depending upon its design. Its weight is easily computed from the weights of the materials of which it is composed. Assuming ballast to be 13 ft. wide and its average depth to be 14 in., the ballast alone, not including the steel plates or concrete underneath it, will weigh approximately 1800 lbs. per linear foot of track, or 900 lbs. per linear foot of stringer. The weight of the stringer must be assumed. An experienced engineer, by taking into account the length of the stringer and the total floor load and live load that it must carry, can estimate very closely what the actual weight will be. This actual weight of the stringer will be small compared with the total load that it will be designed to carry, and therefore any error in the assumption of its weight is usually negligible. For example, a stringer 24 in. deep, composed of web plate, cover plates, and angles, may weigh approximately 120 lbs. per linear foot, and be designed to carry in addition to the flooring a live load, including impact, of 5000 lbs. per linear foot of stringer. In that case the weight of the stringer is approximately only one-fortieth of the live load it must carry.

2. *The dead load for a highway-bridge stringer* consists of a portion of the weight of the flooring (including spiking pieces if used) plus the weight of the stringer itself. The dead load is usually reduced to the load *per linear foot* of stringer, and this load per linear foot will depend upon the spacing of the stringers. If, for example, the distance, center to center, of lines of stringers is 3 ft., each stringer will carry, roughly speaking, 3 sq. ft. of flooring for each foot of its own length. A general idea of the weights of highway-bridge flooring, in pounds per square foot, may be gained from the following values: Four-inch wooden planking, 20 lbs.; a reinforced-concrete sub-floor 4 in. thick and wooden-block wearing surface 3 in. thick, 65 lbs.; a reinforced-concrete sub-floor 4 in. thick and a concrete wearing surface 4 in. thick, 100 lbs. The weight of the stringer itself must be assumed, and, as in the case of a railroad bridge, an error in the assumption of this weight is usually negligible.

3. *The dead load for a floor beam* consists of the loads brought to the beam by the stringers plus the weight of the beam itself. The flooring and stringers are designed before the floor beam is designed, and the stringer loads are therefore known. The size of the floor beam, and therefore its weight, will depend upon its length and the total load (dead and live) that it must carry. Here again, the weight of the beam itself is but a small fraction of the total load for which it is designed, and errors made in assuming its weight are usually negligible, particularly if one is guided by the actual weights of floor beams previously designed.

4. *The dead load for a truss* usually consists of one-half of the weight of the floor system plus one-half of the weight of the bracing plus the weight of the truss itself. If the dead load is assumed to be applied wholly at the loaded chord (127: 4 (*a*)), or two-thirds at the loaded chord and one-third at the unloaded chord (127: 4 (*b*)), there is no necessity for keeping separate the weights of the floor system, the bracing, and the truss. In that case the *total* dead load may be reduced to a uniformly distributed load per linear foot. If this load is expressed as the total weight of bridge per linear foot, one-half of this amount is the dead load per linear foot carried by each truss. The weight of a truss depends upon the length of span, the type of truss, the depth and panel length, and the total load (dead and live) to be carried. Other factors involved are such questions as whether the bridge is for a railroad or a highway; if a railroad bridge, how many tracks there are; if a highway bridge, how wide the roadway is; and whether or not there are street-car tracks or sidewalks. With all of these variables, it is difficult to give even a general idea of the weights of various types of trusses. Empirical formulas and curve diagrams used in estimating dead loads for trusses are, in general, of three types, namely, those that give the weight of the metal in the floor system, those that give the weight of the metal in the trusses, and those that give the total weight

of the metal in the bridge. In all three types, the weights are usually expressed in pounds per linear foot of span. The weight of flooring (rails, ties, ballast, etc., for railroad bridges; planking, pavement, etc., for highway bridges) is not generally included, and this fact should not be overlooked in estimating the total dead load. In many of the formulas for weights of highway bridges, the weights of joists or stringers are not included.

1. *Typical dead loads for bridges.* It is not within the scope of this book to give formulas and data to be used in estimating various dead loads. Such formulas and data belong more properly in a book on structural design. The purpose of the following list of typical weights is primarily to give a general idea of the weights of ordinary bridges and parts of bridges, and also to serve as a guide in specifying dead loads in practice problems which shall be reasonably in accord with dead loads in actual structures.

2. The weights given are in *pounds per linear foot*, except those for the flooring of highway bridges which are per square foot of roadway surface. Weights of floors for railroad bridges are per linear foot of stringer or floor beam. Weights of girder or truss bridges are per linear foot of bridge and include the weight of the floor system except the flooring (ties, rails, planking, etc.), the weight of all bracing, and the weight of the two main girders or trusses.

3. *Weights of Floors*

(Weights of stringers not included.)

Railroad (timber floor), rails, ties, etc., 400 to 500 lbs.
Railroad (ballasted), rails, ties, ballast, etc., 2000 to 2800 lbs.
Highway-bridge flooring, 15 to 200 lbs. *per square foot.*

4. *Weights of Stringers*

Railroad-bridge stringer, 100 to 250 lbs.
Highway-bridge stringer, 15 to 100 lbs.

5. *Weights of Floor Beams*

Railroad, single-track bridge, floor beam 130 to 250 lbs.
Railroad, double-track bridge, floor beam 300 to 450 lbs.
Highway bridge, floor beam 20 ft. long, 50 to 100 lbs.
Highway bridge, floor beam 40 ft. long, 80 to 200 lbs.

6. *Weights of Deck Plate-girder Bridges*

(Weights of rails, ties, ballast, etc., not included.)

Railroad, single-track, span 50 ft., 700 to 1000 lbs.
Railroad, single-track, span 100 ft., 1300 to 1900 lbs.

7. *Weights of Through Plate-girder Bridges*

(Weights include weights of stringers, floor beams, and bracing, but not weight of floor (rails, ties, or other flooring).)

Railroad, single-track, span 50 ft., 1000 to 1600 lbs.
Railroad, single-track, span 100 ft., 1700 to 2400 lbs.
Railroad, double-track, span 50 ft., 2700 to 3300 lbs.
Railroad, double-track, span 100 ft., 3900 to 4600 lbs.
Highway, roadway 20 ft., span 50 ft., 450 to 700 lbs.
Highway, roadway 20 ft., span 100 ft., 650 to 950 lbs.

8. *Weights of Deck Truss Bridges*

(Weights include weights of stringers, floor beams, and bracing, but not weight of floor (rails, ties, or other flooring).)

Railroad, single-track, span 100 ft., 1400 to 2200 lbs.
Railroad, single-track, span 200 ft., 2100 to 3200 lbs.
Railroad, double-track, span 100 ft., 2600 to 4000 lbs.
Railroad, double-track, span 200 ft., 4000 to 5800 lbs.
Highway, roadway 20 ft., span 100 ft., 350 to 750 lbs.
Highway, roadway 40 ft., span 200 ft., 650 to 1400 lbs.

9. *Weights of Through Truss Bridges*

(Weights include weights of stringers, floor beams, and bracing, but not weight of floor (rails, ties, or other flooring).)

Railroad, single-track, span 100 ft., 1300 to 2000 lbs.
Railroad, single-track, span 150 ft., 1600 to 2500 lbs.
Railroad, single-track, span 200 ft., 2000 to 2900 lbs.
Railroad, single-track, span 250 ft., 2400 to 3400 lbs.
Railroad, single-track, span 300 ft., 2700 to 4000 lbs.
Railroad, double-track, span 100 ft., 2400 to 3700 lbs.
Railroad, double-track, span 200 ft., 3700 to 5400 lbs.
Railroad, double-track, span 300 ft., 5000 to 7400 lbs.

Highway, roadway 20 ft., span 100 ft., 300 to 650 lbs.
Highway, roadway 40 ft., span 100 ft., 500 to 1200 lbs.
Highway, roadway 20 ft., span 200 ft., 450 to 1000 lbs.
Highway, roadway 40 ft., span 200 ft., 800 to 1800 lbs.
Highway, roadway 20 ft., span 300 ft., 550 to 1200 lbs.
Highway, roadway 40 ft., span 300 ft., 1000 to 2000 lbs.

1. *Weights of Sidewalks*

When there are sidewalks, add from 60 to 130 lbs. per linear foot of bridge for each sidewalk (exclusive of flooring).

2. **Dead Loads for Roof Structures.** The dead load which a roof truss must carry ordinarily includes (1) the dead loads brought to it by the purlins and (2) the weight of the truss itself. (The weight of bracing is often negligible.)

3. *Purlin dead load.* A purlin must sustain its own weight and a portion of the weight of the roof covering. The area of roof covering supported by one purlin is generally a rectangle the length of which is equal to the length of the bay and the width of which is equal to the distance, center to center, of two adjacent lines of purlins. (Photograph, page 117.) (Analogous to the area of the flooring supported by a highway-bridge stringer.) The weight of roof covering per square foot of roof surface may be found, and from this may be calculated the load per linear foot of purlin. (Purlins may then be selected or designed to carry this load plus other loads, such as snow load and wind load.) The weight of purlin per linear foot will then be known. The weight of roof covering per linear foot of purlin plus the weight of purlin per linear foot (including spiking piece) multiplied by the length of bay is the total dead load brought to an *intermediate* roof truss by a single line of purlins — one purlin over two bays or two purlins in adjacent bays (analogous to the total dead load brought to a floor beam by two stringers in the same line and in adjacent panels). Purlins sometimes support rafters which in turn support roof covering. In such a case the rafters are taken into account in determining the purlin dead load. It is best, if possible, so to design a roof structure that the purlins are connected to the roof trusses at apices only, as previously stated, and in that case the dead load brought to an apex by one line of purlins constitutes a large part of the dead load for that apex.

4. *Note:* It is to be noted that the roof area supported by an *end* roof truss is usually one-half of that supported by an *intermediate* truss, provided all bays are equal in length. The end truss, however, is usually made like the intermediate trusses for simplicity or to provide for future extension. Frequently there is no end truss, the roof being supported by the end of the building.

Note: When a building is in a region where snow falls, the snow may be considered a part of the dead load. In this book, however, snow load is not included in the term dead load.

5. *Weight of roof truss.* The weight of a roof truss is usually reduced to a uniformly distributed load per linear foot just as in a bridge truss, and from this the weight of truss per panel is easily found. This weight is almost always assumed as applied wholly at an upper apex. The weight of a roof truss is not known until it has been designed, and therefore it must be *assumed*, as in the case of a bridge truss. Weights of roof trusses vary with the lengths of span, the type, design, and pitch of the truss, and the loads to be carried. The loads vary primarily with the lengths of bays. The weights of different roof trusses vary much less than the weights of bridge trusses, since there is not such a variation in dead and live loads. It is easier, therefore, to assume correctly weights of roof trusses. Past experience in design is the best guide, and, as in the case of bridges, structural designers have data that are very useful in estimating weights. It is rarely that the weight of a roof truss as computed from the design differs enough from the assumed weight to require a revision of the calculation of stresses.

6. *Apex load.* The weight of a roof truss per panel, plus the dead load brought to an apex by purlins, is generally the total apex dead load. If there are lines of purlins in addition to those at the apices, the dead load which these additional purlins bring to the truss must be taken into account. The total dead load at the end apex (at the heel) is usually taken as one-half the load at an intermediate apex, unless the overhang of the roof makes it more nearly a full panel load. In finding dead-load stresses this half load at the heel or support may be ignored, but it must be taken into account in designing the support. (127 : 5.)

7. **Values Used in Calculating or Assuming Dead Loads for Roof Structures.** The weights of the most common materials used for roof covering, in pounds per square foot, are as follows:

Wooden sheathing 1 in. thick	3 to 4
Wooden shingles	$2\frac{1}{2}$ to 3
Asphalt shingles	$1\frac{1}{4}$ to 3
Asbestos shingles	3 to $6\frac{1}{2}$
Slate shingles $\frac{3}{16}$ to $\frac{1}{4}$ in. thick	$7\frac{1}{4}$ to $9\frac{1}{2}$
Tin (with one thickness of felt)	1
Corrugated steel	1 to 3
Tiles, corrugated	8 to 10
Felt and gravel	5 to 10
Tar and gravel	8 to 10

1. *The weight of a purlin per linear foot* will vary from $1\frac{1}{2}$ to 4 lbs., multiplied by the distance in feet, center to center, of lines of purlins. An average value is 3 lbs. for each square foot of roof covering carried by one linear foot of the purlin.

2. *Weights of roof trusses.* The principal variables tnat are involved in estimating the weights of different roof trusses are length of span, length of bay, pitch of truss, and the loads to be carried. No empirical formulas or diagrams for weights of roof trusses will be given. The purpose of the following list of typical weights is primarily to give a general idea of the weights of roof trusses and, incidentally, to serve as a basis in devising practice problems. The weights are based on a carrying capacity of approximately 50 lbs. per square foot of horizontal projection; weights for other capacities are approximately proportional.

3. WEIGHTS OF ROOF TRUSSES

Pitch	Span	Distance c. to c. Trusses	Type of Truss	Weight of Truss per lin. ft.
$\frac{1}{6}$ to $\frac{1}{3}$	30	12	Fink or Fan	30 to 40
"	40	14	" "	45 to 55
"	40	16	" "	50 to 60
"	50	16	" "	55 to 65
"	50	18	" "	60 to 70
"	60	18	" "	70 to 80
"	60	20	" "	75 to 85
Flat	40	16	Warren	55 to 65
$\frac{1}{4}$	50	10	Howe (Timber)	35 to 45

4. **Live Loads for Bridges.** *The live load for railroad bridges,* as usually specified, consists of a locomotive, or of two locomotives coupled together, followed by a train of cars. The distances between axles of the locomotive and the weights on the axles are specified, thus forming a series of concentrated loads at fixed distances apart. Neither the distances nor the weights correspond exactly to those of a real locomotive, but the system of axle loads specified is such that resulting stresses will be at least as great as those which would be caused by locomotives likely to pass over the bridge. The train load is usually specified as a uniformly distributed load per linear foot of track. If the concentrated loads specified are **axle loads,** each load must be divided by two to obtain a **wheel load** or the load on one rail. Similarly, if the train load is **per linear foot of track,** one-half of it is the load **per linear foot of rail.** Some of the ablest bridge engineers regard the use of concentrated or wheel-load systems as an unnecessary and unscientific refinement, and advocate instead the use of uniformly distributed loads which decrease in the amount per linear foot as spans become longer.

5. **An equivalent uniform load** for a given concentrated-load system and a given length of beam or truss is a uniform load that would cause stresses in the beam or truss *approximately* equal to the stresses that would be caused by the given concentrated load system. Tables of equivalent uniform loads for different lengths of span can be prepared once for all for a given concentrated-load system, and these loads can then be used in place of that system in determining stresses. The methods of calculating the equivalent loads for such tables will be explained in **PART III.**

6. **Locomotive excess load** is a uniform load combined with one or two concentrated loads to take the place of locomotives; one concentrated load is placed at the head of the uniform load, and if there is a second concentrated load it is placed about 50 ft. behind the first. Such excess loads are now seldom, if ever, used.

7. *Live loads for highway bridges* may be uniformly distributed loads corresponding to the weights of compact crowds of people, or concentrated axle loads corresponding to road rollers or heavily loaded automobile trucks. Uniformly distributed live loads are greater per square foot for short spans than for long spans, and greater per square foot for city or suburban bridges than for country bridges. The uniform load may be

assumed to cover both roadway and sidewalks, roadway only, or sidewalks only, whichever assumption results in the maximum stress for the member of the bridge under consideration. If a highway bridge carries street cars, a system of concentrated loads is used which corresponds to the axle loads of the street cars.

1. *Live loads for floor systems.* Concentrated loads are generally used in calculating maximum live-load stresses in the stringers and floor beams of a railroad bridge. The maximum stresses in the stringers of a highway bridge are usually caused by concentrated loads, but the maximum live-load stresses in a floor beam may be caused by concentrated loads, by uniform load, or by a combination of both.

2. *Distribution of live loads.* The maximum live-load stresses in certain members of a truss will occur when the bridge is *fully* loaded from end to end, whereas, in other members, maximum live-load stresses occur when the bridge is *partially* loaded. Certain maximum stresses in a beam occur when live loads are near the center of the beam, whereas others occur when the loads are near an end. Live loads are distributed by the track or flooring to stringers, by the stringers to the floor beams, and by the floor beams to panel points of the truss, just as the weight of the floor system is distributed; but the dead load is immovable and always the same for a given beam or truss, whereas the live load changes its position and consequently changes in amount. The determination of the positions and amounts of the live load that will cause maximum stress in a beam or truss is a subject in itself and will be left for treatment in **PART III.** The customary values for live loads will also be given in **PART III.**

3. **Live Loads for Roof Trusses.** The majority of roof trusses do not carry live loads unless wind pressure be considered a live load. Trusses in a mill building may carry moving concentrated loads of a special character, such, for example, as movable hoists applied to the lower chord. If a floor, as well as a roof, is supported by a truss, moving loads may come upon this floor and consequently cause live-load stresses in the truss. There are, however, no standard live loads or methods of distributing live load analogous to those described for bridges. A roof truss that supports live loads is nearly always a special case in which the amount and distribution of live load must be determined by special requirements.

4. **Wind Pressure.** Wind pressures are usually estimated on the assumption that the direction of the wind is horizontal, but the actual pressure on a structure depends upon whether the exposed surface is a plane and vertical like the side of a building, a plane and inclined like a sloping roof, or a curved surface like that of a cylindrical tank.

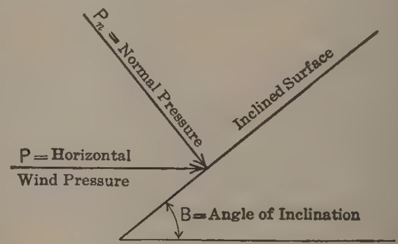
5. *Wind pressure on a vertical surface* varies, theoretically, as the square of the velocity or

$$\text{Pressure} = \text{Constant} \times (\text{Velocity})^2.$$

There is no absolute agreement, however, as to the value of the constant, but that most used by engineers is 0.004. When this value is used the formula gives the following results for pressures on vertical surfaces.

Velocity Miles per Hour	Pressure Pounds per Square Foot
50	10
60	14.4
70	19.6
80	25.6
90	32.4
100	40.0

6. *Wind pressure on an inclined surface* is generally assumed as exerted only in a direction normal to the surface, since the friction of the wind along a comparatively smooth surface is so small as to be negligible. *The normal pressure, however, cannot be derived by resolving the horizontal wind pressure into two components,* one parallel and the other normal to the inclined surface. The normal pressure can only be determined by experiment. Classic investigations are those of Hutton in 1787–88, of Duchemin in 1829, and of Langley in 1888–90. Langley's experiments verified the assumption that the wind pressure is exerted only in a direction



normal to an inclined surface, and the results agreed within about 2 per cent with those deduced from Duchemin's empirical formula:

$$P_n = P \times \frac{2 \sin B}{1 + \sin^2 B},$$

in which P is the unit pressure in pounds per square foot on a vertical surface, B is the angle that the inclined surface makes with the horizontal, and P_n is the normal pressure in pounds per square foot.

1. *Difficulties in estimating wind pressure.* Experiments to determine wind pressure on vertical and inclined surfaces, though many, have necessarily been made on relatively small areas. Moreover, the classic experiments previously referred to were made on *thin* plates. The intensity of high wind pressure is not uniform over a large area, and the effect of deflected currents of air and suction on an actual structure is quite different from the effect on thin plates. The wind pressure on irregular surfaces, or on portions of a structure that are partly behind other portions, is uncertain. Wind pressure is greater per square foot on small structures than on large ones, and greater on structures in exposed places than on those in sheltered places. Finally, there is the question of how great a wind velocity a structure should be expected to withstand. It should be designed for the maximum wind velocity that is likely to occur, but this does not mean that it must be capable of resisting a tornado. A good engineer assumes wind pressures which are reasonable, and, knowing these pressures and their effect on the structure to be uncertain, he will avoid useless and unscientific refinements in the determination of stresses due to wind. The usual factor of safety in design will provide for most emergencies.

2. *Wind pressure on bridges.* In addition to the wind pressure on the exposed surface of a bridge, there is the pressure on the exposed surface of a load crossing the bridge, such, for example, as the pressure on the side of a train of cars. The wind exerts its maximum pressure when blowing in a direction at right angles to the axis of the bridge. Wind pressures are usually specified in one of two ways, namely, as a given amount per square foot of exposed surface or as a given amount per linear foot of chord. In the latter case, the amount specified for the loaded chord is greater than the amount specified for the unloaded chord, to provide for the pressure on a load crossing the bridge. The subject of wind pressures on bridges will be treated in detail in **PART III.**

3. *Values for wind pressures on buildings.* There is considerable variation in the wind pressures specified in the building codes of cities. For vertical sides and ends of buildings that are partially protected from exposure to wind, a pressure of from 20 to 25 lbs. per square foot is most frequently specified; the corresponding maximum pressure for exposed vertical surfaces is 30 lbs. per square foot. These pressures are theoretically equivalent to a wind velocity of approximately 70 miles per hour for 20 lbs. and of 90 miles per hour for 30 lbs. The latter velocity is practically that of the wind in an ordinary hurricane, and the corresponding pressure is enough to overturn empty freight cars. The pressure generally used for roof surfaces is a *normal* pressure per square foot of *roof surface* derived by Duchemin's formula from a unit wind pressure P on a vertical surface. (132 : 6.) Building codes and specifications vary considerably, however, in the value to be used for P. Common values range from 20 to 30 lbs. per square foot. Normal pressures corresponding to these two values of P have been calculated by Duchemin's formula for typical pitches and slopes, and tabulated below.

4. NORMAL PRESSURES FOR ROOF SURFACES IN POUNDS PER SQUARE FOOT OF ROOF SURFACE

Pitch	Slope	Normal Pressure	
		$P = 20$	$P = 30$
$\frac{1}{6}$	4″ in 12″	11.5	17.2
$\frac{1}{5}$	4.8″ in 12″	13.1	19.8
$\frac{1}{4}$	6″ in 12″	14.9	22.4
	30°	16.0	24.0
$\frac{1}{3}$	8″ in 12″	17.0	25.5
$\frac{1}{2}$	45°	18.9	28.3

The wind is generally assumed to act on the windward side of the roof only, and no account is taken of the suction on the leeward side or of the variation in the distribution of pressure, except as the original assumption of pressure is made with these uncertainties in mind.

1. The normal pressure per square foot of inclined roof surface having been determined, it is merely necessary to multiply this pressure by the product of the length of bay by the distance, center to center, of purlins to determine the wind load brought to an intermediate roof truss by a single line of purlins. If purlins intersect trusses at apices only, the load thus obtained is the wind load for an apex.

2. *Note:* The usual method of determining the wind load per apex is to calculate the panel load without regard to the spacing of purlins. The panel load is the wind pressure on a rectangle whose length is that of a bay and whose width is the distance between adjacent apices on the main rafter of the truss. The wind load at the heel and at the peak is usually one-half that at an intermediate apex.

3. If a roof truss is not symmetrical, or if it is free to move horizontally at one of the supports, the wind should be assumed to act in one direction on one side of the roof and then in the opposite direction on the other side, in order to find the maximum stresses. To this extent, the wind pressure then becomes a moving load.

4. *Note:* For ordinary roof trusses a short-cut method is frequently used in which the normal wind loads are replaced by equivalent vertical loads and these vertical loads are then combined with the snow load or with the dead load or with both. This short-cut method will be explained later.

5. **Snow Load.** The weight of freshly fallen snow may be as great as 10 or 12 lbs. per cubic foot; when it becomes saturated the weight may exceed 40 lbs. per cubic foot. The snow load to be assumed depends upon the type of structure and its geographical location. The greatest snow falls occur in northern latitudes and at high altitudes. Less snow will remain on a surface with a steep slope than on one that is comparatively flat; if the slope is greater than 45° the snow load is often negligible.

6. *Snow load for roofs.* Good average values (in pounds per square foot of roof surface) for the latitude of New York and for moderate altitudes are 30 lbs. per square foot for flat or nearly flat roof surfaces, 20 lbs. per square foot for surfaces with a slope of approximately 6 in 12, and 10 lbs. per square foot for steep slopes up to 12 in 12. These values may be increased about 50 per cent for localities of exceptionally heavy snow fall, and decreased by the same amount or even more for localities of light snow fall. In using the above values, however, the type of structure, the purpose for which it is to be used, and the combined effect of snow load and other loads must all be taken into account.

7. *Snow load for bridges* is usually a small percentage of the total dead and live load, and its effect on design is therefore relatively unimportant. Moreover, it is not probable that the maximum live load will be on the bridge simultaneously with the load caused by an occasional extremely heavy snow fall. There is, therefore, compensating action between live load and snow load. An open type of floor, such as that in many railroad bridges, does not retain much snow, whereas a solid floor retains the maximum amount; but even in the latter case, snow is removed, as a rule, before heavy loads cross the bridge.

8. *Combined snow and wind loads for roofs.* The snow load is greatest for a flat roof and least for a roof with a steep slope, but the reverse is true of wind pressure. It is not likely, moreover, that the maximum snow load and the maximum wind pressure will occur at the same time, because the wind would blow the snow off. The designer is often justified, therefore, either in assuming from one-third to one-half of the snow load to act when the wind pressure is a maximum, or from one-third to one-half of the wind pressure to act when the snow load is a maximum. Frequently a sleet load of 10 lbs. per square foot is assumed. The unit working stress used for designing members to resist wind stresses should also be taken into account in any combination of loads. Maximum wind pressure will occur only occasionally, if at all, and then for only a short period; hence either unit working stresses for wind loads may be considerably greater than those used for dead and snow loads, or, if the working stress is the same for all loads, a reduction in wind pressure may be assumed when it is to be combined with dead and snow loads. The combination of different kinds of loads, or of stresses due to different kinds of loads, is really a question of design, and will not be discussed further, except to explain in general a short-cut method of determining stresses by a combination of loads.

9. *Short-cut method of combining wind loads on a roof with other loads.* The short-cut method is based on the fact that for simple trusses the stresses determined from wind loads normal to the roof, first on one side and then on the other, are not greatly different from the stresses determined from the same loads assumed to act vertically and simultaneously on *both* sides of the roof. For example, the stresses in an ordinary truss due to a

normal pressure of 20 lbs. per square foot of roof surface, acting first on one side and then on the other of the roof, are not greatly different from the stresses due to a vertical load of 20 lbs. per square foot of roof surface acting on the entire roof. Moreover, such differences in stresses as may result from these two assumptions of wind loads are usually negligible in their effect on the *total* stresses in the truss obtained by combining stresses due to dead load, snow load, and wind load.

1. The first step in the short-cut method is to select a vertical load (**equivalent combined wind and snow load**) that will result in stresses approximately equal to those obtained by a reasonable combination of wind and snow-load stresses determined separately by the more exact methods. The magnitude of this equivalent load depends largely on the slope of the roof. The minimum recommended by the Building Code Committee of the Bureau of Standards, United States Department of Commerce, is given in the following paragraph:

2. "Roofs having a rise of 4 inches or less per foot of horizontal projection shall be proportioned for a vertical live load of 30 pounds per square foot of horizontal projection applied to any or all slopes. With a rise of more than 4 inches and not more than 12 inches per foot a vertical live load of 20 pounds on the horizontal projection shall be assumed. If the rise exceeds 12 inches per foot no vertical live load need be assumed, but provision shall be made for a wind force acting normal to the roof surface (on one slope at a time) of 20 pounds per square foot of such surface."

3. *Note:* Roof loads are often determined by the building laws which the engineer must observe, and these laws vary greatly in different municipalities. The trend in practice, as shown by the specifications just quoted, is toward the use of the equivalent vertical load when the slope of the roof is not greater than 45°.

4. The equivalent combined wind and snow load having been selected, the corresponding vertical loads per apex are calculated, and the stresses due to these apex loads are then determined by methods to be explained later. The short-cut method needs in its application the use of the same judgment based on experience that is required in combining, by any other method, stresses due to dead, wind, and snow loads.

5. **External Forces Due to Live Loads in Motion. Impact.** The maximum live-load stress in any part of a floor system or in any member of a truss is found by placing the live load in the position in which it will cause that maximum stress, and then determining the stress as if the live load were *standing still* in that position. The stress thus determined may be much less than the actual stress that exists when the live load is in the same position *but moving*, particularly if the member is one upon which the live load acts more or less suddenly or directly, such, for example, as a stringer or a hanger at the end of a floor beam. Allowance for this increase in stresses due to the rapid motion of the live load is called **allowance for impact.** The term *impact* is somewhat misleading, since the allowance is not merely for the impact of the load on the structure but for the total effect of a variety of causes having their origin in the motion of the load. The allowance for impact is usually expressed in an empirical formula based upon experiments with moving loads on bridges; this formula gives the percentage, in any given case, by which the live-load stress should be increased to provide for "impact." The allowance is greatest for short spans, decreasing rapidly as the span increases, and greatest for members upon which the live load acts suddenly or directly.

6. **Centrifugal force.** If in a railroad bridge the track is curved, centrifugal force becomes an external force on the structure and hence the cause of stresses in the structure.

7. **Tractive forces,** due to acceleration or retardation of a train, are also external forces which must sometimes be taken into account. Certain structures, such for example as viaduct towers, are often designed to withstand a longitudinal (tractive) force as great as 20 per cent of the weight of the live load, on the assumption that such a force might be caused at the tops of the rails by the application of brakes on a train.

8. **Reactions.** Let one end of a horizontal beam rest on a support as shown in Fig. 135. Assume that the upper surface of the support is a horizontal plane that is in contact with the lower plane surface of the beam. The **plane of contact,** or the **plane of bearing,** is then horizontal. Assume that the beam is perfectly free to move along the plane of contact, i.e., that there is no connection to the support or any friction that would prevent such a movement. The lines of action of forces exerted on the support by the beam must be *normal to the plane of contact*, i.e., vertical, and, likewise, the lines of action of the forces exerted by the support on the beam (reactions) must also be normal.

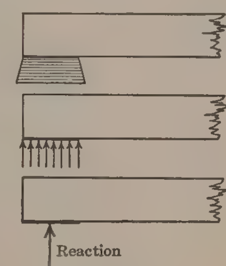


Fig. 135.

(Why?) The end of a beam that is free to move along the plane of contact is called a **free end.**

1. *Note:* It is to be noted that the assumptions made concerning the free end are never perfectly fulfilled. No top of a support or bottom of a beam is a perfect plane, nor are the two planes in perfect contact, nor is the plane of contact perfectly level, nor can friction be perfectly eliminated.

2. It is customary to assume that the pressure on the support from the free end of a beam is uniformly distributed over the area of contact, and that, consequently, the indefinite number of reactions may be replaced by one resultant force, called the **reaction,** acting at the center of the area of contact or **center of bearing.** This assumption would be approximately correct if the beam and support were absolutely rigid, but no material is absolutely rigid. As the beam deflects, the pressure on the support becomes greater toward its inner edge (edge toward the center of the beam), but, as the support yields slightly, the pressure is redistributed more uniformly; the resultant reaction, however, will be between the center of bearing and the inner edge of the support. (Why?) The assumption that the reaction acts at the center of bearing is on the safe side in designing the beam, but not in designing the support. (Why?)

3. It will be understood henceforth that if a beam merely *rests* on a support, the corresponding end of the beam is a free end. The important thing to remember concerning such a condition is that *the reaction is normal to the plane of contact.* Thus, for example, if the beam in Fig. 136 (a) is not fastened to the support and friction is disregarded, the reaction R is normal to the plane of contact. Since R is not parallel to W, the beam would slide off the support, impelled by its own weight, unless restrained by some third force, such as, for example, a force at another support. In Fig. 136 (b), however, R, normal to the plane of contact, is vertical, consequently the weight of the beam acts parallel to R and does not tend to make the beam slide on its support.

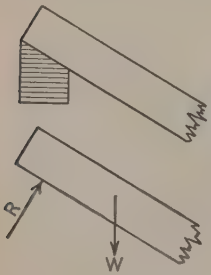


Fig. 136 (a).

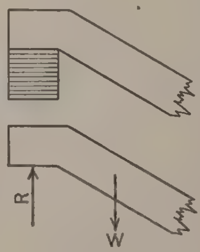


Fig. 136 (b).

4. *Note:* There are various methods of designing the connection of one end of a beam or truss to a support so that the end will be as free as possible to move along the plane of contact. One of the most common methods is to provide a nest of rollers upon which the end may move with a minimum amount of friction and thus be for all practical purposes a free end. There are also different devices for fixing a single point in the line of action of the reaction, such, for example, as a pin in the shoe about which the beam or truss is free to turn. The center of this pin may then be taken as a point P in the line of action of the reaction; if the shoe is on rollers and if the plane of contact is horizontal, the line of action of the reaction may then be assumed for practical purposes as a vertical line through P. One end of the plate girder in Fig. 136 (c), for example, is free to turn on a pin, but in place of rollers a bell-shaped casting is designed to take care of motion in a horizontal direction. The links shown in the photograph were to support the casting in a vertical position while the girder was being placed in position; afterward these temporary erection links were removed.

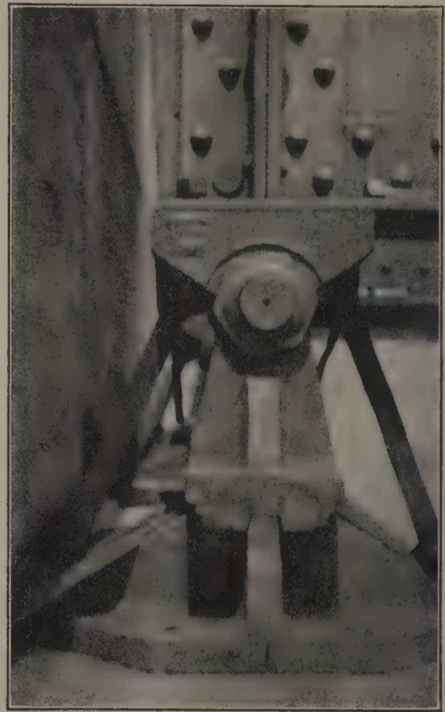


Fig. 136 (c).

5. *If all of the loads on any horizontal beam are vertical and the plane of bearing at each support is horizontal, the reactions at both supports will be vertical even though neither end of the beam is a free end.* A reaction that is not vertical must have a horizontal component, but it can have no horizontal component unless there is some other external force with a horizontal component. (If the plane of bearing at one end of a beam is inclined, thus making the reaction at that end inclined, would the reaction at the other end necessarily be inclined?)

1. If there is friction in a plane of bearing, the corresponding end of a beam is not free. If there is a horizontal or inclined external force acting on a horizontal beam, there will always be, theoretically, friction in the horizontal plane of bearing. To reduce this friction to a negligible quantity, a shoe plate is sometimes fastened to the bottom of a beam. This shoe plate slides on a bed plate or other plate fastened to the support. In heavier structures, if a free end is desired, a nest of rollers may be placed between the beam and its support, or some other provision may be made for a free horizontal movement, as already explained in 136 : 4.

2. A **fixed end** is one that is not free to move along the plane of bearing but can rotate freely in the plane of the external forces. A **restrained end** is one in which no motion of any kind is possible; such an end renders the corresponding reaction indeterminate. In order that the end of a horizontal beam or truss may be free to revolve in a vertical plane and still be fixed, some form of **hinged joint** may be necessary at the support, such, for example, as a pin connection at the shoe; the beam is free to turn on the pin although the shoe is anchored to the support. A hinged joint, moreover, distributes the pressure on the support more uniformly over the area of bearing and thus makes the actual conditions more in accord with the usual assumption of uniform bearing. (See Fig. 111 (*b*) and also Fig. 136 (*c*).)

3. *Note:* The terms *fixed end* and *restrained end* are used in a relative sense only. Thus, for example, the end of a beam so set in a wall that it cannot rotate is a restrained end, though theoretically the elasticity of the material will permit rotation. On the other hand, the end of a beam fastened to a support in any way is not entirely free to rotate, even though the connection is a hinged joint.

4. Since a truss is a special form of beam, the fundamental conceptions and assumptions concerning the ends of beams, just explained, hold true for the ends of trusses also. A structure with more than two supports, or one in which the ends are restrained, is statically indeterminate. Unless otherwise stated, it will be understood henceforth that there are not more than two supports, and that the end of a beam or truss is free to rotate in a vertical plane, even though it is a fixed end.

5. *Note:* Many beams do not rest on planes of bearing at the ends, but are connected, more or less rigidly, to the supports. Examples of such beams are a stringer connected to the floor beams that support it and a floor beam connected to the trusses that support it. When the ends of beams are thus connected, they are partially but not wholly restrained. It is customary to treat them as if they were perfectly free to revolve in a vertical plane. The stresses in a beam determined on this assumption may be greater, but not smaller, than the actual stresses, and the assumption is therefore on the safe side.

6. *Note:* A truss may ordinarily be assumed to be supported at each end on a horizontal plane of bearing. It may not rest directly on such a plane, but the effect on the reaction is usually the same as if it did. In any case, the assumption, as in the case of a beam, is on the safe side.

7. *Note:* Since most loads are due to the action of gravity, the great majority of external forces, including reactions, act in vertical planes. In discussing reactions, therefore, it is usually assumed that the reactions and the loads that cause them lie in the same vertical plane. The fundamentals are the same, however, for reactions in a horizontal plane, such, for example, as those for a horizontal lateral system in which the wind loads and reactions are all in the same horizontal plane.

8. Loads and corresponding reactions constitute a system of co-planar external forces which hold a beam or a truss in equilibrium. In determining reactions, therefore the first step is to consider, as the body in equilibrium, the beam *as a whole*, or the truss *as a whole*. This body, whatever it may be, merely fixes the spatial relations of the external forces, and when these are once known, the body may be ignored. The size or shape of the beam or truss is immaterial except as it fixes these spatial relations. This will become evident from the following illustrations:

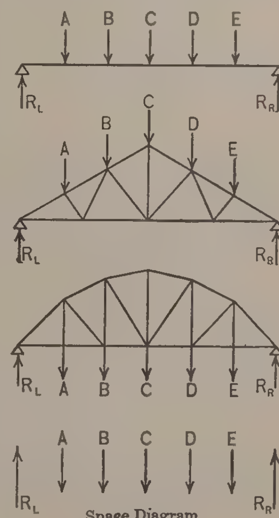


Fig. 137.

9. *Illustrations:* In Fig. 137 are shown a simple beam, a roof truss, and a bridge truss, all of the same length of span and each divided into six equal panels. Let A, B, C, D, and E be known panel loads, the same in magnitude for each truss as for the beam. The spatial relations (in this case the distances between the lines of action of the loads and reactions) are the same for the two trusses of different shape as for the beam (as shown in the space diagram); and, once these relations are known, the reactions may be determined algebraically or graphically from the space diagram regardless of whether the system of external forces is acting on the beam or on one of the trusses.

1. In Fig. 138 are shown the top and bottom chords of a roof truss and a beam of the same length of span as the truss. Let A, B, C, D, and E be wind loads normal to one top chord of the truss. Let the loads on the beam be the same in magnitudes as those on the truss, and spaced as indicated by the dotted lines. The directions of the lines of action of the reactions will depend upon whether the truss or beam has one or both ends fixed. Assuming that the condition in this respect at each end of the beam is the same as at the corresponding end of the truss, each of the two reactions on the beam will be the same in both magnitude and direction as the corresponding reaction on the truss. The space diagram is the same for the beam as for the truss, and again indicates that the beam or the truss may be ignored once the spatial relations of the given loads and the required reactions are known. Notice that if both ends of the truss or beam are fixed, the only element known of either reaction is a point in the line of action.

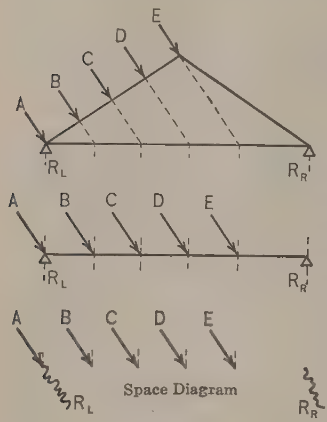


Fig. 138.

2. *Summary of fundamental assumptions and conceptions concerning reactions.*

3. A reaction at a point of support of a beam or truss is the resultant of all forces exerted on the beam or truss by the support; it is as truly an external force as any load or any other external force.

4. A reaction is generally assumed to act at the *center* of bearing, or at the center of the pin at the corresponding end of the beam or truss.

5. When an end is a *free* end, the corresponding reaction is normal to the plane of bearing; for the usual case of a horizontal plane of bearing, the reaction is vertical.

6. *Note:* When an end is a free end, the plane of bearing determines the inclination of the line of action of the corresponding reaction; one does not need to consider anything else. For example, if the plane of bearing for any free end is horizontal, the first thought should be: "The reaction is vertical." One does not need to consider whether or not there is any inclined external force, or whether or not the beam (or truss) is inclined to the horizontal.

7. When an end is a *fixed end*, the corresponding reaction will not be normal to the plane of bearing unless all other external forces (including the other reaction) are either normal to the same plane of bearing or produce no motion of translation parallel to that plane.

8. *Note:* When an end is a fixed end, the corresponding reaction should never be assumed normal to the plane of bearing without careful consideration of all of the other external forces that are acting on the beam or truss. Until after such consideration, nothing is known concerning the line of action of the reaction except a *single point* in that line. If any other external force is inclined to the plane of bearing, the reaction will also be inclined unless the tendency of the inclined force to produce motion of translation parallel to the plane of bearing is exactly counteracted by another inclined external force or by the resultant of two or more such forces. This is equivalent to saying that the reaction cannot be normal to the plane of bearing at a fixed end unless the algebraic sum of the components, parallel to that plane, of all the other external forces is zero. For example, if the plane of bearing at a fixed end is horizontal, the corresponding reaction cannot be vertical unless the algebraic sum of the H components of all other external forces is zero. This will be the case if all other external forces are vertical, i.e., normal to the plane of bearing, but it is not likely to be true if any of the external forces are inclined (though it may be). The first thought one should have, therefore, when considering the reaction at a fixed end of a horizontal beam is: "Are there any loads that are not vertical — if so, the chances are that the reaction will not be vertical."

9. *Note:* It is worth remembering that the line of action of the reaction is always known at a free end, but that frequently only a single point of the line of action of the reaction is known at a fixed end.

10. When the plane of bearing at each of two supports is horizontal and all loads are vertical, both reactions will be vertical, regardless of whether either end of the beam (or truss) is free or fixed.

11. *Note:* In this case the loads can cause no horizontal motion of translation (motion parallel to the planes of bearing), consequently it is immaterial whether either end is free or fixed. Note that when a beam is inclined to the horizontal, both reactions will still be vertical, provided the planes of bearing at the supports are horizontal and all loads are vertical. (136 : 3.)

12. Since the two reactions and the loads that cause them constitute a system of forces that holds the entire beam (or truss) in equilibrium, the beam (or truss) should usually be considered as a whole, when determining reactions.

13. The beam (or truss) that is held in equilibrium by a system of external forces merely determines the spatial relations of the given loads and the required reactions. Once these spatial relations are known, the beam (or truss) may be ignored in determining reactions. (137:8 and 9.)

14. **Reactions on Horizontal Beams with Vertical Concentrated Loads.** If both ends of a horizontal beam are free and the planes of bearing are horizontal, both reactions will be vertical (138 : 5).

When the concentrated loads are vertical and counterbalanced (126 : 3) (when the beam is symmetrically loaded), each reaction is equal to one-half the total load. When the loads are all vertical but not counterbalanced, the problem of determining the reactions is one in parallel forces, and no matter how many loads there may be or how they may be placed on the beam, the problem is always essentially the same. The general analysis is as follows:

Body in equilibrium: The entire beam.
Known: Each load completely and the line of action of each reaction.
Unknown: Magnitude of each reaction (M and M, Case B′). (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma M = 0$, or $\Sigma M = 0$ and $\Sigma V = 0$. (69 : 12.)

1. A problem in parallel forces in which the unknown elements are the magnitudes of two forces falls under Case B′. (38 : 4.) This is by far the most common problem in reactions and its solution is almost always the preliminary step that must be taken in determining stresses in an ordinary truss. The fundamental method of solution and practical suggestions have already been given in CHAPTER VIII. Before studying the illustrative problems which follow, it may be well to review that chapter. It is particularly important to keep in mind the answers to the following questions:

What equation must be used at least once and used first? (38 : 6.)
What two combinations of equations are possible? (39 : 1.)
In using $\Sigma M = 0$ where should the points of moments be taken? (39 : 3.)
In which equation does the algebraic sign of the result give the *sense* of the reaction directly, and in which does it give it indirectly? (41 : 10.)

2. *Note:* It is neither necessary nor desirable to assume the *sense* of a reaction even though it may be obvious; for reasons which will become clear later on, it is better to form the habit of letting the algebraic sign determine the sense.

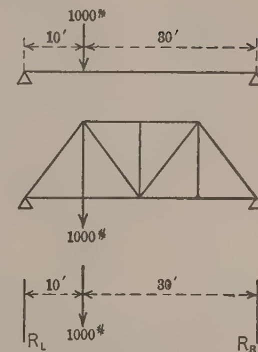
3. If two moment equations are used, results may be checked by $\Sigma V = 0$. If $\Sigma M = 0$ and $\Sigma V = 0$ are used, results may be checked by a second moment equation. (44 : 2.)

4. *Note:* In the following illustrative problems the weight of the beam or truss is not taken into account. All planes of bearing are assumed to be horizontal. For the notation used in the analysis see 20 : 2.

5. *Reactions for a single concentrated load.* (Fig. 139.)
Body in equilibrium: Entire beam (or truss).
Known: Load = 1000 lbs., R_L(L), and R_R (L).
Unknown: R_L(M) and R_R(M) (Case B′).
Equations: $\Sigma M = 0$ and $\Sigma M = 0$.

$$\Sigma M = -1000 \times 30 + R_L \times 40 = 0$$
$$R_L = +750 \text{ (up)}$$
$$\Sigma M = 1000 \times 10 + R_R \times 40 = 0$$
$$R_R = -250 \text{ (up)}$$

Check: $\Sigma V = -1000 + 750 + 250 = 0$



Space Diagram
Fig. 139.

6. From the above illustration the following general principle is evident: *When a beam (or truss) is supported at each end and all external forces (including reactions) are vertical, the reaction at either support caused by a single concentrated load is equal in magnitude to a fraction of the load, and that fraction is equal to the distance of the load from the other support divided by the length of span.* For example, $R_L = \frac{30}{40} \times 1000$, and $R_R = \frac{10}{40} \times 1000$. (Fig. 139.)

7. *Of the two reactions due to a single concentrated load, the greater will be the one nearer the load, i.e., the load causes the greater reaction at the nearer support.* This fact should be used as a rough check to guard against the mistake of interchanging values of the two reactions.

8. *Reactions from several vertical concentrated loads.* (Fig. 140 (*a*).)
Body in equilibrium: Entire beam (or truss).
Known: A, B, C, R_L(L), and R_R(L).
Unknown: R_L(M) and R_R(M) (*Case B′*).
Equations: $\Sigma M = 0$ and $\Sigma M = 0$.

$$\Sigma M = -1800 \times 8 - 800 \times 12 - 1200 \times 20 + R_L \times 24 = 0$$
$$R_L = +2000 \text{ (up)}$$
$$\Sigma M = 1200 \times 4 + 800 \times 12 + 1800 \times 16 + R_R \times 24 = 0$$
$$R_R = -1800 \text{ (up)}$$

Check:

$$\Sigma V = -1200 - 800 - 1800 + 2000 + 1800 = 0$$

The method of solving this problem is exactly the same as that of solving the preceding one — there are merely more forces involved.

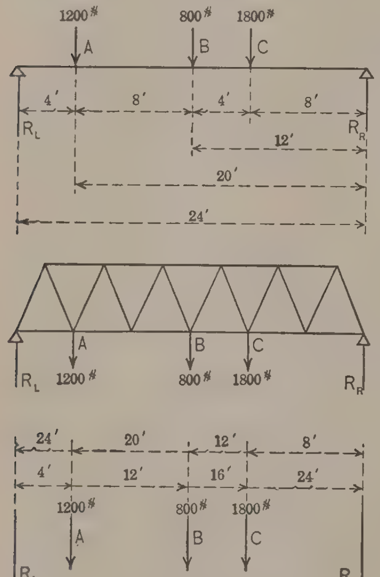


Fig. 140 (a).

1. From the above illustration the following general principle is evident: When a beam (or truss) is supported at each end and all external forces (including reactions) are vertical, *the magnitude of the reaction at one support is equal to the sum of the moments of all of the other external forces about any center of moments in a vertical line through the other support, divided by the length of span.*

2. *Reactions for several vertical concentrated loads on a horizontal beam with two free overhanging ends.* (*Double cantilever.*) (Fig. 140 (*b*).)

Body in equilibrium: Beam.

Known: A, B, R_L(L), C, E, F, R_R(L), G and H.

Unknown: R_L(M) and R_R(M). (*Case B′.*)

Equations: $\Sigma M_m = 0$ and $\Sigma V = 0$.

Check: $\Sigma M_n = 0$.

$$\Sigma M_m = -A \times 8 - B \times 4 + R_L \times 0 + C \times 6 + E \times 16 + F \times 21 + R_R \times 24 + G \times 27 + H \times 30 = 0$$

$$= -600 \times 8 - 600 \times 4 + 0 + 800 \times 6 + 1200 \times 16 + 1600 \times 21 + R_R \times 24 + 400 \times 27 + 400 \times 30 = 0$$

$$24R_R = -73{,}200 \circlearrowleft$$

$$+3050\# = R_R = \frac{73{,}200}{24} \uparrow$$

$$\Sigma V = -A - B + R_L - C - E - F + R_R - G - H = 0$$

$$= -600 - 600 + R_L - 800 - 1200 - 1600 + 3050 - 400 - 400 = 0$$

$$+2550\# = R_L \uparrow$$

$$\Sigma M_n = -A \times 32 - B \times 28 + R_L \times 24 - C \times 18 - E \times 8 - F \times 3 + R_R \times 0 + G \times 3 + H \times 6 = 0$$

$$= -600 \times 32 - 600 \times 28 + R_L \times 24 - 800 \times 18 - 1200 \times 8 - 1600 \times 3 + 0 + 400 \times 3 + 400 \times 6 = 0$$

$$24R_L = +61{,}200 \circlearrowright$$

$$+2550\# = R_L = \frac{61{,}200}{24} \uparrow \quad \text{(Check.)}$$

3. The method of solving this problem is exactly the same as that of solving the preceding one. The fact that some of the loads are outside of the supports merely affects certain algebraic signs in the moment equations. Any one of these outside loads, if great enough, could cause one of the reactions to act downward, and it would then be necessary to anchor the beam at the corresponding support.

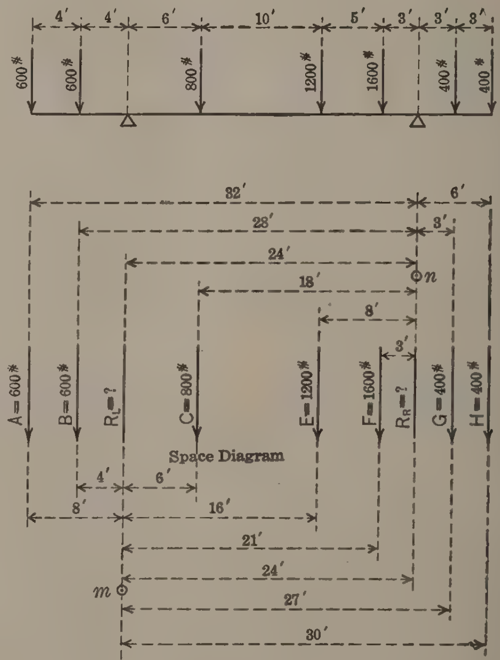


Fig. 140 (*b*).

4. As in the preceding example, the magnitude of a reaction at one support is equal to the *algebraic* sum of the moments of all of the loads about any point in a vertical line through the other support, divided by the length of span.

5. The three problems just solved have been analyzed in detail to emphasize the fact that, for horizontal beams resting on two supports (two free ends) and carrying vertical loads, problems in determining re-

actions *are essentially the same,* regardless of the number of loads and regardless of whether the beam is supported at each end, or whether it overhangs one or both supports. The problem is always one in parallel forces (*Case B′*) for which there is a standard solution. The same method is used for finding reactions on vertical or inclined beams and trusses, provided all of the loads and the two reactions are parallel, but such problems are less common than those for horizontal beams.

1. **Reactions on Horizontal Beams with Uniform Load.** For the purpose of *finding reactions,* a load uniformly distributed over the entire length of a beam may be replaced by an equivalent concentrated load applied at the center of the beam, whether or not the beam has overhanging ends. For example, in Fig. 141 (*a*) the uniform load of 100 lbs. per linear foot, distributed over the entire length of the beam which is 22 ft. long, may be replaced by a load of 2200 lbs. acting 11 ft. from either end, as shown in the space diagram. The problem is then reduced to one of concentrated loads.

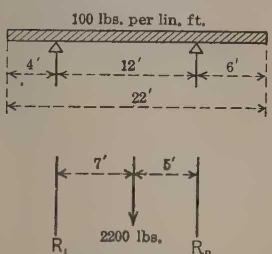


Fig. 141 (*a*).

2. If there are loads uniformly distributed over certain portions of a beam with free ends but not on other portions, the load on each portion may be replaced by an equivalent concentrated load and the problem then becomes one of concentrated loads. Two examples of such a problem, in which the different uniform loads and the weight of the beam itself uniformly distributed are replaced by equivalent concentrated loads, are given here.

3. *Note: Conventional methods of representing beams and loads.* In this book a beam without load is represented by a *single line.* When a uniform load extends end to end of beam, such, for example, as the weight of the beam itself, a second line is drawn end to end and parallel to the line representing the beam. A uniform load covering a portion of a beam is represented in a similar manner except that the second line extends only over the portion of the beam covered. (See Fig. 141 (*b*).) Concentrated loads are represented by arrows or circles representing wheels. When only concentrated loads are taken into account, the weight of beam being neglected, these loads are shown acting on a *single* line that represents the beam; if the weight of the beam is to be included, the concentrated loads are shown acting on a double line that represents the beam and its uniformly distributed weight.

4. *Reactions on a simple horizontal beam with broken uniform loads.* (Weight of beam is included in the loading.) (Fig. 141 (*b*).)

Body in equilibrium: Beam.

Known: A, B, C, E, F, R_L(L), and R_R(L).

Unknown: R_L(M) and R_R(M). (*Case B′.*)

Equations: $\Sigma M = 0$ and $\Sigma V = 0$. Check $\Sigma M = 0$.

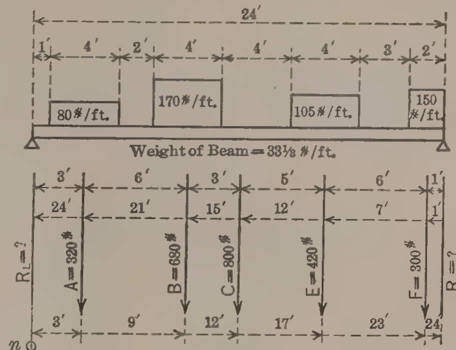


Fig. 141 (*b*).

$$\Sigma M = R_L \times 24 - A \times 21 - B \times 15 - C \times 12 - E \times 7 - F \times 1 = 0$$
$$= R_L \times 24 - 320 \times 21 - 680 \times 15 - 800 \times 12 - 420 \times 7 - 300 \times 1 = 0$$
$$\Sigma V = +R_L - A - B - C - E - F + R_R = 0$$
$$= + \quad -320 - 680 - 800 - 420 - 300 + R_R = 0$$
$$\Sigma M = R_L \times 0 + A \times 3 + B \times 9 + C \times 12 + E \times 17 + F \times 23 + R_R \times 24 = 0$$
$$= 0 + 320 \times 3 + 680 \times 9 + 800 \times 12 + 420 \times 17 + 300 \times 23 + R_R \times 24 = 0$$

5. *Note:* The equations just given are indicated but not solved. For the complete solution see page 44. Read carefully the comments on that page.

6. *Reactions on a double-cantilever horizontal beam with broken uniform loads.* (Weight of beam is included in the loading.) (Fig. 142 (*a*).)

Body in equilibrium: Beam.

Known: A, R_L(L), B, W, R_R(L), C, and E.

Unknown: R_L(M) and R_R(M).

Equations: $\Sigma M = 0$ and $\Sigma V = 0$. Check: $\Sigma M = 0$.

$$\Sigma M_m = -A \times 17 + R_L \times 16 \quad - B \times 8 \quad - W \times 6 + R_R \times 0 + C \times 1 + E \times 7 = 0$$
$$= -800 \times 17 + R_L \times 16 \quad -900 \times 8 - 1050 \times 6 + R_R \times 0 + 320 \times 1 + 480 \times 7 = 0$$

$$\Sigma V = -A + R_L - B \quad -W + R_R \quad -C \quad -E = 0$$
$$= -800 + \quad -900 \quad -1050 + R_R \quad -320 \quad -480 = 0$$
$$\Sigma M_n = -A\times 1 + R_L\times 0 + B\times 8 + W\times 10 + R_R\times 16 + C\times 17 + E\times 23 = 0$$
$$= -800\times 1 + 0 + 900\times 8 + 1050\times 10 + R_R\times 16 + 320\times 17 + 480\times 23 = 0$$

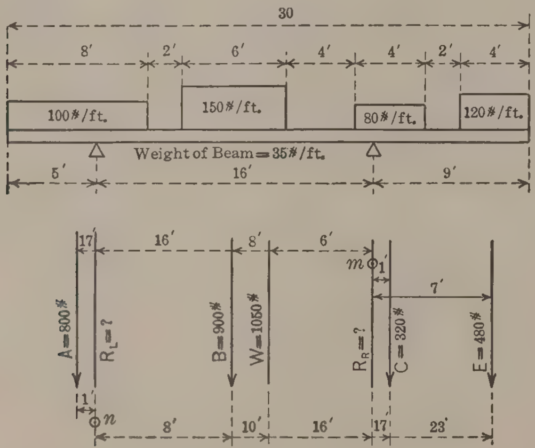


Fig. 142 (a).

1. *Note:* The general method for this problem is exactly the same as that for the preceding one. The equations are indicated but not solved. Solve these equations, paying particular attention to the algebraic signs of the results, and from these algebraic signs determine whether the reactions act upward or downward. Most of the comments on page 44 apply to this problem as well as to the preceding one.

2. *Note:* It is obvious that when a beam is symmetrically supported and symmetrically loaded, each reaction is equal to one-half of the total load on the beam (including the weight of the beam), regardless of whether loads are concentrated or uniform.

3. **Reactions for Panel Loads.** The dead loads on an ordinary truss are applied at panel points of the loaded chord, and are equal, hence each reaction is equal to one-half of the total dead load. Live loads, however, may be applied at certain panel points only. For example, in Fig. 142 (*b*) only four panel points are loaded. The fraction of a panel load that goes to a support is equal to the distance of that load from the *other* support, divided by the length of span (139 : 6); both the distance and the length of span may be expressed in terms of the panel length p. For example, the fraction of A that goes to the *left-hand* support is the distance of A from the *right-hand* support, divided by the length of span, or $4p \div 7p = \frac{4}{7}$. Assuming that the panel loads are equal, either reaction may be found by observing the fraction of each load that goes to the corresponding support, and then adding these fractions.

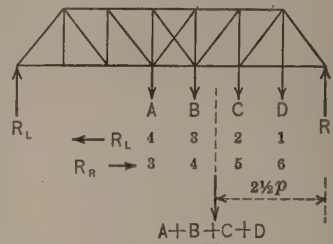


Fig. 142 (*b*).

$$R_L = \frac{4p}{7p}A + \frac{3p}{7p}B + \frac{2p}{7p}C + \frac{1p}{7p}D = \frac{10}{7} \times \textit{panel load}$$

$$R_R = \frac{3}{7}A + \frac{4}{7}B + \frac{5}{7}C + \frac{6}{7}D = \frac{18}{7} \times \textit{panel load}$$

$$\textit{Check:}\quad \frac{10}{7} + \frac{18}{7} = 4 \textit{ panel loads} = R_L + R_R$$

4. This fractional method is an extremely useful short-cut method of calculating reactions from panel loads when a truss is unsymmetrically loaded. It should be kept in mind, however, that this short cut is really an application of $\Sigma M = 0$, with the point of moments in a vertical line through one of the supports.

5. *Note:* To avoid mistakes, it may be well to indicate the numerators of the fractions used in determining reactions as shown in Fig. 142 (*b*). Note that the sum of the two numbers thus indicated at any panel point should equal the number of panels.

6. An alternative method of determining R_L is as follows: The loads A, B, C, and D may be replaced by the resultant of these loads. (Fig. 142 (*b*).) The line of action of this resultant is $2\frac{1}{2}$ panel lengths from the right-hand support, hence

$$R_L = \frac{2.5}{7}(A + B + C + D), \text{ and } R_R = \frac{4.5}{7}(A + B + C + D).$$

7. **Reactions Due to a Concentrated Load Between Panel Points.** Let ag in Fig. 143 (*a*) represent a main girder or truss which supports floor beams at panel points a, b, c, d, e, f, and g. Let the length

of each panel be 24 ft. Let a single concentrated load be on the bridge floor *between* floor beams at c and d, a distance of 18 ft. from d; let $W = 1200$ lbs. represent the portion of this concentrated load that is brought to the girder or truss by the floor beams at c and d. Let W_c and W_d be the portions of W brought to c and d, respectively, by the two floor beams. These floor-beam loads at c and d may be calculated just as if W rested on a beam cd equal in length to one panel or 24 ft.

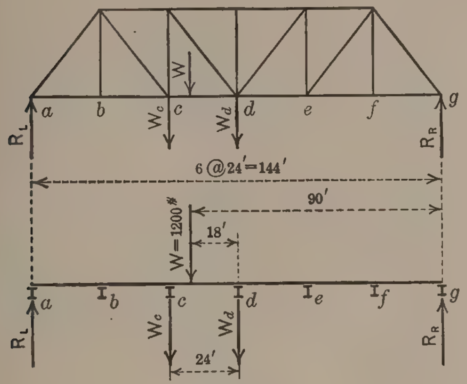


Fig. 143 (*a*).

$$W_c = \frac{1200 \times 18}{24} = 900 \text{ lbs.} \quad W_d = \frac{1200 \times 6}{24} = 300 \text{ lbs.}$$

The reaction

$$R_L = 900 \times \tfrac{4}{6} + 300 \times \tfrac{3}{6} = 750 \text{ lbs.} \quad R_R = 1200 - 750 = 450 \text{ lbs.}$$

1. The reaction R_L may also be determined without regard to floor beams or panel points, just as if the load W rested on a simple beam ag equal in length to the length of the main girder or truss, or 144 ft.:

$$R_L = \frac{1200 \times 90}{144} = 750 \text{ lbs.}$$

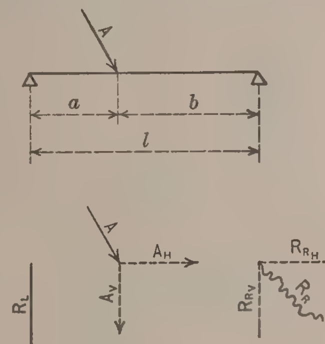
It is evident that the reaction at either end of a main girder or truss due to any number of concentrated loads between panel points may be determined exactly as if there were no floor beams and the loads were on a simple beam whose length is equal to that of the main girder or truss.

Exercise. Give a general proof that the reaction R_L, determined from W_c and W_d, is equal to R_L, determined directly from W.

2. *Graphic methods:* Any of the problems in finding reactions thus far given can be solved by the *graphic* method for Case B′ (page 45). Nothing is gained, however, by the use of this graphic method in the majority of problems that fall under Case B′ — the algebraic method usually involves less work.

3. **Reactions on Horizontal Beams with Inclined Loads.** If an inclined load acts on a horizontal beam, one end, at least, of the beam must be fixed. The method of determining reactions depends, first of all, upon whether one end or both ends are fixed.

4. *Reactions on a horizontal beam, one end free and one end fixed, with a single concentrated inclined load.* (Fig. 143 (*b*).) The reaction at the free end must be vertical (138 : 5), but the line of action of the reaction at the fixed end is unknown (138 : 7). Let the free end be at the left-hand support.



Space Diagram

Fig. 143 (*b*).

Body in equilibrium: Entire beam.

Known: A, R_L(L), and R_R(P).

Unknown: R_L(M) and R_R(M and D) (Case 3).

Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (70 : 3.)

$$\Sigma M = R_L \times l - A_V \times b = 0 \quad \text{or} \quad R_L = A_V \times \frac{b}{l} \text{ (Up.)}$$

$$\Sigma V = R_L - A_V + R_{R_V} = 0 \quad \text{or} \quad R_{R_V} = +A_V - R_L \uparrow \text{ (Up.)}$$

$$\Sigma H = A_H + R_{R_H} = 0 \quad \text{or} \quad R_{R_H} = -A_H \leftarrow \text{(To left.)}$$

$$R_R = \sqrt{(R_{R_V})^2 + (R_{R_H})^2}. \quad \text{Slope of } R_R = R_{R_H} \div R_{R_V}$$

$$\text{Check: } \Sigma M = A_V \times a + R_{R_V} \times l \quad \text{or} \quad R_{R_V} = -A_V \times \frac{a}{l} \text{ (Up.)}$$

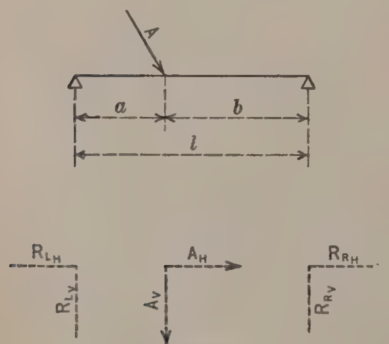
5. Instead of calculating the lever arm of A, it is usually simpler to resolve A into its H and V components (applied at the point in which A intersects the horizontal line between supports), and to use these components in $\Sigma M = 0$ (lever arm of A_H is zero). Note that the fractional part of the V component of A that goes to either support is equal to the

distance to the other support divided by the length of span. This is in agreement with the general principle stated in 139 : 6.

1. It is almost always more advantageous to use the H and V components of a reaction whose line of action is inclined than to work with the reaction itself. This results in changing the problem from one in *Case 3* non-concurrent forces to one in *Case 4*, since the unknowns become three magnitudes, namely, R_L(M), R_{R_H}(M), and R_{R_V}(M). Assuming that the H and V components of A are used in place of A, all forces involved are now either horizontal or vertical, and the same combination of equations is used, namely, $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (Why? (49 : 1).)

2. *Reactions for a horizontal beam, both ends fixed, with a single inclined load.* The unknowns are R_L(M and D) and R_R(M and D). Since there are four unknowns the problem is indeterminate without some assumption. One of two assumptions is usually made: (1) that the H components of the reactions are equal; or (2) that the reactions are parallel to the inclined force and to each other.

3. *Assumption that the H components of the reactions are equal:*



Space Diagram
Fig. 144 (*a*).

Body in equilibrium: Entire beam. (Fig. 144 (*a*).)
Known: A, R_L(P), and R_R(P).
Unknown: R_L(M and D) and R_R(M and D).
Equations: $\Sigma M = 0, \Sigma V = 0, \Sigma H = 0$, and $R_{L_H} = R_{R_H}$

$$\Sigma M = R_{L_V} \times l - A_V \times b = 0$$

or $R_{L_V} = A_V \times \frac{b}{l}$ (up)

$$\Sigma V = R_{L_V} - A_V + R_{R_V} = 0$$

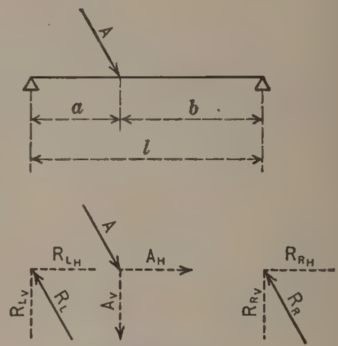
or $R_{R_V} = A_V - R_{L_V}$ ↑ (up)

$$\Sigma H = R_{L_H} + A_H + R_{R_H} = 0$$

$R_{L_H} = R_{R_H} = -\frac{1}{2} A_H$ ← (to left)

The slope of each reaction may be obtained by dividing its H component by its V component.

4. *Assumption that the reactions are parallel to the inclined force:*
Body in equilibrium: Entire beam. (Fig. 144 (*b*).)
Known: A, R_L(L), and R_R(L).
Unknown: R_L(M) and R_R(M). (*Case B'*.)
Equations: $\Sigma M = 0$ and $\Sigma M = 0$.



Space Diagram
Fig. 144 (*b*).

$$\Sigma M = R_{L_V} \times l - A_V \times b = 0$$

or $R_{L_V} = A_V \times \frac{b}{l}$ (up)

(R_L may be calculated from its V component since the angle of inclination is known.)

$$\Sigma M = R_{R_V} \times l + A_V \times a = 0$$

or $R_{R_V} = -A_V \times \frac{a}{l}$ (up)

(R_R may be calculated from its V component.)

5. From a study of the solutions just given for the three different combinations of end conditions, it will be evident that in all three solutions the magnitudes obtained for the *V components* of the two reactions are:

$$R_{L_V} = A_V \times \frac{b}{l} \quad \text{and} \quad R_{R_V} = A_V \times \frac{a}{l}.$$

From these results the following general principle may be stated:

When a horizontal beam, supported at each end, carries an inclined load, the fractional part of the **V component** *of the inclined load that is transmitted to either support is equal to the distance from the load to the other support (measured along the beam), divided by the length of span.* This principle holds true for all three combinations of end conditions, namely, (1) one end free and one end fixed, (2) both ends fixed and H components of the reaction assumed equal, and (3) both ends fixed and reactions assumed parallel to the inclined force. The principle is in accord with that stated in 139 : 6.

6. *Reactions for a horizontal beam with several inclined parallel loads.* Whatever the combination of end conditions may be, the solution is exactly the same as that just given for the same combination of end conditions. Several inclined forces are involved in each equation instead of

one, but since these additional forces are all *known* loads they do not affect the method of solution. Nor would the solution be materially different if the inclined forces were not parallel, since each inclined force could be replaced by its H and V components at the point where its line of action intersects the beam, and the center of moments could then be selected so that all of these H components would pass through it and be eliminated.

1. *Graphic methods of determining reactions when loads are inclined.* For each of the three algebraic methods of calculating reactions on beams with one or more inclined loads, there is a corresponding graphic method. These graphic methods, explained in **PART I,** may all be replaced, however, by a simpler graphic method based on the fact that, no matter which assumption is made concerning end conditions, the fractional part of the V component of the inclined load which goes to either support is the same, and may be found by proportion as follows:

$$R_{L_V} = A_V \frac{b}{l} \quad \text{and} \quad R_{R_V} = A_V \frac{a}{l}$$

$$\frac{R_{L_V}}{A_V} = \frac{b}{l} \quad \text{and} \quad \frac{R_{R_V}}{A_V} = \frac{a}{l}.$$

The graphic method explained here is merely one in which the above proportions are obtained by means of similar triangles.

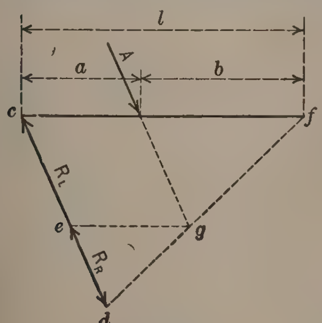


Fig. 145 (*a*).

2. *Assume both ends fixed and the reactions both parallel to the inclined load A.* If the V components of two parallel forces are proportional to a certain quantity, the forces themselves will be proportional to the same quantity. (Why?)

$$\frac{R_{L_V}}{A_V} = \frac{b}{l} = \frac{R_L}{A} \quad \text{and} \quad \frac{R_{R_V}}{A_V} = \frac{a}{l} = \frac{R_R}{A}.$$

3. Lay off *cd* in Fig. 145 (*a*) parallel to the line of action of A, and equal in magnitude to A (using any convenient scale). From f draw a line to d. Find g, the intersection of the line fd and the line of action of A. Through g draw a horizontal line until it intersects *cd* at e.

$$\frac{b}{l} = \frac{gf}{df} = \frac{ec}{dc} = \frac{ec}{A} \quad \text{hence} \quad ec = R_L$$

$$\frac{a}{l} = \frac{de}{dc} = \frac{de}{A} \quad \text{hence} \quad de = R_R$$

4. *Note:* When a reaction is designated by two letters, the sense of that reaction will be indicated by the order in which the two letters are given. For example, the sense of R_L is from e toward c since it is designated by *ec* and not by *ce*. (4 : 5.)

The line *cd*, representing the magnitude of A, has been divided by similar triangles into two parts proportional to a and b, and hence one of these parts (*de*) is equal to R_R and the other (*ec*) to R_L.

5. The reactions R_R and R_L were just determined on the assumption that both reactions are parallel to the inclined force. But the V component of the reaction at either end is the same regardless of end conditions (144 : 5); therefore *mk* and *kc* (Fig. 145 (*b*)) are, respectively, the V components of R_R and R_L, not only when both reactions are parallel to the inclined force, but also when one reaction is vertical and the other inclined, or when the H components of the reactions are equal. Moreover, under any one of the three assumptions of end conditions, the sum of the H components of the two reactions must equal $A_H = dm$.

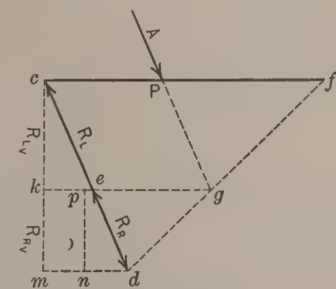


Fig. 145 (*b*).

6. *Both ends fixed, and the H components of the two reactions assumed to be equal.* (Fig. 145 (*b*).) Let n be the middle point of the horizontal line *dm*, and let p be the intersection of a vertical line through n and a horizontal line through e. Since $dm = A_H$, and since by assumption $R_{L_H} = R_{R_H}$, R_{R_H} equals $\frac{1}{2}A_H$, and hence $R_{L_H} = nm = pk$. Since kc must be the V component of R_L, and since R_{L_H} must equal $nm = pk$, the reaction R_L must be equal to *pc*. Similarly, since the V and H components of R_R must be equal, respectively, to *np* (equal to *mk*) and *dn*, the reaction R_R must be equal to *dp*. Note that the three forces in equilibrium, A, R_R, and R_L, form a force triangle *cdpc* in accordance with the principle of 14 : 3.

1. *One end free and one end fixed.* (Fig. 145 (*b*).) Let *c* be the free end. The reaction R_L will be vertical (138 : 5) and equal to its V component, and since this V component must be equal to *kc*, the reaction R_L must be equal to *kc*. The V component of R_R must be equal to *mk*; the H component of R_R must equal the H component of A (Why?), and this is *dm*. The reaction R_R must then be equal to *kd*. Note that the three forces in equilibrium, A, R_R, and R_L, form a force triangle *cdkc*. (14 : 3.)

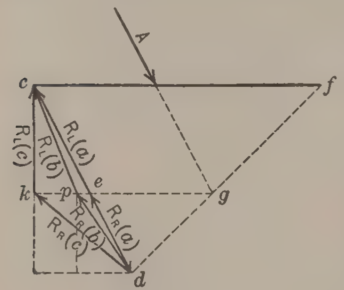


Fig. 146 (*a*).

2. The graphic method just explained may be summarized as follows (references being to Fig. 146 (*a*)).

(*a*) *Reactions parallel:* $R_L(a)$ and $R_R(a)$ found by similar triangles as previously explained.

(*b*) *H components of reaction equal:* $R_L(b)$ and $R_R(b)$.

(*c*) *Left-hand end free, other end fixed:* $R_L(c)$ and $R_R(c)$.

Whatever the end conditions may be, the figure must first be drawn on the assumption that the reactions are parallel in order to obtain *de* and *ec*, the basic values from all three assumptions.

3. *Graphic method applied to a roof truss.* The reactions due to wind pressure on a roof may be quickly determined by the short-cut graphic method just explained. Given the roof and wind loads shown in Fig. 146 (*b*). For the purpose of determining reactions, the wind loads may all be replaced by one resultant force of 4000 lbs.; the line of action of this resultant coincides with that of the middle load and intersects the lower chord at *P*. The problem has now been reduced to one in which the external forces are three, an inclined force and two reactions. It may be solved, therefore, by the method just explained for such a problem. The fact that the three external forces act on a truss instead of on a simple beam is immaterial, once the spatial relations of these forces are known. (137 : 8.) The reactions will be exactly the same as if the inclined load were applied at the point *P* of a simple beam *cf*. To repeat the method of construction already given: Lay off to any convenient scale *cd* parallel and equal to the inclined force (4000 lbs.), and determine the points *g* and *e*. From this point on, the construction will depend on the end conditions, actual or assumed.

(*a*) If the reactions are assumed to be parallel to the inclined load, no further construction is needed since $R_R = de$ and $R_L = ec$.

(*b*) If the H components of the reactions are assumed to be equal, determine the point *p*; then $R_R = dp$ and $R_L = pc$. (145 : 6.)

(*c*) If the left-hand end is free and the other end is fixed, determine the point *k*; then $R_R = dk$ and $R_L = kc$. (146 : 1.)

4. *Note:* Observe that in each case the two reactions form a force triangle with *cd*, the line that represents the resultant force of 4000 lbs. If now *cd* be divided into five parts, the first and last equal to 500 lbs. and the three middle parts each equal to 1000 lbs., these parts will represent the five wind loads, and, instead of a force triangle, there will be a force polygon of seven sides, two of which will represent the two reactions while the other five sides will represent the five wind loads.

5. When one end of the truss is free, the direction of the inclined reaction may be checked by the fundamental principle that when three and only three forces are in equilibrium their lines of action must intersect in a point. In this case the lines of action of the two reactions and the inclined load should intersect in a point; if they do and the inclined force has been laid off correctly in the force triangle, there is also an indirect check on the magnitudes of the reactions. The same check holds good theoretically when the H components of the reactions are equal, but the reactions are then too nearly parallel to permit the use of the check.

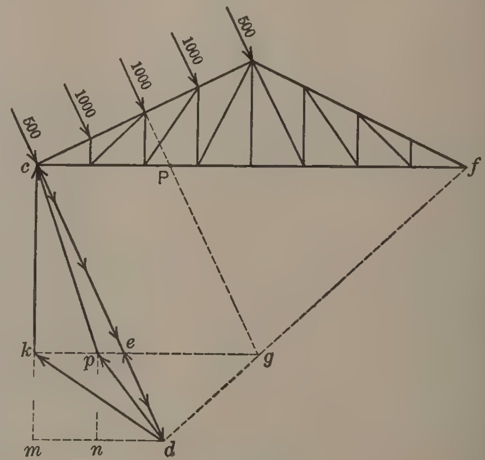


Fig. 146 (*b*).

1. **Summary of Fundamentals Concerning Reactions.** In the following summary it is understood unless otherwise stated:

(*a*) That the beam is a simple horizontal beam supported at each end.

(*b*) That each end is free to revolve in a vertical plane. (137 : 2.)

(*c*) That each end may be regarded as supported on a horizontal plane of bearing. (135 : 8.)

(*d*) That a reaction acts at the center of bearing.

(*e*) That all statements hold true for trusses as well as for beams.

2. In determining reactions the *whole* beam is the body in equilibrium.

3. Reactions are included in the external forces that hold a beam in equilibrium.

4. Once the spatial relations of the external forces are known, it is immaterial what kind of a body they hold in equilibrium — whether it is a simple beam or a truss or some other body. (137 : 8.)

5. When one end is free, the corresponding reaction is vertical; when both ends are free, both reactions are vertical. (138 : 5.)

6. When all loads are vertical, both ends may be free; when one or more loads are inclined, at least one end must be fixed, unless the algebraic sum of the H components of the inclined loads is zero.

7. When one end is fixed, the corresponding reaction is not vertical unless all other external forces, including the other reaction, are also vertical. (138 : 7.)

8. When all loads are vertical, both reactions are vertical, regardless of end conditions — free or fixed.

9. When there is an inclined load or when there is a horizontal force in addition to vertical load, at least one of the reactions must be inclined, and this reaction must be at a fixed end.

10. The line of action of a reaction at a free end is always known (it is vertical), but only *one point* of the line of action of a reaction at a fixed end is known unless all external forces are vertical, or unless the algebraic sum of the H components of the inclined loads is zero.

11. When both ends are fixed the problem of determining reactions is indeterminate unless all external forces are vertical, or unless the algebraic sum of the H components of the inclined loads is zero.

12. When all external forces are vertical, the problem of determining reactions is one in parallel forces (*Case B′*) for which there is a standard solution.

13. When one or more of the external forces are inclined, the problem of determining reactions is one in non-concurrent forces, usually *Case B′* or *Case 3*, for which there is a standard solution.

14. When either unknown reaction is inclined, it is usually best to replace it temporarily by its unknown H and V components. The problem of determining reactions will then be changed to one in *Case 4*. (144 : 1.)

15. Whether a problem in determining reactions is one in parallel forces or one in non-concurrent forces, the standard algebraic solution requires the use of a moment equation ($\Sigma M = 0$) at least once, and usually this equation must be used first. In most cases, the center of moments is taken either at a point of support or somewhere in a vertical line through that point. (Why?)

16. The fraction of a single vertical concentrated load transmitted by the beam to a support is equal to the distance of the load from the other support, divided by the length of span. (From $\Sigma M = 0$ (139 : 6).)

17. The fraction of the V component of a single inclined concentrated load transmitted by the beam to a support is equal to the distance from the other support to the point in which the line of action of the load intersects a horizontal line between the two points of support, divided by the length of span. (This is in accordance with the preceding statement.)

18. When there are several vertical concentrated loads, the reaction at one support is equal to the sum of the moments about any center of moments in a vertical line through the other support divided bv the length of span. (From $\Sigma M = 0$ (140 : 1).)

19. When a beam carries uniformly distributed loads, each of these loads may be replaced by a single equivalent concentrated load (acting at the center of the uniform load); the problem of determining reactions is then reduced to one of concentrated loads. (141 : 2.)

20. When vertical concentrated loads are equally spaced, as in the case of panel loads, but not symmetrically placed, as in the case of a beam partially loaded with live load, either reaction may be found by observing the fraction of each load that is transmitted to the corresponding support and then adding these fractions. (142 : 3.)

21. When one or more loads are inclined and one end of the beam is free, the reaction at the free end is vertical, the reaction at the fixed end is inclined, and the problem is determinate and falls under *Case 3*; by sub-

stituting for the inclined reaction its unknown H and V components the problem may be changed to one in *Case 4*. (144 : 1.)

1. When one or more loads are inclined and both ends of the beam are fixed, and the inclination and magnitude of each reaction is unknown (four unknowns), the problem is indeterminate, and one of two assumptions is commonly made, namely, that the reactions are parallel to the inclined loads or that the H components of the reactions are equal. The problem falls, in the first case, under *Case B′* (parallel forces), and in the second case, under *Case 4* (non-concurrent forces in which the unknowns are the H and V components of the reactions). (144 : 4 and 144 : 3.)

2. When a load is inclined, it may be replaced by its H and V components at any point in its line of action, and in algebraic methods it is often advantageous to work with such components assumed to act at a point in a horizontal line between the two points of support. (When either point of support is taken as a center of moments, all H components will then be eliminated from the corresponding moment equation.)

3. When a beam is inclined and all loads are vertical, the reactions will not be vertical unless the planes of bearing are made horizontal. (136 : 3.)

4. The reaction at either end of a main girder or truss due to one or more loads between floor beams (or panel points) may be determined exactly as if there were no floor beams and the loads were on a simple beam whose length is equal to that of the main girder or truss. (143 : 1)

5. When a single tie or strut acts between a joint of a truss and a support and there is no other connection between the truss and the support, the line of action of the reaction at that support must coincide with the axis of the tie or strut, and the magnitude of the reaction must equal the stress in the tie or strut. (Why?)

6. Any problem in reactions which can be solved algebraically by two *resolution equations* of equilibrium can be solved graphically by the method of the *force polygon*.

7. When the algebraic solution of a problem in reactions requires the use of a *moment equation* of equilibrium, the corresponding graphic solution requires the construction of an *equilibrium polygon*. The algebraic solution is usually preferable to the graphic solution unless the problem is one in which the short-cut graphic method of 145 : 1 can be used to advantage.

8. An algebraic method is usually best for determining reactions when all external forces are vertical; when any or all of them are inclined, the short-cut graphic method explained in 145 : 1 can usually be used to advantage. It follows that the algebraic methods are best for determining reactions due to loads on bridges since these loads are usually vertical, whereas the short-cut graphic method is most useful in determining reactions due to inclined wind loads on roofs.

ASSIGNMENTS

(1) Report on sources of information concerning dead loads for bridges; include tables and diagrams used in estimating dead loads.

(2) Report on sources of information concerning dead loads for roof trusses; include tables and diagrams used in estimating dead loads.

(3) Report on experiments which have been made to determine wind pressure on various surfaces.

(4) Report on different assumptions made regarding the effect of combined wind and snow loads.

(5) Compare wind pressures and snow loads prescribed by the building laws of at least six large cities in different sections of the United States.

(6) Report on failures of roofs due to snow load.

(7) Report on failures of structures due to wind pressure.

(8) Report on different methods of making the conditions at supports such that a structure is statically determinate as regards reactions; include descriptions of devices for distributing pressure uniformly at the support, for making an end free to move on the plane of contact, and for determining a point in the line of action of the reaction.

9. **Miscellaneous Problems in Reactions.** In spite of the great variety of framed structures and the many different methods of supporting them, the problem of determining the reactions at the supports usually falls under one of three cases, namely, *Case B′* or *Case 3* or *Case 4*. Several typical problems in determining reactions will now be analyzed. The student is advised first to check the analysis for each problem in order to become more familiar with the general method of attack, then to go through the problems a second time and actually solve each problem.

10. *Problem 1.* Given: A horizontal beam or truss, carrying vertical loads only, rests on three supports. Required: The magnitude of the vertical reaction at each support. This is a problem in parallel forces, but it is statically indeterminate because there are too many (three) unknown elements. (See 40 : 6 for a complete analysis.)

1. *Problem 2.* Given: A highway Pratt truss bridge on a 5 per cent grade. Number of panels = 6; panel length = 20′- 0$\frac{5}{16}$″; height of truss = 28 ft. Panel loads as shown in Fig. 149 (*a*). Assume that the panel loads are applied vertically, and that the conditions at each support are such that the line of action of each reaction is vertical. Required: The reactions R_L and R_R.

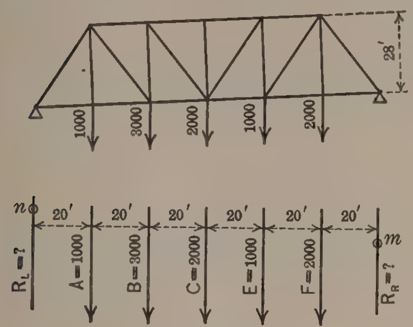


Fig. 149 (*a*).

Analysis

Body in equilibrium: Truss.
Known: R_L(L), A, B, C, E, F, and R_R(L).
Unknown: R_L(M) and R_R(M). (Case B′.) (69 : 12.)
Equations: $\Sigma M = 0$, $\Sigma V = 0$.
From $\Sigma M_m = 0$ determine R_L.
From $\Sigma V = 0$ determine R_R.
From $\Sigma M_n = 0$ check R_R and indirectly R_L.

Answers: R_L = 4500 lbs. R_R = 4500 lbs.

Instead of writing the moment equation $\Sigma M_m = 0$ in the regular form, it may be written as follows:

$$R_L = \tfrac{5}{6} \times 1000 + \tfrac{4}{6} \times 3000 + \tfrac{3}{6} \times 2000 + \tfrac{2}{6} \times 1000 + \tfrac{1}{6} \times 2000. \quad (142:3.)$$

2. *Comments:* The end conditions which cause both reactions to be vertical make the problem similar to that of determining reactions for any simple beam with vertical loads. The only effect that the grade has is to make the horizontal distance between two successive lines of action slightly less than the panel length. This problem is a good illustration of the fact that the only use made of the body in equilibrium is in determining spatial relations. (137 : 8.)

3. *Problem 3.* Given: The truss shown in Fig. 149 (*b*). Each of the lower intermediate joints carries a load of 4 tons, and four of the joints carry in addition 8 tons each as indicated. Required: The reactions R_L and R_R.

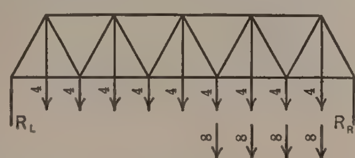


Fig. 149 (*b*)

This problem falls under Case B′, the standard solution for which is: $\Sigma M = 0$ and $\Sigma V = 0$. (69 : 12.) Instead of writing $\Sigma M_m = 0$ in the usual form, however, it may be written as follows:

$$R_L = \tfrac{1}{2} \times (9 \times 4) + 8(\tfrac{1}{10} + \tfrac{2}{10} + \tfrac{3}{10} + \tfrac{4}{10}). \quad (142:3.)$$

It may also be written:

$$R_L = \tfrac{1}{2} \times (9 \times 4) + [(4 \times 8) \times 2\tfrac{1}{2}] \div 10. \quad (142:6.)$$

From $\Sigma V = 0$, $R_R = (9 \times 4 + 4 \times 8) - R_L$.

Answers: R_L = 26 tons; R_R = 42 tons.

4. *Problem 4.* Given: A horizontal beam with four parallel loads inclined at an angle of 30° as shown in Fig. 149 (*c*). Required: The reactions R_L and R_R.

Since there are four unknown elements, namely, R_L(M and D) and R_R(M and D) the problem is indeterminate and some assumption concerning the reactions must be made. Assume that the lines of action of the reactions are parallel to the lines of action of the inclined forces. (144 : 2.)

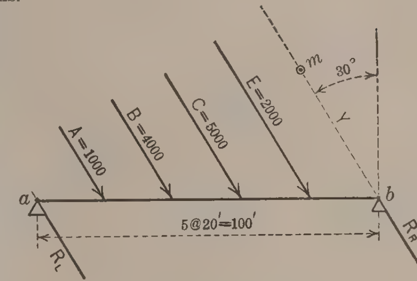


Fig. 149 (*c*)

Analysis

Body in equilibrium: Beam.
Known: A, B, C, E, R_L(L), and R_R(L).
Unknown: R_L(M) and R_R(M). (Case B′.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma Y = 0$.
From ΣM_m determine R_L.
From $\Sigma Y = 0$ determine R_R. (Axis Y parallel to the forces.)

Answers: R_L = 5600 lbs.; R_R = 6400 lbs.

Since the lines of action of the forces divide the beam into five equal panels, the equation $M_m = 0$ may be written as follows:

$$R_L = \tfrac{4}{5} \times 1000 + \tfrac{3}{5} \times 4000 + \tfrac{2}{5} \times 5000 + \tfrac{1}{5} \times 2000 = 5600 \text{ lbs.} \quad (142:3.)$$

Graphic Method

5. The problem may be solved by the graphic method of 145 : 1 as shown in Fig. 150 (*a*). The line of action of the resultant of the four inclined forces is best determined by calculating the distance x from $\Sigma M_a = 0$.

6. *Comment:* Equal panels and parallel reactions make the algebraic solution so simple that nothing is gained by the graphic method. The latter method is more advantageous when the line of action of the resultant of the inclined forces is evident from inspection, particularly if the reactions are not parallel. (See the next problem.)

7. *Problem 5.* Given: The roof truss shown in Fig. 150 (*b*). The left-hand end is a free end and the right-hand end is fixed. The six forces A, B, C, D, E, and F represent

wind loads when the wind blows on the left-hand side of the truss. Required: (a) The reactions R_L and R_R. (b) The reactions R_L and R_R when the same wind loads act on the other side of the truss.

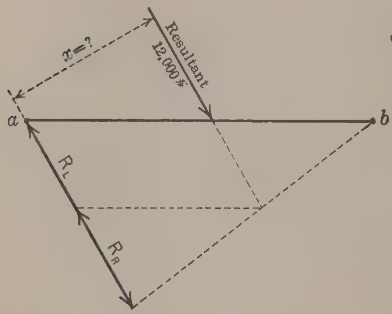


Fig. 150 (a).

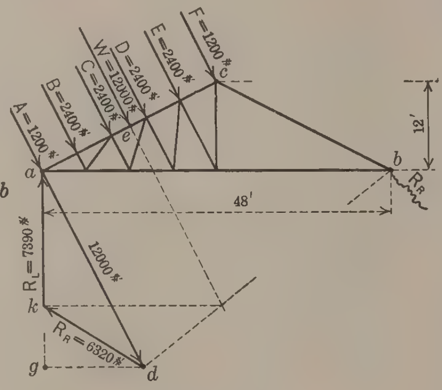


Fig. 150 (b).

Analysis for (a)

1. Let W represent the resultant of the six wind forces. Replace W by W_H and W_V applied at e, and R_R by R_{R_H} and R_{R_V} applied at b. (Why is R_L vertical?) (138 : 5.)

Body in equilibrium: Truss.

Known: W, R_L(L), R_{R_H}(L), and R_{R_V}(L).

Unknown: R_L(M), R_{R_H}(M), and R_{R_V}(M). (Case 4.) (70 : 4.)

Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (70 : 4 (a).)

From ΣM_b determine R_L.

From $\Sigma V = 0$ determine R_{R_V}.

From $\Sigma H = 0$ determine R_{R_H}.

From R_{R_H} and R_{R_V} determine R_R.

Answers: $R_L = 7380$ lbs., $R_R = 6330$ lbs., upward and to the left, $19\frac{3}{16}$ horizontal to 12 vertical.

Graphic Method

2. The graphic method of 145 : 1 is indicated in Fig. 150 (b). The line ad was first laid off equal to $W = 12{,}000$ lbs. to a convenient scale. Since the end of the truss at a is free the reaction R_L is equal to the vertical line ka. The reaction R_R is equal to dk. The values for R_L and R_R obtained by the graphic method are equal within 10 lbs. to those obtained by the algebraic method.

3. *Comment:* Note that gd the H component of R_R is equal and opposite to the H component of W. This must be true regardless of whether the wind loads act on one side or the other of the truss since the reaction R_L at a is vertical, i.e., cannot have an H component.

Analysis for (b)

4. The analysis for the algebraic method is exactly the same as that just given for the wind on the left-hand side.

Answers: $R_L = 3350$ lbs., $R_R = 9120$ lbs., upward and to the right, $8\frac{3}{4}$ horizontal to 12 vertical.

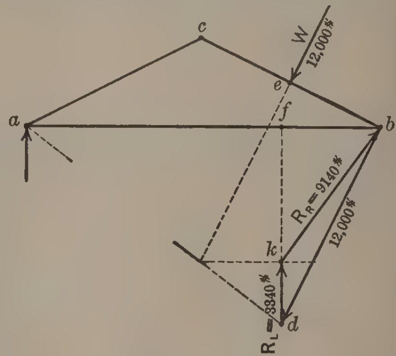


Fig. 150 (c).

Graphic Method

5. The line bd in Fig. 150 (c) was laid off equal to $W = 12{,}000$ lbs. to a convenient scale, and the graphic method of 145 : 1 was used as indicated in the figure. The values for R_L and R_R obtained by the graphic method are equal within 20 lbs. to those obtained by the algebraic method.

6. *Comment:* If the graphic method shown in Fig. 150 (b) is completed first, it is unnecessary to carry through the entire construction shown in Fig. 150 (c) since the point k in the latter figure may be determined by laying off fk equal to ak in Fig. 150 (b). (Why must kf the V component of R_R in Fig. 150 (c) equal ka the V component of R_L in Fig. 150 (b)?) Note that the H component fb of R_R is equal and opposite to the H component of W, just as it was when the wind forces were on the left-hand side of the truss, i.e., fb in Fig. 150 (c) is equal to gd in Fig. 150 (b).

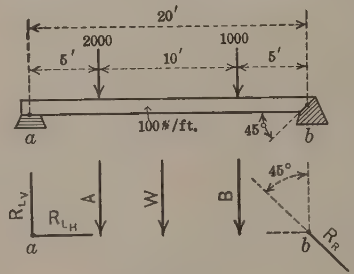


Fig. 150 (d).

7. *Problem 6.* Given: A beam weighing 100 lbs. per lin. ft. and carrying two concentrated loads as shown in Fig. 150 (d). The left-hand end is fixed, but the right-hand end is free to move along the plane of contact which is inclined at an angle of 45°. Required: The reactions R_L and R_R.

The unknown elements are R_L(M and D) and R_R(M). This is Case 3, which may be changed to Case 4 by replacing R_L by R_{L_H} and R_{L_V}, applied at a. (Why is the line of action of R_R known?) (136 : 3 and 138 : 5.)

Analysis

Body in equilibrium: Beam.

Known: A, W, B, R_{L_H}(L), R_{L_V}(L), and R_R(L).

Unknown: R_{L_H}(M), R_{L_V}(M), and $R_R(M)$. (Case 4.) (70 : 4.)

Replace R_R by its H and V components applied at b. All forces will then be either horizontal or vertical, and hence the combination of equations will be:

Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (70 : 4 (a).)

From $\Sigma M_b = 0$ determine R_{L_V}.

From $\Sigma V = 0$ determine R_{R_V}.

From R_{R_V} determine R_{R_H}.

From $\Sigma H = 0$ determine R_{L_H}.

From their components determine R_L and R_R.

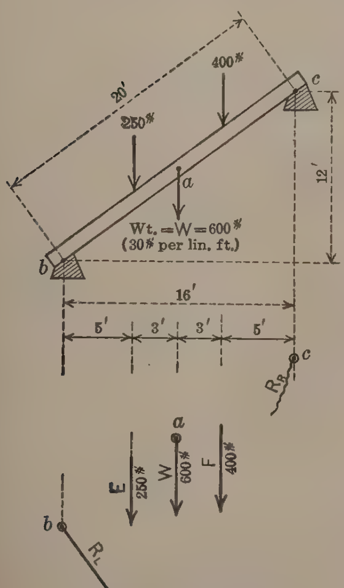


Fig. 151 (*a*).

Answers: $R_L = 3550$ lbs., upward toward the right, $9\frac{13}{16}$ horizontal to 12 vertical; $R_R = 3180$ lbs.

1. *Comment:* By determining the lever arm of R_R with respect to a that reaction could have been calculated directly from $\Sigma M_a = 0$.

2. *Exercise:* Write the analysis for the problem, assuming the left hand end free and the right hand end fixed.

3. *Problem 7.* Given: An inclined beam weighing 30 lbs. per lin. ft., and carrying two concentrated loads as shown in Fig. 151 (*a*). The surface of contact between the beam and each support is parallel to the axis of the beam. Required: The reactions R_L and R_R.

4. Unless some assumption is made concerning the reactions, the problem is indeterminate since there are four unknown elements, namely, M and D for each reaction. The analysis will be given first for the assumption that the upper end of the beam is fixed, and the lower end free to move along the plane of contact.

Analysis

Body in equilibrium: Beam.

Known: E, W, F, R_L(L), R_R(P). (Fig. 151 (*a*).)

Unknown: R_L(M) and R_R(M and D). (Case 3.) (70 : 3.)

Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.

From $\Sigma M_c = 0$ determine R_L.

From $\Sigma H = 0$ determine R_{R_H}.

From $\Sigma V = 0$ determine R_{R_V}.

From R_{R_H} and R_{R_V} determine R_R.

Answers: $R_L = 480$ lbs.; $R_R = 910$ lbs. upward and toward the right, 4 horizontal to 12 vertical.

5. The analysis will now be given for the assumption that the components of the reactions parallel to the planes of contact are equal, i.e., if the X coördinate axis is taken parallel to the axis of the beam, $R_{L_X} = R_{R_X}$. Replace each reaction by two components, parallel respectively to the coördinate axes X and Y. (Fig. 151 (*b*).)

Analysis

Body in equilibrium: Beam.

Known: E, W, F, R_{L_X}(L), R_{L_Y}(L), R_{R_X}(L), and R_{R_Y}(L).

Unknown: R_{L_Y}(M), R_{R_Y}(M), and R_{L_X}(M) $= R_{R_X}$(M). (Case 4.) (70 : 4.)

Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma X = 0$.

From ΣM_c determine R_{L_Y}.

From ΣM_b determine R_{R_Y}.

From $\Sigma X = 0$ determine R_{L_X} and R_{R_X}.

From their X and Y components determine R_L and R_R.

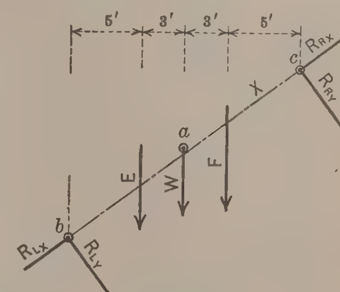


Fig. 151 (*b*).

Answers: $R_L = 610$ lbs., $\frac{1}{4}$ horizontal to 12 vertical,

$R_R = 640$ lbs., $\frac{1}{4}$ horizontal to 12 vertical,

R_L is upward to the right; R_R is upward to the left.

1. *Problem 8.* Given: The truss shown in Fig. 152 (*a*), supported at joints *b* and *f*. There is a load of 1000 lbs. at each lower joint except at *b* and at *f*. Required: The reactions R_L and R_R.

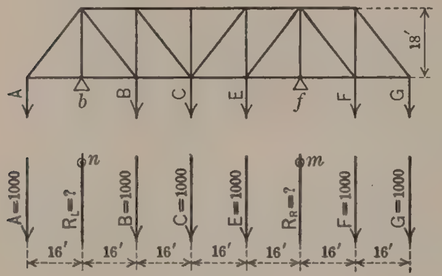


Fig. 152 (*a*).

Analysis

Body in equilibrium: The truss.
Known: A, B, C, E, F, G, R_L(L), and R_R(L).
Unknown: R_L(M) and R_R(M). (Case B′.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma V = 0$.
From $\Sigma M_m = 0$ determine R_L. $\Sigma M_m = 0$ may be written:

$$R_L = \tfrac{5}{4}A + \tfrac{3}{4}B + \tfrac{2}{4}C + \tfrac{1}{4}E - \tfrac{1}{4}F - \tfrac{2}{4}G$$
$$= 1000\left(\tfrac{5}{4} + \tfrac{3}{4} + \tfrac{2}{4} + \tfrac{1}{4} - \tfrac{1}{4} - \tfrac{2}{4}\right).$$

From $\Sigma V = 0$ determine R_R.

Answers: $R_L = 2000$ lbs., $R_R = 4000$ lbs.

2. *Exercise:* Assume that in addition to the given loads in Fig. 152 (*a*) a concentrated load of 8000 lbs. is placed at G, what change will this load make in each reaction?

Answers: R_L due to the 8000 lbs. is $-8000 \times 2 \div 4 = -4000$ lbs. (downward). Hence R_L will be changed from 2000 lbs. upward as found above to 2000 lbs. downward. R_R will be changed from 4000 lbs. upward to 16,000 lb. upward.

3. *Problem 9.* Given: The cantilever truss *abcd* shown in Fig. 152 (*b*), supported by two ties *ce* and *be* and by a hinged joint at *a*. Required: The reaction on the truss at *a* and the forces exerted on the truss by the two ties.

Let K and N represent the forces in the ties *ce* and *be*, respectively, and R the reaction at *a*. Unless some assumption is made concerning R, the problem is indeterminate, since there are four unknown elements, namely, K(M), N(M), and R(M and D). Assume that the V components of the reactions at *a* and at *e* are equal.

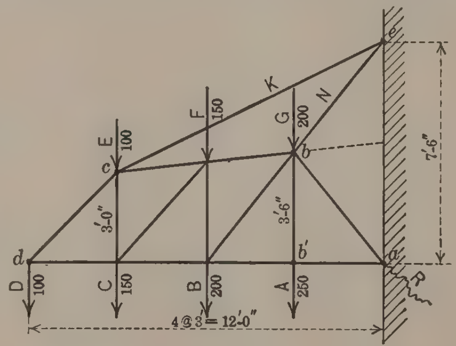


Fig. 152 (*b*).

Let T represent the resultant of the forces K and N. The line of action of this unknown resultant must pass through *e*, hence K and N may be replaced by T_H and T_V applied at *e*. Replace R by R_H and R_V.

Analysis

Body in equilibrium: Truss *abcd*.
Known: A, B, C, D, E, F, G, R_H(L), R_V(L), T_H(L), and T_V(L).
Unknown: R_H(M), T_H(M), and R_V(M) = T_V(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$. (70 : 4 (a).)
From $\Sigma M_e = 0$ determine R_H.
From $\Sigma H = 0$ determine T_H.
From $\Sigma V = 0$ determine $R_V = T_V$ (by assumption).
From R_H and R_V determine R.
Answer: $R = 1080$ lbs., upward and to the left, $19\tfrac{3}{16}$ horizontal to 12 vertical.

4. It remains to determine K and N. Let T' represent the equilibrant of K and N, i.e., $T' = -T$.

Analysis

Body in equilibrium: Particle *e*.
Known: T_H', T_V', K(L), and N(L).
Unknown: K(M) and N(M). (Case B.) (69 : 10.)
Equations: $\Sigma H = 0$ and $\Sigma V = 0$.
From $\Sigma H = 0$ and $\Sigma V = 0$ determine K and N.
Answers: $K = 610$ lbs.; $N = 390$ lbs. (both tension).

5. *Comments:* (a) In place of the algebraic method just given, the corresponding graphic method could be used to advantage for determining K and N. (24 : 4 and page 29.)

6. *Problem 10.* Given: The tower bent shown in Fig. 152 (*c*), anchored to each of the two supports *a* and *b*. Required: The reactions R_L and R_R due to the four horizontal forces A, B, C, and D.

Unless some assumptions are made concerning reactions, the problem is indeterminate. (Why?) Assume that the H components of the two reactions are equal. Replace each reaction by its H and V component.

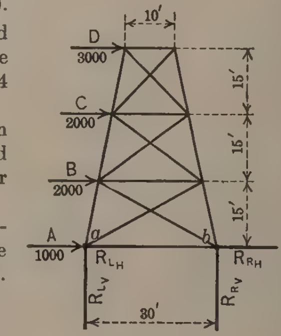


Fig. 152 (*c*).

Analysis

Body in equilibrium: Bent.
Known: A, B, C, D, R_{L_H}(L), R_{L_V}(L), R_{R_H}(L), and R_{R_V}(L).
Unknown: R_{L_V}(M), R_{R_V}(M), and R_{L_H}(M) = R_{R_H}(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$. (70 : 4 (a).)
From $\Sigma M_b = 0$ determine R_{L_V}.
From $\Sigma H = 0$ determine $R_{L_H} = R_{R_H}$.
From $\Sigma V = 0$ determine R_{R_V}.
Determine R_L and R_R from their H and V components.
Answers: $R_L = 8500$ lbs., downward and to the left, $6\tfrac{13}{32}$ horizontal to 12 vertical.
$R_R = 8500$ lbs., upward and to the left, $6\tfrac{13}{32}$ horizontal to 12 vertical.

1. *Problem 11.* Given: The cantilever roof truss shown in Fig. 153 (*a*). Required: The reactions R_L and R_R at *a* and *b* respectively.

The structure is composed of three trusses, the central truss (suspended span) *ded'*

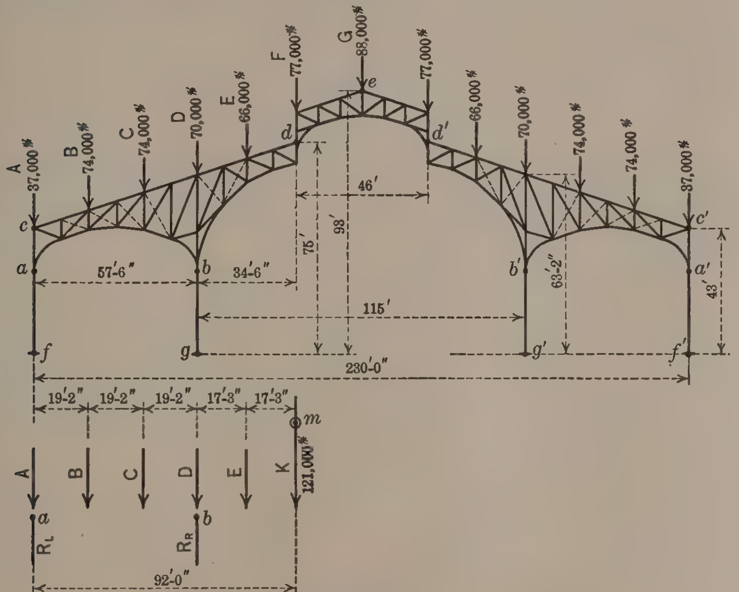


Fig. 153 (*a*).

supported at *d* and *d'* and two cantilever trusses, each supported by two columns. The required reactions are those for the cantilever truss *bacd.*

The load *K* which the central truss brings to the cantilever truss at *d* is $\frac{1}{2} \times (2 \times 77{,}000 + 88{,}000) = 121{,}000$ lbs. The space diagram for the external forces that act on the cantilever truss is shown in the lower portion of the figure.

Analysis

Body in equilibrium: Truss *bacd.*
Known: *A*, *B*, *C*, *D*, *E*, *K*, R_L(L), and R_R(L).
Unknown: R_L(M) and R_R(M). (Case B'.) (69 : 12.)
Equations: $\Sigma M = 0$ and $\Sigma V = 0$.
From $\Sigma M_b = 0$ determine R_L.
From $\Sigma V = 0$ determine R_R.
Answers: $R_L = 18{,}600$ lbs., $R_R = 423{,}400$ lbs. (both upward).

2. *Comment:* If the given loads were on a *horizontal* beam 92 ft. long with one support at the left hand end and the other $34\frac{1}{2}$ ft. from the right hand end, the problem would be exactly the same. This is another illustration of the fact that the only use made of the body in equilibrium is in determining the spatial relations of the forces. (137 : 8.)

3. *Problem 12.* Given: The saw-tooth roof truss shown in Fig. 153 (*b*). The end of the truss at support *a* is free to move horizontally; the end at support *d* is fixed. Required: The reactions R_L and R_R at *a* and *d* respectively. Replace the three horizontal forces by their resultant $A = 1000$ lbs., and the three inclined forces by their resultant $B = 600$ lbs. Replace *B* by its H and V components. (See the space diagram in the lower portion of the figure.)

Analysis

Body in equilibrium: Truss.
Known: *A*, B_H, B_V, R_L(L) and R_R(P).
Unknown: R_L(M) and R_R(M and D). (Case 3.) (70 : 3.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$.
From $\Sigma M_d = 0$ determine R_L.
From $\Sigma H = 0$ determine R_{R_H}.
From $\Sigma V = 0$ determine R_{R_V}.
From R_{R_H} and R_{R_V} determine R_R.
(The algebraic solution is given in full on p. 58.)

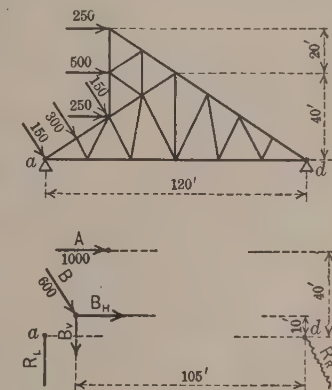


Fig. 153 (*b*).

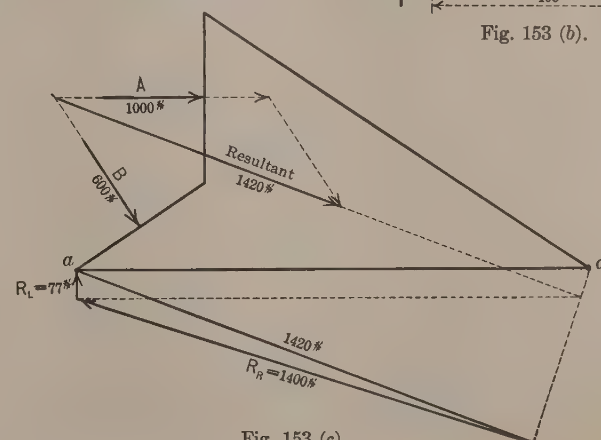


Fig. 153 (*c*).

1. *Comments:* Note that this problem could be changed to Case 4 by replacing R_R by its H and V components. The combination of equations and the solution would be the same, however, as that just given, as will be evident from a comparison of the above analysis with the analysis for *Problem 5*, page 150, which is of the same general character.

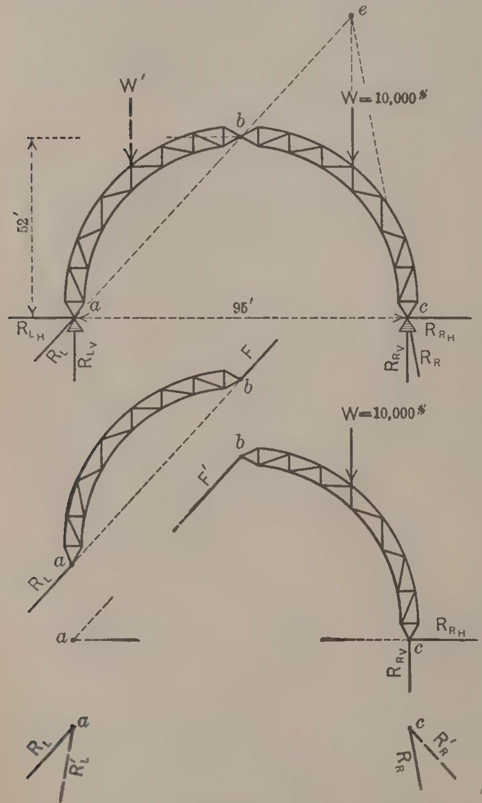


Fig. 154 (*a*).

Graphic Method

2. The graphic method of 146 : 3 applied to the problem just analyzed is shown in Fig. 153 (*c*). The first step is to determine graphically the resultant of *A* and *B* as indicated. The reactions caused by this resultant are then determined graphically by the usual geometrical construction.

3. *Problem 13.* Given: The three-hinged arch shown in Fig. 154 (*a*), i.e., an arch with hinged joints at *a*, *b*, and *c*. Required: The reactions R_L and R_R at *a* and at *c*, respectively, caused by a single concentrated load *W*. The line of action of *W* intersects the horizontal line between *a* and *c* in a point 15 ft. from *c*.

4. Three structures, each complete in itself, are involved in this problem, namely:

The left-hand half of the arch *ab*. The right-hand half of the arch *bc*. The entire arch *abc*.

5. The half arch *ab* is held in equilibrium by two external forces, namely, the reaction R_L at *a*, and the pressure *F* at *b*, the latter being the force exerted on the half arch *ab* by the half arch *bc*. These two forces must have a common line of action, and be equal in magnitude but opposite in sense. (71 : 1.) This principle determines the line of action (*ab*) of both R_L and *F*.

6. Let F' represent the pressure of the half arch *ab* on the half arch *bc*. The forces *F* and F' have the same line of action, are equal in magnitude but opposite in sense.

7. The analysis which follows is for the *entire* arch, and applies, therefore, to the upper portion of Fig. 154 (*a*). Replace R_R by R_{R_H} and R_{R_V} applied at *c*.

Analysis

Body in equilibrium: The entire arch *abc*.
Known: W, R_L(L), R_{R_H}(L), and R_{R_V}(L).
Unknown: R_L(M), R_{R_H}(M), and R_{R_V}(M). (Case 4.) (70 : 4.)
Equations: $\Sigma M = 0$, $\Sigma H = 0$, $\Sigma V = 0$.
Replace R_L by R_{L_H} and R_{L_V} applied at *a*.
From $\Sigma M_c = 0$ determine R_{L_V}.
From R_{L_V} determine R_{L_H} (65 : 11).
From $\Sigma H = 0$ determine R_{R_H}.
From $\Sigma V = 0$ determine R_{R_V}.
From their H and V components determine R_L and R_R.
Answers: $R_L = 2140$ lbs., upward and to the right, $10\frac{15}{16}$ horizontal to 12 vertical.
$R_R = 8540$ lbs., upward and to the left, $2\frac{1}{16}$ horizontal to 12 vertical.

Comments

8. Since there are only three external forces on the arch, namely, *W*, R_L and R_R, the lines of action of these three forces must meet in a point, i.e., in point *e*. (70 : 3.) This may be used as a check.

9. From $\Sigma H = 0$, the H components of the two reactions must be equal in magnitude.

10. Assume that, in addition to the load *W*, a load $W' = W$ is placed on the half arch *ab* as indicated in the figure. The reaction at *a* due to this load W' would be R'_L (see lower part of Fig. 154 (*a*)) equal in magnitude to R_R the reaction at *c* due to *W*; the slope of R'_L would be the same as that of R_R except opposite in direction. The reaction at *c* due to W' would be R'_R equal in magnitude to R_L the reaction at *a* due to *W*; the slope of R'_R would be the same as that of R_L except opposite in direction. The total reaction at *a* due to both *W* and W' would be the resultant of R_L and R'_L, and the total reaction at *c* would be the resultant of R_R and R'_R.

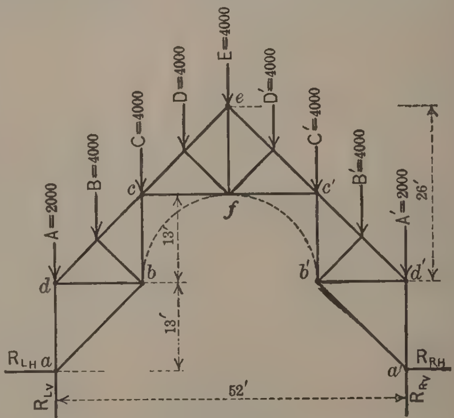


Fig. 154 (*b*).

1. The reactions can be determined by taking the half arch bc as the body in equilibrium. The unknown elements are F'(M), R_{R_H}(M), and R_{R_V}(M). (Case 4.) Replace F' by F'_H and F'_V applied at b, and the analysis becomes identical with that previously given except that F' is involved in place of R_L.

2. *Problem 14.* Given: The hammer-beam roof truss shown in Fig. 154 (b). Required: The reactions R_L and R_R at a and b, respectively. Joints a, c, c' and a' may be considered as hinged joints.

3. The structure is composed of three parts, namely, the central truss cec' and the two hammer-beam frames $abcd$ and $a'b'c'd'$.

4. Since the whole structure is symmetrically loaded, $R_{L_V} = R_{R_V} = 16{,}000$ lbs. It remains to determine R_{L_H} and R_{R_H}. Take the hammer beam frame $abcd$ as the body in equilibrium. The central truss brings to this frame a vertical load W of 10,000 lbs. applied at c. Let X represent the unknown horizontal force exerted on the frame at c.

Analysis

Body in equilibrium: Hammer-beam frame $abcd$. (Fig. 155.)

Known: A, B, W, R_{L_V}, R_{L_H}(L), and X(L).

Unknown: R_{L_H}(M) and X(M). (This is Case 2 (70 : 2), but the usual combination of equations $\Sigma H = 0$ and $\Sigma V = 0$ for that case cannot be used since the two unknown forces are both horizontal.)

Equations: $\Sigma M = 0$ and $\Sigma V = 0$.

From ΣM_c determine R_{L_H}.
From $\Sigma H = 0$ determine X.
From R_{L_H} and R_{L_V} determine R_L.

Answer: $R_{L_H} = -X = 6000$ lbs. $R = 17{,}090$ lbs., upward and to the right, $4\frac{1}{2}$ horizontal to 12 vertical.

5. Applying $\Sigma H = 0$ to the entire structure (Fig. 154 (b),) $R_{L_H} = -R_{R_H}$ or $R_{R_H} = 6000$ lbs. toward the left. $R_R = 17{,}090$ lbs. upward and to the left, $4\frac{1}{2}$ horizontal to 12 vertical.

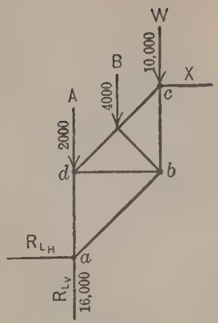


Fig. 155.

ASSIGNMENT

Report on the graphic method of determining reactions by means of an equilibrium polygon when a truss is unsymmetrical or unsymmetrically loaded.

CHAPTER XIII

SHEARS AND BENDING MOMENTS

Stresses in a beam or a truss are caused by external forces. "Shear" is a term for the algebraic sum of certain external forces, and "bending moment" is a term for the algebraic sum of certain moments of these forces. Stresses may be determined in many cases directly from shears and bending moments. A knowledge of how to determine shears and bending moments should, therefore, be acquired early in the study of stresses. Methods of calculating shears and bending moments due to static loads are explained in this chapter; methods of calculating shears and bending moments due to moving loads are explained in CHAPTER XIX.

1. **SECTION AND SEGMENT.** Let the line *ab* in Fig. 156 represent a horizontal beam resting on two supports. Conceive the beam to be cut at *c* by a plane perpendicular to the axis of the beam. The cross section of the beam which lies in the cutting plane is called the **section,** and the portion of the beam on either side of the section is called a **segment.** The left-hand segment will be called the "L segment," and the right-hand segment the " R segment."

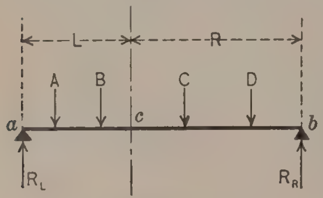


Fig. 156.

2. *Note:* Unless otherwise stated, the forces acting on a beam will be treated as co-planar. The line *ab* represents the horizontal axis of the beam, and the vertical line at *c* represents a vertical line or axis in which the cutting plane of the section intersects the plane of the forces. For co-planar forces, therefore, the section is a *line* instead of a plane, and it will be so considered throughout this book.

3. *Note:* The portion of the beam on either side of the cut is often called a "section" of the beam instead of a "segment." To avoid ambiguity, "section" will be used in this book only to denote the imaginary cut.

4. **VERTICAL SHEAR. Vertical shear** is merely a convenient term for the algebraic sum of the vertical external forces on either segment of the beam. For example, in Fig. 156 (neglecting the weight of the beam) the vertical shear for the L segment is $R_L - A - B$, and the vertical shear for the R segment is $-C - D + R_R$, plus being used for upward forces and minus for downward. If the algebraic sum is plus the shear is **positive**; if minus, the shear is **negative.** The shear for the L segment plus the shear for the R segment equals zero, since it is identical with $\Sigma V = 0$ applied to the beam as a whole. For example, in Fig. 156, L shear + R shear $= (R_L - A - B) + (-C - D + R_R) = \Sigma V = 0 = R_L - A - B - C - D + R_R$. If the algebraic sum of two quantities is zero the two quantities must be equal in magnitude but of opposite algebraic signs. Hence, the L shear must be equal in magnitude to the R shear, but if the L shear is positive the R shear is negative and *vice-versa.* If the vertical shear for either segment is positive, it means that the vertical *external* forces on that segment (the loads and reaction taken together) tend to move it upward; if the shear is negative, the vertical external forces on the segment tend to move it downward. The symbol used in this book for vertical shear is " V."

5. *Note:* If there is only one external force on a segment, this force is the shear. In this case the general definition of shear must be modified since there is no *sum* of forces.

6. *Note: Shearing forces* on a segment are those external forces which act on the segment *parallel to the section.* When these forces are horizontal, as, for example, in

the case of wind pressure, the section is assumed to be horizontal, and the algebraic sum of the shearing forces on a segment is the *horizontal shear*. In like manner there may be *diagonal* or *oblique shear*. In the great majority of problems in stresses, however, the shearing forces are vertical loads and vertical reactions acting on a horizontal beam or truss, and in such problems the shear is *vertical shear*. Throughout this book "shearing forces" on any segment will be considered as co-planar external forces on that segment which are perpendicular to the longitudinal axis of the beam or truss and parallel to the line that represents the section. Since in most cases the shearing forces are vertical, the term "shear" is generally understood to mean *vertical shear* unless it is otherwise stated.

1. **Shearing Stress.** Assume that the shear for the L segment in Fig. 157 (a) is positive, i.e., the shearing forces R_L, A, and B, acting together, tend to move the L segment upward, R_L being greater than $A + B$. Since the segment is in equilibrium there must be forces at work to resist this tendency upward. These resisting forces are *within* the beam — *internal* forces, and hence *stresses*. The sum total of these stresses is the **shearing stress,** and may be represented by a single force S_L acting downward *in the plane* of the section.

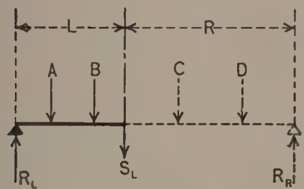


Fig. 157 (a).

2. If the shear for the L segment is positive, that for the R segment must be negative. Hence when the R segment is considered as the body in equilibrium, the shearing stress may be represented by a single force S_R, Fig. 157 (b), acting upward in the plane of the section.

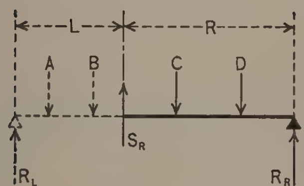


Fig. 157 (b).

3. It is to be noted that a segment is never held in equilibrium by external forces alone, but by external forces and internal forces acting together. When the L segment in Fig. 157 (a) is considered as a body in equilibrium, the shearing stress S_L is the sum total of all of the vertical *internal* forces which the R segment exerts on the L segment to keep the external forces on the L segment from moving it upward, and *vice-versa*, when the R segment is considered as the body in equilibrium, S_R is the sum total of all of the vertical internal forces which the L segment exerts on the R segment to keep the external forces on the R segment from moving it downward. It is to be noted, however, that the section may be assumed at such a point that the shear for the L segment will be negative and the shear for the R segment positive. For example, if the section were between the load D and the right-hand support, the external forces on the L segment would tend to move it downward, whereas the only external force on the R segment (the reaction) would tend to move that segment upward.

4. *Note:* In an ordinary beam the innumerable shearing stresses at a section are distributed with approximate uniformity all over the cross section, but instead of acting parallel to the axis of the beam as in the case of tension or compression they act at right angles to that axis. In a truss, which is a special form of beam, the shearing stresses are carried by the vertical and inclined members, since these are the only members capable of carrying vertical stresses or stresses with vertical components.

5. *Illustration:* In Fig. 157 (c) a beam is represented as carrying three concentrated loads. Let a section be assumed at ab. The *shear* for the L segment (weight of beam neglected) is: $1250 - 900 - 600 = -250$ lbs. (downward), hence the shearing stress S_L must be $+250$ lbs. (upward) or

$$\begin{aligned} \text{Shear} + \text{Shearing Stress} &= 0 \\ -250 + 250 &= 0 \end{aligned}$$

If the R segment is considered as the body in equilibrium, shear $= -300 + 550 = 250$ lbs. (upward) and S_R must be -250 lbs. (downward) or

$$\begin{aligned} \text{Shear} + \text{Shearing Stress} &= 0 \\ 250 \quad + (-250) &= 0 \end{aligned}$$

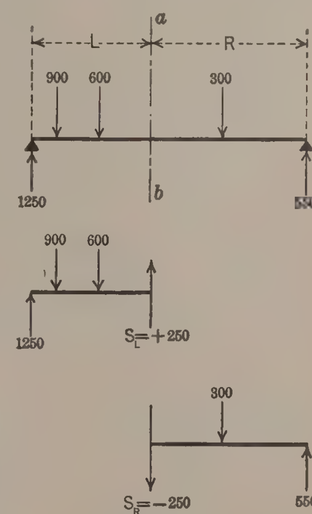


Fig. 157 (c).

6. *Note:* In the case of a beam with vertical loads, the algebraic sign for the shear for a segment will be positive or negative according to whether the reaction on the segment is greater or less than the sum of the loads on the segment. Whichever the algebraic sign of the shear for the left-hand segment may be, the sign of the shear for the right-hand segment will be opposite; and whichever the sign of the *shear* for either segment may be, the sign of the *shearing stress* for that segment will be opposite.

1. The expression "shear *at* a section," so generally used, is misleading. This expression is correct if by "shear" is meant "shearing stress," that is, the *internal* force at the section. If, however, shear is defined as the algebraic *sum* of certain *external* forces, it cannot be *at* a section since this sum is not a force any more than the sum of twenty-five cents and ten cents is a thirty-five-cent piece. Even if shear is considered as the *resultant force* of certain external forces, this resultant force would not ordinarily act *at* the section. The term *shear* should be used solely for external forces or solely for internal forces, not indiscriminately for both. In this book it will be used solely in connection with external forces. As thus used, shear is not a force but merely an algebraic *sum.* Shear is equal to the *magnitude* of the *resultant* of the shearing forces, but a resultant force implies a *line of action* whereas shear does not. Shear is simply the measure of the tendency, in the case of vertical forces, for a segment to move upward or downward — a tendency which is resisted by shearing stress. The expression "shear *for* a section," or better still "shear for a segment," implies no line of action, hence no single force, and for this reason is preferable to "shear *at* a section."

2. The term *shearing force* is sometimes used for the algebraic sum in place of shear, but this too is misleading since it implies a line of action. The same term is sometimes loosely used in place of "shearing stress," as, for example, the "shearing force at a section."

3. **Shears for Different Segments.** In the calculation of stresses in beams or trusses, a section is assumed at any point at which the shearing stress may be desired. For example, in a plate girder the vertical shear may be required at a web splice in order to design the splice. It is the purpose in this chapter to explain how to find the shear for either segment when a section is taken at random. Later it will be shown where to assume sections for special purposes.

4. *Shear due to a reaction.* When the section is assumed at such a point that the only external force acting on the segment is a reaction at a support, that reaction is the shear for that segment. This is a very common case.

5. *End shear.* When the section is assumed indefinitely close to a support of a beam supported at each end, the only force that can act on the indefinitely short segment is the reaction, and this is called the **end shear.** For such a beam, the end shear at one of the supports is the *maximum shear,* that is, the shear for no other section is greater than that for a section indefinitely close to one of the supports.

6. **Shear Due to Concentrated Loads.** In calculating the shear for any segment due to concentrated loads, the following should be kept in mind:

7. If the reaction on the segment due to *all* of the concentrated loads on the *whole* beam is not known, it must be found by considering the *whole* beam as the body in equilibrium, and applying one of the methods for finding reactions explained in the preceding chapter.

8. The shear for any segment is found by merely adding algebraically the reaction and the concentrated loads *on the segment.*

9. *Note:* Once the reaction is known, consider only external forces *on the segment,* ignoring forces elsewhere on the beam. It will be helpful at first to make a separate sketch showing the segment by itself, with all external forces on it clearly indicated.

10. For any section, the shear for the L segment is equal to the shear for the R segment but with opposite algebraic sign.

11. The shears are equal for all sections between two successive external forces.

12. A load on either segment tends to cause *negative* shear for that segment, since the load (down) is greater than any reaction (up) which the load can cause, unless the load is at the support.

13. If a section is assumed indefinitely close to a load and the greatest positive shear is desired for one of the segments, the load must be considered as on the other segment. (This follows from 158 : 12 above.)

14. If a section is taken *at* a load and the load is assumed to act first on one segment and then on the other, the transference of the load from one segment to the other will affect the shear *for the segment to which the load is transferred* (by an amount equal to the load itself) in one of three ways, namely, (1) it will increase a negative shear, (2) it will decrease a positive shear, or (3) it will change a positive shear to a negative shear.

15. If a reaction is the only external force on a segment, it is the shear for that segment.

16. If for every load on the L segment of a simple beam there is an equal load similarly located on the R segment, i.e., if the beam is symmetrically loaded with respect to its center, the shear for a section at the center is zero.

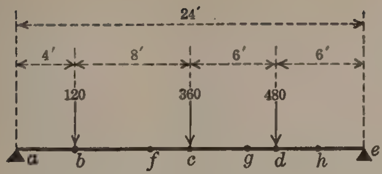


Fig. 159 (*a*).

1. *Illustration:* Given the beam and concentrated loads shown in Fig. 159 (*a*). Required to find the end shears and the shears for sections assumed at points *f*, *c*, *g*, and *h*.

2. Consider the beam as a whole, and find $R_L = 400$, $R_R = 560$ lbs. End shear at left = 400; at right = 560 lbs. The shears for both the L segment and the R segment will be calculated in each case.

3. *Section at f* (Fig. 159 (*b*)).

Shear for L segment *af* = 400 − 120 = 280 lbs. (up).
Shear for R segment *fe* = −360 − 480 + 560 = −280 lbs. (down).

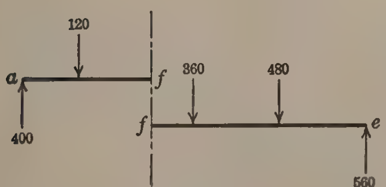


Fig. 159 (*b*).

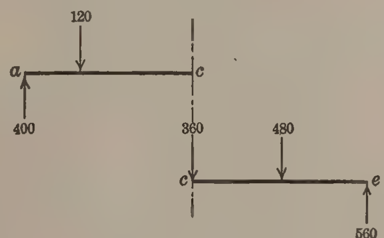


Fig. 159 (*c*).

4. *Section at c* (Fig. 159 (*c*)). If the load of 360 lbs. at *c* is considered on the R segment:

Shear for L segment *ac* = 400 − 120 = 280 lbs. (up).
Shear for R segment *ce* = −360 − 480 + 560 = −280 lbs. (down).

If the load of 360 lbs. at *c* is considered on the L segment:

Shear for L segment *ac* = 400 − 120 − 360 = −80 lbs. (down).
Shear for R segment *ce* = −480 + 560 = 80 lbs. (up).

5. *Section at g* (Fig. 159 (*d*)).

Shear for L segment *ag* = 400 − 120 − 360 = −80 lbs. (down).
Shear for R segment *ge* = −480 + 560 = 80 lbs. (up).

6. *Section at h* (Fig. 159 (*e*)).

Shear for L segment *ah* = 400 − 120 − 360 − 480 = −560 lbs. (down).
Shear for R segment *he* = reaction = 560 lbs. (up).

Comments

7. For each of the four sections, the shear for the L segment is equal to the shear for the R segment, but one is positive (up) and the other negative (down). (158 : 10.)

8. A load on either segment tends to cause negative shear for that segment (158 : 12). For example, if the section is taken at *g*, the load at *b* (Fig. 159 (*a*)) tends to move the segment *ag downward* with a force of 120 lbs. This is partially offset by the upward action of that portion of the reaction which is due to the load, but this portion is only *a fraction* of the load, namely, $\frac{20}{24} \times 120 = 100$. The net result is that the load of 120 lbs. at *b* and the portion of the reaction at *a* due to that load, added algebraically, equal −120 + 100 = −20 lbs., i.e., the shear for segment *ag* due to a load on it (at *b*) is *minus*. (Prove a similar statement for the right-hand segment *ge* and the load at *d*.) (Fig. 159 (*a*).)

9. The effect of transferring a load at a section from one segment to the other (158 : 14) may be illustrated as follows: Assume a section at *b* (Fig. 159 (*a*)). If the load of 120 lbs. at *b* is considered to be on the L segment *ab*, the shear for the R segment *be* is −280 lbs.; but if the load is transferred to the R segment *be*, the negative shear for that segment is increased from −280 lbs. to −400 lbs. If the load of 120 lbs. at *b* is considered to be on the R segment *be*, the shear for the L segment *ab* is +400 lbs.; but if the load is transferred to the L segment *ab*, the positive shear for that segment is decreased from 400 to 280 lbs. Assume a section *c* (Fig. 159 (*a*)). If the load of 360 lbs. at *c* is considered to be on the R segment *ce*, the shear for the L segment *ac* is +280 lbs.; but if the load is transferred to the L segment *ac*, the shear for that segment is changed from 280 lbs. (positive) to −80 lbs. (negative).

10. The shear is constant for all sections between two successive external forces. (158 : 1.) For example, for *any* section between the reaction of 400 lbs. at *a* and the load of 120 lbs. at *b* (Fig. 159 (*a*)), the magnitude of the shear for either the L or R segment is constant (400 lbs.); for *any* section between the load of 120 lbs. at *b* and the load of 360 lbs. at *c*, the magnitude of the shear for either segment is 280 lbs.; and for *any* section between the load of 480 lbs. at *d* and the reaction of 560 lbs. at the right-hand support, the magnitude of the shear for either segment is 560 lbs.

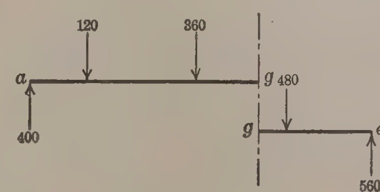


Fig. 159 (*d*).

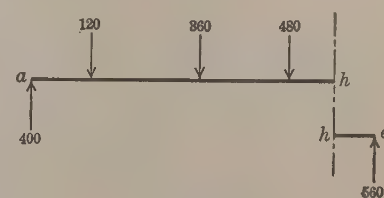


Fig. 159 (*e*).

11. In practice, the shear is usually calculated for one segment only. If both reactions are known, that segment which has on it the smaller number of forces will involve less

arithmetical work than the other segment. For example, for a section at h no calculation is necessary in finding the shear for the R segment since it is equal to the reaction R_R or 560 lbs., whereas to get the same result using the L segment, three magnitudes must be added and this sum subtracted from a fourth magnitude.

1. **Shear Diagrams for Concentrated Loads.** The beam and loads of the illustration just preceding are shown in the upper part of Fig. 160. The length of the beam and the distances between loads are laid off to any convenient scale. A horizontal base line $a'b'c'd'e'$ is laid off to the same scale. From 158 : 15, the shear for the *left-hand* segment for any section between a and b is the reaction R_L = 400 lbs. To any convenient scale, lay off the ordinate $a'A$ = 400 lbs., and draw the horizontal line AB. If now a section is taken at *any* point between a and b, the ordinate in the diagram directly below this point represents the magnitude and algebraic sign of the shear for the corresponding *left-hand* segment, namely, 400 lbs. *up* (or plus). From 159 : 3, the shear for the *left-hand* segment for any section between b and c is 280 lbs., hence the horizontal line $B'C$ is so drawn that all ordinates to it between b' and c' are each equal to 280 lbs. *up* (or plus). From 159 : 5, the shear for the *left-hand* segment for any section between c and d is −80 lbs., hence the horizontal line $C'D'$ is so drawn that all ordinates to it between c' and d' are each equal to 80 lbs. *down* (or minus). From 159 : 6, the shear for the *left-hand* segment for any section between d and e is −560 lbs., hence the horizontal line DE is so drawn that all ordinates to it between d' and e' are each equal to 560 lbs. *down* (or minus).

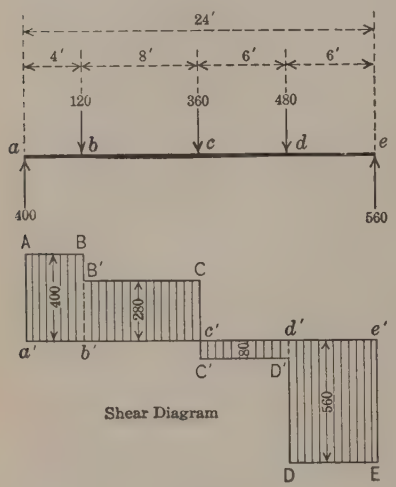


Fig. 160.

2. The broken line $ABB'Cc'C'D'DE$ is the **shear curve** for *left-hand* segments, and the whole diagram is the **shear diagram** for *left-hand* segments. If a section is assumed at *any* point on the beam ae, the ordinate in the shear diagram directly below this point represents the magnitude and algebraic sign of the shear for the corresponding *left-hand* segment, i.e., the segment from a to the point.

3. *Note:* It is important to note that a shear curve may be drawn for left-hand segments or for right-hand segments, but that no one shear curve can be drawn for both left- and right-hand segments. For example, the shear curve for right-hand segments for the beam in Fig. 160 would be similar to that shown in the shear diagram except that it would be inverted, that is, the portion $ABB'Cc'$ would be below the base line instead of above, and the portion $c'C'D'DE$ would be above the base line instead of below. In this book shear diagrams will be for left-hand segments unless it is otherwise stated.

Since a shear diagram shows the shears for all possible sections, it pictures all changes in shear as the length of the left-hand segment varies from zero to the length of the beam. It shows at what points sections should be taken in order to obtain maximum positive or maximum negative shear, and it shows at what point the shear changes from positive (up) to negative (down) or *vice-versa.* A shear diagram for concentrated loads, such as that in Fig. 160, shows that the shear remains constant for all sections between two successive external forces.

Free-hand shear diagrams. A shear diagram drawn to scale may be used to determine the magnitude of shear for any section; but shear diagrams, particularly those for concentrated loads, are not generally used for this purpose. More often, the chief purpose is merely to picture changes in shear, as stated in the preceding paragraph. When such is the purpose, it is usually sufficiently accurate to plot the shear curve free-hand though approximately to scale. This statement applies also to shear diagrams for uniform loads, to be explained later.

Exercise: Prove that the plus area in the shear diagram in Fig. 160 is equal to the minus area.

4. **Shear Due to Uniformly Distributed Loads.** The weight of a beam is usually a uniformly distributed dead load extending from end to end of the beam. There may be other uniformly distributed dead loads covering either the whole beam or certain portions of the beam. The shear for any segment may be calculated as follows:

First: Find the reaction on the segment due to the uniform load or loads, if it is not already known.

Second: Replace that portion of the uniform load which is *on the segment* by an equivalent concentrated load, and thus reduce the problem to one of finding the shear for concentrated loads. If there are two or more separate portions of uniform load *wholly on the segment*, each may be replaced by a concentrated load; if only a part of such a portion is on the segment, that part only can be replaced. (See *Caution* below.)

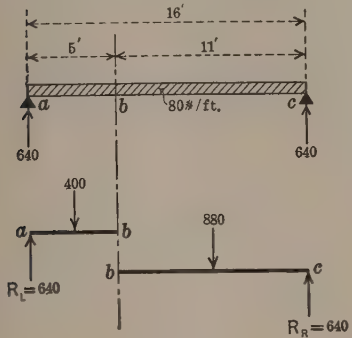


Fig. 161 (*a*).

1. *Illustration:* A beam 16 ft. long (Fig. 161 (*a*)) weighs 80 lbs. per lin. ft. What is the shear for an L segment 5 ft. long?

Shear for L segment *ab*
$= 640 - 400 = 240$ lbs. (up).

Shear for R segment *bc*
$= -880 + 640 = -240$ lbs. (down).

2. *Caution:* A common mistake is to determine the reaction (one-half of the total load), and call this reaction the shear, forgetting the uniform load that is on the segment. For example, to say that the shear for *ab* is 640 lbs. is to overlook 400 lbs. of uniform load. This mistake is not likely to occur if, in finding the shear, one will think of the *segment* only, and make sure that *all* of the external forces on that segment, and *only those forces*, are included in the algebraic sum.

3. *Illustration:* A beam 32 ft. long (Fig. 161 (*b*)) rests on supports 20 ft. apart, 6 ft. of the beam overhanging at each end. The beam weighs 100 lbs. per lin. ft. Between supports it carries an additional uniform load of 300 lbs. per lin. ft., while on each overhanging end there is a uniform load of 200 lbs. per lin. ft. What is the shear for a section 7 ft. to the right of the left-hand support. (L segment = 13 ft.) The reactions R_L and R_R are each 5800 lbs.

A = uniform load on the overhang $= 6 \times 200 = 1200$ lbs. B = weight of the L segment $= 13 \times 100 = 1300$ lbs. C = portion of the uniform load between supports which is on the L segment $= 7 \times 300 = 2100$ lbs.

$$\begin{aligned}\text{Shear for the L segment} &= -A + R_L - B - C\\ &= -1200 + 5800 - 1300 - 2100 = 1200 \text{ lbs. (up).}\end{aligned}$$

D = portion of the load between supports which is on the R segment $= 13 \times 300 = 3900$ lbs. E = weight of R segment $= 19 \times 100 = 1900$ lbs. F = load on the overhang $= 6 \times 200 = 1200$ lbs.

$$\begin{aligned}\text{Shear for the R segment} &= -D - E + R_R - F\\ &= -3900 - 1900 + 5800 - 1200 = -1200 \text{ lbs. (down).}\end{aligned}$$

4. *Note:* Let it be required to find the shear for an L segment 4 ft. long, that is, when the section is 2 ft. to the left of the L support in Fig. 161 (*b*). In this case the only external forces on the segment are the two uniform loads amounting to $4 \times (100 + 200) = -1200$ lbs., and this is the shear. The shear for a section between the unsupported end of a beam and a support may thus be found without determining the reaction.

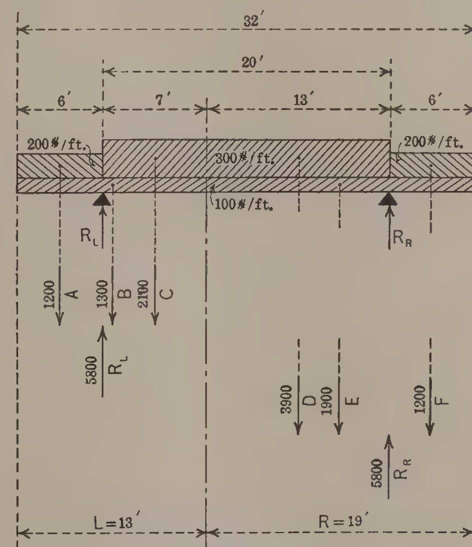


Fig. 161 (*b*).

5. **Shear Diagrams for Uniformly Distributed Loads.** Let l = length of beam (Fig. 162 (*a*)), w = uniform load per lin. ft., and x = length of the left-hand segment. The shear for the L segment is

$$\text{Shear} = \frac{wl}{2} - wx.$$

This is the equation of a straight line. When $x = 0$ the shear is a maximum $\left(\frac{wl}{2}\right)$. When $x = \frac{l}{2}$ the shear is zero. When $x = l$ the shear is a

maximum but negative $\left(-\frac{wl}{2}\right)$. This line plotted gives the shear diagram in the figure. The ordinate at any point indicates the shear for the L segment for a section at that point. The positive shear for an L segment is a maximum at the L support a and decreases as sections are taken nearer and nearer the center c, until for a section at the center (L segment $= \frac{1}{2}l$) the shear becomes zero. As sections are taken beyond the center between c and b, the shear for the L segment becomes negative; it increases as sections are taken nearer and nearer the R support until for a section at the R support the negative shear for an L segment becomes a maximum and equal to the maximum positive shear for an L segment (section at the L support).

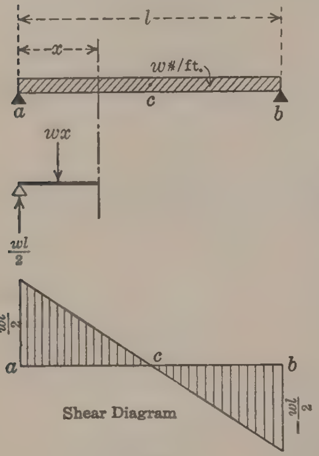


Fig. 162 (*a*).

1. Notice that, unlike the shear curve for concentrated loads, the shear curve for uniform loads has no horizontal portions, that is, the magnitude of the shear is not constant between any two sections.

2. *Question:* What would be the shear diagram for right-hand segments?

3. *Exercise:* Draw the shear diagram for a uniform load which does not cover the entire beam but extends equal distances each side of the center of the beam.

4. *Exercise:* Draw the shear diagram for a beam partially loaded with uniform load when the load extends a greater distance on one side of the center of the beam than on the other.

5. *Exercise:* Draw the shear diagram for left-hand segments for the beam and loads of Fig. 161 (*b*).

6. *Note:* It is evident that for uniformly distributed load, as well as for concentrated loads, (1) the shear for an L segment is equal to the shear for the corresponding R segment but with opposite algebraic sign; (2) any load on either segment tends to cause negative shear for that segment; (3) when a beam supported at each end is symmetrically loaded with respect to its center, the shear for a section at the center is zero.

7. **Bending Moment.** Shearing forces on any segment have been considered in this chapter as co-planar external forces which are perpendicular to the longitudinal axis of the beam and parallel to the line which represents the section. Assuming *any* point in this line as a center of moments, the **bending moment** for a segment is merely a convenient term for the *algebraic sum* of the moments of the shearing forces on that segment. If this algebraic sum is plus it indicates clockwise rotation, that is, the external forces on the segment tend to rotate it clockwise about the assumed point of moments and the bending moment is **positive.** If the algebraic sum is minus, it indicates counter-clockwise rotation and the bending moment is **negative.** The bending moments of L and R segments are equal in magnitude but opposite in algebraic sign (direction of rotation). The symbol used in this book for bending moment is "M_B."

8. In Fig. 162 (*b*) a horizontal beam is shown with three loads A, B, and C. The section is at s. Let M_B represent the bending moment. Then:

M_B for the L segment
$$= R_L \times r - A \times a$$
M_B for the R segment
$$= B \times b + C \times c - R_R \times r'.$$

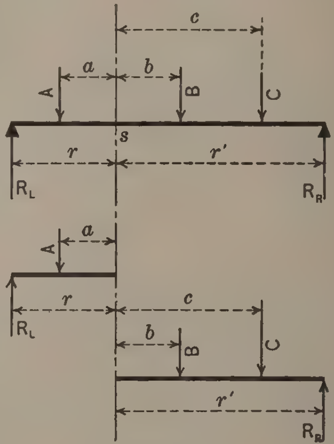


Fig. 162 (*b*).

These two bending moments must be equal in magnitude but opposite in sign, because when added together they become the algebraic sum of the moments of *all* of the external forces on the beam, and this sum must equal zero for *any* point in the plane of those forces. ($\Sigma M = 0$.)

$$\Sigma M_s = M_B \text{ for the L segment} + M_B \text{ for the R segment} = 0$$
$$= (R_L \times r - A \times a) + (B \times b + C \times c - R_R \times r') = 0.$$

Notes

9. Shear is the algebraic sum of forces on a segment; bending moment is the algebraic sum of the *moments* of these same forces when the center of moments is at the section.

10. Bending moment is sometimes loosely defined as positive when the beam bends downward. This is true for one segment only — the bending moment for the other segment must be negative. In the system of algebraic signs adopted in this book, the bending moment for a *left-hand* segment is positive if the bending moment tends to bend the beam downward, provided the beam is a simple beam supported at each end.

1. The bending moment for an L segment is the measure of the tendency of the shearing forces on that segment to rotate it clockwise or counter-clockwise; it is also a measure of the tendency of the L segment to rotate the R segment in the same direction as the L segment itself tends to rotate. A similar statement can be made concerning the bending moment for an R segment.

2. In the great majority of problems in stresses, the axis of the beam or truss is horizontal, and bending moments are found from *vertical* forces. If the axis is vertical, the bending moment will be found from horizontal forces; if the axis is inclined, the bending moment will be found from forces normal to the inclined axis.

3. In mechanics of materials it is usually advisable to assume the point of moments at the center of gravity of the cross section of the beam. In stresses the point of moments is usually elsewhere, as, for example, in a top or bottom chord of a truss.

4. **Resisting Moment.** The bending moment for any section of a simple beam cannot be zero except for a section at a support; for any other section, the segment on either side would rotate were the *external* forces on that segment the only forces acting on it. Assume, for example, that an L segment is considered by itself as the body in equilibrium (Fig. 163), and that the bending moment (from *external forces*) is positive, tending to produce rotation in the direction of M_B. The *internal* forces, whatever they are, can alone resist this moment by tending to produce rotation in the opposite direction, namely, in the direction of M_R. These internal forces may be innumerable, but the algebraic sum of the moments of all of them must equal in magnitude the bending moment, else the L segment would not be in equilibrium. This sum of the moments of the internal forces is called the *resisting moment* (M_R). In a simple horizontal beam the resisting internal forces are horizontal and vary in magnitude from zero at the neutral axis to a maximum for the extreme top or bottom fiber — that is, they are not of equal magnitudes throughout the cross section as is the case in shearing stresses. It is not within the scope of this book to explain how the resisting moment is determined for a beam of given cross section. (Consult books on mechanics of materials or structural design.) The bending moments for trusses are resisted by horizontal forces in the top and bottom chords, as will be explained later.

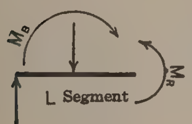


Fig. 163.

5. *Note:* The internal force in each fiber cut by a vertical section through a horizontal beam may be resolved into H and V components. The V component can resist shear for the section, but it cannot resist bending moment because its lever arm for a center of moments in the section is zero. This leaves only the H component to resist bending moment. It may be said, therefore, that the resisting internal forces that enter into a resisting moment are horizontal in the case of a horizontal beam.

6. *Note:* To avoid ambiguity, either the expression "bending moment for a segment" or "bending moment *for* a section" is preferable to "bending moment *at* a section." (For reasons similar to those given in 158 : 1.)

7. **Bending Moments Due to Concentrated Loads.** In calculating the bending moment for any segment due to concentrated loads, the following should be kept in mind:

8. The reaction on the segment, if not known, must be determined. (In the case of an unsupported end there may be no reaction on the segment.)

9. The bending moment for any segment is merely the algebraic sum of the moments of the external forces *on that segment* when the center of moments is taken anywhere in the section. In the case of a simple beam, the external forces include the reaction on the segment as well as the concentrated loads, if any, that are on the segment.

10. *Note:* Once the reaction is known, consider only this reaction and the loads *on the segment*, ignoring forces elsewhere on the beam.

11. For any section, the bending moment for the L segment is equal to the bending moment for the R segment but with opposite algebraic sign.

12. In the case of a simple beam supported at each end, a load (downward force) anywhere on the beam will cause a positive bending moment for any left-hand segment and a negative bending moment for any right-hand segment. (Prove this.)

13. The maximum bending moment, for a beam supported at each end and carrying concentrated loads, will be for a section at one of the loads; if the loads are equal, or approximately so, the maximum bending moment will be for a section at the load nearest the center of the beam; for such a beam the bending moment for a section at either support is zero.

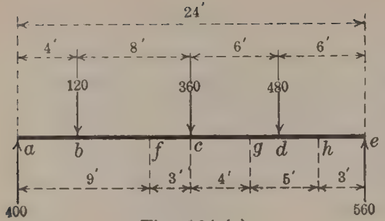


Fig. 164 (*a*).

1. *Illustration.* Given the beam and concentrated loads shown in Fig. 164 (*a*). Required to find the bending moments for sections assumed at points *f*, *c*, *g*, and *h*. (This is the same beam and loading as shown in Fig. 159 (*a*), hence the reactions are $R_L = 400$; $R_R = 560$ lbs.)

2. *Section at f.* (Fig. 164 (*b*).)

Bending moment for L segment *af*:

$$400 \times 9 - 120 \times 5 = 3000 \text{ lb.-ft.}$$

Bending moment for R segment *fe*:

$$360 \times 3 + 480 \times 9 - 560 \times 15 = -3000 \text{ lb.-ft.}$$

3. *Section at c.* (Fig. 164 (*c*).)

Bending moment for L segment *ac*:

$$400 \times 12 - 120 \times 8 = 3840 \text{ lb.-ft.}$$

Bending moment for R segment *ce*:

$$480 \times 6 - 560 \times 12 = -3840 \text{ lb.-ft.}$$

4. *Section at g.* (Fig. 164 (*d*).)

Bending moment for L segment *ag*:

$$400 \times 16 - 120 \times 12 - 360 \times 4 = 3520 \text{ lb.-ft.}$$

Bending moment for R segment *ge*:

$$480 \times 2 - 560 \times 8 = -3520 \text{ lb.-ft.}$$

5. *Section at h.* (Fig. 164 (*e*).)

Bending moment for L segment *ah*:

$$400 \times 21 - 120 \times 17 - 360 \times 9 - 480 \times 3 = 1680 \text{ lb.-ft.}$$

Bending moment for R segment *he*:

$$-560 \times 3 = -1680 \text{ lb.-ft.}$$

Comments

6. For each of the four sections the bending moments for the L segment and the R segment are equal but of opposite algebraic sign, i.e., the bending moment for each of the L segments is positive (clockwise) whereas that for each of the R segments is negative (counter-clockwise).

7. Each figure consists of two space diagrams, one for the L segment and one for the R segment. In calculating the bending moment for any segment it is well to draw a space diagram of all of the external forces on that segment (it is not necessary to represent the segment itself), and place on this diagram the values of the magnitude and lever arm of each force. This simple precaution will help to avoid mistakes and save time in the long run.

8. In practice, the bending moment is calculated for one segment only. If both reactions are known, that segment which has on it the smaller number of external forces

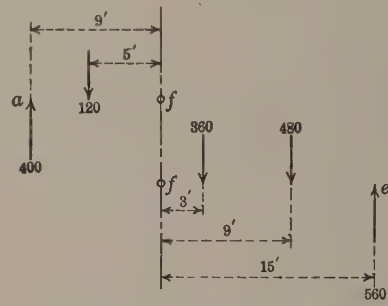


Fig. 164 (*b*).

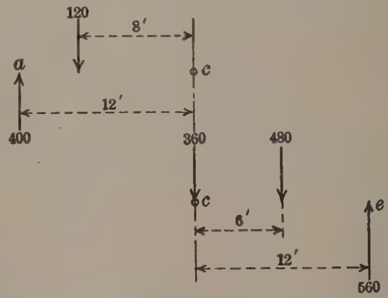


Fig. 164 (*c*).

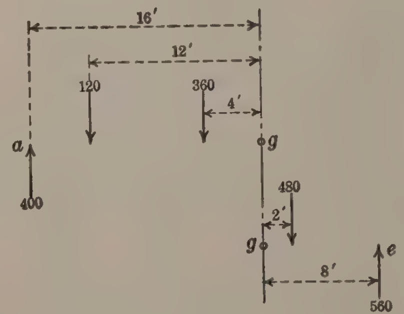


Fig. 164 (*d*).

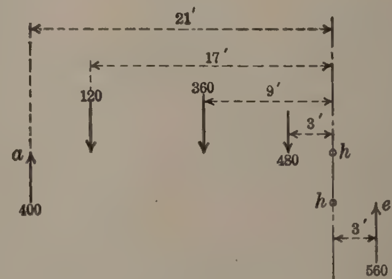


Fig. 164 (*e*)

will involve less arithmetical work than the other segment. For example, for a section at *h*, the calculation of the bending moment for the R segment involves only one multiplication whereas that for the L segment involves four.

1. **Relation Between Bending Moments for Two Different Segments.** Let K (Fig. 165 (a)) represent the L segment for a section at the load B, and X the L segment for *any* section *between* loads B and C, a distance x from B. It is desired to show the relation between the bending moments for segments K and X.

$$M_B \text{ for } K = R_L \times (a + b) - A \times b.$$

$$\begin{aligned} M_B \text{ for } X &= R_L \times (a + b + x) - A \times (b + x) - B \times x. \\ &= [R_L \times (a + b) - A \times b] + (R_L - A - B) \times x. \end{aligned}$$

2. *The bending moment for the segment X is equal to the bending moment for the segment K plus the shear for the segment X multiplied by x.* This statement will hold true regardless of the number of external vertical forces on the segment K, provided none of these forces is to the right of force B. The general principle underlying the statement is important because it affords a method of quickly calculating the bending moment for any section from the known bending moment for the section at the nearest load to the left. A similar principle may be used for right-hand segments. (The general principle just given is the same as that of 32 : 7.)

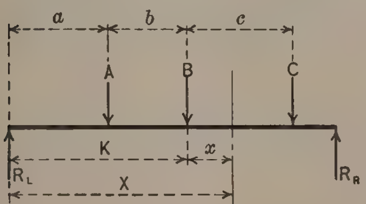


Fig. 165 (a).

3. When $x = c$ the bending moment for a section at C is equal to the bending moment for a section at B plus the constant shear for all sections between B and C multiplied by c. (The load C is not included in the shear — Why?)

4. A more general case than that just explained is that in which there are loads between the two sections as well as elsewhere. Let N (Fig. 165 (b)) represent the L segment for a section at *any* point n (not necessarily at a load) and let O represent the L segment for a section at any other point o between D and E. Let x represent the portion of the beam between the two sections at n and o, then:

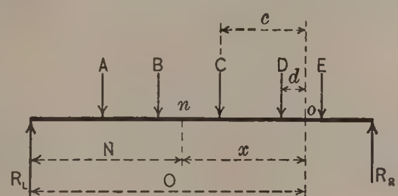


Fig. 165 (b).

$$M_B \text{ for } O = M_B \text{ for } N + (R_L - A - B) \times x - (C \times c + D \times d).$$

5. *The bending moment for the segment O is equal to the bending moment for the segment N plus the shear for the segment N multiplied by the distance x, minus the sum of the moments of all of the loads on the portion x about any point in a vertical line through o.* (Prove this statement.)

6. **Bending-Moment Diagrams for Concentrated Loads.** A bending-moment diagram is similar to a shear diagram (160 : 1) except that an ordinate at any point of a beam represents the bending moment for a section at that point, the ordinate being measured between a base line and the curve for bending moments. The diagram pictures the changes in magnitude of bending moments for left-hand segments (or for right-hand segments) varying in length from zero to the length of the beam. In plotting the curve for bending moments, it is necessary merely to plot the ordinate at each point where a load is applied, and connect successive points on the curve thus obtained by straight lines. (Prove that the equation of the curve for bending moments between any two successive loads is that of a straight line.)

7. *Illustration:* Let it be required to plot the moment diagram for the beam and loads shown in Fig. 165 (c). Bending moments for L segments for sections at b, c, and d, computed from shears as explained in the preceding article, are:

b: $400 \times 4 = 1600$ lb.-ft.

c: $1600 + (400 - 120) \times 8$
$1600 + 280 \times 8 = 3840$ lb.-ft.

d: $3840 + (280 - 360) \times 6$
$3840 - 80 \times 6 = 3360$ lb.-ft.

e: $3360 + (-80 - 480) \times 6$
$3360 - 560 \times 6 = 0$ (check).

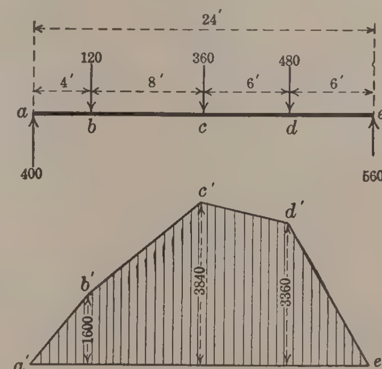


Fig. 165 (c).

From any convenient horizontal base line $a'e'$, and to any convenient scale, ordinates are laid off to represent in pound-feet the bending moments *for L segments* for sections at b, c, and d as shown in the figure. All three ordinates are above the base line since all three bending moments are positive. Since the bending moments

for sections at a and e are zero, and since the equation for bending moments between two consecutive loads is the equation of a straight line, the broken line $a'b'c'd'e'$ is the **bending-moment curve** for left-hand segments. (How would the curve for right-hand segments differ?)

1. *Prove* that the slope of each straight line in the bending-moment curve is proportional to the shear for any section between the two corresponding loads, i.e., for every linear foot measured horizontally $a'b'$ rises 400 lb.-ft. vertically, $b'c'$ rises 280 lb.-ft. vertically, $c'd'$ falls 80 lb.-ft. vertically, and $d'e'$ falls 560 lb.-ft. vertically.

2. *Note:* By making use of the principle that the slope is proportional to the shear, the bending-moment curve can often be sketched free-hand with sufficient accuracy to serve its purpose since such a curve is used not so much to find the magnitudes of bending moments as to indicate rates of change for different sections and to show for which of the sections the bending moment is a maximum.

3. *Prove* that the bending moment for the segment ac is equal to the area of the portion of the shear diagram that corresponds to that segment, and that, in general, a corresponding statement will hold true for any segment.

4. *Prove* that if the line $a'b'$ is produced to intersect the ordinate through c' at a point c'', the intercept $c'c''$ is equal to the moment of the force at b about c, and that, in general, a similar statement may be made concerning any force and the two lines in the moment curve which intersect in the line of action of that force.

5. **Bending Moments Due to Uniformly Distributed Load.** The bending moment for any segment of a beam due to the weight of the beam or to any other uniform load may be found as follows:

First: Determine the reaction on the segment by considering the whole beam as the body in equilibrium.

Second: Replace the portion of the uniform load *which is on the segment* by a single equivalent concentrated load acting at the center of mass of that uniform load, and find the bending moment by the method already explained for concentrated loads. If there are two or more separate portions of uniform load *wholly* on the segment, each should be replaced by an equivalent concentrated load.

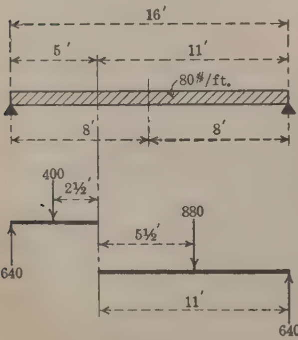


Fig. 166 (*a*).

6. *Illustration:* A beam 16 ft. long weighs 80 lbs. per lin. ft. (Fig. 166 (*a*)). What is the bending moment for an L segment 5 ft. long and for an L segment 8 ft. long?

Bending moment for 5 ft. L segment:

$$640 \times 5 - 400 \times 2\tfrac{1}{2} = 2200 \text{ lb.-ft.}$$

Bending moment for 11 ft. R segment: $880 \times 5\frac{1}{2} - 640 \times 11 = -2200$ lb.-ft. (check).

Bending moment for 8 ft. L segment: $640 \times 8 - 640 \times 4 = 2560$ lb.-ft.

7. *Caution:* The reaction is equal to one-half of the total load on the beam, and is the same whether this load is uniformly distributed or conceived as concentrated at the center. The bending moment, however, is *not* the same for a uniformly distributed load as for a load of equal amount concentrated at the center. For example, assuming the section at the center of the beam in Fig. 166 (*a*), the bending moment for a total load of 1280 lbs. uniformly distributed is 2560 lb.-ft., but for a single load of 1280 lbs. concentrated at the center the bending moment is $640 \times 8 = 5120$ lb.-ft., or twice as great. It is a common mistake in calculating bending moments for uniform load to multiply the reaction (half of the total load) by the length of the segment and *forget to subtract the moment due to that part of the uniform load which is on the segment.* (A similar mistake is often made in calculating shear.) (161 : 2.)

8. *Illustration:* A beam 32 ft. long (Fig. 166 (*b*)) rests on supports 20 ft. apart, 6 ft. of the beam overhanging at each end. The beam weighs 100 lbs. per lin. ft. Between supports it carries an additional uniform load of 300 lbs. per lin. ft., while on each overhanging end there is a uniform load of 200 lbs. per lin. ft. What is the bending moment for a section 7 ft. to the right of the left-hand support? (L segment = 13 ft.) (This is the same beam and loading as shown in Fig. 161 (*b*) and the magnitudes of the reactions and loads are taken from that illustrative example.)

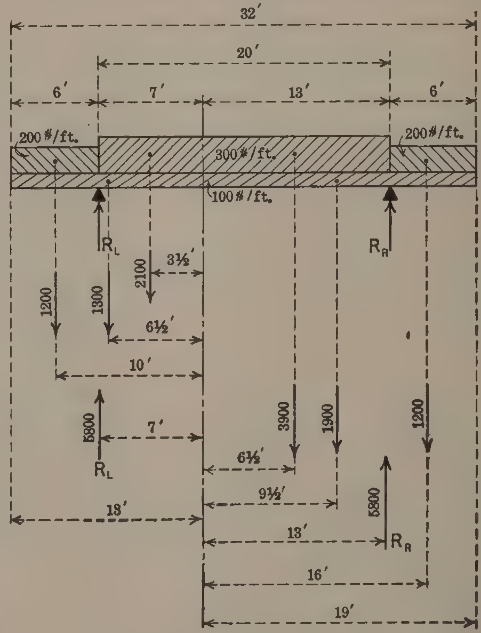


Fig. 166 (*b*).

M_B for L segment:

$$-1200 \times 10 + 5800 \times 7 - 1300 \times 6\tfrac{1}{2} - 2100 \times 3\tfrac{1}{2} = 12{,}800 \text{ lb.-ft.}$$

M_B for R segment:

$$3900 \times 6\tfrac{1}{2} + 1900 \times 9\tfrac{1}{2} - 5800 \times 13 + 1200 \times 16 = -12{,}800 \text{ lb.-ft. (check).}$$

1. *Short-cut method of calculating the bending moment for uniform load.* Let a simple beam of length l be loaded from end to end with a uniformly distributed load of w lbs. per lin. ft. (Fig. 167 (*a*).) Let the length of an L segment be x, and the length of the corresponding R segment be $l - x$. The bending moment for the L segment is:

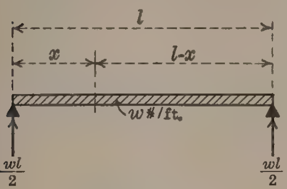


Fig. 167 (*a*).

$$M_B = \frac{wl}{2} \times x - wx \times \frac{x}{2} \text{ or } \frac{w}{2} \times x \times (l - x).$$

Put in the form of a principle: *If a simple beam supported at each end is loaded from end to end with uniformly distributed load, the bending moment for any section is equal to the product of the lengths of the L and R segments (in feet) multiplied by one-half the load per linear foot, — plus for the L segment, minus for the R segment.*

2. *Note.* The above principle affords a very useful short-cut for calculating bending moments for a *simple* beam *fully loaded with uniform load;* it does not hold true for partially loaded beams. Applying the short cut to the illustrative example of Fig. 166 (*a*):

Bending moment for 5 ft. L segment $= 5 \times 11 \times \frac{80}{2} = 2200$ lb.-ft.

Bending moment for 8 ft. L segment $= 8 \times 8 \times \frac{80}{2} = 2560$ lb.-ft.

3. *Prove* that the bending moment for any section due to a single concentrated load at that section is twice as great as the bending moment for the same section due to a load equal in amount to the concentrated load, but uniformly distributed from end to end of the beam.

4. **Bending-Moment Diagrams for Uniformly Distributed Loads.** The equation $M_B = \frac{wlx}{2} - \frac{wx^2}{2}$, given in the preceding article as the equation for bending moments for a beam uniformly loaded from support to support, is the equation of a parabola. When $x = 0$ or l, $M_B = 0$, and when $x = \frac{1}{2}l$, $M_B = \frac{wl^2}{8}$. If, therefore, at the center of $a'b'$ (Fig. 167 (*b*)) an ordinate $c'd'$ be laid off equal to $\frac{wl^2}{8}$ to any convenient scale and a parabola be drawn through a', d', and b', this parabola will be the bending-moment curve for left-hand segments. The ordinate at any section measured from the base line $a'b'$ to the curve will be equal to the bending moment for the L segment for that section. For example, for a section at e the bending moment for the segment ae is equal to the ordinate $e'f'$ measured to the scale used in laying off $c'd'$. For the illustrative example in 166:6 the bending moment for a section at the center was found to be 2560 lb.-ft. If $c'd'$ be laid off to equal 2560 lb.-ft., a parabola be drawn through a', d', and b', and $a'e'$ be laid off equal to 5 ft., then $e'f'$ would be equal to 2200 lb.-ft., the bending moment for the segment ae. (What would be the bending-moment diagram for right-hand segments?)

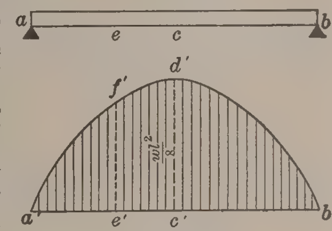


Fig. 167 (*b*).

5. *Exercise.* A uniform load extends on each side of the center of a beam for a distance equal to one-fourth of the length of the beam, thus leaving the beam without load for a distance from each support equal to one-fourth of the length. Sketch the bending-moment curve for left-hand segments.

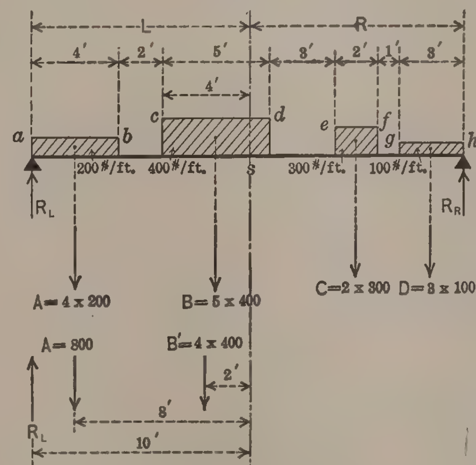


Fig. 167 (*c*).

6. **Broken Uniform Loads.** The loads shown on the beam in Fig. 167 (*c*) represent broken uniform loads which the beam carries in addition to its own weight. For purposes of discussion, the weight of the beam may be ignored. Certain portions of the beam are loaded, and certain portions are not; in this case, moreover, the weight per linear foot varies for different loads as follows: $ab = 200$, $cd = 400$, $ef = 300$, and $gh = 100$. In calculating the reactions, each portion of uniform load may be replaced by a single concentrated load equal in weight to the portion of uniform load and acting at the center of that portion (since

the whole beam is the body in equilibrium); but in calculating either the shear or bending moment for any segment, only loads *on that segment* may be replaced. When a definite portion of uniform load extends partly on one segment and partly on the other, only the part on the segment under consideration may be replaced. For example, in determining the shear or bending moment for a section at s, *all* of the uniform load ab may be replaced by a single load $A = 4 \times 200$ since *all* of ab is on the L segment, but only a portion of the load cd is on the L segment, namely, the portion from c to the section s, i.e., $4 \times 400 = 1600$ lbs.; hence only this portion may be replaced by a single force, namely, $B' = 1600$ lbs. acting midway between c and s. The difference between $B = 2000$ lbs., the force used in calculating the reaction, and the force $B' = 1600$ lbs. used in calculating shear or bending moment is not merely one of magnitude (400 lbs.), but also one in line of action since the line of action of B is one-half a foot nearer s than the line of action of B'.

1. Similarly, in finding the shear or bending moment for the R segment, *all* of the load ef may be replaced by a single load, namely, $C = 600$ lbs., and all of the load gh may be replaced by a single load, namely, $D = 300$ lbs.; but only a portion of the load from c to d may be replaced, namely, the portion from s to d equal to 400 lbs., since only this portion is on the R segment.

2. *Caution:* No mistake is more common in calculating shears and bending moments for uniform load than that of replacing *all* of a uniform load by a single force when only *a portion* of that load is on the segment. To avoid this mistake, think of the segment as separate from the rest of the beam and ask yourself: "What are the loads actually on the segment?"

3. *Exercise:* Calculate the bending moment for the left-hand segment 10 ft. long in Fig. 167 (c), and check the result by calculating the bending moment for the corresponding right-hand segment.

4. **Maximum Shear and Maximum Bending Moment.** For any given segment of a beam, the shear and the bending moment due to a given dead load cannot vary, but for different segments there may be different values for either shear or bending moment. The values most often required in designing are the *maximum* values, whatever the corresponding lengths of segments may be. Maximum shear and maximum bending moment do not occur for the same segment or section; on the contrary, the maximum shear occurs for a section for which the bending moment is zero and, conversely, the maximum bending moment occurs for a section for which the shear is zero. For example, the maximum shear for a beam carrying a uniform load from end to end is for a section indefinitely close to a support, a section for which the bending moment is zero, and the maximum bending moment is for a section at the center of the beam, a section for which the shear is zero. Shear diagrams and moment diagrams show for what segments or sections shears and bending moments are maximum, but for most cases the required sections may be determined more quickly by means of the following principles:

5. *Maximum shear* will usually occur for a section indefinitely close to a support. There may be other sections for which the shear is as great, but no section for which the shear is greater.

6. *Maximum bending moment.* (1) For concentrated loads the maximum bending moment will occur for a section at one of the loads; it will be the section for which the shear changes from positive to negative — passes through zero. For loads that are approximately equal this section will be at the load nearest the center of the beam. For a uniform load, extending from end to end of the beam or for equal distances on each side of the center of the beam, the maximum bending moment will occur for a section at the center — a section for which the shear is zero. For a restricted length of uniform load unsymmetrically placed with respect to the center of the beam, the maximum bending moment will be for a section for which the shear is zero. In general, *the maximum bending moment due either to concentrated loads or to a uniform load of any length will occur for a section for which the shear is zero or passes through zero.* There may be, in some cases, more than one such section.

7. *Note:* Determining maximum shear or maximum bending moment is largely a matter of knowing where to assume the section. Once this is done, the actual calculation is usually simple.

8. **Relations Between Shears and Bending Moments.** In addition to the relation between maximum bending moment and zero shear, just stated, there are other important relations which may be summarized as follows:

9. The bending moments for different sections between two successive concentrated loads increase or decrease in proportion to the constant shear for all sections between those two loads. (166 : 1.) Hence, $y = S \times$

x, in which S represents the constant shear, x the distance of any section from one of the two loads, and y the difference between the bending moment for the section between loads and the bending moment for the section at the load from which x is measured. $y = S \times x$ is the equation of the straight line which is that portion of the bending moment curve that corresponds to bending moments for sections between two given loads. (165 : 6.)

1. If the shear is zero for all sections between any two points of a beam, the bending moments for all of those sections will be equal. (This follows from the principle given in the preceding paragraph.)

2. *Illustration:* Two equal concentrated loads are so placed that the center of a beam is half way between them. The shear for all sections between the loads is zero, and the bending moments for these sections are equal.

3. The bending moment for any segment is equal to the area of that portion of the shear diagram which corresponds to that segment. If part of the area is plus and part minus, the algebraic sum of the areas is equal to the bending moment. (166 : 3.)

4. **Cantilever Beams.** In the calculation of shear or bending moment for a simple cantilever beam, it is not necessary first to determine

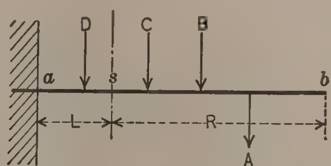


Fig. 169 (*a*).

a reaction; in fact, this reaction is often indeterminate and never a single force. In this respect only do the methods of calculation differ from those for an ordinary beam. In calculating either shear or bending moment for any section of a cantilever beam, select that segment on which there is no reaction (the segment between the section and the unsupported end), regardless of whether it is a right- or left-hand segment. The only forces acting on such a segment are *known*. For example, in Fig. 169 (*a*) the beam is fixed at a but unsupported at b. For any section s the R segment would be selected. The shear for segment R is $-A - B - C$ and the bending moment is the sum of the moments of A, B and C about s. Note that any force on the L segment, such as D, does not affect the shear or bending moment, nor is it involved in any way in the calculations (since it is not on the R segment), nor does it cause any reaction at b as it would if ab were an ordinary beam.

5. If a uniform load is on the beam as in Fig. 169 (*b*), the shear for segment R is equal to the amount of load on segment R, regardless of

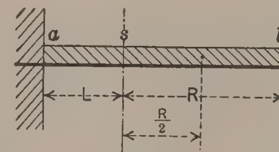


Fig. 169 (*b*).

whether or not any load is on segment L, and the bending moment is equal to the load on R multiplied by $\frac{R}{2}$, i.e., the load *on the segment* may be replaced by an equal load concentrated at its center of gravity as in the case of an ordinary beam.

6. The shear for any segment on the side of the section toward the unsupported end is equal to the total load on that segment, whether it be concentrated or uniform.

7. The maximum shear will be for a section at a, though in the case of the concentrated loads, the shear for *any* section to the left of load D (Fig. 169 (*a*)) will be as great.

8. The maximum bending moment will also be for a section at a.

9. *Exercise:* Draw the shear and bending-moment diagrams for each of the beams shown in Figs. 169 (*a*) and 169 (*b*), using the following data: For the beam in Fig. 169 (*a*), $D = 100$ lbs., $C = 400$ lbs., $B = 200$ lbs., $A = 600$ lbs.; beam 18 ft. long from a to b; L = 8 ft.; distance from a: $D = 6$, $C = 11$, $B = 14$, and $A = 16$ ft. For the beam in Fig. 169 (*b*), uniform load 200 lbs. per lin. ft.; beam 18 ft. long; L = 8 ft. By means of the two shear diagrams and the two bending-moment diagrams, verify the statements that maximum shear and maximum bending moment occur for a section at a.

CALCULATIONS OF SHEARS AND BENDING MOMENTS

Section	Shear		Distance between Sections		Bending Moment	Section
At *1*	0					
	−600					
At *2*	−600	×	4	=	−2400	At *2*
	−600					
At *3*	−1200	×	4	=	−4800	
	+2550				−7200	At *3*
At *4*	+1350	×	6	=	+8100	
	−800				+900	At *4*
At *5*	+550	×	10	=	+5500	
	−1200				+6400	At *5*
At *6*	−650	×	5	=	−3250	
	−1600				+3150	At *6*
At *7*	−2250	×	3	=	−6750	
	+3050				−3600	At *7*
At *8*	+800	×	3	=	+2400	
	−400				−1200	At *8*
At *9*	+400	×	3	=	+1200	
					(Check) → 0	At *9*

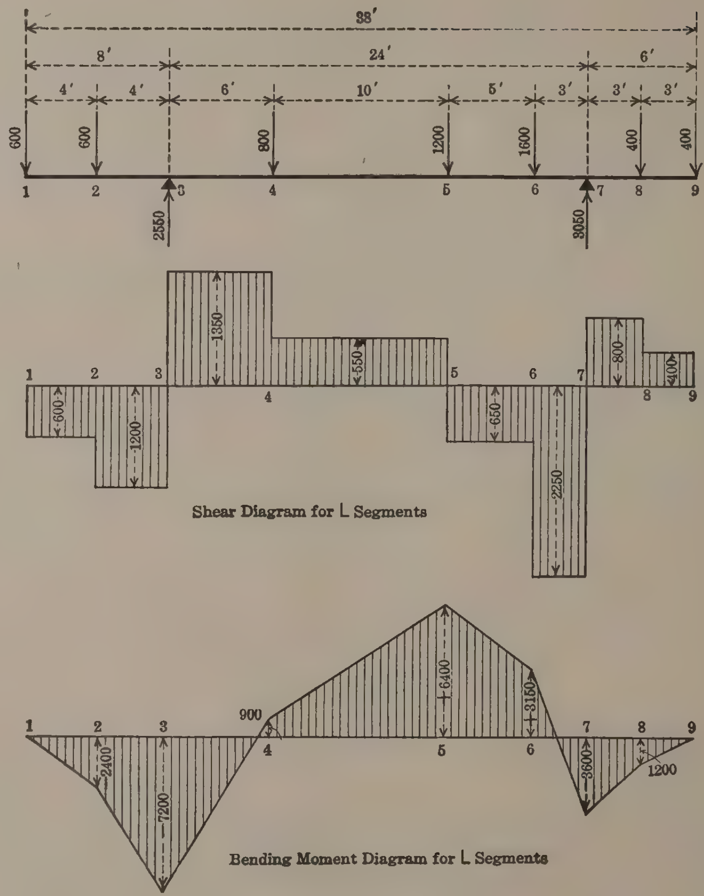


Fig. 170.

1. *Double Cantilever Beam:* The beam with two overhanging or cantilever ends, shown on the opposite page, illustrates exceptionally well a number of principles previously given in this chapter.

The reactions are calculated on page 140.

2. The shear for each section except that at *2* was calculated by adding (algebraically) to the shear for the preceding section the force which acts between those two sections. (Each section is assumed to the left of the corresponding load indefinitely close to that load.)

3. The bending moment for each section except that at *2* was calculated by adding to the bending moment for the preceding section the product of the shear for the section by the distance between sections. For example, the bending moment for the section at *4* is equal to the bending moment for the section at *3* plus the shear for the section at *4* (or for any section between *3* and *4*) multiplied by the distance between *3* and *4*. (165 : 3.)

From inspection of the diagrams the following statements will be evident:

4. The shear for all sections between two successive external forces (two loads or a load and a reaction) is constant. (158 : 11.)

5. The maximum positive shear is for a section indefinitely close to a support (point *3*), and the maximum negative shear is also for a section indefinitely close to a support (point *7*). (158 : 5.) Shears equally great will occur, respectively, for any section between *3* and *4* and between *6* and *7*.

6. The shear changes algebraic sign — passes through zero — for sections at supports *3* and *7*, and also for a section at point *5*.

7. The maximum negative moment is for a section at *3*, and the maximum positive moment is for a section at *5*; both are sections for which the shear passes through zero. (168 : 6.) For the section at *7* for which the shear also passes through zero, the bending moment is the greatest negative bending moment for any section of the right-hand cantilever, though not an absolute maximum for the entire beam.

8. Points of contra-flexure are points just to the left of *4* and just to the right of *6*, at which the bending-moment curve changes from minus to plus or *vice-versa* — passes through zero.

9. The slope of any line in the bending-moment curve is proportional to the constant shear for all sections between the two corresponding loads, and the inclination is downward to the right or upward to the left according to whether the constant shear is negative or positive. (166 : 1.) For example, between *2* and *3* the shear is -1200 lbs. and the slope of the corresponding portion of the bending-moment curve is -1200 downward for every foot horizontal; similarly, the shear between *3* and *4* is $+1350$ lbs. and the slope of the corresponding portion of the bending-moment curve is 1350 upward for every foot horizontal.

10. The bending moment for any segment is equal to the area of that portion of the shear diagram which corresponds to that segment. (166 : 3.) For example, for the segment from *1* to *3* the area of the shear diagram is $-600 \times 4 + (-1200 \times 4) = -7200$ lb.-ft., which is the bending moment for segment *1–3*; similarly, for segment *1–4* the area of the shear diagram is $-7200 + 1350 \times 6 = +900$ lb.-ft., which is the bending moment for segment *1–4*.

11. Neither the bending moment nor the shear for any section outside of the supports is affected by any load between supports, but the shear or the bending moment for any section between supports is affected by each load outside of the supports. (Why?)

12. **Beams with Panels.** In many structures certain loads are applied at panel points only, as, for example, when the dead load of a floor system is transmitted through floor beams to the main girders. These loads from the floor beams are concentrated loads, and the shear or bending moment for any section may be calculated just as for any series of concentrated loads. Since, however, bending moments are usually required in such a case for sections at panel points only, and since the panels are usually of equal length and the loads are of equal magnitude, the short-cut method of calculating bending moments explained in the accompanying illustrative problem may be used to advantage.

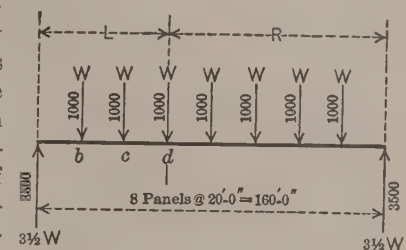


Fig. 171.

13. *Bending Moments for Panel Loads.* Let a beam or truss be divided into panels and a panel load W be applied at each of the panel points except at the supports as shown in Fig. 171. Let p represent the panel length. The bending moment is required for a section at some panel point — for example, at d. The moment may be expressed in terms of W and p.

$$M_B = 3\tfrac{1}{2}W \times 3\,p - W(2p + p) \text{ (for L segment).}$$
$$M_B = W(4p + 3p + 2p + p) - 3\tfrac{1}{2}W \times 5p \text{ (for R segment).}$$

If $W = 1000$ lbs. and $p = 20$ ft., then:

$$M_B = 3500 \times 3 \times 20 - 1000\,(40 + 20) = 150{,}000 \text{ lb-ft. for L segment.}$$
$$M_B = 1000\,(80 + 60 + 40 + 20) - 3500 \times 100 = -150{,}000 \text{ lb.-ft. for R segment.}$$

By finding the sum of the lever arms of *all* the panel loads on a segment, as indicated by the parentheses above, and then multiplying by the panel load, several multiplications may be saved as compared with the method of multiplying each panel load, by its lever arm.

14. An alternative short-cut is to replace all panel loads on a segment by their resultant and calculate the bending moment due to this resultant and the reaction. For example, the resultant of the loads on the L segment is 2000 lbs. acting midway between b and c, and the bending moment due to this resultant and the reaction is $3500 \times 60 - 2000 \times 30 = 150{,}000$ lb.-ft. Similarly, the resultant of the loads on the R segment is 4000 lbs., and M_B for that segment is $4000 \times 50 - 3500 \times 100 = 150{,}000$ lb.-ft.

1. *The maximum shear* for a beam with panels will occur for a section indefinitely near a support, just as in the case of any beam carrying concentrated loads.

2. *The maximum bending moment* will occur for a section at a panel point, since it must occur for a section at a load (168 : 6) and the loads are applied only at panel points. It will be for that section for which the shear passes through zero, just as in the case of any beam carrying concentrated loads. (168 : 6.)

3. The shear is constant for all sections between panel points. (158 : 11.)

4. The bending moment for any section between panel points, if it is desired, may be calculated directly from the forces on the corresponding segment, or it may be found from the bending moment for the section at the nearest panel point on the segment by adding to that bending moment (algebraically) the product of the shear for the segment multiplied by the distance from the panel point to the section, just as for any beam with concentrated loads. (165 : 2.)

5. *Exercise:* Given two beams AB and CD, each 42 ft. long, and each with a uniform load of 200 lbs. per lin. ft. extending from end to end. CD is divided into six panels each 7 ft. long, but AB is a simple beam without panels. Full panel loads of 1400 lbs. are at panel points on CD except at the supports where there are half panel loads. Draw the shear diagram for AB and, superimposed on this, the shear diagram for CD. Draw the bending-moment diagram for AB and, superimposed on this, the bending-moment diagram for CD. Observe: (1) That the bending moments for sections at panel points CD are the same as for corresponding points of AB, but that for sections between panel points the bending moment is less for CD than for AB. (2) That the shear for a section at the center of each panel of CD is equal to the shear for the corresponding point of AB, but for a section at any panel point the shear for CD is greater than for AB, except at the supports where it is less. (The maximum shear is less for CD than for AB.)

6. **Shears and Moments Due to Both Concentrated and Uniform Loads.** It is often necessary to determine the combined shear and the combined bending moment due to fixed concentrated loads and fixed uniform loads. For example, in the case of a girder which supports one or more columns in a building, the weight of the girder is uniformly distributed whereas the loads from the columns are concentrated. It is usually simpler in such cases to calculate separately the shear or the bending moment due to each kind of load, as if there were two beams, one with uniform load and the other with concentrated loads, and then to combine results. An additional reason for separate calculations is that often the uniform load is static whereas the concentrated loads are moving, in which case, as will be shown later, separate calculations are usually necessary; it is well, therefore, from the beginning, habitually to separate calculations for uniform load from those of concentrated loads.

7. *Combined Shear.* The combined shear for any given segment, due to fixed uniform and concentrated loads, is simply the algebraic sum of the results found by determining the shears for each kind of load separately. The *maximum combined shear* is usually the combined shear for a section indefinitely close to a support.

8. *Combined Bending Moment.* The combined bending moment for any given segment is found, as for shear, by merely adding results. The *maximum combined bending moment* is not always as easily determined, since the maximum bending moment due to uniform load and the maximum bending moment due to concentrated load may not occur for the same section. For example, a single concentrated load may be on a beam at some point other than the center. The maximum bending moment due to the weight of the beam is for a section at the center, while the maximum bending moment due to the concentrated load will be for a section at the load. One method of determining the section for which the combined bending moment is greater than for any other section is to plot the bending-moment diagrams for uniform load and that for the concentrated load in a combined diagram, so arranged that the ordinate at any point is equal to the combined bending moment for the corresponding section, and from inspection ascertain for which section the combined bending moment is a maximum. The maximum may also be determined algebraically. It will occur for a segment for which the combined shear due to concentrated loads and uniform load is zero. In actual practice the section for which the combined bending moment is a maximum is usually evident from a comparison of the loads, particularly if the concentrated load is large when compared with the uniform load, as it frequently is.

9. It is incorrect, theoretically, to combine the uniform-load bending moment for one segment with the concentrated-load bending moment for a different segment, but this may often be done in practice without appreciable effect on the design, as, for example, when the bending moment due

to the weight of a beam for a section at the center of the beam is combined with the concentrated-load bending moment for a section at a load quite near the center. The bending moment thus obtained is slightly greater than any that can actually occur, and therefore the result is on the side of safety.

1. **Inclined Loads.** If external forces or loads are not vertical but inclined, the vertical shear and the corresponding bending moment for any segment will be calculated from the V components of the inclined external forces — the H components cannot affect vertical shear or bending moment. The V component of each force is considered as applied at the point in which the line of action of that force intersects the neutral axis of the beam.

SUMMARY OF THE CHAPTER

2. *Section* is the cross section which lies in a cutting plane at right angles to the axis of the beam; for co-planar forces, this section is a straight line. *Segment* is the portion of the beam on one side or the other of the section. (156 : 1.)

SHEAR

3. *Shearing forces* are *external* forces *parallel* to a section that is normal to the longitudinal axis of the beam. Shearing forces include reactions as well as loads. If external forces are not parallel to a section, their components parallel to the section are the shearing forces. The most common shearing forces are vertical since the most common external forces are those due to gravity. Shearing forces on one segment of a beam tend to move that segment parallel to the section, while the shearing forces on the other segment tend to move that segment also parallel to the section but in exactly the opposite direction. (156 : 6.)

4. *Reaction a shearing force which must be determined.* Neither the shear nor the bending moment for any segment can be determined until the reaction on that segment, if there is one, has been determined. (There always is a reaction except in the case of a cantilever beam.) Once the reaction on a segment is known, only the external forces *on that segment* should be considered; forces elsewhere on the beam should be ignored.

5. *Shear* for a segment is the algebraic sum of the shearing forces on that segment. The most common shear is vertical shear. If the shearing forces on a segment of a horizontal beam tend to move that segment *upward*, the shear for that segment is *plus;* if they tend to move the segment *downward*, the shear for the segment is *minus*. If there is but one shearing force on a segment, the shear is equal to that force, regardless of whether the force is a load or a reaction. (156 : 4.)

6. *Shear for right and left segments equal.* For any given beam and section, the shear for one segment is equal in magnitude to the shear for the other segment but of opposite algebraic sign. (156 : 4.)

7. *Shearing stress* at a section is the sum of all *internal* forces that resist the shear for that section. The external forces tend to move a segment up or down; shearing stresses hold the segment in equilibrium. (157 : 1.)

8. *End shear* is the shear for a section indefinitely close to the end of a beam; for a simple beam supported at each end, the end shear is equal to the corresponding reaction. (158 : 5.)

9. *Maximum shear* for a given beam and loading is the shear for a section so chosen that the algebraic sum of the forces on either segment is as great or greater than the shear for any other section. For a simple beam, the end shear is the maximum shear for either concentrated or uniform loads. (158 : 5.)

SHEAR DUE TO CONCENTRATED LOADS

10. Shears for all sections between two successive external forces are equal. (Shear is constant.)

11. A load on either segment tends to cause *negative* shear for that segment.

12. If a section is taken at a load and the load is assumed to act first on one segment and then on the other, the transference of the load from one segment to the other will affect the shear *for the segment to which the load is transferred* in one of three ways, namely, (1) it will increase a negative shear, (2) it will decrease a positive shear, or (3) it will change a positive shear to a negative shear.

13. If a section is assumed indefinitely close to a load and the maximum positive shear is desired for one of the segments, the load must be considered as on the other segment; if the maximum negative shear is desired for a segment, the load must be considered as on the segment.

14. If a simple beam is symmetrically loaded with respect to its center, the shear for a section at the center is zero.

SHEAR DUE TO UNIFORM LOAD

1. That portion of any uniform load *which is on a segment* may be replaced by an equivalent concentrated load, and the problem of finding shear may thus be reduced to one of concentrated loads.

2. If every portion of a beam is covered with uniform load, shear cannot be constant between any two sections.

3. Load on a segment tends to cause negative shear for that segment.

4. When a beam is supported at each end and symmetrically loaded with uniform load, the shear for a section at the center is zero.

SHEAR CURVES AND DIAGRAMS

5. *A shear curve* is a curve such that if at any point of the beam an ordinate is measured between the curve and an assumed base, this ordinate will equal the shear for the section at that point. A shear curve and base (shear diagram) is a graphic representation of the variation in magnitudes of shears for all left-hand segments (or for all right-hand segments) varying in length from zero to the length of the beam. For a given beam and loading, the shear diagram for left-hand segments is the same as the shear diagram for right-hand segments, except that at any point the two curves are on opposite sides of the base (above or below for vertical shear) an equal distance from that base.

6. *For concentrated loads* the shear curve is a series of straight lines. For vertical shear the portion of the shear curve between two successive loads is *horizontal* since the shear between two such loads is constant. (160 : 1.)

7. *For uniformly distributed load* extending the entire length of a beam supported at each end, the shear curve is a straight inclined line which crosses the base at the center of the beam; no portion is horizontal since shear is not constant between any two sections. The ordinate at either support is equal to the reaction at that support. (161 : 5.)

8. The plus area in a shear diagram is equal to the minus area, both for concentrated and for uniform loads.

BENDING MOMENT

9. *Bending moment* for a segment is the algebraic sum of the moments of the shearing forces on that segment about any point in the section. The most common case is that in which the axis of the beam is horizontal and the shearing forces and section are vertical. Bending moments clockwise are plus, — counter-clockwise, minus.

10. *Bending moments for two corresponding segments are equal.* For any given beam and section, the bending moment for one segment is equal in magnitude to the bending moment for the other segment, but of opposite algebraic sign.

11. *Effect of a load on algebraic signs.* In the case of a beam supported at each end, any load, concentrated or uniform, anywhere on the beam tends to cause a positive bending moment for any left-hand segment and a negative bending moment for any right-hand segment.

12. *Bending Moment and Resisting Moment.* Bending moment is the measure of the tendency of the *external* forces on a segment to cause that segment to rotate about an axis of moments at the corresponding section; resisting moment is the resultant moment of certain *internal* forces which resist this rotation. (163 : 1 and 163 : 4.)

13. The bending moment and the resisting moment for a given segment are equal in magnitude but of opposite algebraic sign. Their algebraic sum, therefore, equals zero, i.e., the algebraic sum of the moments of the forces (external and internal) which act on a given segment must equal zero to satisfy the moment equation of equilibrium $\Sigma M = 0$.

14. *Maximum bending moment* for a given beam and loading is the bending moment for a section so chosen that the magnitude of the bending moment for that section is as great or greater than the bending moment for any other section.

BENDING MOMENTS DUE TO CONCENTRATED LOADS

15. For a beam supported at each end, the maximum bending moment will be for a section at one of the concentrated loads; if the loads are equal or approximately equal, this section will be at the load nearest the center of the beam; the bending moment at either support is zero.

16. If N and O are the lengths of two left-hand segments and O is the longer, the bending moment for the segment O is equal to the bending moment for the segment N plus the *product* found by multiplying the constant shear for all segments whose lengths are between N and O by the lever-arm difference O minus N. (165 : 2.) If there are external

forces, loads, or reactions between n, the section for segment N, and o, the section for segment O, the algebraic sum of the moments of these forces about o must be subtracted from the product just referred to. (165 : 4.)

BENDING MOMENTS DUE TO UNIFORM LOAD

1. That portion of any uniform load *which is on a segment* may be replaced by an equivalent concentrated load, and the problem of calculating the bending moment may then be reduced to one of concentrated loads. *Caution:* No uniform load which is not on the segment should be included in an equivalent concentrated load. (168 : 2.)

2. If a beam rests on a support at each end and a uniform load extends the entire length of the beam, the bending moment for *any* section is equal to the product of the lengths of the corresponding L and R segments (in feet) multiplied by one-half the uniform load per linear foot.

3. For a beam supported at each end and loaded its entire length with uniform load, the maximum bending moment will be for a section at the center of the beam; the bending moment at either support is zero.

4. *Bending moments for concentrated and uniform loads compared.* The bending moment for any section due to a single concentrated load at that section is twice as great as the bending moment for the same section due to a load equal in amount to the concentrated load, but uniformly distributed from end to end of beam.

5. *Broken uniform loads* may be replaced by equivalent concentrated loads in calculating shear or bending moment; but if the section is taken through one of the broken portions, only that part of the broken portion which is on the segment may be replaced by an equivalent concentrated load. (167 : 6.)

BENDING-MOMENT CURVES AND DIAGRAMS

6. *A bending-moment curve* is a curve such that if at any point of the beam an ordinate is measured between the curve and an assumed base, this ordinate will equal the bending moment for the section at that point. A bending-moment curve and base (bending-moment diagram) is a graphic representation of the magnitudes of bending moments for all left-hand segments (or for all right-hand segments) varying in length from zero to the length of the beam. For a given beam and loading, the bending-moment curve for left-hand segments is the same as that for right-hand segments except that at any point the curves are on opposite sides of the base.

7. *For concentrated loads, the bending-moment curve* is a series of straight lines between external forces (loads or reactions). No line of the curve is horizontal unless the bending moment is constant for all sections between two successive external forces. The slope of the line between any two external forces is proportional to the shear for any section between those forces. (165 : 6 and 171 : 9.)

8. *For uniformly distributed load* extending the entire length of a beam supported at each end, *the bending-moment curve* is a parabola which passes through the two ends of the base line and through a point (the vertex) above the center of the base line. The ordinate to the vertex is equal to $\frac{wl^2}{8}$, in which w is the load per linear foot and l the length of the beam.

SECTIONS FOR MAXIMUM SHEAR OR BENDING MOMENT

9. *Maximum shear* will occur for a section indefinitely close to a support.

10. *Maximum bending moment* will occur for a section for which the shear is zero or passes through zero. For concentrated loads this section will be at one of the loads, usually one near the center of the beam; for a uniform load completely covering the beam or extending equal distances each side of the center of the beam, the section will be at the center of the beam. For a restricted portion of uniform load unsymmetrically placed with respect to the center, the section will be such that the load on the segment divided by the length of the segment will equal the total load on the beam divided by the length of the beam.

RELATIONS BETWEEN SHEAR AND BENDING MOMENT

11. The maximum bending moment is for a section for which the shear is zero; and the maximum shear occurs, for ordinary beams, for a section at the support (for which the bending moment is zero).

12. The bending moment for different sections between two successive loads increases or decreases in proportion to the constant shear for all sections between those two loads. Hence $y = Sx$, in which y = differ-

ence in bending moment, S = shear, and x = distance between the two sections for which the difference y in bending moments is required.

1. If the shear for all sections between any two points of a beam is zero, the bending moment will be constant for all sections between the same points.

2. The bending moment for any segment is equal to the area of that portion of the shear diagram which corresponds to that segment, this area being considered as the algebraic sum of plus and minus areas. (171 : 10.)

MISCELLANEOUS

3. *Cantilever beam.* The segment used is the segment toward the unsupported end. The shear is simply the load on the segment; the bending moment is the sum of the moments of the loads on the segment, the center of moments being at the section. Neither the shear nor the bending moment for any section is affected by any load between the section and the support. Maximum shear and bending moment both occur for a section at or indefinitely near to a support. (For double cantilever beams see the illustrative example (171 : 1).)

4. *Beams with panels* are merely beams with concentrated loads, usually equal in magnitude and at equal distances apart. The principles pertaining to concentrated loads on an ordinary beam hold true for a beam with panels. A short-cut method of calculating bending moments for sections at panel points is given in 171 : 13 and 14.

5. *Shears and moments due to both concentrated and uniform loads* should be calculated as if there were two beams, one with uniform load and one with concentrated load, and the results may then be combined. Avoid combining the shear or bending moment for one segment with the shear or bending moment for a different segment. (See 172 : 9 for an exception.) (For maximum bending moment due to combined concentrated and uniform loads see 172 : 8.)

6. **Notation and Algebraic Signs.** Throughout this book, *shear* is understood to mean *vertical* shear unless otherwise stated, and is denoted by "V." Bending moment is denoted by "M_B."

7. When only concentrated loads are under consideration, a beam or a segment of a beam is represented by a *single* horizontal line. When a load is uniformly distributed, a *second* horizontal line is drawn slightly above the first, extending over that portion of the beam covered by the load, — the entire length if the uniform load extends from end to end. When any portion of uniform load is replaced by a single equivalent concentrated load, the latter frequently is shown acting above or below the double horizontal line; but in such a case the concentrated load should be used for purposes of calculation, and the uniform load, indicated by the double line, should be ignored.

8. For any given section, the shear for one segment is plus and for the other it is minus. Unless otherwise stated, the algebraic sign given for shear is for the *left-hand* segment. *Positive* shear then means that the shearing forces on the left-hand segment tend to move that segment *upward; negative* shear means that they tend to move it *downward.*

9. For any given section the bending moment for one segment is plus and for the other it is minus. Unless otherwise stated, the algebraic sign given for bending moment is for the *left-hand* segment. *Positive* bending moment then means that the external forces (shearing forces) on the left-hand segment tend to revolve that segment *clockwise* about a center of moments anywhere in the section; *negative* bending moment means that they tend to revolve it *counter-clockwise.*

10. In calculating shear or bending moment, it is efficient to work with whichever segment has the smaller number of external forces acting on it. If this happens to be the right-hand segment, the resulting algebraic sign for shear or bending moment must be reversed if it is desired that the final algebraic sign shall be for the left-hand segment. This is well illustrated in the calculation of the shear and bending moment for segment *ad* in *Problem 6* at the end of this chapter.

11. **Illustrative Problems.** The illustrative problems that are given in the remainder of this chapter offer an opportunity for studying the practical applications of the methods of calculating shear and bending moment and of the methods of drawing shear diagrams and bending-moment diagrams.

12. *The arrangement of the algebraic work* is that already used in the illustrative problems in Part I, and, although it is unusual, it is efficient. Reading from left to right, *results* are given first, then the quantities used in obtaining those results are indicated, and finally what the results stand for is denoted, as, for example, whether the result is a reaction, shear, or

bending moment. This arrangement brings all results under each other where they may be found easily and checked; should a result be questioned, the *method* of obtaining it may then be checked by observing the quantities indicated. This arrangement will be used frequently throughout this book.

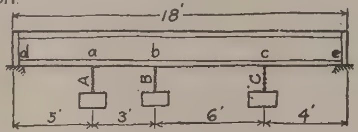
1. In problems in which there are two types of loading, concentrated and uniform, the shears and bending moments are calculated first for one type and then for the other, and the results are then combined in order to obtain the total shear or the total bending moment.

2. A separate sketch is shown for each segment for which the shear or bending moment is calculated. For the beginner this is a wise precaution, but when one has become accustomed to calculating shears and bending moments, such sketches are not necessary.

3. All algebraic signs are for left-hand segments. In studying the illustrative problems, the meaning of these signs, as indicated by the arrows, should be observed carefully.

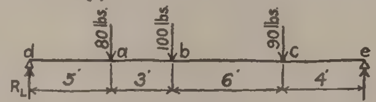
4. The shear diagrams and bending-moment diagrams have been plotted to scale, although, as already stated, it is frequently sufficient to draw such diagrams free-hand.

Problem I. Three chains A, B, and C are attached to a girder *de* as shown in the sketch.



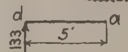
The loads carried by the chains, including their own weights, are A = 80 lbs, B = 100 lbs., and C = 90 lbs. The weight of the beam may be neglected.

Required: (a) The shear and the bending moment for each of the segments *da*, *db*, and *dc*. (b) The shear and bending moment diagrams plotted to scale. (c) The maximum shear and the maximum bending moment.



$133^{\#} = [90\times4+100\times10+80\times13]\div18 = \text{Reaction } R_L$

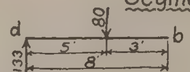
Segment *da* = 5 ft.



$133^{\#} = V\uparrow$

$665^{\#\text{-ft.}} = 133\times5 = M_B$

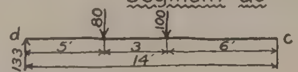
Segment *db* = 8 ft.



$53^{\#} = 133-80 = V\uparrow$

$824^{\#\text{-ft.}} = 133\times8-80\times3 = M_B$

Segment *dc* = 14 ft.



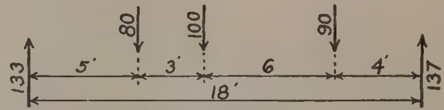
$-47^{\#} = 133-80-100 = V\downarrow$

$542^{\#\text{-ft.}} = 133\times14-80\times9-100\times6 = M_B$

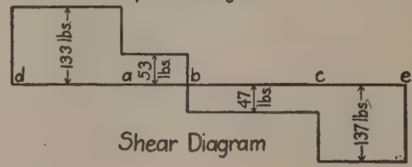
$-137^{\#} = 133-80-100-90 = V$ (for section between *c* and *e*) $\downarrow = -R_R$

SHEARS AND BENDING MOMENTS CHECKED

Section		Shear		Distance between sections		Bending Moment M_B		Section
At *a*	=	133	×	5	=	665	at	*a*
		−80						
At *b*	=	53	×	3	=	159		
		−100				824	at	*b*
At *c*	=	−47	×	6	=	−282		
		−90				542	at	*c*
At *e*	=	−137						

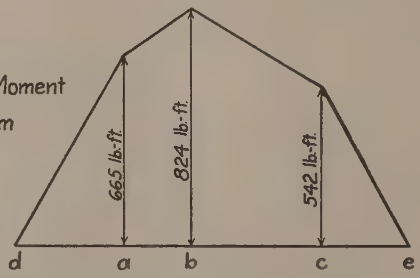


Space Diagram



Shear Diagram

Bending Moment Diagram

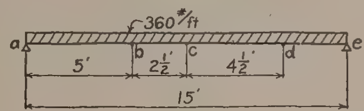


133 lbs. = Maximum positive shear (end shear)

−137 lbs. = Maximum negative shear (end shear)

824 lb-ft = Maximum bending moment (section at b).

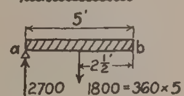
Problem 2. A steel I-beam supports a uniform load, including its own weight, of 360 lbs. per linear foot as shown in the sketch.



Required: (a) The shears and bending moments for the segments ab, ac, and ad. (b) The shear and bending moment diagrams plotted to scale. (c) The maximum shear and the maximum bending moment.

$2700 = 360 \times 15 \div 2 = R_L$

Segment ab = 5 ft.

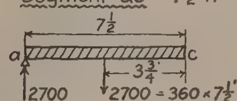


$900^{\#} = 2700 - 1800 = V \uparrow$

$9000^{\#ft} = 2700 \times 5 - 1800 \times 2\tfrac{1}{2} = M_B$

$9000^{\#ft} = \frac{360}{2} \times 5 \times 10 = M_B$ = Check by short-cut method.

Segment ac = $7\tfrac{1}{2}$ ft

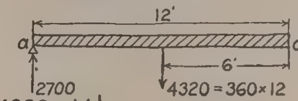


$0 = 2700 - 2700 = V$

$10125^{\#ft} = 2700 \times 7\tfrac{1}{2} - 2700 \times 3\tfrac{3}{4} = M_B$

$10125^{\#ft} = \frac{360}{2} \times 7\tfrac{1}{2} \times 7\tfrac{1}{2} = M_B$ (Check)

Segment ad = 12 ft.

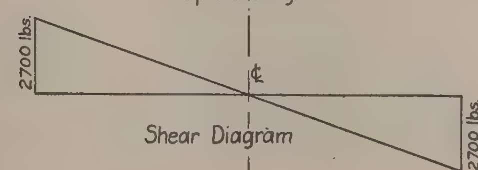


$-1620^{\#} = 2700 - 4320 = V \downarrow$

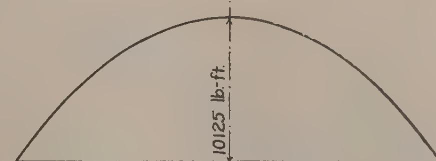
$6480^{\#ft} = 2700 \times 12 - 4320 \times 6 = M_B$

$6480^{\#ft} = \frac{360}{2} \times 12 \times 3 = M_B$ (Check)

Space Diagram



Shear Diagram



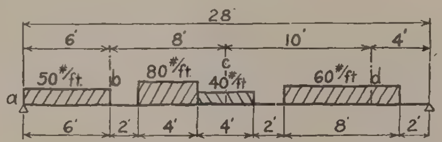
Bending Momemt Diagram

$2700^{\#}$ = Maximum positive shear (end shear)

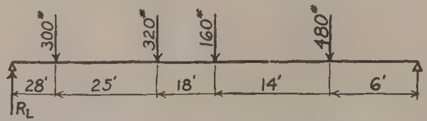
$-2700^{\#}$ = Maximum negative shear (end shear)

$10125^{\#ft}$ = Maximum bending moment (section at c)

Problem 3. A beam supports broken uniform loads as shown in the sketch The weight of the beam may be neglected.

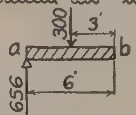


Required: (a) The shears and bending moments for segments ab, ac, and ad. (b) The shear and moment diagrams plotted to scale. (c) The maximum shear and the maximum bending moment.



$656^{\#} = (+300 \times 25 + 320 \times 18 + 160 \times 14 + 480 \times 6) \div 28 = R_L$

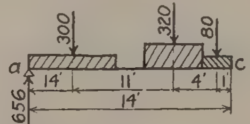
Segment ab = 6 ft.



$356^{\#} = 656 - 300 = V \uparrow$

$3036^{\#\cdot ft} = 656 \times 6 - 300 \times 3 = M_B$

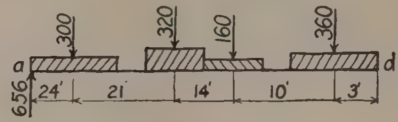
Segment ac = 14 ft.



$-44^{\#} = 656 - 300 - 320 - 80 = V \downarrow$

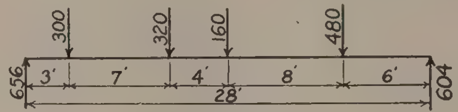
$4524^{\#\cdot ft} = 656 \times 14 - 300 \times 11 - 320 \times 4 - 80 \times 1 = M_B$

Segment ad = 24 ft.

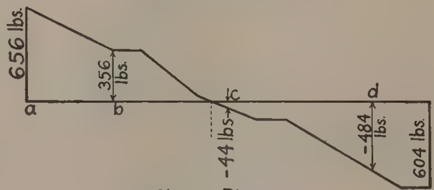


$-484^{\#} = 656 - 300 - 320 - 160 - 360 = V \downarrow$

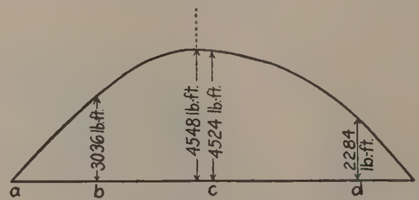
$2284^{\#\cdot ft} = 656 \times 24 - 300 \times 21 - 320 \times 14 - 160 \times 10 - 360 \times 3 = M_B$



Space Diagram



Shear Diagram



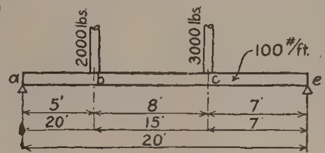
Bending Moment Diagram

$656^{\#}$ = Maximum positive shear (end shear)

$-604^{\#}$ = Maximum negative shear (end shear)

$4548^{\# ft}$ = Maximum bending moment (Occurs at point of zero shear)

Problem 4. A beam weighing 100 lbs per linear foot supports two columns carrying loads respectively of 2000 lbs. and 3000 lbs. as shown in the sketch

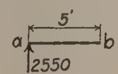


Required: (a) The shears and bending moments for segments ab and ac. (b) The shear and moment diagrams plotted to scale (c) The maximum shear and the maximum bending moment

CONCENTRATED LOADS

$2550^{\#} = (2000 \times 15 + 3000 \times 7) \div 20 = R_L$

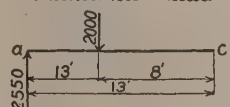
Segment ab = 5 ft.



$2550^{\#} = V \uparrow$

$12750^{\#\text{-ft}} = 2550 \times 5 = M_B$ ↻

Segment ac = 13 ft.



$550^{\#} = 2550 - 2000 = V \uparrow$

$17150^{\#\text{-ft}} = 2550 \times 13 - 2000 \times 8 = M_B$ ↻

UNIFORM LOADS

$1000^{\#} = 100 \times 20 \div 2 = R_L$

$500^{\#} = 1000 - 5 \times 100 = V$ for segment ab. ↑

$3750^{\#\text{-ft}} = \frac{100}{2} \times 5 \times 15 = M_B$ for segment ab. ↻

$-300^{\#} = 1000 - 13 \times 100 = V$ for segment ac. ↓

$4550^{\#\text{-ft}} = \frac{100}{2} \times 13 \times 7 = M_B$ for segment ac. ↻

TOTAL SHEARS

End Shear

$2550^{\#}$ = Concentrated
$1000^{\#}$ = Uniform
$3550^{\#}$ = Total end shear

Segment ab

$2550^{\#}$ = Concentrated
$500^{\#}$ = Uniform
$3050^{\#}$ = Total shear for ab

Segment ac

$550^{\#}$ = Concentrated
$-300^{\#}$ = Uniform
$250^{\#}$ = Total shear for ac

End Shear (Section at e)

$-2450^{\#}$ = Concentrated
$-1000^{\#}$ = Uniform
$-3450^{\#}$ = Total end shear (section at e)

TOTAL BENDING MOMENTS

Segment ab

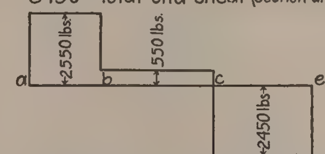
$12750^{\#\text{-ft}}$ = Concentrated
$3750^{\#\text{-ft}}$ = Uniform
$16500^{\#\text{-ft}}$ = Total M_B for ab

Segment ac

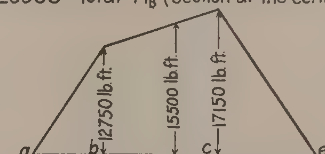
$17150^{\#\text{-ft}}$ = Concentrated
$4550^{\#\text{-ft}}$ = Uniform
$21700^{\#\text{-ft}}$ = Total M_B for ac

Section at the center

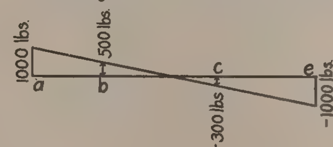
$15500^{\#\text{-ft}} = 2550 \times 10 - 2000 \times 5$ = Concentrated
$5000^{\#\text{-ft}} = \frac{100}{2} \times 10 \times 10$ = Uniform
20500 = Total M_B (section at the center)



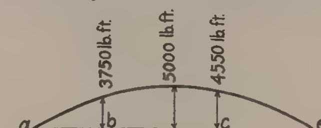
Shear Diagram for Concentrated Loads



Moment Diagram for Concentrated Loads



Shear Diagram for Uniform Load



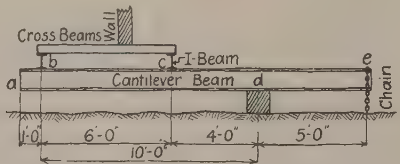
Moment Diagram for Uniform Load

$3550^{\#}$ = Maximum positive shear (end shear)

$-3450^{\#}$ = Maximum negative shear (end shear)

$21700^{\#\text{-ft}}$ = Maximum bending moment (section at c)

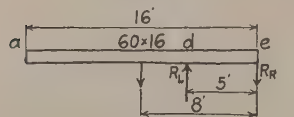
Problem 5 Two I-beams rest on a cantilever beam as shown in the sketch. The cantilever beam rests on a support at d and is anchored down by a chain at e. The I-beams support temporarily, during excavation underneath, a portion of a wall which rests on cross beams at right angles to the I-beams



The loads brought to the cantilever at b and c, including all weights except that of the cantilever beam itself, are respectively 3000 lbs. and 5000 lbs. The cantilever beam weighs 60 lbs. per lin. ft. Required: (a) The shears and bending moments for the segments ac, ad, and ae. (b) The shear and moment diagrams drawn to scale. (c) The maximum shear and the maximum moment

UNIFORM LOAD

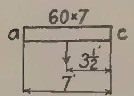
Reaction



$1536^{\#} = (60\times16\times8)\div5 = R_L\uparrow$

$-576^{\#} = +60\times16 - 1536 = R_R\downarrow$

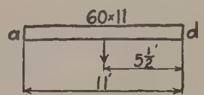
Segment ac = 7 ft.



$-420^{\#} = V\downarrow$

$-1470^{\#\text{-ft.}} = -420\times3\frac{1}{2} = M_B$ ↺

Segment ad = 11 ft.



$-660^{\#} = V\downarrow$

$-3630^{\#\text{-ft.}} = -660\times5\frac{1}{2} = M_B$ ↺

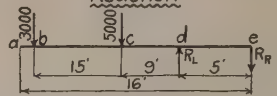
Segment ae = 16 ft.

$576^{\#} = V\uparrow = -(-R_R)\downarrow$

$0 = M_B$

CONCENTRATED LOADS

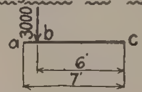
Reaction



$18000^{\#} = (+3000\times15 + 5000\times9)\div5 = R_L\uparrow$

$-10000^{\#} = 3000 + 5000 - 18000 = R_R\downarrow$

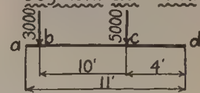
Segment ac = 7 ft.



$-3000^{\#} = V\downarrow$

$-18000^{\#\text{-ft.}} = 3000\times6 = M_B$ ↺

Segment ad = 11 ft.



$-8000^{\#} = -3000 - 5000 = V\downarrow$

$-50000^{\#\text{-ft.}} = -3000\times10 - 5000\times4 = M_B$ ↺

$= -(R_R\times5)$ Check.

Segment ae

$10000^{\#} = V\uparrow = -(-R_R)\downarrow$

$0 = M_B$

TOTAL SHEARS

$-3420^{\#} = -420 - 3000$ for ac ↓

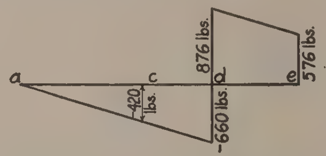
$-8660^{\#} = -660 - 8000$ for ad ↓

$10576^{\#} = 576 + 10000$ for ae ↑

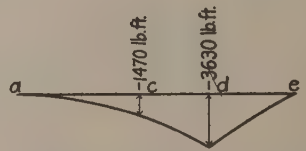
TOTAL BENDING MOMENTS

$-19470^{\#\text{-ft.}} = -1470 - 18000$ for ac ↺

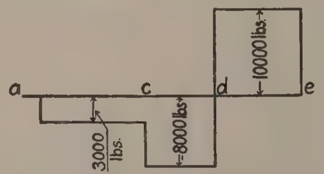
$-53630^{\#\text{-ft.}} = -3630 - 50000$ for ad ↺



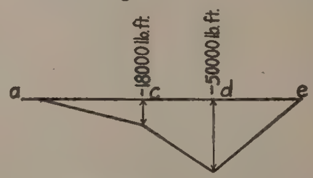
Shear Diagram for Uniform Load



Moment Diagram for Uniform Load



Shear Diagram for Concentrated Loads



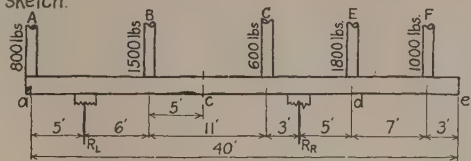
Moment Diagram for Concentrated Loads

$-8660^{\#} =$ Maximum negative shear

$10876^{\#} =$ Maximum positive shear

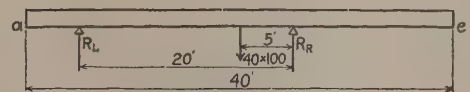
$-53630^{\#\text{-ft.}} =$ Maximum Bending Moment ↺

Problem 6 A double cantilever girder ae supports five columns as shown in the sketch.



The beam weighs 100 lbs. per lin. ft. The column loads are: A=800 lbs., B=1500 lbs., C=600 lbs., E=1800 lbs., and F=1000 lbs. Required: The shears and bending moments for segments ac and ad.

UNIFORM LOADS

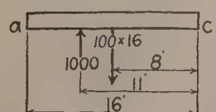


Reactions

$1000^{\#}=[(40\times100)\times5]\div20=R_L$

$3000^{\#}=4000-1000=R_R$

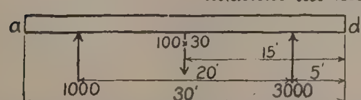
Segment ac = 16 ft.

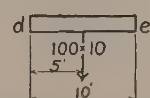


$-600^{\#}=1000-100\times16=V\downarrow$

$-1800^{\#ft}=1000\times11-1600\times8=M_B$ ↻

Segment ad = 30 ft.

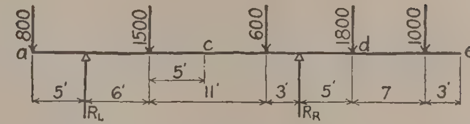




$1000^{\#}=1000-3000+3000=V=-(-100\times10)$ Check $\uparrow$ (Segment de)

$-5000^{\#ft}=1000\times25-3000\times15+3000\times5=M_B=-100\times10\times5$ Check ↻

CONCENTRATED LOADS

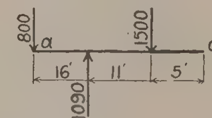


Reactions.

$1090^{\#}=[800\times25+1500\times14+600\times3-1800\times5-1000\times12]\div20=R_L$

$4610^{\#}=[800+1500+600+1800+1000]-1090=R_R$

Segment ac = 16 ft.

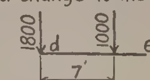


$-1210^{\#}=-800+1090-1500=V\downarrow$

$-8310^{\#ft}=-800\times16+1090\times11-1500\times5=M_B$ ↻

Segment ad = 30 ft.

(Use the segment de and change to the opposite algebraic signs)



$2800^{\#}=-(-1800-1000)=V$ for ad $\uparrow$

$-7000^{\#ft}=-(1000\times7)=M_B$ for ad ↻

TOTAL SHEARS

For segment ac

$-600^{\#}$ = Uniform load

$-1210^{\#}$ = Concentrated load

$-1810^{\#}$ = Total $\downarrow$

For segment ad

$1000^{\#}$ = Uniform load

$2800^{\#}$ = Concentrated load

$3800^{\#}$ = Total $\uparrow$

TOTAL BENDING MOMENTS

For segment ac

$-1800^{\#ft}$ = Uniform load

$-8310^{\#ft}$ = Concentrated load

$-10110^{\#ft}$ = Total ↻

For segment ad

$-5000^{\#ft}$ = Uniform load

$-7000^{\#ft}$ = Concentrated load

$-12000^{\#ft}$ = Total ↻

CHAPTER XIV

ALGEBRAIC METHOD OF SECTIONS

In this chapter is explained the general method of determining stresses by the **algebraic method of sections.** This is the method in which the body in equilibrium is always some rigid portion or segment of the truss.

1. EXTERNAL FORCES. Before the dead-load stresses can be calculated, the external forces (dead loads and reactions) that cause these stresses must be determined.

2. *Dead load.* The dead load, consisting of the weight of the floor system, the weight of the trusses, and the weight of the bracing, may be reduced to weight per linear foot of bridge, one-half of which is the dead load per linear foot of truss. (128 : 4 to 130 : 1.)

3. *Panel load.* When panels are equal in length, as they usually are, the dead load per linear foot of truss multiplied by the panel length is the panel load per truss. The half-panel loads at the supports may be disregarded in the calculation of stresses in the trusses. (127 : 5.) The number of panel loads will then be one less than the number of panels.

4. Throughout this chapter, fictitious panel loads of 1000 lbs. each will be assumed in problems that are used to illustrate various principles. (85 : 3.) These panel loads are applied only at joints of the *loaded* chord unless otherwise stated.

5. *Reactions.* When panels are equal, the reactions due to dead load are usually each equal to the sum of the panel loads divided by two. If for any reason the panel loads vary or are not symmetrically distributed with respect to the center of the truss, the reactions may be calculated by the methods explained in CHAPTER XII, particularly in 142 : 3.

6. *Truss diagram:* Before beginning the calculation of the stresses in any truss, it is desirable to draw a diagram of that truss, and to indicate clearly on that diagram the principal dimensions of the truss; also the lines of action and the magnitudes of all panel loads and reactions. The truss diagram may often be drawn free-hand instead of to scale. Even a free-hand sketch, however, should show a truss properly proportioned, with diagonals inclined at an angle of from 40° to 45° to the vertical. (109 : 5.)

7. SEGMENT OF A TRUSS IN EQUILIBRIUM. In Fig. 184 (a) a ten-panel Pratt truss has been divided into two parts by the omission of the three members in the fourth panel. The part $aBDd$, taken by itself, is a

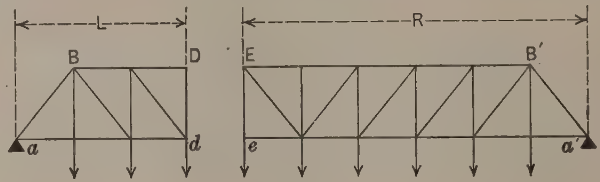


Fig. 184 (a).

rigid portion of the truss, since it is composed of triangles. For the same reason the part $EB'a'e$ is a rigid portion of the truss. The two portions $aBDd$ and $EB'a'e$ will be called, respectively, left-hand and right-hand segments or the **L segment** and the **R segment.**

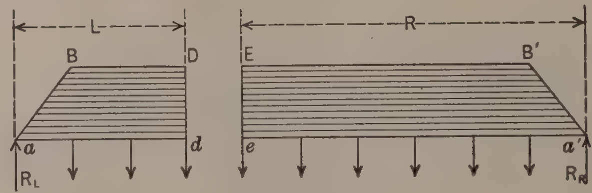


Fig. 184 (b).

8. Either segment may be considered to be a rigid body. For purposes of analysis it is immaterial how many members are within the perimeter of

the rigid body or in what direction they lie, provided they divide the interior space into triangles. It may even be helpful to regard each segment as a solid body, as indicated in Fig. 184 (*b*).

1. It is evident that neither segment as represented in Fig. 184 (*a*) (or in Fig. 184 (*b*)) is in equilibrium. If, however, the three members *DE*,

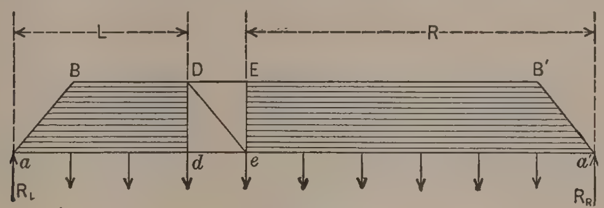


Fig. 185 (*a*).

De, and *de* are inserted, as shown in Fig. 185 (*a*), both segments are in equilibrium if the truss as a whole is in equilibrium. These three members may, therefore, be conceived as holding the L segment in equilibrium against the action of R_L and three panel loads, and as holding the R segment in equilibrium against the action of R_R and six panel loads. First one and then the other of these segments will now be considered as the body in equilibrium.

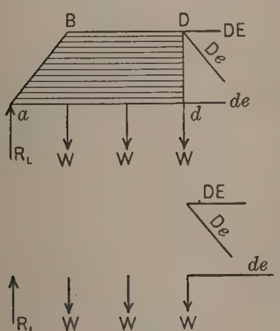


Fig. 185 (*b*).

2. The L segment *aBDd*, represented in Fig. 185 (*b*), is a rigid body held in equilibrium by seven forces external to itself, namely, the reaction R_L, the three panel loads, and the three members *DE*, *De*, and *de*. Four of these forces (the reaction and the three panel loads) are *known* forces, and are external not only to the L segment of the truss but also to the truss as a whole. The other three forces are those exerted by the members *DE*, *De*, and *de*; they are the unknown stresses in those three members, and are external to the segment of the truss but internal when the truss is considered as a whole. The lower portion of Fig. 185 (*b*) is a space diagram pure and simple, in which are represented the entire system of seven non-concurrent forces that hold the L segment in equilibrium.

3. The R segment represented in Fig. 185 (*c*) is a rigid body held in equilibrium by ten forces external to itself, namely, the reaction R_R, the six panel loads, and the three members *DE*, *De*, and *de*. Seven of these forces (the reaction and the six panel loads) are *known* forces and are external not only to the R segment but also to the truss as a whole. The other three forces are those exerted by the members *DE*, *De*, and *de* (the same three members that act on the L segment), and these three forces are the unknown stresses in members *DE*, *De*, and *de*; they are external forces to the R segment but are internal when the truss is considered as a whole. The lower portion of Fig. 185 (*c*) is a space diagram in which are represented the system of ten non-concurrent forces which hold the R segment in equilibrium.

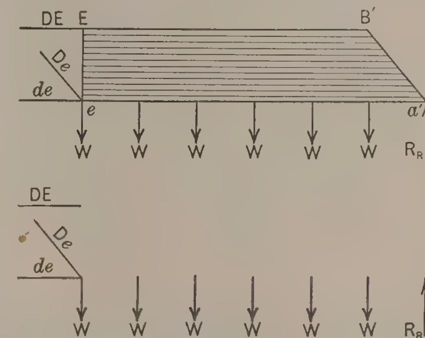


Fig. 185 (*c*).

4. In the space diagram of Fig. 185 (*b*), four of the forces are completely known and the only unknown element of each of the other three forces, *DE*, *De*, and *de*, is *magnitude*. The unknown elements, therefore, are three magnitudes, and hence the problem of determining these magnitudes falls under Case 4 (70 : 4); the problem may be solved by the standard method for that case, as will be shown later.

5. In the space diagram in Fig. 185 (*c*), seven of the forces are completely known, and the only unknown element of each of the other three forces, *DE*, *De*, and *de*, is magnitude. The unknown elements, therefore, are three magnitudes (the same three as in Fig. 185 (*b*)), and hence the problem of determining these magnitudes falls under Case 4.

6. It is important to note, first, that in determining the stresses in the three members *DE*, *De*, and *de*, *either* segment, the L segment or the R segment, may be taken as the body in equilibrium and, second, that whichever segment is taken, the problem is essentially the same, namely, one in

which the unknown elements are three magnitudes (Case 4). It is important to note also that only the *perimeter* of the segment enters into the problem, that all truss members in the space within the perimeter should be ignored, and that the sole use made of the perimeter is in determining the inclinations of the lines of action shown in the space diagram and the positions of these lines with respect to each other. Once the spatial relations of these lines of action are known, it is immaterial whether the segment on which the corresponding forces act is composed of triangles or is solid — in fact, it could be any irregular body or it could be ignored altogether, and only the space diagram used in solving the problem. (50 : 5.) Moreover, the lines of action in the space diagrams are not to be considered as limited in length; on the contrary, each is to be conceived as indefinite in extent.

1. The general method of assuming a segment of a truss as the body in equilibrium, in order to determine the stresses in the three members which act on that segment, is a method that can be used for any type of truss. In nearly all of the illustrative problems in this chapter, the left-hand segment is chosen as the body in equilibrium. In some cases, however, the right-hand segment can be used to better advantage, as will be explained later.

2. *Note:* The two segments of a solid beam or girder are formed by an imaginary cutting plane or section, and are separated only by that plane (as explained in the preceding chapter), whereas the two segments of a truss are formed by removing a whole panel and are, therefore, separated by that panel.

3. **Typical Analysis and Solution of a Problem by the Method of Sections.** Given: The truss in Fig. 186, 120 ft. long and 24 ft. high. Required: The stresses in the three members CD, cD, and cd. The left-hand segment upon which these three members act is $aBCc$.

4. *Note:* The three diagrams marked "(a)", "(b)", and "(c)" in Fig. 186 are not usually drawn. They are shown here for the purpose of emphasizing three fundamental conceptions used in the method of sections. The diagram marked "(a)" corresponds to the first conception, namely, a definite, rigid segment of the truss considered as a body in equilibrium. The diagram marked "(b)" corresponds to the second conception, namely, this segment held in equilibrium by a system of forces which include a reaction, panel loads, and *three* members of the truss. The space diagram marked "(c)" corresponds to the third or final conception, namely, the system of forces *separate* from the segment on which they act. The first two conceptions lead to the third, and this third means that, in the last analysis, it is the system of *forces* and not the truss that must be considered in analyzing a problem. The engineer commonly works directly from the truss diagram without the help of other diagrams, but this truss diagram is in the background merely as a means of determining distances between parallel forces and points in which non-parallel forces intersect; these spatial relations having been determined, the forces stand out in the engineer's mind almost as if the truss diagram did not exist. He sees in his imagination the forces separate from the segment of the truss as in a space diagram, though he does not draw such a diagram. This habit of mind is easily acquired and is most helpful in analyzing problems in stresses.

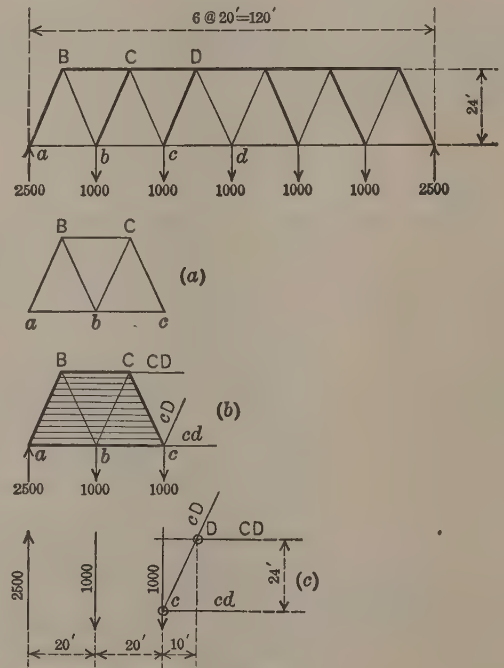


Fig. 186.

5. The problem of determining the stresses in the three members CD, cD, cd, falls under Case 4. (Why?) There are three combinations of the equilibrium equations which may be used for the solution of problems in

Case 4. (70 : 4.) When, as in this problem, there is only *one* inclined force among all the forces that hold a body in equilibrium, the combination of two moment equations and one resolution equation ($\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$) is the best. (70 : 4 (b).) The complete analysis and solution of the problem will now be given.

Analysis

Body in equilibrium: $aBCc$.

Known: Load of 1000 lbs. at each of the joints b and c; the reaction (2500 lbs.), and the lines of action of CD, cd, and cD.

Unknown: Magnitudes of CD, cd, and cD. (Case 4.)

Equations: $\Sigma M_c = 0$, $\Sigma M_D = 0$, and $\Sigma V = 0$.

Solution

$$\Sigma M_c = 2500 \times 40 - 1000 \times 20 + CD \times 24 = 0$$
$$\text{or } CD = -3333 \circlearrowleft \text{ or} \leftarrow \text{(compression).}$$
$$\Sigma M_D = 2500 \times 50 - 1000 \times 30 - 1000 \times 10 + cd \times 24 = 0$$
$$\text{or } cd = -3541 \circlearrowleft \text{ or} \rightarrow \text{(tension).}$$
$$\Sigma V = 2500 - 1000 - 1000 + cD_V = 0$$
$$\text{or } cD_V = -500 \downarrow \text{ and } cD = 541 \swarrow \text{(compression).}$$
$$\text{(Check) } \Sigma H = 0 = -CD + cd - cD_H = -3333 + 3541 - 208 = 0.$$

Comments

1. For each of the moment equations, the center of moments was assumed at the intersection of the lines of action of two forces whose magnitudes were unknown, in order to eliminate these forces from the equation. (34 : 5.)

2. The algebraic sign resulting from a *moment* equation gives the direction of rotation, and from this direction of rotation about the corresponding center of moments it is evident whether the force found from the equation acts toward (compression) or away from (tension) the body (segment) in equilibrium. CD and cd are both minus, but CD is in compression and cd is in tension. The algebraic sign of cD_V found from a *resolution* equation gives the sense of cD_V directly, namely, downward or toward the body in equilibrium (compression). The method of determining from the algebraic sign whether a member is in tension or in compression will be explained more fully later.

3. *Exercise:* (*a*) By the method of sections just explained, check the following stresses for the truss in Fig. 186.

BC = 2080 lbs. (C); bC = 1620 lbs. (C); bc = 2710 lbs. (T).

(*b*) Show by the method of sections that the tension in Cc is equal to the compression in bC.

4. *A second typical problem.* Given: A Pratt truss 120 ft. long; height of truss 24 ft.; panel load 1000 lbs. Required: The stresses in the members CD, Cd, and cd. In the analysis, Fig. (*a*) in Fig. 187 is to be used, and in the solution, the space diagram Fig. (*b*).

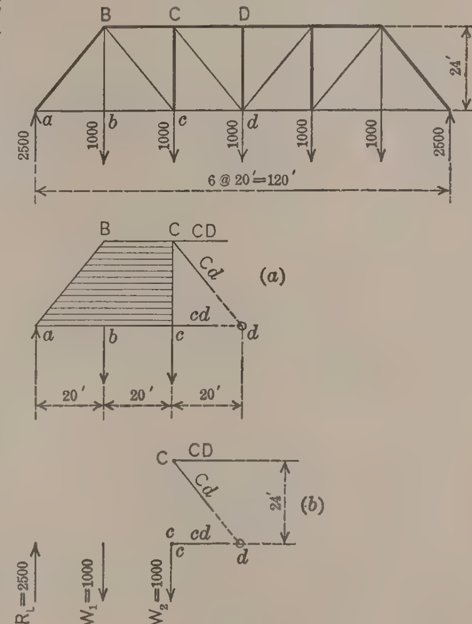


Fig. 187.

Analysis

Body in equilibrium:

Body $aBCc$.

Known: R_L, W_1, W_2, CD(L), Cd(L), and cd(L).

Unknown: CD(M), Cd(M), and cd(M). (Case 4.)

Equations: $\Sigma M_d = 0$, $\Sigma M_C = 0$ and $\Sigma V = 0$.

Solution

$$\Sigma M_d = R_L \times 60 - W_1 \times 40 - W_2 \times 20 + CD \times 24 = 0$$
$$= 2500 \times 60 - 1000 \times 40 - 1000 \times 20 + CD \times 24 = 0$$
$$-3750 = CD \circlearrowleft \text{ about } d \text{ or} \leftarrow \text{(compression).}$$
$$\Sigma M_C = R_L \times 40 - W_1 \times 20 + cd \times 24 = 0$$
$$= 2500 \times 40 - 1000 \times 20 + cd \times 24 = 0$$
$$-3330 = cd \circlearrowleft \text{ about } C \text{ or} \rightarrow \text{(tension).}$$
$$\Sigma V = 2500 - 1000 - 1000 + Cd_V = 0$$
$$-500 = Cd_V \downarrow$$
$$650 = Cd \searrow \text{(tension).}$$

5. *Note:* This problem is exactly like the third illustrative problem in Case 4, page 62; it was made so intentionally in order to emphasize the fact that a problem in

finding stresses by the method of sections is merely a *problem in Case 4*, and that to the student who thoroughly understands the solution of a problem in Case 4, there is little that is new in the method of sections. It may be well at this point to review carefully pages 48 to 51, and to study particularly the illustrative problems (including comments) in Case 4 (pages 61 to 63). This entire chapter on the method of sections is based on Case 4, hence it is important to have fresh in mind the methods for that case before proceeding further.

1. *Exercise:* By the method of sections, check the following stresses for the truss in Fig. 187.

BC = 3330 lbs. (C); Bc = 1950 lbs. (T); $bc = ab$ = 2080 lbs. (T).

2. **Special Cases in the Calculation of Stresses.** In any truss there are always certain stresses which may be determined by methods more simple and direct than the method of sections. The truss in Fig. 187 furnishes several good illustrations.

3. The stress in the inclined end post aB is most easily calculated from its V component, and this V component is equal to the reaction. (Why?) This statement holds true for an inclined end post in practically every type of truss.

4. The stress in the chord ab is equal to the H component of the stress in the end post. (Why?) This statement holds true for chord members in end panels in many other types of trusses.

5. From fundamental principle 71 : 5 the stress in bc is equal to the stress in ab. A similar statement holds true for stresses in chord members at any joint at which there is no inclined web member. From the same fundamental principle the stress in the hip vertical Bb is equal to the load at b. Similar conditions occur in other types of trusses whenever a panel load is transmitted directly to a vertical member.

6. From fundamental principle 71 : 4 there can be no stress in Dd. It will be shown in **PART III** that there may be a live-load stress in that member.

7. The stress in Cc is equal to the V component of the stress in Cd. (Why?) The principle involved (71 : 7) may be applied to many joints in various types of trusses.

8. *Note:* The special cases just given are examples of stresses determined by considering only the concurrent forces at a *joint* of the truss. Such cases will be treated more fully in the next chapter. They are given here to emphasize the fact that the method of sections should not be used for the calculation of any stress which may be determined by one of the principles given for special cases on page 71.

9. **Section and Segments.** Let it be required to calculate the stresses in the three members CD, Cd, and cd (Fig. 188 (a)). The segment selected as the body in equilibrium must be rigid, and it must be one on which the three unknown forces act as *external forces to the segment.* There are only two segments that satisfy these two conditions, namely, the left-hand segment $aBCc$ and the right-hand segment $DBa'd$. (Fig. 188 (b).) These two segments may be considered as being formed by a section that cuts the three members in which the stresses are desired, namely, CD, Cd, and cd (indicated by broken lines in Fig. 188 (b)). This section may be represented by any line at any slope, provided it cuts the three members and *no other member*, as, for example, the line *1–1* (Fig. 188 (a).) The section may also be conceived as removing the entire panel in which the three members CD, Cd, and cd lie. The purpose of the section, however conceived, is merely to make perfectly clear the outline of each of two segments, left and right, either of which may be considered as the body in equilibrium in calculating the stresses in CD, Cd, and cd.

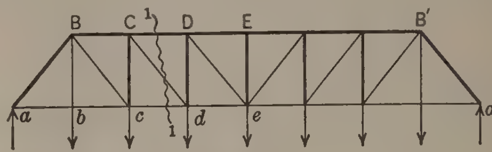


Fig. 188 (a).

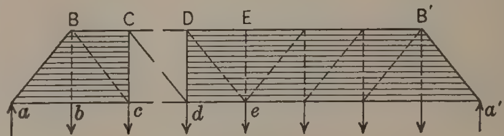


Fig. 188 (b).

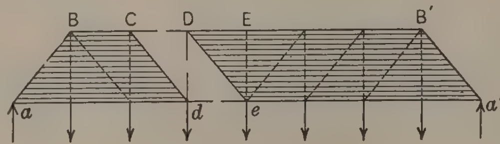


Fig. 188 (c).

10. If the stress in only *one* member is required, the section must cut that member *and not more than two* other members. (Why?) For example, if the stress in CD is required, either the section in Fig. 188 (b) or that in Fig. 188 (c) may be used. Each of these two sections divides the truss into two segments, left and right, and it will be shown later that either a left-hand or a

right-hand segment may be used as the body in equilibrium. Note that in Fig. 188 (c) the three members cut are not all in the same panel; hence, if a portion of the truss is conceived as removed by the section, that portion lies partly in one panel and partly in another. Although this is not exceptional, it is more often the case that all three members cut by a section are in the same panel.

1. In calculating stresses by the method explained in this chapter, it is not essential that a truss be conceived as cut at all; it is more in accord with actual conditions merely to regard some rigid portion of the truss as being held in equilibrium by external forces and by certain members of the truss (usually three) acting with these external forces to form a system of forces in equilibrium. It is helpful, however, in determining what segment to select in any given case, and in keeping this segment in mind during calculations, to conceive the truss as cut by a section. It is from this conception that the method of sections gets its name.

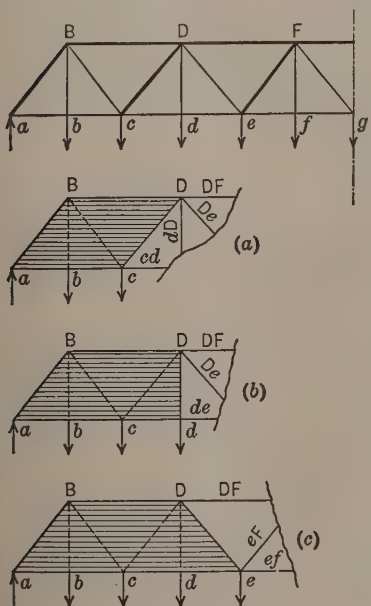


Fig. 189 (a).

2. In some types of trusses it is impossible to cut a given member by a section without cutting more than two other members, but this is rather an exceptional case which will be considered later.

3. **Choice of Section.** It has just been stated that a section should be so chosen that it cuts not more than three members of the truss, including the member or members in which the stresses are required. The reason for this is explained in the following illustration:

4. *Illustration:* In Fig. 189 (a) is shown a diagram of one-half of a twelve-panel truss. It is desired to find the stress DF. If the section shown in Fig. (a) is chosen, there will be *four* members acting on the segment (namely DF, De, Dd, and cd), and unless the magnitude of the stress in one of these members is known, the problem is indeterminate. (69 : 1 (a).) Even if the stress in one of the four members is known, it is better to choose the section shown in Fig. (b), since only three members act on this segment, and the stress in any one may be determined without using in the corresponding equation of equilibrium a magnitude previously calculated. One of the advantages of the method of sections is that usually a stress in a member can be calculated independently of the stresses in other members, and thus the result is not affected by errors, large or small, in the previous calculations of other stresses. It is to be noted that the stress in DF can also be found from the section and segment shown in Fig. (c), though one additional known force will enter into the equilibrium equation. (What force?) This choice between two segments is of common occurrence in the case of chord members but not in the case of web members.

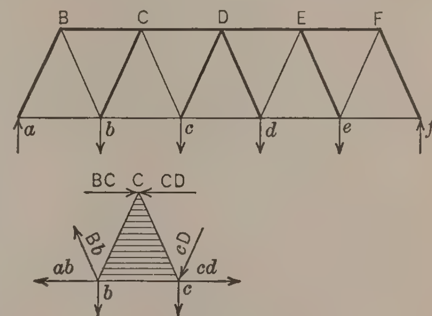


Fig. 189 (b).

5. *Note:* A section divides a truss into two segments, each of which is a rigid portion of the truss in equilibrium. Any other rigid portion of the truss is likewise in equilibrium, but it cannot ordinarily be used as the body in equilibrium because too many truss members are involved. For example, if from the five-panel truss shown in Fig. 189 (b) a rigid portion bCc is chosen, there will be in addition to the loads at b and c six members acting to hold bCc in equilibrium. Unless the stresses in at least three of these members are known, the problem is indeterminate; and even if the stresses in three members are known, the calculation of the stresses in the other three members will be affected by any errors in the three known stresses as previously calculated.

6. *Sections for all members of a truss.* A series of sections is necessary when stresses in all members of a truss are required. For example, in Fig. 190 (a) the stresses in the members BC, Bc, and bc may be determined from either segment formed by the section *1–1*, the stresses in CD, Cd, and cd from either segment formed by the section *2–2*, and the stresses in DE,

De, and *de* from either segment formed by the section *3–3*. For each of the three sections, it would be well to select the left-hand segment as the body in equilibrium, although, as will be explained later, it is sometimes best to select a right-hand segment. Each of the three sections results in a problem which is complete in itself; the calculation of the stresses in all of the members of any truss by the method of sections is really a series of such problems, all alike in form and each a problem in Case 4.

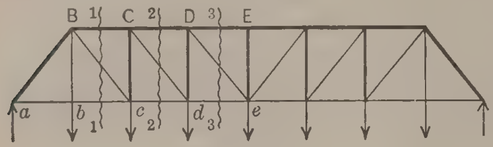


Fig. 190 (*a*).

1. *Note:* It will be shown later that the stress in *cd* is equal to that in *BC*, and that the stress in *de* is equal to that in *CD*, so that for each of the sections *2–2* and *3–3*, only *two* unknown stresses need be calculated.

2. *Questions:* (*a*) Why is a section unnecessary in calculating the stress in *aB*? (188 : 3.) (*b*) What fundamental principle applies to the stress in *Bb*? (188 : 5.) (*c*) If external loads are applied to lower joints only, why is the stress in *Ee* zero? (188 : 6.) (*d*) How should sections be taken to determine the stresses in *Cc* and *Dd*? (These stresses are equal to the V components respectively of *Cd* and *De*. (188 : 7.) It is not, therefore, really necessary to take a section in either case.)

3. **The Forces that Hold a Segment in Equilibrium.** The forces that hold a segment in equilibrium are (1) the reaction at the support, (2) the loads on the segment, and (3) three forces exerted by the three truss members cut by the section. Occasionally, as in the case of a cantilever truss, there may be no reaction on the segment; occasionally, also, there may be a fourth truss member which exerts a force on the segment. It is to be noted that all of these forces may be considered as *external* to the segment, though the three exerted by the truss members are internal when the truss as a whole is considered.

4. Until the method of sections has been thoroughly mastered, it is well to make a small free-hand sketch of each segment used as a body in equilibrium, and then to indicate clearly each force that acts on that segment to hold it in equilibrium, just as if each segment were a separate problem (as it really is). Members within the perimeter of a segment should be ignored, and the segment should be treated as if it were a solid body. It is well to remember that the segment itself is useful only in determining the spatial relations of the forces. (186 : 4.)

5. **Analysis for a Segment.** The form of analysis thus far used in this book may be applied to problems in the calculation of stresses by the method of sections, as illustrated in the problems on pages 186 and 187. The purpose of the analysis is first to gain a clear conception of the forces and *all* of the forces that act on the segment, and then to choose the most efficient combination of equilibrium equations for solving the problem. As the great majority of problems fall under Case 4, it is usually unnecessary to write out an analysis for each segment, but there should always be a corresponding mental analysis in which particular attention should be paid to the forces whose magnitudes are unknown, because upon these depends the choice of equilibrium equations. In this mental analysis it is well to observe carefully also the *number* of forces completely known, and, when the equilibrium equations have been written, to use this number as a check to make sure that no known force has been overlooked — a most common mistake, particularly if separate sketches are not made for different segments.

6. **Choice of Equilibrium Equations.** There are three combinations of equilibrium equations in common use for finding stresses by the method of sections; these combinations are discussed under Case 4 on page 49.

7. *The combination $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$* is the one most commonly used for trusses with parallel chords when the web member which acts on the segment under consideration is a *diagonal*. (49 : 4.) For example, in Fig. 190 (*b*) the web member which acts upon the segment is a diagonal *eF*, the stress in which can be found from $\Sigma V = 0 = R - W_1 - W_2 - W_3 - W_4 + eF_V$. The stress in *eF* could not be calculated as easily from $\Sigma M = 0$. (Why?) *DF* can be found from $\Sigma M_e = 0$, and *ef* from $\Sigma M_F = 0$; the lever arm of *DF* and of *ef* is the height of the truss.

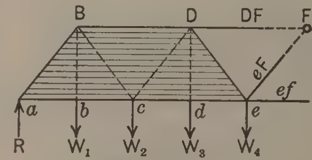


Fig. 190 (*b*).

8. *Note:* Note that this illustration is of the same general type as that given on page 62 as an illustrative problem under Case 4. Study the analysis and the comments on that page and answer the following questions:

Why not use $\Sigma H = 0$ in calculating the stress in eF?

Why assume e as the point of moments in finding DF, and F as the point of moments in finding ef?

Why not use $\Sigma H = 0$ in calculating the stress in ef or DF?

1. *Note:* The stress in one chord member may be determined from the stress in the other chord member and the diagonal cut by the section, but this involves in $\Sigma H = 0$ *two* stresses previously computed. It would seldom, if ever, be advantageous to use this method.

2. *The combination $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$* can be used to advantage in parallel chord trusses when the web member which acts on a segment is vertical, because then there is no inclined force involved. (49 : 1.) For example, in Fig. 191 (a) the web member is the vertical eE. Either of the chord stresses, DE or ef, may be calculated from $\Sigma M = 0$ as in the preceding example. The other may be found from the equation $\Sigma H = 0 = DE + ef$ (since no other force has an H component). This is equivalent to saying that the force in DE is equal in magnitude to the force in ef but opposite in sense. The use of $\Sigma H = 0$ in this case involves a stress previously calculated, but is justified because of the simplicity of the equation. Finally, the stress in eE may be found from $\Sigma V = 0 = R - W_1 - W_2 - W_3 - W_4 + eE$.

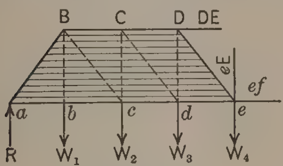


Fig. 191 (a).

3. *Note:* Note that this illustration is of the same general type as that given on page 61 as an illustrative problem under Case 4. Study the analysis and comments on that page. The Pratt truss (Fig. 191 (a)) is one of the few standard types of trusses in which it is possible so to choose a segment that no inclined web member is acting on it.

4. *The combination of three different moment equations* illustrated on page 63 can seldom be used to advantage in the method of sections, except for a truss with a polygonal chord.

5. The following three guiding principles for the choice of equilibrium equations are evident from the two illustrative examples just given; they apply particularly to parallel-chord trusses, but are also useful in applying the method of sections to any truss.

6. *The stress in a chord member is usually best calculated by means of a moment equation; when, however, there is no inclined force acting on the segment, the stress in one chord member may be found most easily from $\Sigma H = 0$.*

7. *The stress in a web member is usually best calculated from $\Sigma V = 0$, since all of the other forces involved in that equation are generally known vertical forces (loads and a reaction).*

8. *Avoid the use of $\Sigma M = 0$ for calculating the stress in a member unless the lever arm for that stress is easily determined.*

9. *Note:* It is not always possible to follow the last rule; if, for example, there are three unknown magnitudes, at least one moment equation must be used. It is well to keep in mind, however, that, as a general rule, it is easier to find the lever arm for a horizontal force, such as that in a horizontal chord member, than for an inclined force, such as that in an inclined web member or an inclined chord member. For example, in Fig. 191 (b), let it be required to find by means of a moment equation the stress in one of the members adjacent to the center of the roof truss. The lever arm for the horizontal chord member cd (point of moments at D the intersection of DE and Dd) is the height of the truss at joint D, a distance more easily calculated than would be the length of the lever arm for either of the inclined members DE or Dd. (What are the lever arms for these two members?)

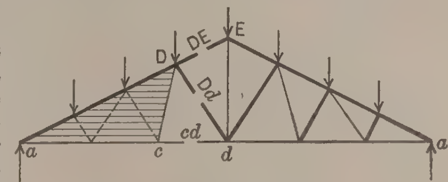


Fig. 191 (b)

10. **To Determine Whether a Member is in Tension or Compression.** In this book, the sense of the force exerted by any member of the truss on a given segment is not assumed, but is determined by the algebraic sign which results from solving an equilibrium equation in which the unknown force was entered with a plus sign. If the resulting algebraic sign of the unknown force is such that that force acts *toward* the segment, the stress is *compression;* if *away* from the segment, *tension.* (Why?) It must be kept in mind that an algebraic sign which results from a resolution equation gives the sense of a force *directly*, whereas one that results from a moment equation determines sense *indirectly* by giving the direction of rotation. (33 : 5 and 6.) Note that the sense of a force which a member exerts on a left-hand segment is opposite to the sense of the force which it exerts on the right-hand segment. For example, in Fig. 191 (b), the member DE acts toward the left on the left-hand segment aDc, but toward the right on the right-hand segment Eda'. An arrow indicating the sense of the force in DE, placed at one end of DE in the truss diagram, would be

opposite in direction from an arrow at the other end of DE. (6 : 6.) (How can a member act in opposite directions at the same time? (5 :4.))

1. *Note:* Many engineers prefer to assume all three members of unknown magnitudes to act *away* from the segment under consideration (equivalent to assuming all three members in tension) and then to give the unknown quantity in an equation the algebraic sign which is in accord with that assumption. If the algebraic sign which results from solving the equation is plus, the original assumption of sense is correct; if minus, the sense should be reversed. For example, in Fig. 192 (*a*) all three forces of unknown magnitude, CD, Cd, and cd, were assumed to act *away* from the segment. The corresponding equations of equilibrium are given below; they are exactly like the equations for the illustrative problem on page 187 except that the algebraic signs for the moment of cd and for the V component of Cd are minus in the equations instead of plus, and the corresponding results are plus instead of minus. (Compare equations with those on page 187.)

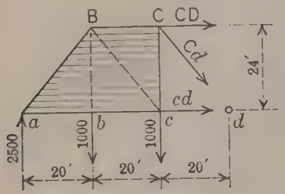


Fig. 192 (*a*).

$$\Sigma M_d = 2500 \times 60 - 1000 \times 40 - 1000 \times 20 + CD \times 24 = 0$$
$$-3750 = CD$$
$$\Sigma M_C = 2500 \times 40 - 1000 \times 20 - cd \times 24 = 0$$
$$+3330 = cd$$
$$\Sigma V = 2500 - 1000 - 1000 - Cd_V = 0$$
$$+500 = Cd_V$$

2. The results for cd and Cd_V are both *positive*, hence the assumption that cd and Cd act *away* from the segment (tension) is correct. The result for CD on the other hand is *negative*, and hence the assumption that CD acts *away from* the segment is incorrect; it must therefore act *toward* the segment (compression). According to this method a *plus* result means tension and a *minus* result compression, and it is customary among many engineers to use these signs to indicate on a stress diagram whether a stress is tension or compression.

3. *Note:* Although the method just explained of assuming the sense of each unknown force to be away from the body in equilibrium is quite generally used, it will not be followed in this book; instead, the method of giving plus signs to unknowns in an equation has been adopted. (This is also equivalent to assuming sense.) To the experienced engineer it makes little difference which method is followed, but for the beginner in the study of stresses there are certain advantages to be gained by the use of the latter method.

4. **Choice Between Two Segments.** In calculating the stress in any truss member, either the left-hand or the right-hand segment may be taken as the body in equilibrium. For example, in calculating the stress in the member CD (Fig. 192 (*b*)) either the left-hand segment $aBCb$ or the right-hand segment $cDFf$ may be taken as the body in equilibrium.

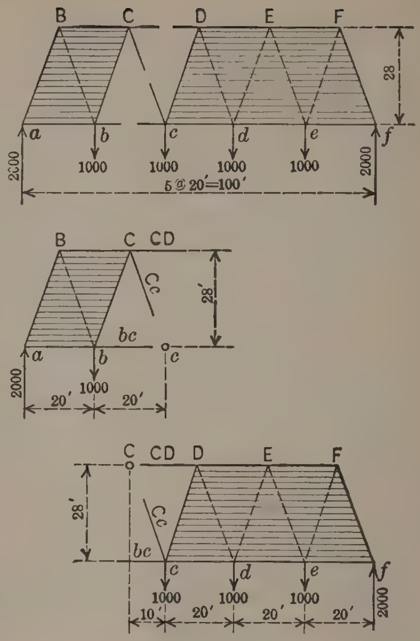


Fig. 192 (*b*).

For the left-hand segment:

$$\Sigma M_c = +2000 \times 40 - 1000 \times 20 + CD \times 28 = 0.$$
$$CD = -60{,}000 \div 28.$$

For the right-hand segment:

$$\Sigma M_c = -2000 \times 60 + 1000 \times 40 + 1000 \times 20 + CD \times 28 = 0.$$
$$CD = +60{,}000 \div 28.$$

The results from the two equations are identical except for the algebraic signs; these are opposite, as they should be. In the first equation the plus sign means clockwise rotation about c, that is, CD must act *toward* the left-hand segment $aBCb$; in the second equation the minus sign means counter-clockwise rotation about c, that is, CD must act *toward* the right-hand segment $cDFf$. Hence in either case CD must be in compression. (Show in a similar manner from equilibrium equations that the stresses for Cc or for bc may be found from either segment.)

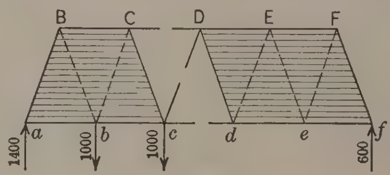


Fig. 192 (*c*).

5. For a truss symmetrically loaded like that in Fig. 192 (*b*) the shorter segment should be used, since this involves the smaller number of loads and hence shorter equilibrium equations. For a truss unsymmetrically loaded, however, that segment

which has on it the smaller number of loads should be used, even though it be the longer segment. For example, in finding the stress in CD, cD, or cd (Fig. 192 (c)), somewhat less arithmetical work is involved if the segment $DFfd$ is chosen than if the segment $aBCc$ is used. (Show that the results are the same for cD or for cd, whichever segment is chosen.)

1. **Summary of the General Method of Sections.** The *general* method of sections, as distinguished from the method of sections based on shear and bending moment (to be explained later), is the method by which the stresses in three members of a truss are determined from a single section which cuts those members. The general method may be summarized as follows:

2. *First:* Determine the external forces, loads, and reactions, and indicate these forces on a truss diagram. (184 : 1 to 6.)

3. *Second:* Divide the truss into two segments, left and right, by a section which cuts the three members in which the stresses are required. The section should not cut any other member if it can be avoided.

4. *Third:* Select as the body in equilibrium the left-hand or the right-hand segment, whichever has on it the smaller number of panel loads. (192 : 4.) Consider the perimeter of the segment as the perimeter of a rigid body, and ignore all truss members within that perimeter. (184 : 8.)

5. *Fourth:* Observe carefully what forces external to the segment hold it in equilibrium. The forces are usually (1) the reaction at the support, (2) the panel loads on the segment, and (3) the forces exerted by the three truss members cut by the section.

6. *Fifth:* Analyze the problem by noting what forces observed in the preceding step are *wholly* known, what forces are *partly* known, and what are all of the unknown elements. From the unknown elements, determine under what case the problem falls. (The unknown elements are usually three magnitudes, i.e., Case 4.)

7. *Sixth:* Choose the combination of equilibrium equations that is the best for determining the three unknowns, write out each equation, make sure that no force has been omitted or incorrectly entered, check the lengths of lever arms in the moment equations, and when reasonably certain that all three equations are indicated correctly, solve each equation. To determine the stress in a web member, use, as a rule, the equation $\Sigma V = 0$. To determine the stress in a chord member, use a moment equation, assuming as a center of moments the joint at which the other chord member and the web member cut by the section intersect. (187 : 1.) If there are no inclined forces among the external forces on the segment, the equation $\Sigma H = 0$ can be used to advantage in place of one of the moment equations. (191 : 2.)

8. *Seventh:* From the algebraic signs of the results found by solving the equations, determine the sense of each unknown force. (The sign obtained from a resolution equation gives sense directly; the sign obtained from a moment equation gives direction of rotation, from which sense will be evident.) From the sense of each force determine whether it acts toward the segment (compression) or away from the segment (tension). (191 : 10.)

9. *Note:* Notice that the method just given follows essentially the method of attack explained in Chapter V, and used for all of the illustrative problems in PART I.

10. **Shear and Resistance to Shear (Parallel-chord Trusses).** Let W represent the panel load for the truss in Fig. 193, and A the angle of inclination of an inclined web member (**truss angle**). Let it be required to calculate the stress in the inclined member cD.

Body in equilibrium: Segment aBc.

Equation: $\Sigma V = 0$. (193 : 7.)

$$\Sigma V = (3\tfrac{1}{2}W - W - W) + cD_V + BD_V + cd_V = 0.$$

In this equation the portion in parentheses is the *shear* for the segment aBc, and the remainder of the equation represents the **resistance to shear.** But since $BD_V = 0$ and $cd_V = 0$, the force cD_V must be equal in magnitude to

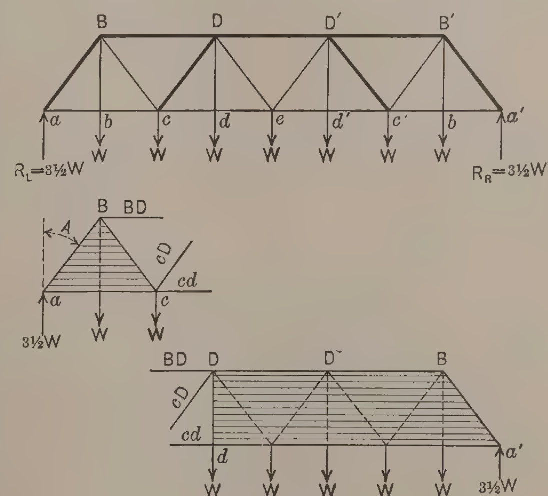


Fig. 193.

the shear but opposite in sense, i.e., the member cD alone may be said to resist the shear. Put in the form of an equation:

$$\textit{Shear} = \textit{Vertical component of inclined web member}$$
$$3\tfrac{1}{2}W - W - W = +1\tfrac{1}{2}W = -cD_v.$$

The shear is positive and therefore tends to move segment aBc upward, hence the member cD must act downward or toward the segment (compression).

1. The shear for one segment is equal to the shear for the other segment but is opposite in sense, hence the same result should be obtained from the right-hand segment $DB'a'd$.

$$\Sigma V = (3\tfrac{1}{2}W - W - W - W - W - W) + cD_V = 0.$$
$$\textit{Shear} = \textit{Vertical component of the inclined web member}$$
$$3\tfrac{1}{2}W - 5W = -1\tfrac{1}{2}W = +cD_V.$$

The shear is negative and therefore tends to move segment $DB'a'd$ downward, hence the member cD must act upward or *toward* the segment (compression).

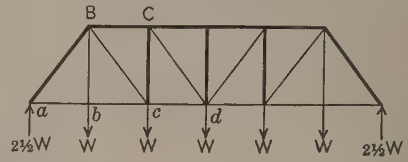
2. Whichever segment is taken, the web member cD resists the shear for that segment; moreover, it is the only member that can resist this shear or the tendency of the external forces (reaction and loads) on the segment to move the segment up or down, since the horizontal chord members have no vertical components and cannot therefore resist this tendency.

3. *Note:* Unless otherwise stated, the term "shear," as used in this book, will mean shear for a *left-hand segment.*

4. For the ordinary parallel-chord truss, only one web member will be cut by a section, and hence it will be the only web member acting on either segment. The V component of the stress in that member will equal in magnitude the shear for either segment, and the stress itself may be found from its V component by multiplying that component by the secant of the angle which the web member makes with the vertical (trigonometric method (10 : 1)), or by the geometric method (10 : 2). Hence:

5. *If a parallel-chord truss is divided into two segments by a section that cuts only one web member, the stress in that member is equal to the shear for either segment multiplied by the secant of the angle that the member makes with a line at right angles to the chords of the truss.*

6. *If the shear tends to move a segment up (positive shear), the web member will act downward on the segment; and, vice-versa, if the shear tends to move a segment down (negative shear), the web member will act upward on the segment.* The direction in which the web member acts (toward or away from the segment) will determine whether the member is in compression or in tension.



7. *Note:* The first principle just given is in the most general and comprehensive form, and applies to a web member regardless of whether it is inclined or vertical. If it is vertical, however, the secant of the angle is unity, and the stress in the member is equal in magnitude to the shear. For example, the stress in the vertical web member Cc (Fig. 194 (*a*)) is equal to the shear, i.e., $2\tfrac{1}{2}W - W - W = \tfrac{1}{2}W$. The second principle may be applied as follows: Since the shear tends to move the segment aBc upward, the member Cc must act downward or toward the segment. (Compression.)

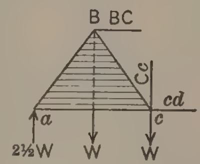


Fig. 194 (*a*).

8. *Note:* There are certain types of parallel-chord trusses in which there are web members which cannot be cut by any section that does not at the same time pass through another web member. In such a case, the shear is resisted by both web members acting together, and the two principles just given must be modified. For example, in the truss shown in Fig. 194 (*b*) the principles would apply to the member $f'g$ since this is the only web member cut by the section hh, but they would not apply to the member Ef' since the web member ef' is also cut by the section kk which cuts Ef'. These two members would act together to resist the shear on the segment $aBEe$, though not necessarily with equal force.

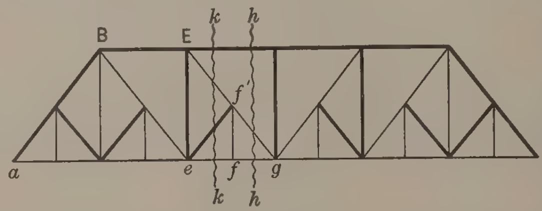


Fig. 194 (*b*).

9. *Note:* The principle that the V component of the stress in a web member is equal to the shear is merely another method of expressing the condition of equilibrium, $\Sigma V = 0$. If there were a vertical component of the stress in either the top or the bottom chord member cut by the section, the principle would not hold true, since this V component

of the chord member would enter into the equation $\Sigma V = 0$. In other words, an inclined chord member would act with the web member to resist the shear. Hence the two principles given do not apply to any web member if a chord member cut by the section is inclined. For example, in the truss shown in Fig. 195 (*a*) the rule would apply to the web member *De*, since both chord members *DE* and *de* cut by the section *hh* are horizontal; but the rule would not apply to the web member *Cd*, since one chord *CD* cut by the section *kk* is inclined, and would therefore act with *Cd* to resist the shear on the left-hand segment *aBCc* or on the corresponding right-hand segment *DB′a′d*.

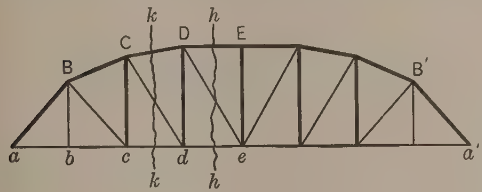


Fig. 195 (*a*).

1. *Check.* To resist positive shear (on a left-hand segment) a diagonal must always act downward at its left-hand end, regardless of whether that end is at an upper or a lower joint; if it is at a lower joint, it must, from the position of the diagonal, be compression; if at an upper joint, tension. This and similar checks on the sense of a stress, based on a clear conception of how the forces are acting, become instinctive to one experienced in the calculation of stresses.

2. **Bending Moment and Resistance to Bending Moment (Parallel-chord Trusses).** The truss shown in Fig. 195 (*b*) is the same as that used in explaining shear. (193 : 10.) Let the panel length be represented by p and the height of truss by h. Let it be required to calculate the stress in the chord member *BD*.

Body in equilibrium: Segment *aBc*.
Equation: $\Sigma M_c = 0$. (193 : 7.)

$$\Sigma M_c = (3\tfrac{1}{2}W \times 2p - W \times p - W \times 0) + BD \times h + cD \times 0 + cd \times 0.$$

In this equation, the portion in parentheses is the *bending moment* for the segment *aBc*, and the remainder of the equation represents the **resistance to bending moment.** But since, for the center of moments chosen, namely, *c*, the moment of each of the two members *cD* and *cd* is zero, the moment of the stress in *BD* must be equal in magnitude to the bending moment about the same center, but must be opposite in algebraic sign or direction of rotation, i.e., the member *BD* alone may be said to resist the bending moment for the segment *aBc* when *c* is the center of moments. Put in the form of an equation:

Bending Moment = Resisting moment of the chord member BD.

$$3\tfrac{1}{2}W \times 2p - W \times p = BD \times h.$$

The bending moment tends to revolve the segment *aBc* clockwise (positive bending moment), and hence the member *BD* must tend to cause counterclockwise rotation about *c*; to do this *BD* must act toward the segment (compression).

3. The bending moment for one segment is equal to the bending moment for the other segment but is opposite in direction of rotation, hence the same result should be obtained from the right-hand segment *DB′a′d*.

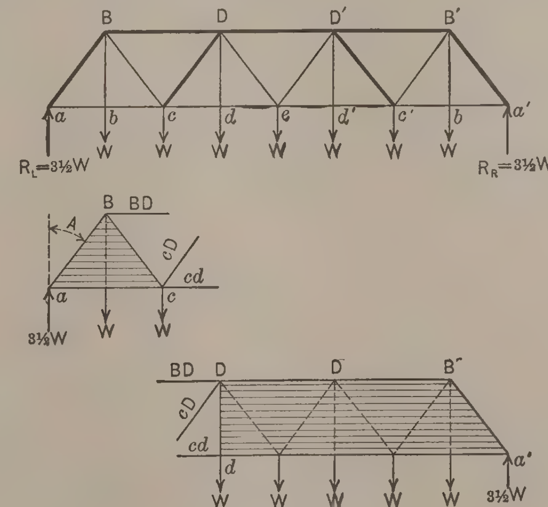


Fig. 195 (*b*).

(Prove that the bending moment for this segment is equal in magnitude to $6W \times p$ and is counter-clockwise; that the member *BD* must tend to revolve the segment *DB′a′d* clockwise about the center of moments *c*, and hence must act toward that segment (compression).)

1. Whichever segment is taken, the chord member BD resists the bending moment for that segment about c as a center; moreover, it is the only member that can resist this bending moment or tendency of the external forces (reactions and loads) on the segment to revolve the segment about c, since the moment about c of each of the other two members, cD and cd, which act upon the segment, is zero.

2. From the equation last given, it is evident that the stress in BD is equal to the bending moment divided by h. It can be easily shown that, similarly, the stress in any chord member, top or bottom, may be found by dividing a bending moment by the height of truss in accordance with the following principle:

3. *If a parallel-chord truss is divided into two segments by a section which cuts two chord members and a web member, the stress in either chord member is equal to the bending moment for either segment divided by the height of truss, provided the center of moments for the bending moment is taken at the intersection of the web member and the other chord member* (*the chord member for which the stress is not being calculated*).

4. *If the bending moment tends to revolve a segment clockwise* (*positive bending moment*), *the chord member must act in such a direction, to the right or to the left, as to tend to revolve the segment counter-clockwise about the same center of moments; and, vice-versa, if the bending moment tends to revolve a segment counter-clockwise* (*negative bending moment*), *the chord member must act in such a direction as to tend to revolve the segment clockwise.* The direction in which the chord member acts (toward or away from the segment) will determine whether the member is in compression or in tension.

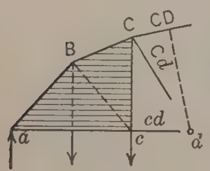


Fig. 196.

5. *Note:* The first principle (196 : 3) is not in the most general form. It may be modified to apply to an inclined chord member by substituting for "divided by the height of truss" the phrase "divided by the lever arm of the chord member." For example, in Fig. 196 the stress in the chord member CD is equal to the bending moment for the segment $aBCc$ about d as a center of moments, divided by the lever-arm distance from d to CD. The second principle (196 : 4) holds good for either a horizontal or an inclined chord.

6. *Note:* The bending moment may be taken about some other point than the intersection of two members; but in that case each of the three members cut by the section will act with the other two members to resist the bending moment, unless the member passes through the center of moments. To eliminate two of the three members, the center of moments for bending moment is taken at a joint of either the top or the bottom chord.

7. *Remark:* Just as the rule for calculating the stress in a *web member* from *shear* (194 : 5) is merely another method of expressing a *resolution equation of equilibrium* ($\Sigma V = 0$), so the rule for calculating the stress in a *chord member* from *bending moment* is merely another method of expressing a *moment equation of equilibrium* ($\Sigma M = 0$). It is better to understand and remember this than to remember the rules in their exact form.

8. *Note:* In calculating a bending moment, it is well either to replace all of the panel loads on the segment by their resultant or to multiply a single panel load by the sum of the lever arms of all of the panel loads on the segment. In either case the lever arm may be expressed in terms of panel lengths. (171 : 13 and 14.)

9. **Short Cuts in Calculating Stresses by the Method of Sections.** The problem of finding the stresses in *three* members of a truss cut by a section is always a problem in Case 4 — three unknown magnitudes. The stresses in *all* of the members of the truss may be found by calculating first the stresses in three members cut by one section, then the stresses in three members cut by another section, and so on until all stresses are known. For each section the method of Case 4 is repeated, and hence there are as many problems in Case 4 as there are sections. A more efficient method of calculation than this will now be given. In using this method, however, one should understand and remember that it is merely a modification of the standard method of solution of a problem in Case 4. The modified method differs from the standard method mainly in the order in which the calculations are made.

10. *Stresses in web members.* The stress in a web member may be determined directly from shear. (194 : 5.) When panel loads are equal, as they usually are assumed to be in calculating dead-load stresses for ordinary bridges, the shear may be expressed in terms of panel loads.

11. When a section cuts a *vertical* web member but no inclined member, the stress in the vertical web member is equal to the shear for either segment of the truss to the left or right of the section, hence this stress may be expressed in terms of panel loads. Moreover, the stresses in all such web members may be expressed in this way, one after the other.

12. *Illustration:* The left-hand half of a Pratt truss of ten panels is represented in Fig. 197 (a). Let W represent a panel load. Let each

of the vertical members except Bb be cut by a section which does not cut any inclined member. Then:

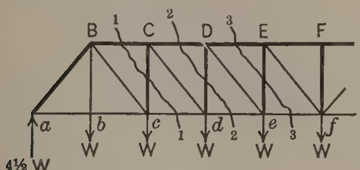


Fig. 197 (a).

Member	Section	Shear = Stress
Cc	1–1	$2\frac{1}{2}W =$
Dd	2–2	$1\frac{1}{2}W =$
Ee	3–3	$\frac{1}{2}W =$
Ff		$= 0$

1. The calculation of the stresses in these vertical members will consist merely in substituting the value of W and making the necessary multiplications. Why cannot the stress in Bb be expressed in a similar manner in terms of shear? To what is the stress in Bb equal? (188 : 5.) Why is the stress in Ff zero? (188 : 6.)

2. When a section cuts an inclined web member and no other inclined or vertical member, the stress in the inclined member may be expressed in terms of panel loads multiplied by the secant of the angle of inclination. (194 : 5.) This angle of inclination is usually the same for all inclined web members of a parallel chord truss, hence the stresses in all such inclined web members may be expressed, one after another, in the same general terms as shown in the following illustration:

3. *Illustration:* The stresses for the inclined web members of the truss shown in Fig. 197 (b) are as follows:

Member	Section	Shear × secant A = Stress
Bc	1–1	$3\frac{1}{2}(W \times \sec A) =$
Cd	2–2	$2\frac{1}{2}(W \times \sec A) =$
De	3–3	$1\frac{1}{2}(W \times \sec A) =$
Ef	4–4	$\frac{1}{2}(W \times \sec A) =$

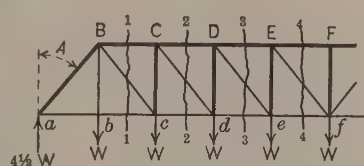


Fig. 197 (b).

The quantity ($W \times \sec A$) may be computed once for all; the calculations for the stresses in these inclined web members will then consist merely in multiplying this quantity by $3\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$, and $\frac{1}{2}$. (Why is the stress in aB equal to the reaction multiplied by the secant of A?)

4. *Check for magnitude of the shear.* If a truss is symmetrically loaded with respect to the center, the magnitude of the shear is equal to the sum of the panel loads between the section and the center; in a truss with an even number of panels, one-half of the panel load at the center should be included in the summation. (Apply this check to Fig. 197 (b).)

5. *When only two web members meet at a joint of a parallel-chord truss and there is no external load at that joint, the V components of the members are equal in magnitude and opposite in sense.* This is merely equivalent to stating that $\Sigma V = 0$ for the forces at the joint; it is also an application of the principle for the special case given in 71 : 6.

6. From the principle just given, the stress in an intermediate post of a Pratt truss is equal in magnitude but opposite in sense to the V component of the diagonal that meets that post *at a joint of the unloaded chord.* For example, the stress in Cc in Fig. 197 (a) is equal in magnitude to the V component of the stress in Cd, but opposite in sense. If, therefore, the stress in a post is known, the stress in the diagonal may be determined directly from the stress in the post by multiplying by the secant of the truss angle; and, conversely, if the stress in the diagonal is known, the stress in the post may be calculated directly from the stress in the diagonal by dividing by the secant of the truss angle. (Compare equations in 196 : 12 and 197 : 3.)

7. Applying the same general principle to a Warren truss *without verticals,* the stress in a given diagonal is equal in magnitude but opposite in character to the stress in that diagonal that meets the given diagonal at a joint of the unloaded chord.

8. *Stresses in chord members.* The stress in a chord member may be determined directly from bending moment. In a parallel-chord truss, the stress in any chord member cut by a section that passes through only two other members, a web member and a second chord member, is equal to the bending moment for either segment to the right or left of the section divided by the height of truss (lever arm of the chord member), provided the center of moments is assumed at the joint at which the web member and the second chord member meet. (196 : 3.) The bending moment may be expressed in terms of the panel load W and the panel length p, hence the stress in a chord member may be expressed in terms of W, p, and h (the height of truss). (195 : 2.) The stresses in all of the chord members

may be expressed, one after the other, in terms of W, p, and h, and the calculations may then be made as shown in the following illustration:

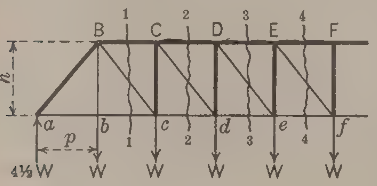


Fig. 198.

1. *Illustration:* In the following illustration the bending moments used in determining stresses in upper chord members have been expressed by indicating the *summation of panel lengths* before multiplying by W (171 : 13), whereas the bending moments used in determining stresses in lower chord members have been expressed by indicating the *resultant* of the loads on the segment multiplied by its lever arm. (171 : 14.) This was done in order that these two short-cut methods of calculating bending moments might be compared.

Member	Section	Center of Moments	Bending Moment (M_B)	$M_B \div h$
BC	1–1	c	$4\frac{1}{2}W \times 2p - W \times p$	$8(Wp \div h) =$ Stress
CD	2–2	d	$4\frac{1}{2}W \times 3p - Wp(1+2)$	$10\frac{1}{2}(Wp \div h) =$
DE	3–3	e	$4\frac{1}{2}W \times 4p - Wp(1+2+3)$	$12(Wp \div h) =$
EF	4–4	f	$4\frac{1}{2}W \times 5p - Wp(1+2+3+4)$	$12\frac{1}{2}(Wp \div h) =$
bc	1–1	B	$4\frac{1}{2}W \times p - W \times 0$	$4\frac{1}{2}(Wp \div h) =$
cd	2–2	C	$4\frac{1}{2}W \times 2p - W \times p$	$8(Wp \div h) =$
de	3–3	D	$4\frac{1}{2}W \times 3p - 2W \times 1\frac{1}{2}p$	$10\frac{1}{2}(Wp \div h) =$
ef	4–4	E	$4\frac{1}{2}W \times 4p - 3W \times 2p$	$12(Wp \div h) =$

2. The quantity $Wp \div h$ may be computed once for all; the calculations for the stresses in these chord members will then consist merely in multiplying this quantity by 8, $10\frac{1}{2}$, 12, $12\frac{1}{2}$, and $4\frac{1}{2}$.

3. It is to be noted that the last three calculations, i.e., those for the stresses in *cd*, *de*, and *ef*, are unnecessary, since the stresses in these three lower chord members are equal, respectively, to the stresses in the upper chord members *BC*, *CD*, and *DE*. (Why?) (191 : 2.) Why is the stress in *ab* equal to the stress in *bc*? (188 : 5.)

4. *Note:* The calculations for the stresses in web members and in chord members by the methods just explained may all be *indicated* in tabular form *before any calculations are made*, and the stresses may then be calculated and inserted in the last column. (See illustrative problem, page 207, for a complete example.) This results not only in greater speed in calculations, but in fewer mistakes and in the saving of time which otherwise would be lost in finding and correcting mistakes. For example, in the tabulation on page 207 it is evident that the shear decreases by an amount equal to W as one moves one's eye down the column, and any deviation from this variation is likely to be noticed; similarly, in the column for bending moments on page 207, the lever arms of the reaction and the moments of the panel loads increase in a regular progression as one scans the column downward.

5. *Note:* The short-cut methods of calculating stresses by the method of sections, explained in this article, are equivalent to the *method of coefficients*, which is quite generally used for the calculation of stresses in parallel-chord trusses with equal panel loads. This method of coefficients will be explained in the next chapter.

6. **Summary of the Method of Calculating Stresses from Shears and Bending Moments.** The method of calculating stresses from shears and bending moments is merely a modification of the general method of sections summarized in 193 : 1. Assuming that a parallel-chord truss is divided into two segments by a section which cuts only three members, one web member and two chord members, the following statements hold true:

7. The web member resists the entire shear for either segment. If the member is vertical, the stress in it is equal to the shear; if the member is inclined, its V component is equal to the shear. If the shear tends to move the segment up (positive shear), the web member acts downward on the segment; if the shear tends to move the segment down (negative shear), the web member acts upward on the segment.

8. Either chord member resists the entire bending moment for either segment, provided the center of moments is taken at the intersection of the other chord member and the web member. The stress in a chord member is equal to the corresponding bending moment divided by the lever arm of the chord member, which in a parallel-chord truss is equal to the height of the truss. If the bending moment tends to revolve the segment clockwise about the center of moments (plus), the chord member will act in such a direction, right or left, as to tend to revolve the segment counter-clockwise; if the bending moment is counter-clockwise (minus), the sense of the force exerted by the chord member must be such that the force tends to revolve the segment clockwise about the center of moments.

1. The method of calculating stresses based on these principles may be summarized as follows:

2. *First:* Determine the external forces, loads, and reactions, and indicate these forces on a truss diagram. (184 : 1.)

3. *Second:* Calculate the shear resisted by each web member, i.e., the shear for either segment formed by a section that cuts the web member and only two other members (chords). (193 : 10.)

4. *Note:* In the case of ordinary trusses, the shear is best expressed in terms of panel loads and left in this form, the value to be calculated later. (196: 12 and 197: 3.)

5. *Note:* When only two web members meet at a joint of the *unloaded* chord, the stress in one can be calculated directly from the stress in the other (197 : 5), so that it is necessary to use the method of sections only in determining one of the two stresses. (See 197 : 6 and 197 : 7 for application to Pratt trusses and Warren trusses.)

6. *Third:* Calculate the bending moment resisted by each chord member, i.e., the bending moment for either segment formed by a section that cuts the chord member and only two other members, a second chord member and a web member. (Center of moments at the intersection of these last two members.)

7. *Note:* In the case of parallel-chord trusses, the bending moment is best expressed in terms of panel loads and panel length, and left in this form to be calculated later.

8. *Fourth:* From the shears calculated in the second step, determine the stresses in the web members. (196 : 10.) The stress in a vertical web member is equal to the shear that the member resists (196 : 11); the *vertical component* of the stress in an inclined member is equal to the shear that the member resists, hence the stress itself is equal to that shear multiplied by the secant of the angle of inclination. (197 : 2.)

9. *Note:* If W is the panel load and A the angle of inclination, the quantity $W \sec A$ may be calculated once for all. The stress in any inclined web member will then be equal to that quantity multiplied by the number of panel loads indicated in the corresponding expression for shear. (197 : 3.)

10. *Fifth:* From the bending moments calculated in the third step, determine the stresses in the chord members. (197 : 8.) The stress in any chord member is found by dividing the corresponding bending moment by the lever arm of the chord member.

11. *Note:* If W is the panel load, p the panel length, and h the height of truss, for a parallel chord truss, the quantity $(W \times p) \div h$ may be calculated once for all. The stress in any chord member will then be equal to that quantity multiplied by the numeral which appears in the corresponding expression for bending moment when that expression has been reduced to the lowest terms (in terms of W, p, and h). (198 : 1.)

12. *Sixth:* Determine the nature of the stress, tension or compression, for each member. In the majority of cases the nature of the stress is obvious.

13. *Note:* In an ordinary truss supported at each end, members in the upper chord are usually in compression and those in the lower chord are usually in tension. If any doubt exists as to the nature of stress, it is generally in connection with the stress in a web member. When in doubt, it is well to remember that a truss member, in order to resist either shear or bending moment, must act in such a direction as to produce an opposite effect, i.e., opposite motion of translation (up or down) in case of shear, and opposite rotation (clockwise or counter-clockwise) in case of bending moment.

14. **Effect of Dead Load Applied at Joints of the Unloaded Chord.** Throughout this chapter, each panel load has been considered as applied at a floor-beam joint or panel point, i.e., to a joint of the loaded chord. This is in accord with the first assumption in 127 : 4. When a portion of the dead load is considered as applied at joints of the unloaded chord, the stresses in certain members will differ from those determined for the first assumption of loading. For trusses with verticals, only the stresses in those verticals, as a general rule, are affected. For example, if two-thirds of the total dead panel load is concentrated at the joint at the foot of a vertical and one-third at the joint at the top (127 : 4 (*b*)), the stress in the vertical is either one-third of a panel load greater or one-third less than it would be if the entire panel load were concentrated at the foot, depending on whether the third at the top is carried through the vertical to the bottom, as in a post of a Pratt truss, or the two-thirds at the bottom is carried through the vertical to the top, as in a vertical of a Warren truss with verticals. Under such conditions, the stresses in a truss are probably best determined by proceeding as if the entire dead load were applied at the joints of the loaded chord, with the understanding, however, that the stresses in verticals will later be corrected to accord with the assumption that a portion of the dead load is applied at joints of the unloaded chord.

15. When a truss has a triangular web system without verticals, as, for example, the web system of a simple Warren girder, the assumption that a portion of the dead load is applied at joints of the unloaded chord affects

not only the stresses in the web members but the stresses in some of the chord members as well. In such a case it is probably better, at least for the beginner, to indicate the fractional loads just as they are assumed to be applied at the various joints of the truss diagram, and then to determine by the usual equilibrium equations the stresses due to these loads. This is the method followed in the illustrative problem at the end of this chapter in which stresses are calculated for a Pratt truss with one-third of each panel load applied at a joint of the upper chord; but in this case, since only stresses in the vertical posts are affected, the method first explained, which involves merely the correction of stresses in the verticals, could just as well have been used.

1. *Exercise:* Given: The Warren truss with verticals shown in Fig. 193. The load per panel is 12,000 lbs. If the stresses are calculated first for 12,000 lbs. at each lower apex, and then for 8000 lbs. at each lower apex and 4000 lbs. at each upper apex, in which members will the stresses be different for the two assumptions? Show what the difference for each member is.

2. *Exercise:* Given: The Warren truss without verticals shown in Fig. 186. The panel load is 15,000 lbs. instead of 1000 lbs. as indicated in the figure. The first assumption is that an entire panel load is applied at each lower joint, and the second assumption is that two-thirds of a panel load is applied at each lower joint and one-third at each upper joint. Show for each of the four members of the truss that meet at joint D what difference, if any, there is between the stress in the member calculated for the first assumption and the stress calculated for the second.

3. **Special Cases in the Method of Sections.** In certain types of trusses there are members which cannot be cut by any section that does not also intersect at least *three* other members. The cutting of four members, in all of which the stresses are unknown, makes the problem of determining these four stresses either actually or apparently statically indeterminate. Three of the most common of these special cases will be considered.

4. *Two diagonals in the same panel.* It frequently happens that two diagonals occur in the same panel, as, for example, in the center panel of a Pratt truss with an odd number of panels. Assuming that the two diagonals act simultaneously, the stresses caused in them by loads on the truss cannot be of the same character; one must be tension, the other compression. (Why?) Frequently each diagonal is designed to take tension only. Under these conditions, only one diagonal can act at a time. When the shear for the panel containing the diagonals is such that one of the corresponding segments tends to move upward, the diagonal that can *pull* downward on that segment will be in action; and when the shear is such that the same segment tends to move downward, the other diagonal will *pull* upward. In such a case, the problem may be solved by merely omitting or ignoring the diagonal not in action. In the center panel of a truss with an odd number of panels, the shear for dead load is zero; consequently, neither diagonal is in action, but the problem arises in connection with live-load stresses in the two diagonals, as will be explained later.

5. Frequently each diagonal is designed to take either tension or compression (" stiff " diagonal). Under these conditions the problem becomes statically indeterminate, since the two diagonals can then act simultaneously. The assumption usually made, in order to render the problem determinate, is that the shear for the panel is equally divided between the two diagonals, and this means, in parallel-chord trusses, that the stresses are equal in magnitude but opposite in character. It is this case that will now be considered.

6. Let $eEFf$ in Fig. 200 represent a typical panel with two " stiff " diagonals. The stresses in the diagonals may be determined by $\Sigma V = 0$ and by the additional assumed condition that the V components are equal. The stress in EF may be determined from $\Sigma M = 0$ with m as a center of moments, and the stresses in ef from $\Sigma M = 0$ with m' as a center of moments. In each of these moment equations the moment of the stress in one diagonal will cancel the moment of the *equal* stress in the other, since the lines of action are equally distant from the center of moments and the stress in one diagonal tends to cause rotation in a direction opposite to that which the stress in the other diagonal would cause. It is to be noted that it is only necessary to calculate the stress in one of the chord members, since the stresses in the two chord members are equal. (Prove this from $\Sigma M = 0$, and also from $\Sigma H = 0$.)

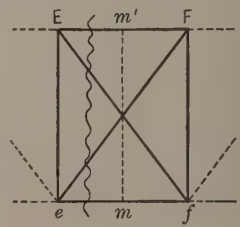


Fig. 200.

7. *The K-truss.* Another special case is that of the K-truss. Let $eEFf$ in Fig. 201 (*a*) represent a typical panel of a K-truss. The H components of the web members $e'F$ and $e'f$ must be equal in magnitude but

opposite in sense. (71 : 6.) If these two members have equal inclinations as is usually the case in a K-truss, the stresses in them will be equal. These stresses may be found from $\Sigma V = 0$, in which the $e'F_V = -e'f_V$, i.e., the shear is equally divided between the two members. The stress in EF may be found from $\Sigma M = 0$ with e as a center, regardless of whether the inclinations of the two members $e'F$ and $e'f$ are equal or unequal. Likewise, the stress in ef may be found from $\Sigma M = 0$ with E as a center. The stresses in EF and ef are equal.

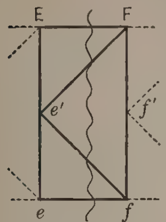


Fig. 201 (*a*).

1. *Exercise:* Prove the statements made in the last three sentences. (Suggestion: Replace the stresses in $e'F$ and $e'f$ by their H and V components applied at e'.)

2. *The Baltimore truss.* A third special case in which a section cuts four members occurs in the Baltimore truss. Let $eEGg$ in Fig. 201 (*b*) represent a typical double panel of a Baltimore truss. The stresses in EG, $f'g$, and fg may be determined exactly as in a Pratt truss, since a section 1–1 may be taken through these three members without cutting a fourth member. But this is not true of the stresses in Ef', ef', and ef, since a section 2–2 through these three members cuts a fourth member EG. The stress in EG, found from section 1–1, could, however, be substituted in the equilibrium equations, thus making the problem determinate. Moreover, the stress in ef is obviously equal to that in fg. This leaves only two unknowns, Ef' and ef'. These stresses can be determined, but not by the simple equation $\Sigma V = 0$, since both unknowns would appear in that equation. It is desirable, however, to use $\Sigma V = 0$ in order that the stress in Ef' may be determined directly from the shear. In order to do this the stress in ef' must be known, and it will now be proved that the V component of this stress (ef'_V) is always equal to one-half of the load at the panel point f, unless some other member not shown in the figure is acting at f', such, for example, as a counter $f'G$.

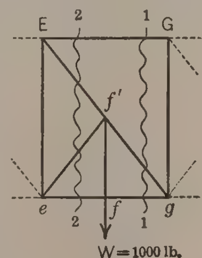


Fig. 201 (*b*).

3. In Fig. 201 (*c*) is shown joint f' taken by itself with the four members $f'E$, ef', $f'f$, and $f'g$ acting at the joint. Let $W = 1000$ lbs. be the load at the panel point f. Then the stress in $f'f$ is obviously W or 1000 lbs. (71 : 5.) To prove that $ef'_V = \frac{1}{2}W = 500$ lbs. $= \frac{1}{2}f'f$, let $f'a$ represent the stress in $f'f = 1000$ lbs. to any convenient scale. Draw bf' normal to $Ef'g$, and ac normal to bf'. Then cf' is the component normal to $Ef'g$ of the force $f'a$. But since the components normal to $Ef'g$ of the stresses in $f'f$ and ef' must be equal in magnitude (71 : 7), cf' is also the component normal to $Ef'g$ of the stress in ef'. The magnitude of the stress itself (in ef') is therefore represented by $c'f'$. The V component of $c'f'$ is $f'd$, and it may easily be proved from similar triangles that $f'd = \frac{1}{2}f'a$, i.e., the V component of $c'f'$, the stress in ef', is one-half of $f'a$, the stress in $f'f$, or, in this case, the V component of the stress in ef' is 500 lbs. It is to be noted that the stress in ef' must be opposite in sense to the stress in $f'f$.

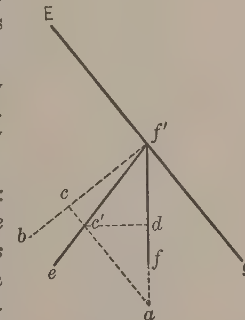


Fig. 201 (*c*).

4. *Exercise:* As another method of proof, show by means of a moment equation that ef'_V must equal $\frac{1}{2}f'f$. (Suggestion: Replace ef' by ef'_V and ef'_H applied at any point of ef' as c' or e.)

5. The essential thing to remember is that: *The V component of a sub-diagonal in a Baltimore truss is equal in magnitude to one-half of the stress in the corresponding sub-vertical but opposite in sense.* This general statement holds true for live-load stresses as well as for dead-load stresses, and for sub-ties as well as for sub-struts. The statement is the key to the solution of any apparently indeterminate problem that may arise in the calculation of stresses in a Baltimore truss, and in certain trusses of a similar type with subdivided panels.

6. **Advantages and Disadvantages of the Algebraic Method of Sections.** The calculation of the stresses in the three members cut by a section involves, in the great majority of cases, the use of two moment equations of equilibrium. If the top and bottom chords of the truss are parallel, the length of the lever arm of the unknown force in each moment equation is equal to the height of the truss; but if the chords are not parallel, the lever arm for the unknown force in an inclined chord must be determined. For this reason, the method of sections is more easily applied to parallel-chord trusses than to non-parallel-chord trusses. For a truss of the latter type, such, for example, as a non-parallel-chord roof truss, the

graphic method of successive joints, explained in the next chapter, can often be used to better advantage. It may be noted, however, that in many cases unknown lever arms may be scaled instead of being computed if the truss diagram has been drawn carefully to scale.

1. The short cuts for the method of sections (explained in 196 : 9) render this method particularly applicable to parallel-chord trusses with equal panel loads; but the method of coefficients, explained in the next chapter, is equally good, if not better, for calculating stresses in such trusses. When panels or panel loads are unequal, the method of sections is usually the best algebraic method that can be used. Panel loads due to locomotive-wheel loads, for example, are unequal, and live-load stresses due to this kind of loading are almost invariably calculated from shears and bending moments, which, in turn, are determined by the method of sections.

2. The stress in any member of a truss may be calculated by the method of sections without involving the stress in any other member. For this reason, the method is frequently used for checking *all* the stresses in a truss by checking the stress in a single member. For example, when the stresses in a parallel-chord truss are to be determined by the method of coefficients (to be explained later), the coefficients for all members in the truss can be checked by ascertaining whether or not the coefficients for two or three members near the center of the truss are correct, and the coefficients for these two or three members may be checked most easily by the method of sections. Again, when the stresses in a roof truss have been determined by the graphic method of successive joints (to be explained later), any error made in obtaining the stress for a member near the end of the truss will usually affect the stresses in all members between that member and the center of the truss; consequently, by checking the stress in a member near the center by the method of sections, one can check the stresses in all of the other members.

3. **Graphic Method of Sections.** The graphic method which corresponds to the algebraic method of sections is seldom used for determining stresses, and hence it will not be explained here. As an optional exercise, however, it may be well to determine the stresses in members *CD*, *Cd*, and *cd* (Fig. 187) by this graphic method. (See graphic solution for Case 4, page 60.)

4. **Illustrative Problems.** The remainder of this chapter is devoted to illustrative problems. In all but one of these problems, the analysis and equations are given for determining the stresses in *only three* truss members, namely, those cut by a typical section. The student should check each statement and each equation in order to fix in mind the general method of sections and to acquire the habit of analyzing a problem for the purpose of applying a standard method of solution. The last problem in the chapter illustrates the method of calculating the stresses in *all* members of a truss by a systematic application of the short-cut methods explained in 196 : 9.

5. In checking the problems, one after the other, the student should realize that different types of trusses and conditions of loading do not really present different problems — that all problems are fundamentally the same; that having mastered the method for one type of truss he need fear no problem in any other type. The simplicity of the method thus becomes real to him, and he gains confidence in his ability to use it. In order to emphasize the statement that the method of sections is essentially the same for all types of parallel-chord trusses, equations are given for determining the stresses in three corresponding members in four types of trusses. A comparison of these equations will show that not only is the general method the same for all four types of trusses, but that, with few exceptions, the equation for determining the stress in a given member in one type is identical with the equation for determining the stress in the corresponding member in one or more of the other types. (Figs. on p. 203.)

To find A in Fig. (*a*), (*b*), or (*d*): $\Sigma M_m = 0 = 2500\times40 - 1000\times20 + A\times24$

To find C in Fig. (*c*): $\Sigma M_n = 0 = 2500\times40 - 1000\times20 + C\times24$

To find B in all four figures: $\Sigma V = 0 = 2500 - 1000 - 1000 + B_V$

To find C in Fig. (*b*), or (*d*): $\Sigma M_n = 0 = 2500\times60 - 1000\times40 - 1000\times20 + C\times24$

To find A in Fig. (*c*): $\Sigma M_m = 0 = 2500\times60 - 1000\times40 - 1000\times20 + A\times24$

To find C in Fig. (*a*): $\Sigma M_n = 0 = 2500\times50 - 1000\times30 - 1000\times10 + C\times24$

Question: In which of the four problems would it have been better to use $\Sigma H = 0$ in place of one of the moment equations?

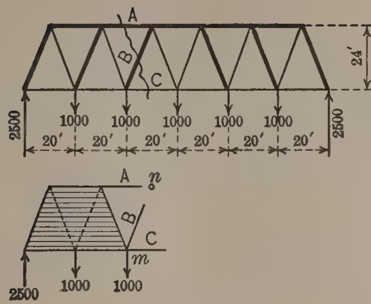


Fig. (a).

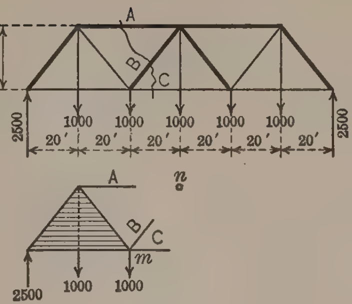


Fig. (b).

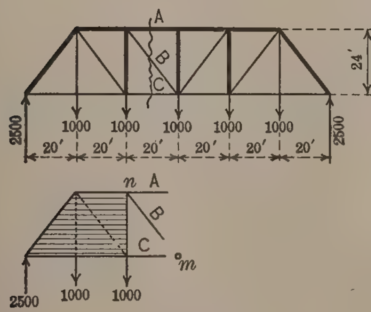


Fig. (c).

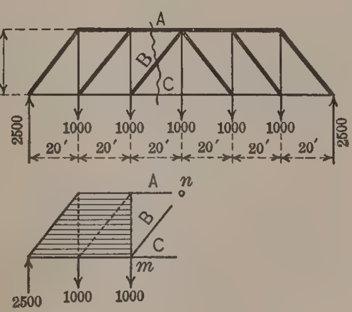


Fig. (d).

In the equations just given, panel loads are entered as separate forces; but in most of the illustrative problems that follow, all of the panel loads that act on the segment in equilibrium are replaced by a resultant load in moment equations. (171 : 14.)

The notation used is as follows: w = the panel load; W = the resultant of panel loads on the segment; R_L and R_R = the reactions; p = the panel length; h = the height of truss.

The analysis and the equations of equilibrium are written in terms of the symbols just given. By using these symbols and reducing the corresponding equations to their lowest terms before substituting actual values, the work of computation is reduced to a minimum.

Problem 1. Given: The pony truss shown in the photograph on page 95. Panel length = 15′-0″; height of truss = 7′-6″; panel load w = 6000 lbs. Required: The stresses in CD, Cc, and bc. (Fig. 203 (a).)

$R_L = 2w$. $W = w = 6000$ lbs.

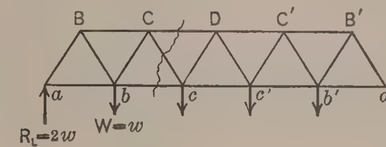


Fig. 203 (a).

Body in equilibrium: $aBCb$.
Known: R_L, W, CD(L), Cc(L), and bc(L).
Unknown: CD(M), Cc(M), and bc(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

$$\Sigma M_c = R_L \times 2p - W \times p + CD \times h = 0 = 3w \times p + CD \times h$$

36,000 lbs. (C) $= CD$.

$$\Sigma M_C = R_L \times 1\tfrac{1}{2}p - W \times \tfrac{1}{2}p + bc \times h = 0 = 2\tfrac{1}{2}w \times p + bc \times h$$

30,000 lbs. (T) $= bc$.

$$\Sigma V = R_L - W + Cc_V = 0.$$

8,500 lbs. (T) $= Cc$.

Question: Why is the stress in bC equal in magnitude to the stress in Cc but opposite in character? (197 : 7.)

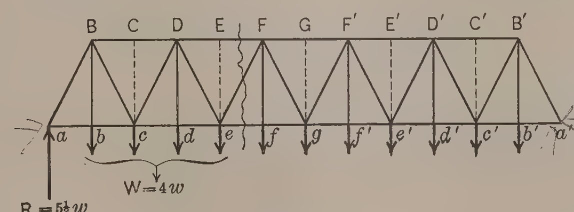


Fig. 203 (b).

Problem 2. Given: The Warren truss with verticals shown in the photograph on page 98. Panel length = 16′-0″; height of truss = 32′-0″; panel load w = 24,000 lbs. Required: The stresses in EF, eF, and ef. (Fig. 203 (b).)

$R_L = 5\tfrac{1}{2}w$. $W = 4w = 96{,}000$ lbs.

Body in equilibrium: $aBDe$.
Known: R_L, W, EF(L), eF(L), and ef(L).
Unknown: EF(M), eF(M), and ef(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

$$\Sigma M_e = R_L \times 4p - W \times 1\tfrac{1}{2}p + EF \times h = 0 = 16w \times p + EF \times h$$

192,000 lbs. (C) $= EF$.

$$\Sigma M_F = R_L \times 5p - W \times 2\tfrac{1}{2}p + ef \times h = 0 = 17\tfrac{1}{2}w \times p + ef \times h$$

210,000 lbs. (T) $= ef$.

$$\Sigma V = R_L - W + eF_V = 0$$

40,300 lbs. (C) $= eF$.

Problem 3. Given: The deck Pratt truss partially shown in the photograph on page 104. Number of panels = 8; panel length = 20′-3″; height of truss = 24′-3″; panel load w = 24,300 lbs. Required: The stresses in CD, Dd, and de. (Fig. 204 (*a*).)

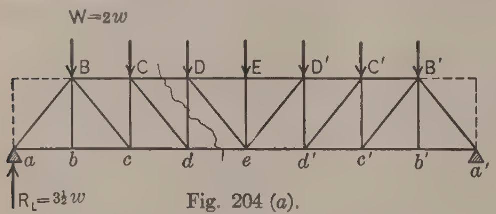


Fig. 204 (*a*).

$R = 3\frac{1}{2}w.$
$W = 2w = 48{,}600$ lbs.

Body in equilibrium: $aBCd$.
Known: R_L, W, CD(L), Dd(L), and de(L).
Unknown: CD(M), Dd(M), and de(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (70 : 4 (a).)

$\Sigma M_d = R_L \times 3p - W \times 1\frac{1}{2}p + CD \times h = 0 = 7\frac{1}{2}w \times p + CD \times h$
152,200 lbs.(C) $= CD$.
$\Sigma V = R_L - W + Dd = 0 = 1\frac{1}{2}w + Dd$
36,500 lbs.(C) $= Dd$.
$\Sigma H = -CD + de = 0$
152,200 lbs.(T) $= de$.

Question: Why is the stress in Cd equal in magnitude to the stress in Dd multiplied by the secant of the truss angle. (197 : 6.)

Problem 4. Given: A Warren truss with verticals shown in Fig. 204 (*b*). Panel length = 20 ft.; height of truss = 26 ft.; panel load w = 6000 lbs. In addition to the seven panel loads, there is a concentrated load at each of the joints b, c, and d as shown. Required: The stresses in DD', De, de, and Dd.

The right-hand segment $eD'B'a'$ has the smaller number of external forces acting on it, hence that segment will be taken as the body in equilibrium (192 : 5.)

$R_R = 3\frac{1}{2}w + \frac{1}{8} \times 4000 + \frac{2}{8} \times 8000 + \frac{3}{8} \times 2000 = 24{,}250$ lbs. $W = 4 \times w = 24{,}000$ lbs.

Body in equilibrium: $eD'B'a'$.
Known: R_R, W, DD'(L), De(L), and de(L).
Unknown: DD'(M), De(M), and de(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

$\Sigma M_e = -24{,}250 \times 80 + 24{,}000 \times 30 + DD' \times 26 = 0$
46,900 lbs.(C) $= DD'$.
$\Sigma M_D = 24{,}250 \times 100 + 24{,}000 \times 50 + de \times 26 = 0$
47,100 lbs.(T) $= de$.
$\Sigma V = 24{,}250 - 24{,}000 + De_V = 0$
320 lbs.(C) $= De$.
8000 lbs.(T) $= 2000 + 6000 = Dd$ (71 : 5.)

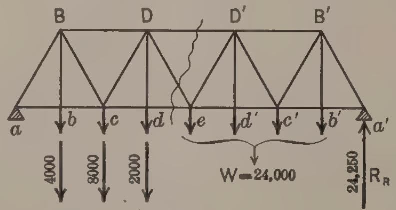


Fig. 204 (*b*).

Problem 5. Given: The Pratt truss shown in the photograph on page 92. Panel length = 22′-0″; height of truss 32′-0″; panel load w = 24,000 lbs. one fourth of which is assumed to act at the upper apex. Required: The stresses in BC, Cc, and cd. (Fig. 204 (*c*).)

$R_L = 2\frac{1}{2}w;\ W = 2 \times \frac{3}{4}w = 1\frac{1}{2}w;\ W' = \frac{1}{4}w.$

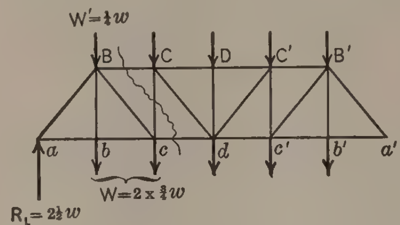


Fig. 204 (*c*).

Body in equilibrium: aBc.
Known: R_L, W, W', BC(L), Cc(L), and cd(L).
Unknown: BC(M), Cc(M), and cd(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma V = 0$, and $\Sigma H = 0$. (70 : 4(a).)
From $\Sigma M_c = 0$ determine BC.
From $\Sigma V = 0$ determine Cc.
From $\Sigma H = 0$ determine cd.

Answers: BC = 66,000 lbs.(C); Cc = 18,000 lbs.(C); cd = 66,000 lbs.(T).

Question: Why is the stress in Cd equal in magnitude to (18,000 − 6000) multiplied by the secant of the truss angle? (197 : 6 and 199 : 14.)

Problem 6. Given: The cantilever truss shown in Fig. 204 (*d*). Panel length = 12 ft.; height of truss = 14 ft.; panel load w = 2000 lbs. Required: The stresses in BC, Bc, and bc.

It is not necessary to determine the reactions at a and f. $W = 3w$.

Body in equilibrium: $cCDe$.
Known: W, BC(L), Bc(L), and bc(L).
Unknown: BC(M), Bc(M), and bc(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.
From $\Sigma M_c = 0$ determine BC.
From $\Sigma M_B = 0$ determine bc.
From $\Sigma V = 0$ determine Bc.

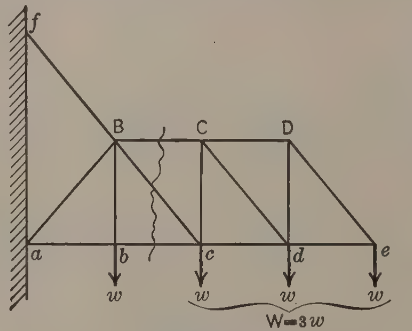


Fig. 204 (*d*).

Answers: BC = 5100 lbs.(T); bc = 10,300 lbs.(C); Bc = 7900 lbs.(T).

Problem 7. Given: The truss shown in Fig. 205 (*a*), supported at joints *b* and *f*. Panel length = 16 ft.; height of truss = 18 ft.; panel load *w* at each lower joint except *b* and *f* is 3500 lbs. Required: The stresses in *de*, *Ee*, and *EF*.

$$R_L = w(\tfrac{5}{4} + \tfrac{3}{4} + \tfrac{2}{4} + \tfrac{1}{4} - \tfrac{1}{4} - \tfrac{2}{4}) = 2w. \quad (152:1.)$$

Body in equilibrium: *aBEd*.

Known: R_L, loads at *a*, *c*, and *d*, *de*(L), *Ee*(L), and *EF*(L).

Unknown: *de*(M), *Ee*(M), and *EF*(M). (Case 4.)

Equations: $\Sigma M = 0$, $\Sigma H = 0$, and $\Sigma V = 0$. (70 : 4(a).)

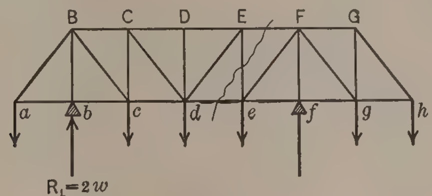


Fig. 205 (*a*).

$$\Sigma M_e = -w(4p + 2p + p) + R_L \times 3p + EF \times h = 0 = -w \times p + EF \times h$$

3110 lbs. = *EF* ⤸ Tension.

$$\Sigma H = EF + de = 0$$

-3110 lbs. = *de* ← Compression.

$$\Sigma V = -3w + R_L + Ee = 0 = -w + Ee$$

3500 lbs. = *Ee* ↑ Compression.

Problem 8. Given: The tower bent shown in Fig. 205 (*b*). The bent is anchored at *a* and *a′* and the horizontal external forces due to wind pressure are those indicated. Required: The stresses in *ab*, *ab′*, and *a′b′*. It is not necessary to determine the reactions R_L and R_R.

Body in equilibrium: *bdd′b′*.

Known: *A*, *B*, *C*, *D*, *ab*(L), *ab′*(L), and *a′b′*(L).

Unknown: *ab*(M), *ab′*(M), and *a′b′*(M). (Case 4.)

Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma H = 0$.

From $\Sigma M_{b'} = 0$ determine *ab*.

From $\Sigma M_a = 0$ determine *a′b′*.

From $\Sigma H = 0$ determine *ab′*.

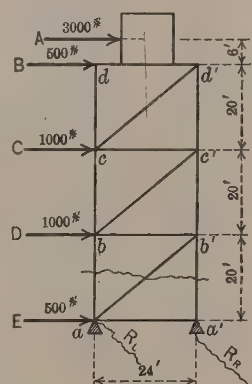


Fig. 205 (*b*).

Answers: *ab* = 7400 lbs.(T); *a′b′* = 12,000 lbs.(C); *ab′* = 7200 lbs.(T).

Problem 9. Given: A roof truss with inclined loads as shown in Fig. 205 (*c*). *A* = *F* = 1200 lbs.; *B* = *C* = *D* = *E* = 2400 lbs. Assuming that the stresses in all members have been determined by a graphic method (to be explained later), it is required to check the stress in the member *de* by the method of sections. (Fig. 205 (*d*).)

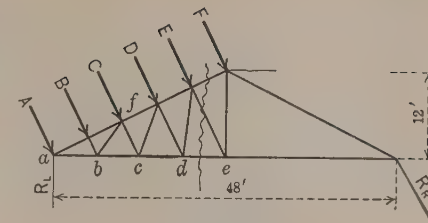


Fig. 205 (*c*).

R_L = 7380 lbs. (Calculated in *Problem 5*, page 149.) *A* = 1200 lbs.; *W* = 9600 lbs. is the resultant of *B*, *C*, *D* and *E* applied at *f*. Resolve *A* and *W* into their H and V components.

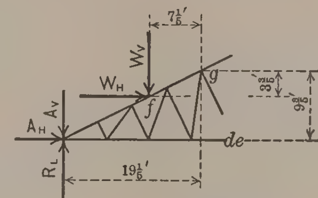


Fig. 205 (*d*).

$$\begin{aligned}\Sigma M_g &= R_L \times 19\tfrac{1}{5} - A_H \times 9\tfrac{3}{5}\\ &\quad - A_V \times 19\tfrac{1}{5} - W_H \times 3\tfrac{3}{5}\\ &\quad - W_V \times 7\tfrac{1}{5} + de \times 9\tfrac{3}{5}\\ &= 0\end{aligned}$$

4000 lbs. (T) = *de*.

Problem 10. Given: The deck Baltimore truss shown in the photograph on page 106. Panel length = 20 ft.; depth of truss = 40 ft.; panel load *w* = 36,000 lbs.

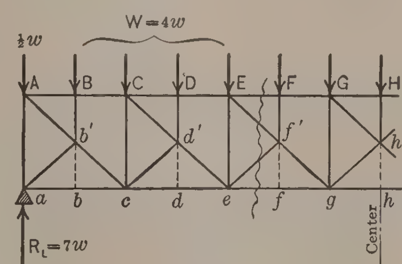


Fig. 205 (*e*).

Required: The stresses in *EF*, *Ef′*, *ef′*, and *ef*. (Only one half of the truss is shown in Fig. 205 (*e*).)

The half diagonal *ef′* is in compression, and the V component of the stress in *ef′* is $\frac{1}{2}w$. (201 : 2 and 5.)

$R_L = 7w$; $A = \frac{1}{2}w$; $W = 4w$; $ef'_V = \frac{1}{2}w$; $ef' = \frac{1}{2}w \times \sec 45° = 25{,}450$ lbs.(C).

Body in equilibrium: *aAEe*.
Known: R_L, A, W, ef', EF(L), Ef'(L), and ef(L).
Unknown: EF(M), Ef'(M), and ef(M). (Case 4.)
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

Replace ef' by ef'_H and ef'_V applied at e.

$$\Sigma M_g = R_L \times 6p - \tfrac{1}{2}w \times 6p - W \times 3\tfrac{1}{2}p - ef'_V \times 2p + EF \times h = 24wp + EF \times h$$

432,000 lbs.(C) = EF.

Replace ef' by ef'_H and ef'_V applied at G.

$$\Sigma M_E = R_L \times 4p - \tfrac{1}{2}w \times 4p - W \times 1\tfrac{1}{2}p + ef'_V \times 2p + ef \times h = 0 = 21wp + ef \times h$$

378,000 lbs.(T) = ef.

$$\Sigma V = R_L - \tfrac{1}{2}w - W - ef'_V + Ef'_V = 2w + Ef'_V$$

101,800 lbs.(T) = Ef'.

Note: Instead of determining the stresses in EF and ef directly from moment equations as just indicated, it is somewhat simpler to use the following method: Assume $aAFf$ as the body in equilibrium. From $\Sigma M_g = 0$ determine FG and from $\Sigma M_E = 0$ determine fg. Then $EF = FG$ and $ef = fg$.

Problem 11. Given: A K-truss with parallel chords. (One-half of the truss is shown in Fig. 206 (*a*).) Panel length = 16 ft.; height of truss = 32 ft.; panel load = 20,000 lbs. Required: The stresses in DE, $d'E$, $d'e$, and de. The magnitude of the stress in $d'E$ is equal to the magnitude of the stress in $d'e$. (200 : 7.)

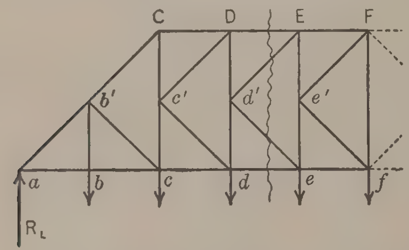


Fig. 206 (*a*).

Body in equilibrium: *aCDd*.
Equations: $\Sigma V = 0$, $\Sigma M = 0$, and $\Sigma H = 0$.

From $\Sigma V = 0$ determine $d'E = d'e$ (both downward). (Why?)

From $\Sigma M_d = 0$ determine DE. (Note that $d'E$ and $d'e$ cancel each other in this equation.)

From $\Sigma H = 0$ determine de. (Note that $d'E_H$ and $d'e_H$ cancel each other in this equation.)

Answers: DE = 105,000 lbs.(C) = de(T); $d'E$ = 21,200 lbs.(C) = $d'e$(T).

Question: Why is the stress in Ee' equal in magnitude to the stress in $d'E$ divided by the secant of the angle $d'Ee'$? (197 : 5.)

Problem 12. Given: A Pratt truss on a 5% grade with loads as shown in Fig. 206 (*b*). Panel length = 20′-0$\frac{5}{16}$″; height of truss = 28′-0″; length of span = 120′-0″. Required: The stresses in the members CD, Cd, and cd.

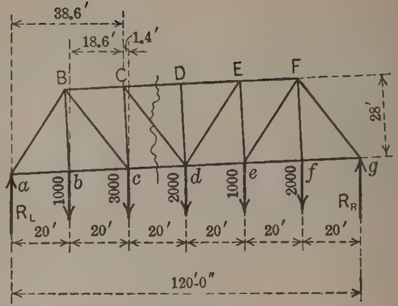


Fig. 206 (*b*).

R_L = 4500 lbs. (calculated in *Problem 2*, page 149).

Body in equilibrium: *aBCc*.
Equations: $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$.

$$\Sigma M_d = R_L \times 60 - 1000 \times 40 - 3000 \times 20 + CD \times 28 = 0$$

6100 lbs.(C) = CD.

$$\Sigma M_C = R_L \times 38.6 - 1000 \times 18.6 + 3000 \times 1.4 + cd \times 28 = 0$$

5700 lbs.(T) = cd.

$$\Sigma V = R_L - 1000 - 3000 - CD_V + cd_V + Cd_V = 0$$

600 lbs.(T) = Cd.

Problem 13. In *Problem 13*, page 207, it was required to calculate the stresses in all members in one-half of a deck Pratt truss similar to that shown in the photograph on page 104. The method of procedure was as follows:

(1) The truss diagram was drawn and the loads and reaction were indicated on this diagram.

(2) The preliminary calculations were indicated and then these calculations were made.

(3) The tabular forms for calculating stresses were prepared, the second column being left blank. Equations for stresses were written in terms of W the panel load, p the panel length, h the height of truss, and the secant of the truss angle A. (196 : 9 to 198 : 3.) After these equations had been reduced to the lowest terms, they were checked roughly by inspection. It is to be noted that W represents a panel load and not the resultant of panel loads on a segment as in the preceding problems.

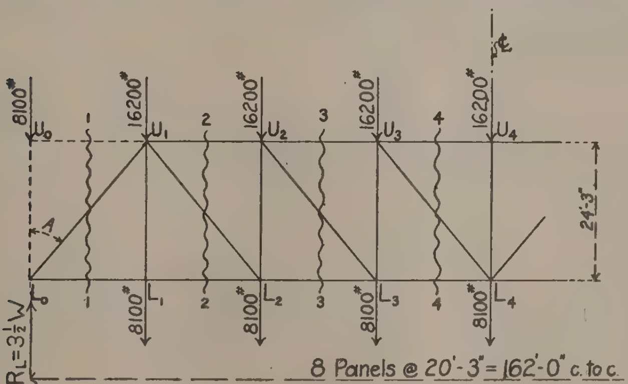
(4) The stresses were calculated and inserted in the second column.

Note: In the preliminary calculations and in the calculation of stresses, the actual work of calculation was left until this work had been completely indicated. (Why? (198 : 4).)

PROBLEM 13

ILLUSTRATIVE PROBLEM IN THE CALCULATION OF THE DEAD-LOAD STRESSES IN A DECK PRATT TRUSS BY THE METHOD OF SECTIONS.

Given: A deck Pratt truss railroad bridge. Number of panels = 8. Panel length, p, = 20'-3"; height of truss, h, = 24'-3". Dead load per lin. ft. of bridge = 2400 lbs., distributed one third to the lower chord, two thirds to the upper. Required: The dead-load stresses in all members of the truss.



Preliminary Calculations

$24\,300^{\#} = (2400 \times 20\frac{1}{4}) \div 2 = W =$ Dead load per panel per truss

$8\,100^{\#} = \frac{1}{3} \times 24\,300 = \frac{1}{3}W =$ Portion of panel load at lower apex

$16200^{\#} = \frac{2}{3} \times 24\,300 = \frac{2}{3}W =$ " " " " " upper "

$20300^{\#} = (24\,300 \times 20\frac{1}{4}) \div 24\frac{1}{4} = (W \times p) \div h$

$1.303 = \sqrt{(20\frac{1}{4})^2 + (24\frac{1}{4})^2} \div 24\frac{1}{4} =$ Secant of truss angle A

$31\,700^{\#} = 24\,300 \times 1.303 = W \times \sec A$

Stresses in Chord Members

Member	Stress	Section	Center of Moments	Bending Moment	Stress in Terms of $M_B \div h$
U_1U_2	$121800^{\#}$(C)	2-2	L_2	$3\frac{1}{2}W \times 2p - W \times p$	$\overbrace{6(W \times p)}^{M_B} \div h = 6 \times \overbrace{20300^{\#}}^{W \times p \div h}$
U_2U_3	$152300^{\#}$(C)	3-3	L_3	$3\frac{1}{2}W \times 3p - Wp(1+2)$	$7\frac{1}{2}(W \times p) \div h = 7\frac{1}{2} \times$ "
U_3U_4	$162400^{\#}$(C)	4-4	L_4	$3\frac{1}{2}W \times 4p - Wp(1+2+3)$	$8(W \times p) \div h = 8 \times$ "
$L_0L_1L_2$	$71\,100^{\#}$(T)	1-1	U_1	$3\frac{1}{2}W \times p$	$3\frac{1}{2}(W \times p) \div h = 3\frac{1}{2} \times$ "
L_2L_3	$121\,800^{\#}$(T)	Stress	in L_2L_3	equal to stress in	U_1U_2
L_3L_4	$152300^{\#}$(T)	"	" L_3L_4	" " " "	U_2U_3

Stresses in Diagonal Members

Member	Stress	Section	Shear × Secant A
L_0U_1	$111\,000^{\#}$(C)	1-1	$\overbrace{3\frac{1}{2}}^{\text{Shear}}W \times \sec A = 3\frac{1}{2} \times \overbrace{31700^{\#}}^{W \times \sec A}$
U_1L_2	$79300^{\#}$(T)	2-2	$2\frac{1}{2}W \times \sec A = 2\frac{1}{2} \times$ "
U_2L_3	$47600^{\#}$(T)	3-3	$1\frac{1}{2}W \times \sec A = 1\frac{1}{2} \times$ "
U_3L_4	$15\,900^{\#}$(T)	4-4	$\frac{1}{2}W \times \sec A = \frac{1}{2} \times$ "

Stresses in Vertical Members

Member	Stress	
U_1L_1	$8\,100^{\#}$(T)	Lower chord panel load = $\frac{1}{3}W$
U_2L_2	$52\,700^{\#}$(C)	$(U_2L_3)_v + \frac{2}{3}W = 1\frac{1}{2}W + \frac{2}{3}W = 36\,500^{\#} + 16\,200^{\#}$
U_3L_3	$28400^{\#}$(C)	$(U_3L_4)_v + \frac{2}{3}W = \frac{1}{2}W + \frac{2}{3}W = 12\,200^{\#} + 16\,200^{\#}$
U_4L_4	$16\,200^{\#}$(C)	Upper chord panel load = $\frac{2}{3}W$

Stresses in Supplementary Members

U_0U_1	0	
U_0L_0	$8\,100^{\#}$(C)	One half upper chord panel load = $\frac{1}{3}W$

CHAPTER XV

ALGEBRAIC METHOD OF SUCCESSIVE JOINTS

In this chapter is explained the general method of calculating stresses in which each joint of the truss in turn is considered as a particle held in equilibrium by the forces that act at that joint. Some of these forces are *internal* forces (stresses) exerted by members of the truss that meet at the joint, whereas others may be *external* forces, such as loads or reactions.

1. **An Illustrative Example of the Use of the Method of Successive Joints** will be given first, and then the general method will be summarized. Let the triangular truss of six panels shown in Fig. 208 (a) be loaded with panel loads of 1000 lbs. applied at lower apices. The reactions are each 2500 lbs.

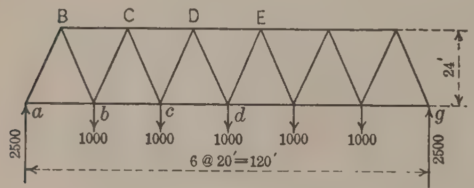


Fig. 208 (a).

2. Any joint of the truss may be considered as a particle held in equilibrium by the forces which act at that joint. For example, joint b (Fig. 208 (b)) may be considered as a particle held in equilibrium by the four members (ab, Bb, bC, and bc) and the load of 1000 lbs. If not more than two of the magnitudes of the forces exerted by the members are unknown, the two unknown magnitudes may be found by the method of Case B (concurrent forces). (69 : 10.) If there are more than two unknown magnitudes, the problem is indeterminate. Each joint is a separate problem in Case B, and this problem is independent of the problems at other joints except that it usually involves the magnitudes of two or three stresses which were determined at other joints. In order to find the stresses in all of the members in the truss, it is necessary to solve as many problems as there are joints in one-half of the truss, namely, joints a, B, b, C, c, and D. The standard algebraic solution for a problem in concurrent forces in which the unknown elements are two magnitudes (Case B) involves the use of only the two resolution equations $\Sigma H = 0$ and $\Sigma V = 0$. The algebraic method of successive joints, therefore, consists merely in applying to each joint of the truss in succession the condition of equilibrium represented by these two equations, namely, that there shall be no motion of translation. If there is no such motion at any joint of the truss, then no joint can revolve about any other joint, and hence, taking the truss as a whole, there can be no motion of rotation. Since there can be no motion either of translation or rotation, the truss as a whole must be in equilibrium. It is to be noted that the method of successive joints does not involve the use of the moment equation of equilibrium.

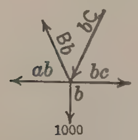


Fig. 208 (b).

3. *Note:* The method of successive joints, and therefore this entire chapter, is based on Case B. Before proceeding further, it may be well to review Chapter VI, particularly the general method of procedure for algebraic solutions (24 : 1) and the illustrative problem in Case B on page 28, including the various comments.

4. *Calculation of stresses.* The calculation of the stresses in the truss shown in Fig. 208 (a) will now be given in detail. The first joint attacked must be one at which there are not more than two magnitudes unknown. There are only two such joints in the whole truss, namely, joints a and g

at the supports. It is immaterial which of these two joints is taken as the first particle in equilibrium. If a is chosen, the joints in the left-hand half of the truss will be attacked in order, one after the other, beginning at a and proceeding toward the center of the truss; if g is chosen, the joints in the right-hand half of the truss will be attacked in a similar way. In the calculations which follow, the joints in the left-hand half of the truss are taken as the particles in equilibrium.

1. *Note:* By applying certain fundamental principles to joint d at the middle of the bottom chord it would be possible to begin at that joint and proceed toward either end, but it is simpler to begin with joint a or g.

2. Let A represent the truss angle (inclination of a diagonal web member to the vertical); the trigonometric functions to be used in calculating components may be found as follows:

$$\sqrt{10^2 + 24^2} = 26 = \text{length of diagonal}$$
$$26 \times \sin A = 10, \text{ or } \sin A = .385$$
$$26 \times \cos A = 24, \text{ or } \cos A = .923$$

Joint a. (Fig. 209 (*a*).)

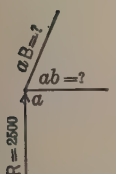


Fig. 209 (*a*).

Known: R, aB(L), and ab(L).
Unknown: aB(M) and ab(M). (Case B.) (69 : 10.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.

$$\Sigma V = aB_V + 2500 = 0$$
$$= aB \times .923 + 2500 = 0 \quad aB = -2710 \text{ lbs.} \downarrow \text{(C).}$$
$$\Sigma H = aB_H + ab = 0$$
$$= -2710 \times .385 + ab = 0 \quad ab = +1040 \text{ lbs.} \rightarrow \text{(T).}$$

The result for aB from $\Sigma V = 0$ is minus, hence aB acts downward or toward a (compression). The H component of aB will be minus. From $\Sigma H = 0$, ab is plus, hence acts toward the right or away from a (tension).

3. The next joint attacked must be at the other end (from a) of one of the members aB or ab in which the stress has just been found, otherwise there would be more than two unknown magnitudes; it must therefore be one of the joints b or B. It cannot be b, since even with the magnitude of ab known there will still be three unknown magnitudes, namely, bB, bC, and bc. But at joint B there are only two unknown magnitudes, namely, BC and Bb, the magnitude of aB having been determined at joint a. Since aB has already been found to be in compression, it must act toward joint B, or upward and to the right. (72 : 3.) Its V component at B is therefore plus, and its H component is also plus. (65 : 3.)

Joint B. (Fig. 209 (*b*).)

Known: aB, BC(L), and Bb(L).
Unknown: BC(M) and Bb(M). (Case B.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.

$$\Sigma V = aB_V + Bb_V = 0$$
$$= 2500 + Bb \times .923 = 0 \quad Bb = -2710 \text{ lbs.} \downarrow \text{(T).}$$
$$\Sigma H = aB_H + Bb_H + BC = 0$$
$$= 1040 + 1040 + BC = 0 \quad BC = -2080 \text{ lbs.} \leftarrow \text{(C).}$$

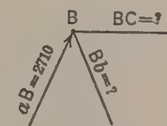


Fig. 209 (*b*).

The algebraic signs show that Bb acts downward or away from B (tension), whereas BC acts to the left or toward B (compression).

4. The joint b may now be attacked, since there are only two unknowns, namely, the magnitudes of bC and bc.

Joint b. (Fig. 209 (*c*).)

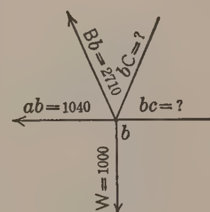


Fig. 209 (*c*).

Known: W, ab, Bb, bC(L), and bc(L).
Unknown: bC(M) and bc(M). (Case B.)
Equations: $\Sigma V = 0$ and $\Sigma H = 0$.

$$\Sigma V = -W + Bb_V + bC_V = 0$$
$$= -1000 + 2500 + bC \times .923 = 0$$
$$bC = -1630 \text{ lbs.} \downarrow \text{(C).}$$
$$\Sigma H = -ab - Bb_H - bC_H + bc = 0$$
$$= -1040 - 1040 - 630 + bc = 0$$
$$bc = +2710 \text{ lbs.} \rightarrow \text{(T).}$$

Both ab and Bb have already been found to be in tension, hence they may be shown in Fig. 209 (c) as acting away from joint b. From algebraic signs of the results obtained from the equations, bC acts toward b (compression) and bc acts away from b (tension).

5. It is evident that the problem for any one of the three joints thus far attacked was a very simple problem under Case B. By attacking in a

similar manner successive joints in the order C, c, and D, the stresses in the remaining members of the left-hand half of the truss may be calculated. Since the truss is symmetrical and symmetrically loaded, the stress in any member in the right-hand half of the truss is equal to the stress in the corresponding member in the left-hand half; it is unnecessary, therefore, to proceed beyond joint D.

1. **General Method of Successive Joints Summarized:** The general method of calculating stresses by the method of successive joints may be summarized as follows:

2. *First:* Draw a diagram of the truss (or of half of the truss if the truss is symmetrical and symmetrically loaded), calculate the reactions, and indicate on the truss diagram all of the external forces that act on the truss.

3. *Second:* Begin by attacking the joint at one of the supports, and find the stresses in the two truss members which meet at that joint by the standard method for a problem in concurrent forces, Case B. ($\Sigma V = 0$ and $\Sigma H = 0$.)

4. In trusses supported at both ends, usually only *two* members of the truss meet at the joint at a support. When a truss has one supported and one unsupported end, usually only two members meet at the joint at the unsupported end, and, by beginning at that joint, stresses may often be found in members throughout the truss without first determining reactions.

5. *Third:* The stresses in two members of the truss were calculated in the second step. These two members extend from the joint considered in the second step to two other joints, one at the other end of each member. At one of these two joints there will be only two unknowns, usually the unknown magnitudes of the stresses in two members; attack that joint next, using the equations $\Sigma V = 0$ and $\Sigma H = 0$ as in the second step. The sense of a *known* stress in any member at the joint will be opposite to the sense at the other end of that member.

6. *Fourth:* Repeat the general method of the third step, proceeding from joint to joint and making use at each joint of the stresses in members at that joint which have previously been calculated. Treat each joint as a separate problem in concurrent forces (usually Case B). If the truss is symmetrical and symmetrically loaded, it is only necessary to calculate the stresses in the members in one-half of the truss.

Suggestions

7. Articles 184 : 1 to 5 on external forces at the beginning of Chapter XIV applies to this chapter also, as does article 184 : 6 on drawing truss diagrams.

8. Usually not more than one force *external* to the truss acts at a joint, and this force is completely known; all of the other forces are exerted by truss members. Under the conventional systems of loading, there are often no loads or other external forces at many of the joints, such, for example, as the joints of the upper chord of a through-bridge truss.

9. When attacking one joint, ignore completely the rest of the truss and concentrate on the forces that act at that joint. It may be helpful at first to make a separate sketch for each joint, but this should soon become unnecessary.

10. An unknown is given a plus sign in an equation. Since the equations used are resolution equations, the algebraic sign of a result gives the sense of the corresponding force directly. (Rotation is not involved.) From the sense of a force it is evident whether that force acts toward or away from the joint under consideration, and hence whether it is in compression or tension.

11. Remember that the sense of the force which a member exerts on a joint at one end of that member is opposite to the sense of the force which the member exerts on a joint at the other end. It is well to place two arrows on each member in the truss diagram as soon as the stress in that member has been determined, one arrow at each end. (These two arrows will be opposite in direction.)

12. The two unknown magnitudes at a joint may often be determined by inspection from a fundamental principle as, for example, when the lines of action of all of the forces at a joint lie in one or the other of two straight lines. (71 : 5.) (See 188 : 2 for other special cases.)

13. In parallel-chord trusses the diagonal web members usually are inclined at the same angle (truss angle), and the trigonometric functions of this angle necessary for determining components should be calculated at the start from the panel length and height of truss. (It is not necessary to determine the angle itself.) When inclined members which meet at a joint have different angles of inclination, their H and V components are more easily found by the geometric method in which the lengths of the members are used. (65 : 2.)

14. Occasionally it is of advantage to use components parallel to co-ordinate axes which are not horizontal or vertical, but parallel or perpendicular to some member acting at the joint. For example, in Fig. 210 is shown a joint in the top chord of a roof truss. The co-ordinate axes could advantageously be assumed, one parallel and the other perpendicular to the member BC (or CD). (See illustrative problem on p. 28 and problem 28 : 7.)

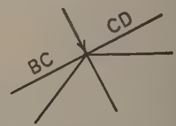


Fig. 210.

15. **Analysis of Joints from Inspection.** The number of forces which ordinarily act at a joint of a truss is so small, rarely exceeding five or six, that many elements concerning these forces may be determined by

inspection. Analysis from inspection will frequently render unnecessary the calculation of certain stresses, and, in any event, will serve as a rough check on stresses which have been calculated. There is no excuse for letting a calculated result stand as correct when a quick and easy analysis of the forces from inspection would show that result to be thousands of pounds in error.

1. If all of the forces which meet at a joint are resolved into H and V components, (1) the sum of the components that act to the right must equal the sum of those that act to the left (from $\Sigma H = 0$), and (2) the sum of the components that act upward must equal the sum of those that act downward (from $\Sigma V = 0$). By applying first one and then the other of these principles, it is usually easy to determine by the mere inspection of a joint not only which forces act toward and which away from the joint (compression and tension), but also what each of the two unknown magnitudes must be, in terms of the known forces. Not only is this analysis from inspection useful as a check on calculated stresses, but it is the basis of the method of coefficients, to be explained later. The analysis is very simple in parallel-chord trusses, since usually at least half of all of the forces at a joint are either horizontal or vertical; moreover, not more than three or four ordinarily have V components and not more than four have H components.

2. Analysis by inspection should become a fixed habit from now on, in any method of determining stresses. Its importance justifies the insertion of two illustrative examples in which joints are analyzed in order in each of two typical trusses. In the figure for each joint, each of the forces already determined from other joints is shown with the sense indicated by an arrow, while the line of action of each of the two forces of unknown magnitude is shown without an arrow. In analyzing the forces at any joint, it is well to begin with the forces that have V components, particularly at a joint where there is an external force. From the external force it will be evident in many cases how the other forces that have V components act.

3. In the first illustrative example equations are given in which the magnitudes of the two stresses required at each joint are expressed in terms of panel loads and the secant or tangent of the truss angle. These equations are of secondary importance at present, since the object is to illustrate how one can get a clear conception from inspection of the *relative* magnitudes of the forces at each joint and the directions in which these forces act. It is suggested, therefore, that the student ignore the equations just referred to until he has been through the analysis of each joint for *both* illustrative examples; he should then go back and check each of the equations.

4. *First illustration of the analysis of joints from inspection:* In Fig. 211 one-half of a six-panel triangular truss is shown.

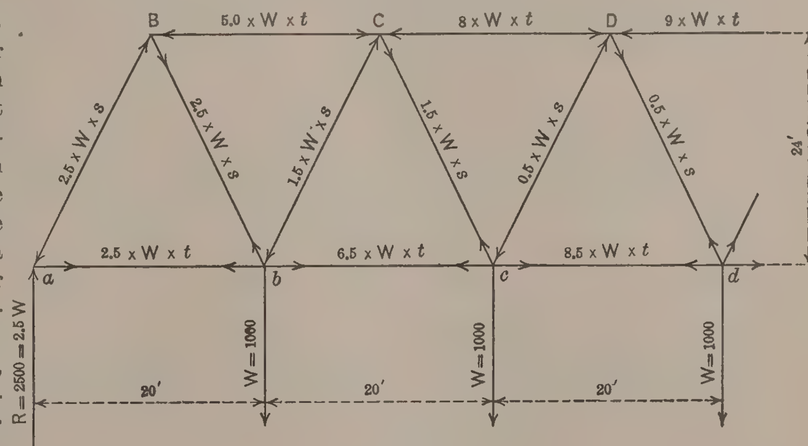


Fig. 211.

Panel length = 20′. Height of truss = 24′.
Secant of the truss angle = s = 1.083.
Tangent of the truss angle = t = .417.
Panel load in thousands of pounds = W.

5. *Note:* In checking numerical values, keep in mind the following trigonometric relations:

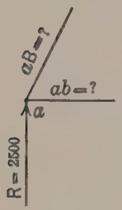
$$F = F_V \times \sec A. \quad (65:8.)$$
$$F_H = F \times \sin A = F_V \times \sec A \times \sin A = F_V \times \tan A. \quad (65:10.)$$

6. *Note:* When the sense of a stress in a member at a given joint is known, it is sometimes well in designating the stress to make the letter at the joint the *first* letter

of the two that represent the stress if that stress acts *away* from the joint, — the *second* letter if the stress acts toward the joint. For example, *Ba* indicates that the stress *Ba* acts away from joint *B*, whereas *aB* indicates that the stress acts toward joint *B*. (4 : 5.) In the analysis and solution of problems, however, it is doubtful if anything is gained by this distinction, particularly as the sense of each of the two unknown stresses at a joint is, strictly speaking, unknown.

Joint a. (Fig. 212 (*a*).)

1. Since *aB* and *R* are the only forces with V components, and since *R* acts upward, *aB* must act downward toward *a* (compression), and its V component must equal *R*.



Joint a
Fig. 212 (*a*).

$$aB_V = -R = -2.5W.$$
$$aB = -2.5 \times W \times s.$$

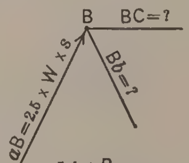
Since *aB* and *ab* are the only forces with H components, and *aB* acts toward the left, *ab* must act toward the right away from *a* (tension), and be equal to the H component of *aB*.

$$ab = aB_H \text{ or } ab = aB_V \times t.$$
$$ab = 2.5 \times W \times t.$$

Joint B. (Fig. 212 (*b*).)

2. Since *aB* and *Bb* are the only forces with V components and *aB* acts upward toward *B* (compression), *Bb* must act downward away from *B* (tension) and its V component must equal the V component of *aB*. Since the inclination is the same for both forces, it follows that the force *Bb* must equal *aB*.

$$Bb_V = -aB_V \text{ or } Bb = -aB.$$
$$Bb = -2.5 \times W \times s.$$



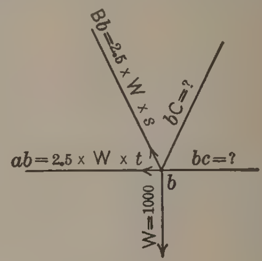
Joint B
Fig. 212 (*b*).

Since *aB* and *Bb* both act toward the right, *BC*, the remaining force at *B*, must act toward the left or toward *B* (compression) and be equal to the sum of the H components of *aB* and *Bb*.

$$BC = BC_H = -(aB_H + Bb_H) = -2 \times (2.5 \times W \times t).$$
$$BC = -5.0 \times W \times t.$$

Joint b. (Fig. 212 (*c*).)

3. Since *Bb*, *W*, and *bC* are the only forces with V components, and since *Bb*, acting upward, has a greater V component ($2.5W \times s$) than *W*, acting downward, *bC* must act downward toward *b* (compression) and its V component must equal the difference between the V components of *Bb* and *W*.



Joint b
Fig. 212 (*c*).

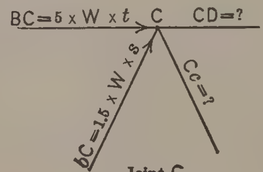
$$bC_V = -(Bb_V - W) = -2.5W + W = -1.5W.$$
$$bC = -1.5 \times W \times s.$$

Since *W* has no H component, and since *ab*, *Bb*, and *bC* all act toward the left, *bc* must act toward the right or away from *b* (tension) and be equal to the sum of the H components of *ab*, *Bb*, and *bC*.

$$bc = ab + Bb_H + bC_H = 2.5 \times W \times t + 2.5 \times W \times t + 1.5 \times W \times t.$$
$$bc = 6.5 \times W \times t.$$

Joint C. (Fig. 212 (*d*).)

4. Since *bC* and *Cc* are the only forces with V components and *bC* acts upward, *Cc* must act downward away from *C* (tension) and be equal in magnitude to *bC*. (The angle of inclination of *bC* and *Cc* is the same.)



Joint C
Fig. 212 (*d*).

$$Cc_V = -bC_V \text{ and hence } Cc = -bC.$$
$$Cc = -1.5 \times W \times s.$$

Since *BC*, *bC*, and *Cc* all act toward the right, *CD*, the remaining force at *C*, must act toward the left or toward *C* (compression) and be equal to the sum of the H components of *BC*, *bC*, and *Cc*.

$$CD = -(BC + bC_H + Cc_H) = -(5.0 \times W \times t + 1.5 \times W \times t + 1.5 \times W \times t).$$
$$CD = -8 \times W \times t.$$

5. It is left to the student to analyze in a similar manner first joint *c* and then joint *D*. The final results for all members are indicated on the truss diagram (Fig. 211) in terms of thousands of pounds and *s* (secant) or *t* (tangent) of the truss angle. By substituting the values for *W*, *s*, and *t*, check the stresses in several typical members by comparing with results obtained on page 209.

1. *Second illustration of the analysis of joints from inspection:* In Fig. 213 is shown one-half of an eight-panel Pratt truss. In the analysis given for each joint, only the action of the forces with respect to each other will be considered; no attempt will be made to express the magnitudes of the forces (stresses), although this could easily be done by methods exactly the same as those used in the preceding illustration.

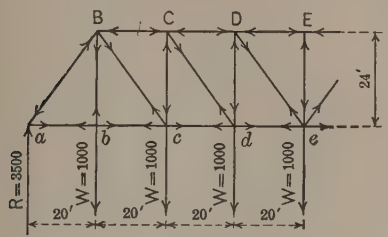


Fig. 213.

2. As an optional exercise, complete the analysis for each joint by writing the expressions for stresses which may be determined at that joint in terms of panel loads and the secant or tangent of the truss angle. Panel length = 20 ft.; height of truss = 24 ft.

Joint a

3. The analysis is exactly the same as that for joint a of the preceding illustration.

Joint b

4. This joint must be analyzed before joint B. (Why?) From fundamental principle $Bb = -W = 1000$ lbs. tension, and $bc = -ab$. (71 : 5.)

Joint B

5. At joint a it was found that aB is a compression member, and at joint b it was found that Bb is a tension member, hence at joint B the force aB acts upward and Bb acts downward. The V component of Bc must therefore be equal to the difference of the V components of aB and Bb, and since the V component of aB is greater than the V component of Bb, the force Bc must act downward or away from B (tension).

$$Bc_V = -(aB_V - Bb).$$

Since Bb has no H component and both aB and Bc act toward the right, BC must act toward the left or toward B (compression), and be equal to the sum of the H components of aB and Bc.

$$BC = -(aB_H + Bc_H).$$

Joint c

6. Members bc and Bc are each in tension, as determined respectively at joints b and B, hence both act away from c.

Since Bc acts upward at c and W acts downward, and since the V component of Bc is greater than W, it follows that Cc must act downward toward c (compression), and be equal to the difference of the V components of Bc and W. (No other forces have V components.)

$$Cc = -(Bc_V - W).$$

Since neither W nor Cc has an H component, and since both bc and Bc act toward the left, cd must act toward the right (tension), and be equal to the sum of the H components of Bc and bc.

$$cd = -(-bc - Bc_H) = bc + Bc_H.$$

Joint C

7. Since Cc and Cd are the only forces with V components, and since Cc acts upward (Why?), the force Cd must act downward (tension), and its V component must be equal to Cc.

$$Cd_V = -Cc.$$

Since Cc has no H component, and since both BC and Cd act toward the right, CD, the only other force with an H component, must act toward the left or toward C (compression), and be equal to the sum of the H components of BC and Cd.

$$CD = -(BC + Cd_H).$$

It is left to the student to analyze in a similar manner joints d and D.

Joint E

8. No diagonal is shown at E, since under symmetrical loading no diagonal is acting at E. Under these conditions the stress in Ee is zero from fundamental principle (71 : 4), and $EF = -DE$.

9. *Practice questions in the analysis of joints:* The questions that follow are to be answered by analyzing joints from inspection by the methods just explained. In each case, the truss is assumed to be symmetrically loaded, and therefore only one-half of the truss is shown in the diagram. In each question, the sense of a stress with respect to the joint under consideration is indicated by the order of the two letters used in designating that stress. (211 : 6.)

Truss in Fig. 214 (a)

1. *Joint C:*
   Why must the stress in bC = the stress in Cc?
   Which magnitude is greater, DC or BC, and why? How much greater, in terms of stresses in web members and the truss angle A?

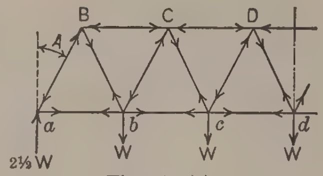


Fig. 214 (*a*).

2. *Joint c:*
   How much greater is the stress in cC than the stress in Dc, in terms of a panel load and the truss angle?

Truss in Fig. 214 (b)

3. *Joint B:*
   Why must Bc be in tension?
   Is the stress in Bc greater or less than the stress in aB, and by what amount in terms of a panel load and the truss angle?
4. *Joints a and b:*
   Why must ab and bc be in tension?
   What is the stress in bc, in terms of the reaction and the truss angle?
5. *Joint d:* What is the stress in dD?

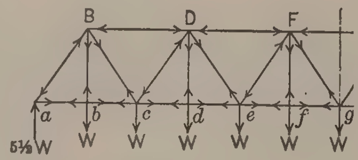


Fig. 214 (*b*).

Truss in Fig. 214 (c)

6. *Joint E:*
   Is there any stress in Ee? Why?
7. *Joint D:*
   What is the stress in dD, in terms of the stress in De and the truss angle?
8. *Joint c:*
   How much greater is the stress in cB than the stress in Cc, in terms of panel loads and the truss angle?
9. *Joint C:*
   How much greater is the stress in DC than the stress in BC, in terms of the stress in Cd and the truss angle?

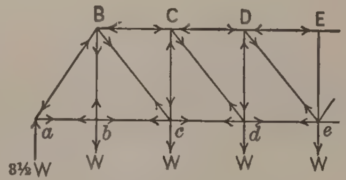


Fig. 214 (*c*).

Truss in Fig. 214 (d)

10. *Joint E:*
    If E is at the center of the truss, why must the stress in dE equal the stress in fE, and what is the magnitude of each in terms of a panel load and the truss angle? Why are both in compression?

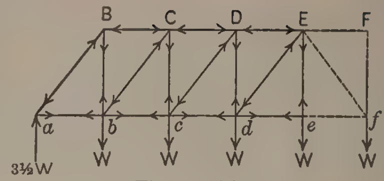


Fig. 214 (*d*).

11. *Joint d:*
    Assuming Ed in compression, why must dD be in tension, and what is the stress in dD in terms of panel loads?
12. *Joint C:*
    What is the stress in bC in terms of the stress in Cc and the truss angle?

Truss in Fig. 214 (e)

13. *Joint E:*
    What is the stress in eE in terms of a panel load? Why?
14. *Joint e:*
    What is the stress in eD in terms of a panel load and the truss angle?
15. *Joint C:*
    How much greater is the stress in cC than the V component of Cd?

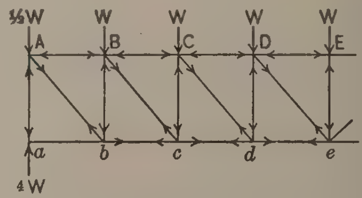


Fig. 214 (*e*).

16. *Joint d:*
    How much greater is the stress in de than the stress in dc, in terms of the stress in dC and the truss angle?
17. *Joint a:*
    What can be said about the magnitude of the stress in ab?
18. *Joint b:*
    What is the stress in bc in terms of the stress in bA?

Truss in Fig. 214 (f)

19. *Joint d:*
    What is the stress in dd' equal to?
20. *Joint d':*
    Why must the stress in cd' be equal to one-half of a panel load multiplied by the secant of the truss angle? (201 : 3.) In what other members are stresses of the same magnitude as the stress in cd'?

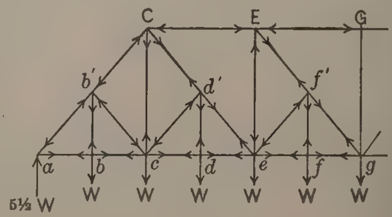


Fig. 214 (*f*).

1. *Joint b′:*
   Why must cb' be in compression? Does cb' act to increase or decrease the stress in ab'?
2. *Joint c:*
   Assuming $b'c = d'c$ and both in compression, what is the stress in cC in terms of W?
3. *Joint f′:*
   How much greater in terms of a panel load and the truss angle is the stress in $f'E$ than the stress in $f'g$?
4. *Joint e:*
   Does the member $f'e$ act to increase or decrease the stress in ef and by how much in terms of W and the truss angle?

Exercise: From inspection of the various joints, check the direction of all arrows on the six truss diagrams, beginning with Fig. 214 (*a*).

5. **Method of Coefficients:** In the illustrative example on pages 197 and 198, the stress in each web member was written in terms of a panel load and the secant of the truss angle, and the stress in each chord member in terms of a panel load and the tangent of the truss angle. This method of indicating stresses is the basis of a short cut for the method of successive joints, called the **method of coefficients.** Since this method is the one most generally used for parallel-chord trusses, it will be explained in detail, first for web members and then for chord members. The first essential step is to indicate correctly stresses in web members; once this is done it is comparatively easy to indicate stresses in chord members. If one has mastered the method of sections explained in the preceding chapter and the method of analyzing joints from inspection, one should find the method of coefficients easy to understand and very simple in its application.

6. **Coefficients for Web Members.** Two methods of determining coefficients for web members will be explained, namely, (1) from the shear for a panel, and (2) from the analyses of joints of the truss.

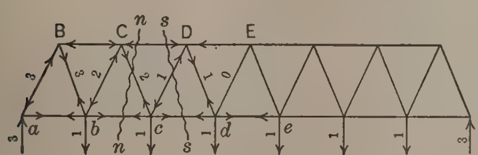


Fig. 215 (*a*).

7. *To determine the coefficients of web members from shears.* In the explanation of the method of sections, it was shown that the stress in any vertical web member of a parallel chord is equal to the *vertical shear*, and for any inclined web member the stress is equal to the *vertical shear multiplied by the secant of the truss angle.* (194 : 5.) By vertical shear is meant simply the algebraic sum of the vertical external forces (usually panel loads and a reaction) which act on that segment of the truss on which the web member under consideration also acts. (193 : 10.) The vertical shear may be expressed in terms of a panel load. (It will be equal to the algebraic sum of a reaction and a certain number of panel loads, but the reaction can be expressed in terms of a panel load.) In Fig. 215 (*a*), for example, let each panel load equal unity; then the reaction = 3. The shear for the member cD is the shear for the section s–s or for the segment $aBCc$, and is equal to $3 - 1 - 1 = 1$, hence the stress in $cD = 1 \times$ panel load $\times$ secant of truss angle. The shear for Cc, i.e., the shear for the section n–n or for the segment $aBCb$, is $3 - 1 = 2$, and the stress in Cc is $2 \times$ panel load $\times$ secant of truss angle. On each of the remaining web members is shown the numerical value of the shear from which the corresponding stress may be found. (Numerical value multiplied by the product of one panel load by the secant of the truss angle.) Henceforth these numbers will be called **coefficients.** Keep in mind that each coefficient represents the corresponding number of panel loads, regardless of how many pounds or tons constitute such a load. In Figs. 215 (*b*) and 215 (*c*), coefficients are shown for the web members of two

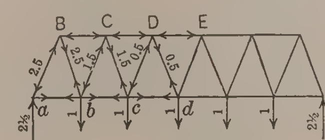


Fig. 215 (*b*).

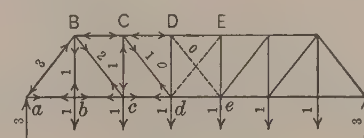


Fig. 215 (*c*).

other trusses. Before proceeding further, check, by the method of sections, all coefficients in each of these two figures as well as those in Fig. 215 (*a*).

8. Note that in proceeding from the center of a truss toward a support, coefficients increase by one (panel load) whenever a loaded joint is passed; this is because there is one less panel load to subtract from the reaction in finding the shear. Keeping this fact in mind, one can begin at

the center and write the coefficients on web members without deriving each coefficient from the shear, though the shear should be used as a check when one is in doubt as to the correctness of any coefficient.

1. Note also that coefficients for web members of a truss with an *even* number of panels involve a fraction ($\frac{1}{2}$ or 0.5) while those for a truss with an odd number of panels are expressed in whole numbers.

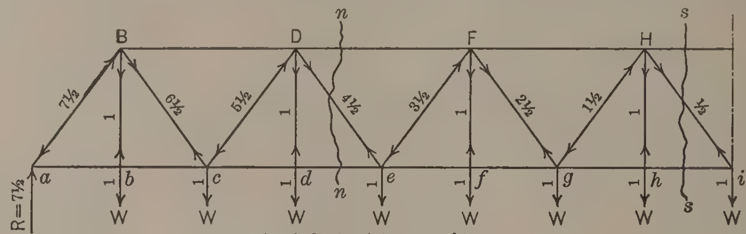
2. The coefficient for an end post aB is always equal to the reaction (expressed in panel loads), and this fact should be used as an important check on all coefficients for web members. If coefficients are written in order from the center toward the end and the last coefficient (that for the end post) is correct, all the others are probably correct.

3. The coefficient for Bb in Fig. 216 is unity, from inspection of joint b. (Why?) The coefficient for any similar member which meets either the top or bottom chord at right angles and is the only web member at the joint is better determined from inspection of the forces at the joint than from shear.

4. In Fig. 215 (c) the shear for a section through the center panel is zero, therefore there can be no stress in either diagonal, De or dE. For trusses with an odd number of panels the dead-load coefficients for the web members of the center panel are always zero, provided the truss is symmetrically loaded. (This rule applies to ordinary parallel-chord trusses of simple type.) The coefficient for the diagonal in an adjacent panel will then be unity.

5. *To determine the coefficients for web members from analyses of joints.* In determining the coefficient for any web member from the shear for a section through that member, one is applying $\Sigma V = 0$ to a *segment* of the truss. The coefficients for web members may be determined by another method in which $\Sigma V = 0$ is applied to each joint of the truss in succession, beginning at the center and proceeding toward the end. The method of procedure at each joint is the same. *Ignore all horizontal members* and apply $\Sigma V = 0$ in the following manner: The sum of the coefficients of all *downward* forces must equal the sum of the coefficients of all *upward* forces. In determining coefficients for web members by this joint-to-joint method, it is best to obtain the coefficient for the first web member (that nearest the center of the truss) from shear, and to use the method of shear as a check whenever one is in doubt as to the correctness of any web-member coefficient. The joint-to-joint method will now be illustrated.

6. At joint b in Fig. 216, by fundamental principle, the stress in Bb is equal to a panel load, hence the coefficient for Bb, and similarly for each of the other vertical members, is "1." These coefficients and that for the reaction are placed on the diagram at the start. The coefficient for Hi, the web member nearest the center, is determined best from the



One half of a sixteen-panel truss

Fig. 216.

shear for section s–s, i.e., $(7\frac{1}{2} - 7 \times 1) = +\frac{1}{2}$. Shear is positive, hence Hi acts downward on the segment (at H), and its coefficient is $\frac{1}{2}$. The other coefficients are determined by applying $\Sigma V = 0$ to each joint in succession. Each line in the tabulation that follows indicates the *mental* analysis of the corresponding joint of the truss; the *required* coefficient obtained at that joint is enclosed in brackets.

Joint	*Down*		*Up*
H	Hi and Hh	=	[gH]
	$\frac{1}{2} + 1$	=	[$1\frac{1}{2}$]
g	Hg and W	=	[gF]
	$1\frac{1}{2} + 1$	=	[$2\frac{1}{2}$]
F	Fg and Ff	=	[eF]
	$2\frac{1}{2} + 1$	=	[$3\frac{1}{2}$]
e	Fe and W	=	[eD]
	$3\frac{1}{2} + 1$	=	[$4\frac{1}{2}$]
D	De and Dd	=	[cD]
	$4\frac{1}{2} + 1$	=	[$5\frac{1}{2}$]

Joint	*Down*		*Up*
c	Dc and W	=	$[cB]$
	$5\frac{1}{2} + 1$	=	$[6\frac{1}{2}]$
B	Bc and Bb	=	$[aB]$
	$6\frac{1}{2} + 1$	=	$[7\frac{1}{2}]$
a	Ba	=	R
	$7\frac{1}{2}$	=	$7\frac{1}{2}$ (Check)

1. The work indicated above is done mentally, and once one has become accustomed to the method one can proceed very rapidly from joint to joint. It is well, as one proceeds in this way, to apply now and then the check by shear. For example, the member De is about half way between the center and the end, and it is well to check its coefficient, $4\frac{1}{2}$, by shear for the section n–n, i.e., $(7\frac{1}{2} - 3 \times 1) = 4\frac{1}{2}$.

2. *Note:* It is to be noted that at each of the loaded joints, c, e, and g, the required coefficient for the next web member at that joint is larger by one than the coefficient for the preceding web member at the same joint, as already noted in connection with determining coefficients from shear. This statement holds true in this particular truss, not only for the lower joints c, e, and g, where floor beams bring the loads to the trusses, but also at upper joints B, D, F, and H, since the load brought by the floor beam to the joint directly below any one of these joints is carried by a hanger to the upper joint.

3. *Exercise:* Beginning at the center of the truss and working toward the left-hand end, check by inspection of joints the coefficients shown for each of the four trusses that follow. When in doubt, use the method of sections (shear).

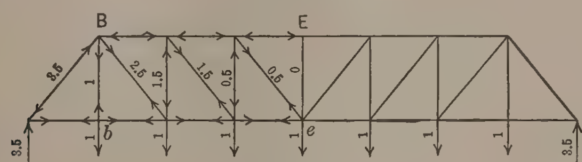


Fig. 217 (*a*).

4. Why, in Fig. 217 (*a*), is the coefficient for Ee zero and the coefficient for Bb unity?

5. In Fig. 217 (*b*) the coefficient for De is zero. (Why?) In checking the coefficient for Dd by shear, use the segment $aBDc$ and ignore De since its stress is zero.

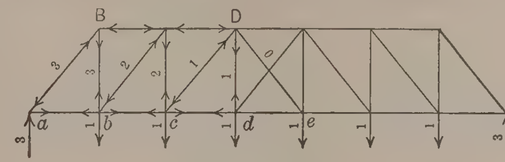


Fig. 217 (*b*).

6. In Fig. 217 (*c*) the coefficient "1" is written first on each vertical web member. (Why?) The coefficient for any diagonal is easily checked from the shear.

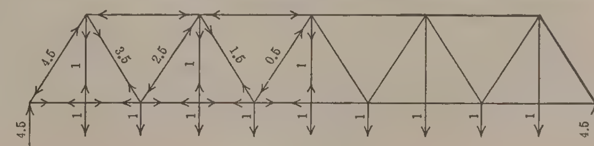


Fig. 217 (*c*).

7. In Fig. 217 (*d*) one-half only of the truss is shown. First the coefficient "1" is put on each sub-vertical, bb', dd', and ff', and "0.5" is then put on each sub-diagonal,

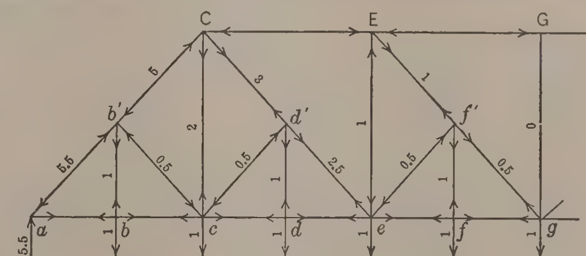


Fig. 217 (*d*).

$b'c$, cd', and ef'. (201 : 3.) The sub-verticals are in tension and the sub-diagonals are in compression. This should be kept in mind in analyzing such a joint as d', in order to get the coefficient for an upper diagonal, such as Cd'. ($d'd$ and $d'e$ act downward, while cd' acts upward, hence $2.5 + 1 - 0.5 = 3$ is the coefficient for Cd'.) Likewise, in checking the coefficient for Cd' by shear, the member cd', as well as the member Cd', acts on the segment aCc, hence $Cd' + cd' = Cd' + 0.5 = shear = 5\frac{1}{2} - 1 - 1$ or $Cd' = 3$ (check).

1. *To determine whether a web member is in tension or in compression.* For a symmetrical truss symmetrically loaded, the shear for any segment less than one-half of the truss is *positive*, that is, if only the reaction and the loads were acting on the segment it would move *upward.* The web member or members which act on the segment must, therefore, act *downward* to maintain equilibrium. By keeping this fact in mind it is easy to determine whether a web member is in tension or compression by merely noting whether in acting downward it acts away from or toward the segment. (If by any chance the shear is negative, as in the case of the unsupported end of a cantilever, this rule is reversed.)

2. In any type of truss for which the method of coefficients is generally used, it is usually evident at the start which members are in tension and which in compression. It is well to place arrows near each joint on the truss diagram to show which members act toward and which away from the joint. If this is done for one-half of the truss *before* beginning to determine coefficients, the work is simplified and mistakes are less likely to occur. In any case, these arrows should be put on the web members before attempting to determine coefficients for chord members.

3. *Partial panel loads at joints.* When partial panel loads are applied at both upper and lower joints, some of the coefficients will be changed, but the general method of determining coefficients remains the same. For example, in Fig. 218 two-thirds of a panel load is considered as applied at each lower joint and one-third at each upper joint.

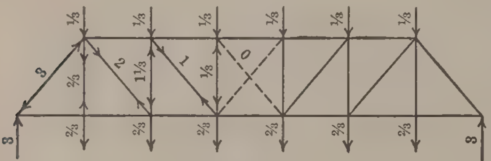


Fig. 218.

The only members for which the coefficients are different from what they would be if the entire panel load were assumed as applied at the lower joint are the vertical web members. (Check the coefficients for these members by the method of sections (shear).)

Note that when a partial load is at a joint at which there are only *two* web members, the coefficient for one of the web members is greater than the coefficient for the other by an amount equal to the partial load, i.e., by a *fraction* of a panel load.

Note: When partial loads are transmitted directly to vertical members as, for example, to posts of a Pratt truss or to verticals of a Warren truss with verticals, only the coefficients for those posts or verticals are different from what they would be were the loads applied only at joints of the loaded chord. When, however, partial loads are transmitted directly to diagonals, as in the case of a Warren truss without verticals, not only are the coefficients for the diagonals affected but the coefficients for chord members as well (199 : 15).

Exercise. Given: A Warren truss without verticals; the number of panels is eight. The first assumption is that an entire panel load is applied at each lower joint and the second assumption is that two-thirds of a panel load is applied at each lower joint and one-third at each upper joint. Required: (*a*) To show on one truss diagram the coefficients for the first assumption and on another diagram the coefficients for the second assumption. (*b*) To compare the two sets of coefficients for the purpose of determining which members are affected by the partial loads.

4. *To calculate the stress in a web member from its coefficient.* If W is the magnitude of a panel load, the stress in any *vertical* web member is equal to the coefficient for that member multiplied by W, and the stress in any *inclined* web member is equal to the coefficient for that member multiplied by ($W \times$ *secant* A), in which A is the truss angle.

5. **Coefficients for Chord Members.** Coefficients for horizontal chord members are best determined from a mental analysis of the forces at successive joints, though the method of sections is useful in checking coefficients thus determined.

6. It is best to begin at one end of the truss and proceed, joint by joint, toward the center, determining first the coefficients for all members of the upper chord and then for all members of the lower chord, or *vice-versa.* Since a chord member in parallel-chord trusses is a horizontal member, its stress or its coefficient will be obtained from the equation $\Sigma H = 0$. Hence, in the mental analysis at each joint, *ignore all vertical members* and consider only horizontal and inclined members. The mental analysis will then consist merely in applying $\Sigma H = 0$ as follows: The sum of the coefficients for all members that act toward the *right* must equal the sum of the coefficients for all members that act toward the *left.* Beginning at the end and working toward the center of a truss, it will be found as a general rule that, if the chord member in which the coefficient is required acts toward the left at a joint under consideration, all other members at that joint which have H components will act toward the right, and *vice-versa,* i.e., the required coefficient will be equal to the sum of the coefficients in all the other members at the joint which have H components.

This is not intended as a statement of a fundamental principle, for there are exceptions to the rule, but as a statement of a fact, evident from inspection, and worth remembering in determining coefficients in chord members.

1. *Caution:* The student should not abandon the mental analysis of forces at a joint, even while keeping in mind such a rule as that just given. Note that the statement does not apply to *any* chord member but only to the particular chord member for which the coefficient is required, working from the end of the truss toward the center. This implies that the coefficients for the inclined web members and the preceding chord member at any joint have already been determined.

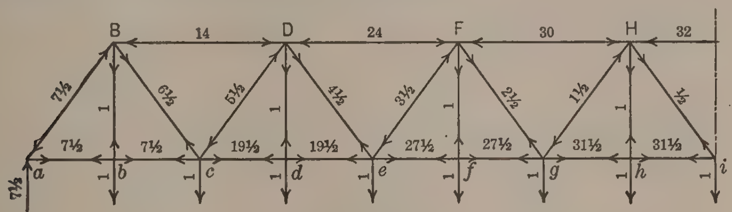


Fig. 219.

2. The half truss in Fig. 219 is the same as that in Fig. 216. Assuming that the coefficients for all web members have already been determined, it is required to find the coefficients for the chord members by applying $\Sigma H = 0$ to each joint in succession, first for the top chord and then for the bottom chord. Each line in the tabulation that follows indicates the *mental* analysis for the corresponding joint of the truss; the *required* coefficient obtained at a joint is enclosed in brackets.

Upper Chord

Joint	*To the right*		*To the left*
B	*aB* and *Bc*	=	[*DB*]
	$7\frac{1}{2} + 6\frac{1}{2}$	=	[14]
D	*BD*, *cD*, and *De*	=	[*FD*]
	$14 + 5\frac{1}{2} + 4\frac{1}{2}$	=	[24]
F	*DF*, *eF*, and *Fg*	=	[*HF*]
	$24 + 3\frac{1}{2} + 2\frac{1}{2}$	=	[30]
H	*FH*, *gH*, and *Hi*	=	[*HH*]
	$30 + 1\frac{1}{2} + \frac{1}{2}$	=	[32]

Lower Chord

Joint	*To the left*		*To the right*
a	*Ba*	=	[*ab* = *bc*]
	$7\frac{1}{2}$	=	[$7\frac{1}{2}$]
c	*cb*, *cB*, and *Dc*	=	[*cd* = *de*]
	$7\frac{1}{2} + 6\frac{1}{2} + 5\frac{1}{2}$	=	[$19\frac{1}{2}$]
e	*ed*, *eD*, and *Fe*	=	[*ef* = *fg*]
	$19\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2}$	=	[$27\frac{1}{2}$]
g	*gf*, *gF*, and *Hg*	=	[*gh* = *hi*]
	$27\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2}$	=	[$31\frac{1}{2}$]

3. *Note:* Observe that at each of the joints in the upper chord the chord member for which the coefficient is required acts toward the left, whereas all other horizontal or inclined members at that joint act toward the right. At each of the joints in the lower chord the chord member for which the coefficient is required acts toward the right, whereas all other horizontal or inclined members at that joint act toward the left. This is in accord with the general rule stated in 218 : 6 — a rule, however, which should not be followed blindly. (219 : 1.)

4. *To calculate the stress in a chord member from its coefficient.* The stress in any chord member is equal, in the last analysis, to the sum of the H components of the stresses in certain inclined web members. (Why?) The H component of the stress in any inclined web member is equal to that stress multiplied by the sine of the truss angle. The stress in any web member, expressed in terms of the coefficient for that member, a panel load W, and the truss angle A, is coefficient $\times$ ($W \times$ secant A). The corresponding H component obtained by multiplying by the sine is: coefficient $\times$ ($W \times$ secant $A \times$ sine A) or coefficient $\times$ ($W \times$ tangent A). It follows, then, that *the stress in any chord member is equal to its coefficient multiplied by the product of a panel load by the tangent of the truss angle.* This stress is compression or tension according to whether the member, in acting toward the right or toward the left, acts toward or away from the joint.

5. *Note:* The two products obtained by multiplying a panel load by the tangent and by the secant of the truss angle are two constants; the coefficient for each horizontal member is multiplied by the first and the coefficient for each inclined member by the second.

1. *Illustration:* The truth of the statement that the stress in a chord member is equal to its coefficient multiplied by the product of a panel load W by the tangent of the truss angle A may be illustrated by the chord member DF.

$$DF = BD + cD_H + De_H \text{ (Joint } D\text{)}$$
$$BD = Bc_H + aB_H = (7\tfrac{1}{2} + 6\tfrac{1}{2}) \times W \times \tan A$$
$$cD_H = 5\tfrac{1}{2} \times W \times \tan A$$
$$De_H = 4\tfrac{1}{2} \times W \times \tan A$$
$$DF = (7\tfrac{1}{2} + 6\tfrac{1}{2} + 5\tfrac{1}{2} + 4\tfrac{1}{2}) \times W \times \tan A = 24 \times W \times \tan A$$

2. *Note:* The distinction between the meaning of a coefficient for a web member and the meaning of a coefficient for a chord member should always be kept clearly in mind. The stress in a web member is obtained by multiplying its coefficient by the product $W \times \sec A$, whereas the stress in a chord member is obtained by multiplying its coefficient by the product $W \times \tan A$. For example, in Fig. 219 the members aB and ab have the same coefficient, but the stresses are far from being the same as might erroneously be assumed.

3. *Exercise.* Beginning at the end of the truss and working toward the center, check by inspection the coefficients for the chord members in each of the four trusses, assuming that the coefficients for the web members are correct. One-half only of each truss is shown. Assume a panel load to be applied at each lower joint.

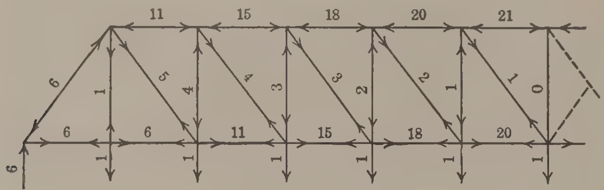


Fig. 220 (*a*).

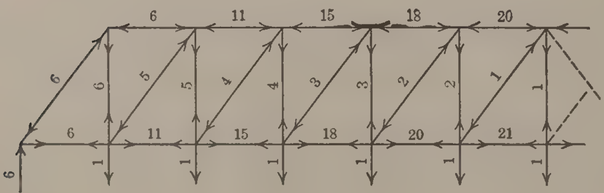


Fig. 220 (*b*).

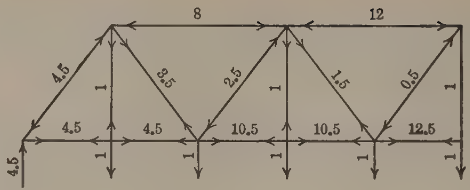


Fig. 220 (*c*).

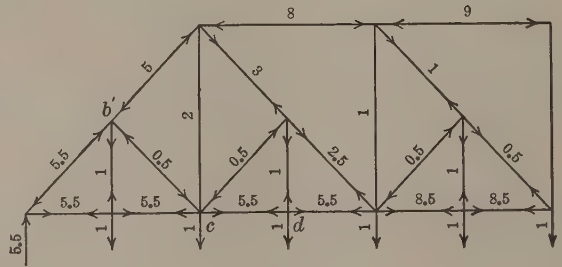


Fig. 220 (*d*).

4. *Note* that joint c in Fig. 220 (d) is an exception to the general rule since the member cd for which the coefficient is required is not acting in a direction opposite to that of all other members at the joint which have H components ($b'c$ also acts toward the right), and hence the coefficient of cd is not equal to the sum of the coefficients for all other horizontal or inclined members at the joint. This is an illustration of the danger of blindly following a rule without regard to fundamental principles. (219 : 1.)

5. *Coefficients for a truss unsymmetrically loaded.* Let the truss shown in Fig. 221 be loaded with equal panel loads applied at joints f, g, and h only. Required: Coefficients for all members.

6. The general methods of determining coefficients for web members (215 : 6) and for chord members (218 : 5) hold good. The reaction is $\frac{6}{8}\,W$. For any web member to the left of f this reaction is the shear, and hence the coefficients for every web member to the left of f is $\frac{6}{8}$. The coefficients for each of the web members to the right of f may be checked by shear. Note that the vertical member Ff is in tension and the diagonal

eF is in compression (Why?), whereas all other verticals, except the hip verticals, are in compression, and all other diagonals except the end posts are in tension, as is usually the case in a Pratt truss. Note also that the largest coefficient for any member in the top chord is not for a center

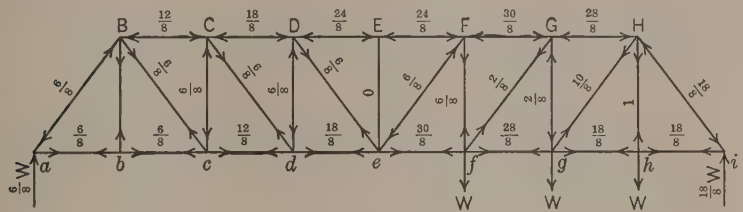


Fig. 221.

member, DE or EF, as in the case of a symmetrically loaded truss, but occurs for the member FG.

Note: The method of determining coefficients for the web members of a partially loaded truss is the basis of the method, to be explained in **PART III**, of determining live-load coefficients for web members.

1. **Comparison between the Method of Sections and the Method of Coefficients.** In calculating the stresses in a parallel-chord truss with equal panel loads by the method of sections, the stress in a vertical member may be expressed in terms of panel loads. (196 : 11.) Likewise, the stress in an inclined web member may be expressed in terms of panel loads and the secant of the truss angle. (197 : 2.) In either case, the expression for the stress in any member is identical with that obtained for the same member by the method of coefficients.

2. *Exercise:* Show that the expressions in the column headed "Shear" in 196 : 12 and the expressions in the column headed "Shear × secant" in 197 : 3 are identical with the expressions for the stresses in the corresponding members as obtained by the method of coefficients.

In calculating the stresses in the chord members of a parallel-chord truss by the method of sections, the stress in each member may be expressed in terms of panel loads and the tangent of the truss angle. (197 : 8.) Such an expression is identical with that obtained for the same member by the method of coefficients, except that in the method of sections "$p \div h$" is used instead of its equivalent the *tangent of the truss angle* used in the method of coefficients.

3. *Exercise:* Show that the expressions in the column headed "$M_B \div h$" in 198 : 1 are identical with the expressions for the corresponding members as obtained by the method of coefficients, except that "$p \div h$" is used in place of its equivalent the *tangent of the truss angle.*

4. **Methods of Checking Coefficients.** Coefficients should be checked by one or more of the following methods:

5. *To check web-member coefficients by shear.* ($\Sigma V = 0$.) The coefficient for any web member should equal the shear (expressed in panel loads) for a section which cuts that member but no other inclined or vertical member.

6. This check should *always* be applied at the start, i.e., to the coefficient for the web member nearest the center of the truss. If the coefficient for this member is incorrect, it is probable that the coefficients for all other web members will be incorrect. It is well also to apply the check whenever one is in doubt as to the correct coefficient for any web member.

7. *To check web-member coefficients by the reaction.* The coefficient for the end post should be the same as the coefficient for the reaction.

8. If, beginning at the center of the truss and proceeding toward one end, the coefficient for each web member is obtained from the coefficient for the preceding web member, as explained in 216 : 5, the coefficient for the end post involves the coefficients for all other web members, hence if that coefficient is correct it is probable that the coefficients for the other web members are correct. *This check should always be applied.* The coefficients for chord members are obtained from the coefficients for web members, and hence every precaution should be taken to make the latter correct before beginning work on the former.

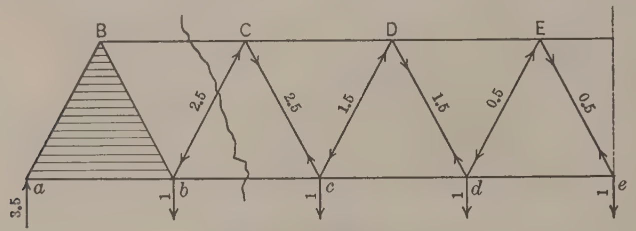
9. *To check chord-member coefficients by $\Sigma H = 0$.* If a top-chord member, a bottom-chord member, and a web member are cut by a section that passes through no other member, the sum of the coefficients for the two cut members which act in the same direction, right or left, on either segment formed by the section, must equal the coefficient for the third cut member. (This is equivalent to applying $\Sigma H = 0$ in the method of sections.)

10. *Note:* This is a check, incidentally, for the web member cut by the section as well as for the two chord members. When all coefficients have been placed on a truss dia-

gram, this check can be applied *at a glance* to the three members in any panel. When there are four members in a panel, a similar check based on $\Sigma H = 0$ may be used.

1. *Illustration:* In checking coefficients for web members from shear ($\Sigma V = 0$) and coefficients for chord members from $\Sigma H = 0$, one is really applying the method of sections. Examples of the use of each of these checks will now be given.

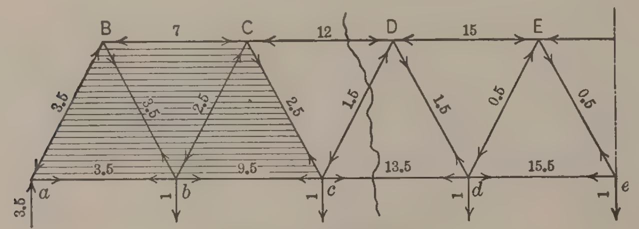
2. Assume that, in placing coefficients on the web members in Fig. 222 (*a*), the member *bC* has been reached, and that the coefficient for *bC* has been found to be *2.5*. Is this coefficient correct? Cut *bC* by a section and apply the check from the shear for segment *aBb*.



Half of an eight-panel truss

Fig. 222 (*a*).

$$\text{Shear} = bC_V.$$
$$3.5 - 1 = 2.5 \text{ (check)}.$$



Half of an eight-panel truss

Fig. 222 (*b*).

3. Assume that all of the coefficients have been placed on the truss diagram in Fig. 222 (*b*). Are the coefficients for members *CD*, *cD*, and *cd* correct? Cut these three members by a section, and take the segment *aBCc*, thus formed, as the body in equilibrium. Apply $\Sigma H = 0$:

$$CD + cD_H \text{ (to the left)} = cd \text{ (to the right)}.$$
$$12 + 1.5 \qquad = 13.5 \text{ (check)}.$$

4. Assume that in placing the coefficients on the web members in Fig. 222 (*c*) the member *Cc* has been reached and that the coefficient for *Cc* has been found to be *2*. Is this coefficient correct? Cut *Cc* by a section and apply the check from the shear for segment *aBc*.

$$\text{Shear} = Cc.$$
$$4 - 1 - 1 = 2 \text{ (check)}.$$

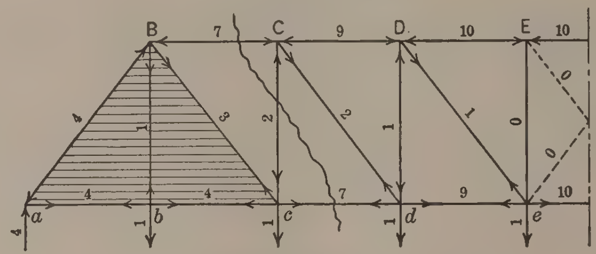


Fig. 222 (*c*).

5. Using the segment *aBc*, check the coefficients for *BC* and *cd* from $\Sigma H = 0$.

$$BC \text{ (to the left)} = cd \text{ (to the right)}.$$
$$7 \qquad = 7 \text{ (check)}.$$

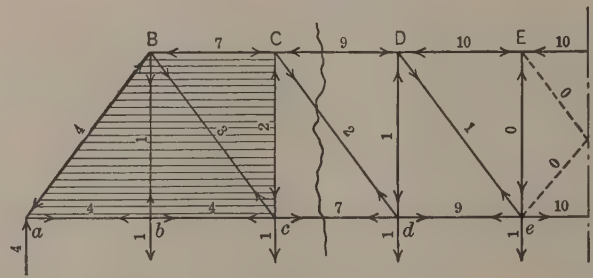


Fig. 222 (*d*).

6. Assume that all coefficients have been placed on the truss diagram in Fig. 222 (*d*). Are the coefficients for members *CD*, *Cd*, and *cd* correct? Cut these three members by a section and take the segment *aBCc* as the body in equilibrium. Apply $\Sigma H = 0$.

$$Cd_H + cd \text{ (to the right)} = CD \text{ (to the left)}.$$
$$2 + 7 \qquad = 9 \text{ (check)}.$$

1. *To check a chord-member coefficient by* $\Sigma M = 0$. Cut the chord member by a section, and find the stress in the member from $\Sigma M = 0$, expressed in terms of a unit panel load, panel length, and height of truss, exactly as explained in connection with the method of sections. (195 : 2.)

2. *Illustration:* Let p and h represent, respectively, the panel length and the panel height of a truss, and A the truss angle. It is required to check the coefficient for the stress in DE in Fig. 222 (*b*) from $\Sigma M = 0$. Cut the member DE by a section, and take the segment $aBDc$ as the body in equilibrium.

$$\Sigma M_d = 3.5 \times 3p - 1 \times p - 1 \times 2p + DE \times h = 0.$$
$$DE = 15 \times (0.5p \div h) = 15 \tan A \text{ (check).}$$

3. Required to check the coefficient for the stress in DE in Fig. 222 (*d*) from $\Sigma M = 0$. Cut DE by a section and take the segment $aBDd$ as the body in equilibrium.

$$\Sigma M_e = 4 \times 4p - 1 \times (p + 2p + 3p) + DE \times h = 0.$$
$$DE = 10 \times (p \div h) = 10 \times \tan A \text{ (check).}$$

4. *Note:* The check just given is a more positive check for the coefficient of a chord member than any other check, since it involves no other coefficient, i.e., no previous calculation.

5. *To check a chord-member coefficient by the summation of the coefficients in inclined web members.* To check the coefficient for any top-chord member, add the coefficients for all inclined members that meet the top chord between the *center* of the chord member and the *nearer* end of the truss. The sum should equal the coefficient for the chord member. Similarly, to check the coefficient for a bottom-chord member, add the coefficients for all inclined members that meet the bottom chord between the *center* of the chord member and the *nearer* end of the truss.

6. *Illustration:* Are the coefficients for DE and de in Fig. 222 (*b*) correct?

The inclined members that meet the top chord between the *center* of DE and the nearer end of the truss are: Dd, cD, Cc, bC, Bb, and aB. Adding the coefficients for these members: $1.5 + 1.5 + 2.5 + 2.5 + 3.5 + 3.5 = 15$ (check).

7. The inclined members that meet the bottom chord between the *center* of de and the nearer end of the truss are dE, Dd, cD, Cc, bC, Bb, and aB. Adding the coefficients for these members: $0.5 + 1.5 + 1.5 + 2.5 + 2.5 + 3.5 + 3.5 = 15.5$ (check).

8. Are the coefficients for DE and de in Fig. 222 (*d*) correct?

For DE: $1 + 2 + 3 + 4 = 10$ (check).
For de: $2 + 3 + 4 = 9$ (check).

9. *Question:* Upon what principle is the check just illustrated based? (219 : 4.)

10. *Note:* Notice that in checking the coefficient for a center member of either the top or the bottom chord, one is also checking, to a certain extent, the coefficients for other members in that chord.

11. *Note:* In checking the coefficient for a chord member by the summation of the coefficients of the inclined members, one is really finding the sum of the H components of the inclined members. The assumption in this summation that all coefficients are plus, is equivalent to the assumption that the H components of the inclined members all act toward the right, whereas in the case of the lower chord they all act toward the left. This inconsistency may be ignored since the object is to check the magnitude of the coefficient and not the character of the stress. (It is usually obvious whether a chord member is in compression or tension.) In any case the summation of the coefficients for inclined members implies that these members all act in the *same* direction, right or left, on the top chord or the bottom chord, whichever is under consideration. This is generally the case, but there are exceptions, as explained in 220 : 4. If, however, in these exceptional cases, the coefficients for inclined members are given plus and minus signs, according to whether the corresponding members act to the right or to the left, and are then added *algebraically*, the check will still hold good. For example, in the bottom chord of the truss in Fig. 220 (*d*) it is required to check the coefficient for the bottom-chord member in the sixth panel. The algebraic signs for all inclined members that meet the bottom chord will be minus, except that for $b'c$, hence:

$$-0.5 \ -2.5 \ -0.5 \ +0.5 \ -5.5 = -8.5 \text{ (check).}$$

12. Note in this illustration the importance of excluding from the summation the coefficient of any inclined member that does not meet the chord between the *center* of the chord member and the *nearer* end of the truss. For example, it would be incorrect to include in the summation the coefficient -0.5 for the diagonal in the sixth panel.

13. *Exercise:* An excellent test of one's ability to check coefficients may be had in applying all four methods of checking to the unsymmetrically loaded truss shown in Fig. 221.

14. Four methods of checking coefficients have now been given. The ease with which each can be applied, almost at a glance, should result in their constant use, and in the elimination or quick correction of all mistakes in coefficients.

15. **Trusses with Two Diagonals in the Same Panel.** The application of the method of sections to trusses in which there are two diagonals in the same panel was explained in 200 : 4. The method of coefficients as applied to such trusses does not differ materially from the method of sections.

16. *Trusses with two "stiff" diagonals in each panel.* (Fig. 200.) As stated in 200 : 4, the problem of determining stresses is statically inde-

terminate without some assumption. The assumption usually made is that the shear for the panel is equally divided between the two diagonals. The coefficients for the diagonals in each panel may easily be determined on this assumption, and these coefficients should be placed on the truss diagram at the start. All other coefficients can then be determined by the usual methods.

1. *Note:* The coefficient for each diagonal of the center panel of a truss with an odd number of panels is zero, provided the truss is symmetrically loaded with respect to the center. (Why?)

2. *The K-truss.* No assumption is necessary in the case of the K-truss, since, from fundamental principle 71 : 6, the coefficients of the two diagonals in each panel must be equal. The method of procedure is practically the same as that outlined in the preceding paragraph. (See 200 : 7 and also *Problem 7* of the illustrative problems at the end of this chapter.)

3. *The Baltimore truss.* The coefficient for each half diagonal is "0.5" and these coefficients should be placed on the truss diagram at the start. All other coefficients can then be obtained by the usual methods. (See 217 : 7; also *Problem 6* of the illustrative problems at the end of this chapter.)

4. **Tabulation for the Calculation of Stresses.** When all of the coefficients have been placed on the truss diagram and checked, some convenient form of tabulation should be used in which all calculations are indicated. The work of calculation may then be done by slide rule, or in some other way, all at one time. A form similar to that used in the two complete illustrative problems at the end of this chapter is recommended.

5. **Method of Coefficients Summarized.**

6. *First:* Draw a diagram of the truss (or one-half of the truss) of sufficient size to provide for the coefficients which will be written on the different members. Indicate the loads, determine the reaction, and indicate the coefficient for the reaction. Place arrows, one at each end of each member, to indicate whether a member acts toward or away from a joint (compression or tension).

7. *Second:* Beginning at the center of the truss and working toward an end, *joint by joint*, place coefficients on web members (215 : 6); as each loaded joint is passed, the coefficient is increased by one (or by a fraction if only a fraction of a panel load is applied at the joint, as is the case when loads are divided between upper and lower chord joints). Check the coefficient for the end member (usually an end post) by seeing if it equals the coefficient for the reaction; this serves also as a partial check on all web members.

8. In beginning at the center of the truss, obtain the first coefficient *from the shear*, and make sure that it is correct — if it is incorrect all other coefficients will probably be incorrect. In an ordinary truss with an even number of panels, the first coefficient is "0.5" whereas in an ordinary truss with an odd number of panels it is "1." (In the latter truss the coefficient for a diagonal in the center panel is "0.") It is not best, however, to depend on this rule. When in doubt concerning coefficients in web members at the center or elsewhere, apply $\Sigma V = 0$, which means in most cases that the coefficient is equal to the vertical shear.

9. In trusses which have hip verticals, vertical hangers, or sub-diagonals, place coefficients (determined by inspection) on these members first.

10. In addition to checking the end member by the reaction and other web members by shear, check members at each joint as it is passed, by applying $\Sigma V = 0$, that is, by observing if the sum of the coefficients of members at the joint that act downward equals the sum of the coefficient of members that act upward.

11. Make sure that the coefficients for web members are correct before beginning to determine coefficients for chord members.

12. *Third:* Beginning at an end of the truss and working toward the center, *joint by joint*, place coefficients first on each member of one chord (top or bottom) and then on each member of the other chord. At any joint, the sum of the coefficients of members which act toward the right must equal the sum of the coefficients of members which act toward the left (from $\Sigma H = 0$). This usually, though not always, means that the required coefficient for a chord member is equal to the sum of the coefficients for all other horizontal or inclined members at the joint under consideration. (Ignore vertical members.)

13. When the coefficients for all of the members of one chord have been determined, the coefficients for members of the other chord may be checked by applying $\Sigma H = 0$ to the proper segment (221 : 9); this may be done for each chord member as soon as its coefficient is obtained, or as frequently as may be considered advantageous.

14. The coefficients for both chord members in a center panel should be checked by the summation of the coefficients for the inclined members. (223 : 5.) Either coefficient may also be checked with certainty by applying $\Sigma M = 0$ to the proper segment. (223 : 1.)

In checking, by either of these two methods, the coefficients for the chord members in a center panel, one is also checking, to a considerable extent, the coefficients for all other members.

1. *Fourth:* Multiply a panel load, first, by the secant of the truss angle, and, second, by the tangent of the truss angle. Multiply each of the coefficients for inclined members by the first product and each of the coefficients for horizontal members by the second product, and thus obtain the corresponding stresses. The stress in a vertical member is simply its coefficient multiplied by a panel load. (Why?) Adopt some convenient form of tabulation for calculations, such as that recommended in 224 : 4, or else work directly from a truss diagram of such a size that the magnitude of each stress may be written along the corresponding member.

2. **Advantages and Disadvantages of the Algebraic Method of Successive Joints.** The algebraic method of successive joints, as explained in the first part of this chapter, is rarely used for the calculation of stresses in an entire truss, partly because of cumulative errors which occur from using at one joint stresses calculated at a preceding joint, but mainly because other methods are more efficient. Frequently, however, it is advantageous to apply $\Sigma V = 0$ and $\Sigma H = 0$ to a single joint, or to several joints, in order to find certain stresses that are more easily determined by this method than by any other.

3. For calculating stresses in parallel-chord trusses, the method of coefficients (a short cut for the algebraic method of successive joints) is the most efficient method that can be used. It is quickly and easily applied, it does not involve the cumulative errors just referred to, and it offers abundant opportunity for the frequent checking of results. The method may be made still more efficient by the preparation and use of tables of coefficients for different types of trusses and for different numbers of panels for each type, as will be explained in PART III.

4. When the top and bottom chords of a truss are not parallel, the application of the method of coefficients is not as simple, and in many cases the method of sections can be used to better advantage. The graphic method of successive joints (explained in the next chapter) is, however, exceptionally efficient for determining stresses in trusses with inclined chords. The method of coefficients is not advantageous when panel loads are unequal.

5. *Note:* Stress coefficients may be calculated and tabulated for any type of truss with an inclined chord, as, for example, for standard types of roof trusses, and it is often advantageous to do this; but these stress coefficients are not the simple coefficients used in this chapter, and are determined in quite a different manner.

6. Neither the algebraic method of successive joints nor the method of coefficients is as useful in checking stresses found by graphic methods as is the algebraic method of sections, since by the latter method the stress in any one member may be found quickly and easily, independently of stresses in other members.

It frequently happens that the stresses in some of the members of a truss are best calculated by the method of sections, whereas the stresses in other members are best calculated by applying the resolution equations of equilibrium to one or more joints of the truss. One should be on the alert to combine the two methods in this way instead of adhering blindly to either method.

7. **Illustrative Problems.** In each of the first seven illustrative problems at the end of this chapter, the data are exactly the same as those in a corresponding problem at the end of Chapter XIV. Each problem, therefore, affords an opportunity to compare the method of coefficients with the method of sections.

8. The following method of procedure in studying each problem is recommended:

1. Check each of the coefficients on the truss diagram.
2. Check the checks for coefficients. (221 : 4 to 223 : 14.)
3. Check the equations from which the stresses are calculated.
4. Compare the equations with those in the corresponding problem in Chapter XIV.

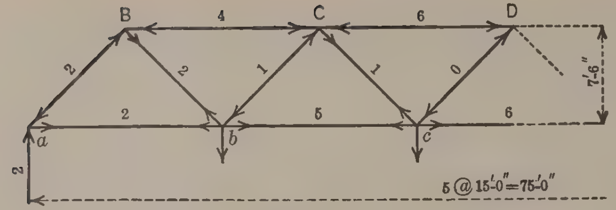


Fig. 226 (*a*).

Problem 1. The data are the same as for *Problem 1*, page 203.
Check for coefficients: $aB = 2 = R_L$; $CD = 2 + 2 + 1 + 1 = 6$; $cd = -DC - Dc = 6$.
$1.0 = 7.5 \div 7.5 = \tan A$. $1.41 = \sec A$.
Panel load $= 6000$ lbs.

Stresses in CD, bc, and Cc

36,000 lbs.(C) $= 6 \times 6000 \times 1 = CD$
30,000 lbs.(T) $= 5 \times 6000 \times 1 = bc$
8,500 lbs.(T) $= 1 \times 6000 \times 1.41 = Cc$

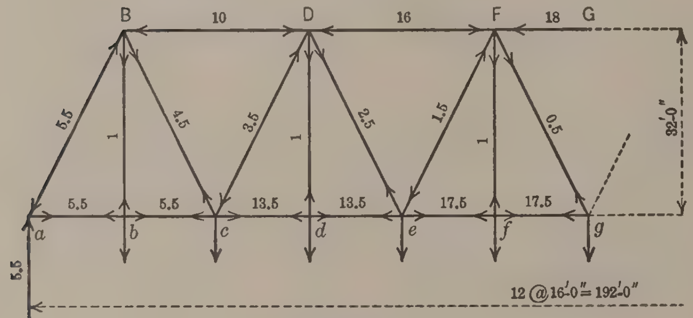


Fig. 226 (*b*).

Problem 2. The data are the same as for *Problem 2*, page 203.
Check for coefficients: $aB = 5.5 = R_L$; $FG = 5.5 + 4.5 + 3.5 + 2.5 + 1.5 + 0.5 = 18$;
$fg = -GF + Fg = -18 + 0.5 = -17.5$.
$0.5 = 16 \div 32 = \tan A$. $1.12 = \sqrt{16^2 + 32^2} \div 32 = \sec A$.
Panel load $= 24,000$ lbs.

Stresses in DF, ef, and eF

192,000 lbs.(C) $= 16 \times 24,000 \times 0.5 = DF$
210,000 lbs.(T) $= 17.5 \times 24,000 \times 0.5 = ef$
40,300 lbs.(C) $= 1.5 \times 24,000 \times 1.12 = eF$

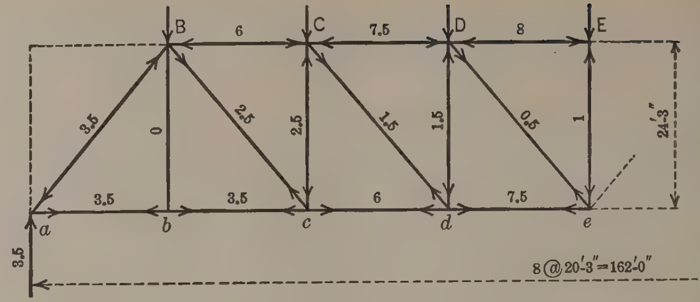


Fig. 226 (*c*)

Problem 3. The data are the same as for *Problem 3*, page 204.
Check for coefficients: $aB = 3.5 = R_L$; $DE = 3.5 + 2.5 + 1.5 + 0.5 = 8$; $de = -CD$.
$0.835 = 20.25 \div 24.25 = \tan A$.
Panel load $= 24,300$ lbs.

Stresses in CD, Dd, and de

152,200 lbs.(C) $= 7.5 \times 24,300 \times 0.835 = CD$
36,500 lbs.(C) $= 1.5 \times 24,300 = Dd$
152,200 lbs.(T) $= de = -CD$

Problem 4. The data are the same as for *Problem 5*, page 204.
Check for coefficients:
$aB = 2.5 = R_L$; $CD = 2.5 + 1.5 + 0.5 = 4.5$;
$cd = -BC$.
$0.688 = 22 \div 32 = \tan A$.
$1.21 = \sqrt{22^2 + 32^2} \div 32 = \sec A$.
Panel load $= 24,000$ lbs.; $\frac{1}{4}$ panel load at upper joint.

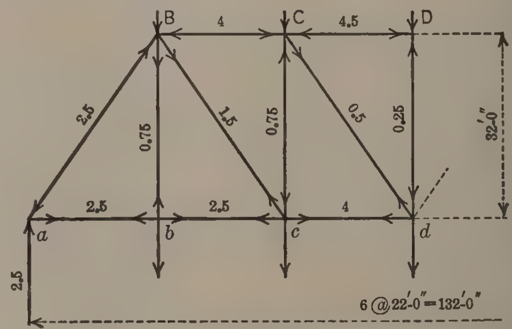


Fig. 226 (*d*).

Stresses in BC, Cc, cd, and Cd

66,000 lbs.(C) $= 4 \times 24,000 \times 0.688 = -BC = cd$
18,000 lbs.(C) $= 0.75 \times 24,000 = Cc$
14,500 lbs.(T) $= 0.5 \times 24,000 \times 1.21 = Cd$

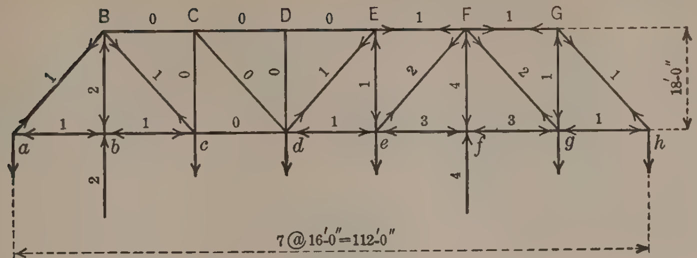


Fig. 227 (*a*).

Problem 5. The data are the same as for *Problem 7*, page 205.

$$R_L = \frac{5}{4} + \frac{3}{4} + \frac{2}{4} + \frac{1}{4} - \frac{1}{4} - \frac{2}{4} = 2; \; R_R = 6 - 2 = 4$$

Check for coefficients: $aB = 1 =$ load at a; $FG = -1 + 1 - 1 - 2 + 2 = -1$; $EF + eF_H - ef = 1 + 2 - 3 = 0$.

$\frac{8}{9} = 16 \div 18 = \tan A$. Panel load $= 3500$ lbs. (None at b and f.)

Stresses in de, Ee, and EF

3110 lbs.(C) $= 1 \times 3500 \times \frac{8}{9} = de = -EF$(T). 3500 lbs.(C) $= 1 \times 3500 = Ee$(C).

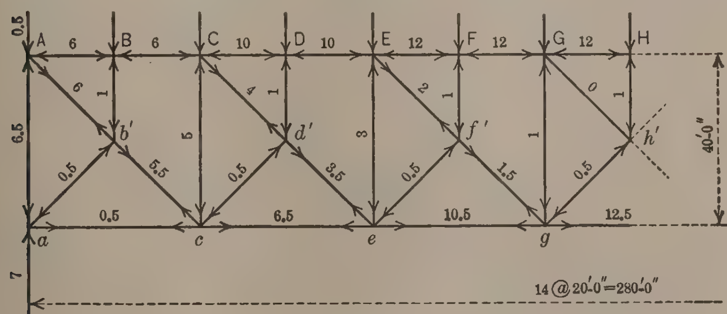


Fig. 227 (*b*).

Problem 6. The data are the same as for *Problem 10*, page 205.

Check for coefficients: $-Aa - ab' + R_L = -6\frac{1}{2} - \frac{1}{2} + 7 = 0$; $GH = 6 + 4 + 2 + 0 = 12$; $-HG - h'g + gg = -12 - 0.5 + 12.5 = 0$

$1.0 = \tan A$. $1.414 = \sec A$. Panel load $= 36{,}000$ lbs.; $\frac{1}{2}$ panel load at A.

Stresses in EF, Ef', ef', and eg

432,000 lbs.(C) $= 12 \times 36{,}000 \times 1 = EF$. 25,500 lbs.(C) $= 0.5 \times 36{,}000 \times 1.414 = ef'$.

101,800 lbs.(T) $= 2 \times 36{,}000 \times 1.414 = Ef'$. 378,000 lbs.(T) $= 10.5 \times 36{,}000 \times 1 = eg$.

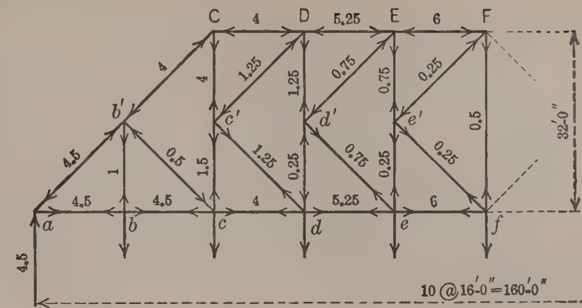


Fig. 227 (*c*).

Problem 7. The data are the same as for *Problem 11*, page 206.

Check for coefficients: $ab' = 4.5 = R_L$; $EF = 4 + 1.25 + 0.75 = 6 = -ef$

$1.0 = \tan A$. $1.414 = \sec A$. Panel load $= 20{,}000$ lbs.

Stresses in DE, $d'E$, $d'e$, and de

105,000 lbs.(C) $= 5.25 \times 20{,}000 \times 1 = DE = de$(T)

21,200 lbs.(C) $= 0.75 \times 20{,}000 \times 1.414 = d'E = d'e$(T)

Problems 8 and 9. The truss in *Problem 8*, page 228, is a Warren truss with verticals similar to that shown in the photograph on page 98. The truss in *Problem 9*, is a deck Pratt truss similar to that shown in the photograph on page 104. In each of these problems it was required to calculate by the method of coefficients the stresses in all members in one-half of the truss. The method of procedure in each problem was as follows:

(1) The truss diagram was drawn and the loads and reaction were indicated on this diagram in terms of W, the panel load.

(2) The arrow heads were placed on the members in the truss diagram to indicate tension or compression and the coefficients were then placed on the diagram.

(3) The coefficients were checked from inspection. (The methods of checking are indicated underneath the truss diagram for the benefit of the student, but in practice all this work of checking would be done mentally.)

(4) All of the preliminary calculations were first indicated and then these calculations were made.

(5) The first, third, and fourth columns of the tabular forms were completed in that order.

(6) The stresses were calculated and inserted in the second column.

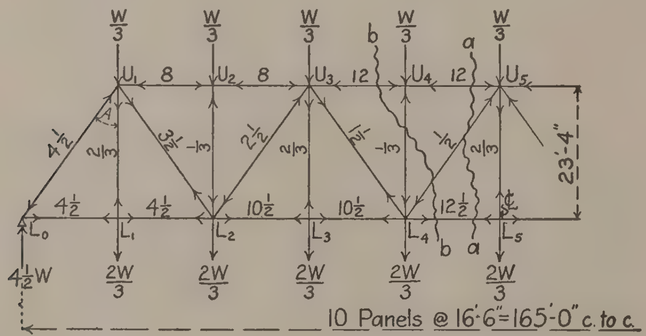
Note: In the preliminary calculations and in the calculation of stresses, the actual work of calculation was left until this work had been completely indicated. (Why? (198 : 4.))

The truss in *Problem 9* is the same as that in *Problem 13*, page 207. The student is advised to compare carefully the problem on page 229 with the problem on page 207 since these two problems afford an excellent comparison of the method of coefficients with the method of sections.

PROBLEM 8.

ILLUSTRATIVE PROBLEM IN THE CALCULATION OF THE DEAD-LOAD STRESSES IN A THROUGH WARREN TRUSS WITH VERTICALS BY THE METHOD OF COEFFICIENTS.

Given: A through Warren truss with verticals. Number of panels = 10. Panel length, p, = 16'-6"; height of truss, h, = 23'-4". Dead load per lin. ft. of bridge = 900 lbs., distributed one third to the upper chord, two thirds to the lower. Required: The dead-load stresses in all members of the truss.



Checks for Coefficients (from inspection)

$L_0U_1 = 4\frac{1}{2} = R_L$ (Check)

$L_4L_5 = 12\frac{1}{2} = +L_0U_1 + U_1L_2 + L_2U_3 + U_3L_4 + L_4U_5 = +4\frac{1}{2} + 3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$ (Check)

$\Sigma H = 0 = -U_4U_5 - L_4U_5 + L_4L_5 = -12 - \frac{1}{2} + 12\frac{1}{2}$ (Check) (Section a-a)

$\Sigma V = 0 = +R_L - \frac{11}{3}W - L_4U_4 - L_4U_5 = +4\frac{1}{2} - \frac{11}{3} - \frac{1}{3} - \frac{1}{2}$ (Check) (Section b-b)

Preliminary Calculations

$7425^{\#} = (900 \times 16\frac{1}{2}) \div 2 = W$ = Dead load per panel truss

$0.707 = 16\frac{1}{2} \div 23\frac{1}{3} = \tan A$

$1.225 = \sqrt{(16\frac{1}{2})^2 + (23\frac{1}{3})^2} \div 23\frac{1}{3} = \sec A$

$5250^{\#} = 7425 \times 0.707 = W \times \tan A$

$9100^{\#} = 7425 \times 1.225 = W \times \sec A$

Stresses in Chord Members.

Member	Stress	Coefficient	W × tan A
$U_1U_2U_3$	42000#(C)	8	5250#
$U_3U_4U_5$	63000#(C)	12	"
$L_0L_1L_2$	23600#(T)	$4\frac{1}{2}$	"
$L_2L_3L_4$	55100#(T)	$10\frac{1}{2}$	"
L_4L_5	65600#(T)	$12\frac{1}{2}$	"

Stresses in Diagonal Members.

Member	Stress	Coefficient	W × sec A
L_0U_1	41 000#(C)	$4\frac{1}{2}$	9100#
U_1L_2	31 900#(T)	$3\frac{1}{2}$	"
L_2U_3	22 800#(C)	$2\frac{1}{2}$	"
U_3L_4	13 700#(T)	$1\frac{1}{2}$	"
L_4U_5	4 600#(C)	$\frac{1}{2}$	"

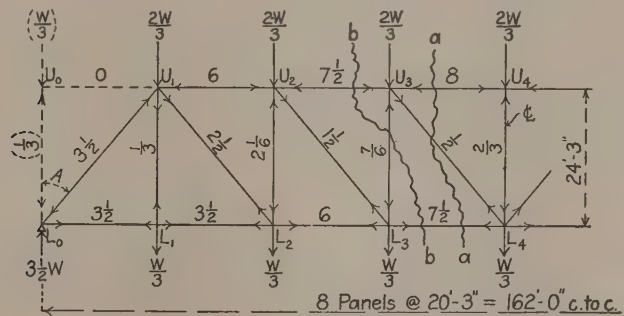
Stresses in Vertical Members.

Member	Stress	Coefficient	W
U_1L_1, U_3L_3, U_5L_5	5000#(T)	$\frac{2}{3}$	7425#
U_2L_2, U_4L_4	2500#(C)	$\frac{1}{3}$	"

PROBLEM 9

ILLUSTRATIVE PROBLEM IN THE CALCULATION OF THE DEAD-LOAD STRESSES IN A DECK PRATT TRUSS BY THE METHOD OF COEFFICIENTS

Given: A deck Pratt truss railroad bridge Number of panels = 8. Panel length, p, = 20'-3"; height of truss, h, = 24'-3". Dead load per lin. ft. of bridge = 2400 lbs., distributed one third to the lower chord, two thirds to the upper. Required: The dead-load stresses in all members of the truss.



Checks for Coefficients (from inspection)

$L_0U_1 = 3\frac{1}{2} = R_L$ (Check)

$U_3U_4 = 8 = +L_0U_1 + U_1L_2 + U_2L_3 + U_3L_4 = +3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$ (Check)

$\Sigma H = 0 = -U_3U_4 + U_3L_4 + L_3L_4 = -8 + \frac{1}{2} + 7\frac{1}{2}$ (Check) (Section a-a)

$\Sigma H = 0 = -U_2U_3 + L_3L_4 = -7\frac{1}{2} + 7\frac{1}{2}$ (Check) (Section b-b)

$\Sigma V = 0 = 3\frac{1}{2}W - \frac{7}{3}W - U_3L_3 = 3\frac{1}{2} - \frac{7}{3} - \frac{7}{6}$ (Check) (Section b-b)

Preliminary Calculations

$24\,300^{\#} = (2400 \times 20\frac{1}{4}) \div 2 = W$ = Dead load per panel per truss

$0.835 = 20\frac{1}{4} \div 24\frac{1}{4} = \tan A$

$1.303 = \sqrt{(20\frac{1}{4})^2 + (24\frac{1}{4})^2} \div 24\frac{1}{4} = \sec A$

$20\,300^{\#} = 24\,300 \times 0.835 = W \times \tan A$

$31\,700^{\#} = 24\,300 \times 1.303 = W \times \sec A$

Stresses in Chord Members

Member	Stress	Coefficient	W×tan A
U_1U_2	121800#(C)	6	20300#
U_2U_3	152300#(C)	$7\frac{1}{2}$	"
U_3U_4	162400#(C)	8	"
$L_0L_1L_2$	71100#(T)	$3\frac{1}{2}$	"
L_2L_3	121800#(T)	= stress in U_1U_2.	
L_3L_4	152300#(T)	= " " U_2U_3.	

Stresses in Diagonal Members

Member	Stress	Coefficient	W×sec A
L_0U_1	111000#(C)	$3\frac{1}{2}$	31700#
U_1L_2	79300#(T)	$2\frac{1}{2}$	"
U_2L_3	47600#(T)	$1\frac{1}{2}$	"
U_3L_4	15900#(T)	$\frac{1}{2}$	"

Stresses in Vertical Members

Member	Stress	Coefficient	W
U_1L_1	8100#(T)	$\frac{1}{3}$	24300#
U_2L_2	52700#(C)	$2\frac{1}{6}$	"
U_3L_3	28400#(C)	$\frac{7}{6}$	"
U_4L_4	16200#(C)	$\frac{2}{3}$	"

Stresses in Supplementary Members

U_0U_1	0	
U_0L_0	8100#(C)	One half upper chord panel load $= \frac{1}{3}W$

CHAPTER XVI

GRAPHIC METHOD OF SUCCESSIVE JOINTS

The *graphic* method of successive joints is exactly the same in principle as the *algebraic* method of successive joints explained in the first part of the preceding chapter. The graphic method is explained in detail in this chapter, but its most practical and common application, namely, its use in determining stresses in roof trusses, is treated as a subject in itself in the next chapter.

It is seldom advantageous to use the graphic method of successive joints in determining stresses in parallel-chord trusses, but in order that this method may be compared, not only with the algebraic method of successive joints which it closely parallels, but also with the algebraic method of sections, the same simple parallel-chord trusses used in preceding chapters to illustrate the two algebraic methods are used in this chapter to illustrate the graphic method.

1. **Introductory Statements Concerning the Graphic Method.** *The first step* in the graphic method is to draw a *truss diagram.* In an algebraic solution this diagram may be drawn free-hand, but in the graphic method it must be drawn carefully to scale. *Bow's notation* (24 : 8) is used for the truss diagram.

2. *The second step* is to represent on the truss diagram all loads that are on the truss. The line of action of each load should be drawn exactly in its correct position, but the magnitude of the force is not laid off to scale, i.e., the line of action of a load may be of any convenient length.

3. *The third step* is to determine the *reactions.* For this purpose, either an algebraic or a graphic method may be used, whichever is most efficient.

4. *The fourth step* is to draw a *force polygon* for the *external forces*, loads and reactions. (If the entire truss is taken as a rigid body in equilibrium, the force polygon for all of the forces that hold it in equilibrium should close (14 : 8).) If the reactions are determined graphically, it will be necessary to combine the third step with this fourth step.

5. These first four steps are all preliminary to the actual determination of the stresses. In the algebraic method of successive joints, stresses were determined by attacking each joint in succession, as a problem in itself. Each joint was considered as a particle held in equilibrium by the forces at that joint, regardless of whether any one of the forces was a load, a reaction, or a force exerted by a member of the truss. The problem at each joint was one in concurrent forces in which the unknown elements were two magnitudes, i.e., forces exerted by two members of the truss. (Case B.) The statements just made concerning the algebraic method of successive joints hold true in every respect for the graphic method. The difference between the two methods is merely this: In the algebraic method, the two unknown magnitudes at any joint are determined by means of the two equations $\Sigma H = 0$ and $\Sigma V = 0$; in the graphic method, the two unknown magnitudes are determined by a force polygon, which is the graphic equivalent of the two resolution equations of equilibrium. (14 : 6.) Hence:

6. *The fifth step* is to complete for *each joint* of the truss in succession a *force polygon* for all forces at that joint, including all known forces, external or internal, and the two unknown internal forces (stresses).

7. Each of the five steps just outlined will now be explained in detail.

8. *Note:* This whole chapter and the next are based on the graphic method of solution for Case B. It may be well, therefore, to review the portion of Chapter VI in which that method is explained (24 : 4), particularly the illustrative problem on page 29.

1. **The Truss Diagram.** The truss diagram must be drawn carefully to scale because it serves as a *space diagram*, from which is determined the inclination or slope of every truss member and of the line of action of every external force. In all of the force polygons, each line will be drawn parallel to the corresponding line in the space diagram; if the inclinations in the space diagrams are not correct, the force polygons will not be correct, and consequently the required stresses scaled from the force polygons will not be correct. Suggestions for accuracy in drawing will be given later.

2. **Use of Bow's Notation.** Bow's notation, explained on page 24, will be used throughout this chapter. The most important fact to remember concerning this notation is that every force in a space diagram is designated by two letters, one on either side of its line of action and, conversely, there must be a line of action between any two adjacent letters in a space diagram, otherwise, at least one of the letters is superfluous. For example, in Fig. 231 the panel loads are *GF*, *FE*, *ED*, *DC*, and *CB*; the left-hand reaction is *BA*, but, since there are no external forces on the upper portion of the truss, one letter *A* is all that is needed clear around to *G*. The right-hand reaction is therefore *AG*, not *HG*. If *H* had been placed to the right of the right-hand reaction, it would imply that there was another external force between *H* and *A*, but there is no such force. The members of the truss are: *Top Chords:* *Ab*, *Ad*, *Af*, *Ah*, and *Aj*. *End Posts:* *Aa* and *Ak*. *Bottom Chords:* *aB*, *cC*, *eD*, *gE*, *iF* and *kG*. *Web Members:* *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gh hi*, *ij*, and *jk*.

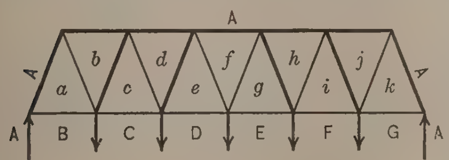


Fig. 231.

3. It is well to begin at one end of the truss and place capital letters in alphabetical order around the outside of the truss to denote loads and reactions, and then to place lower-case letters within the truss, one in each triangle, also in alphabetical order. A line of action designated by two capital letters will then be that of an *external* force; by one capital and one lower-case letter, that of a member in the *perimeter* of the truss; and by two lower-case letters, that of a *web* member. If preferred, numerals may be used in place of lower-case letters; then any line of action designated by two letters will be that of an external force; by one letter and a numeral, that of a member in the perimeter; and by two numerals, that of a web member. Many engineers prefer to use capital letters exclusively for the truss diagram, and the corresponding lower-case letters for the force diagram.

4. *Note:* When one has become accustomed to the graphic method, one is likely to adopt some modification of the notation suggested. For example, one may prefer to use the same letters for the right-hand half of the truss that were chosen for the left-hand half (or the same letters with primes), in order that any member in one half will be designated by the same two letters as those used for the corresponding member in the other half. Several different systems of notation will be used in the illustrative problems in the next chapter.

5. **Force Polygon for External Forces.** Before stresses can be determined by attacking each *joint* of the truss, it is usually necessary to attack the *truss as a whole*, and to draw the force polygon for the *external* forces, namely, loads and reactions. First, draw as much as possible of this polygon by laying off in succession the known loads. This portion of the polygon is sometimes called the **load line,** and this load line is usually straight. The next step is to find the reactions and thus to complete the polygon for external forces. In many cases, the reactions are each equal to one half of the total load on the truss. If, however, the truss is unsymmetrical or is unsymmetrically loaded, the reactions cannot be assumed as equal; and if any load is inclined, both reactions cannot be vertical, — one at least must be inclined. In either case the reactions must be determined by one of the methods explained in Chapter XII. The most efficient method should be used, regardless of whether it is an algebraic or a graphic method. In most cases an algebraic method is preferable. In any event, it is folly to begin to determine internal forces (stresses) until one is reasonably certain that the force polygon for external forces (loads and reactions) is correctly drawn. If the reactions and loads are *parallel*, this force polygon will lie wholly in a straight line.

6. *Note:* In special cases, as, for example, when a truss is supported at one end only, stresses may be determined graphically without first finding reactions and completing the force polygon for external forces.

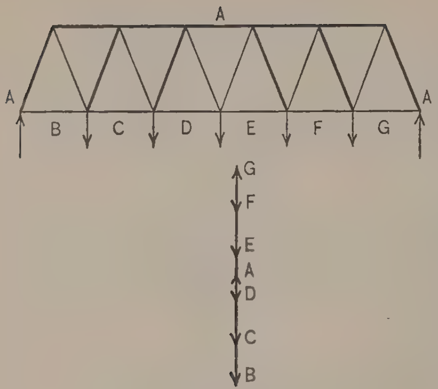


Fig. 232 (a).

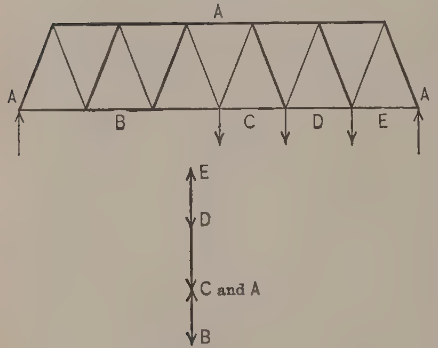


Fig. 232 (b).

1. *Illustration:* In Fig. 232 (a) there are five equal panel loads. Beginning with GF, these five loads were laid off in order, thus forming the load line from G to B. The reactions are equal, hence A is half way between B and G, thus making the two remaining sides of the polygon, BA and AG, equal. The polygon ($GFEDCBAG$) lies in a straight line, since all external forces are parallel; is seven-sided, since there are seven external forces; and closes at G, since the external forces are in equilibrium.

2. *Illustration:* In Fig. 232 (b) three joints only are loaded. Assume these loads to be equal. The load line will extend from E to B. Find the reactions (algebraically): BA will equal $\frac{1}{6} + \frac{2}{6} + \frac{3}{6}$ of a panel load or one load, and AE will equal $\frac{5}{6} + \frac{4}{6} + \frac{3}{6}$ or two panel loads. (142 : 3.) Hence A in the force polygon must fall at C in order to make BA equal one panel load and AE equal two. The polygon $EDCBAE$ lies in a straight line (Why?), is five-sided (Why?), and closes at E (Why?).

3. *Illustration:* In Fig. 232 (c) six equal inclined loads act on the upper joints of the truss. The load line will extend from A to G. To complete the polygon for external forces the two reactions must be determined. Let it be assumed that the left-hand reaction HA is vertical. Its magnitude may be determined by one of the methods explained in CHAPTER XII. Once this magnitude is known, the point H in the force polygon may be plotted, and the two closing lines of the polygon corresponding to the two reactions will be GH and HA. If the two reactions had been assumed parallel to the loads, the force polygon would have fallen in a straight line.

4. *Illustration:* In each of the three illustrative examples just given, the two reactions formed two adjacent sides of the force polygon. In Fig. 232 (d) the two reactions JA and GH will not be adjacent in the force polygon since they are not adjacent in the space diagram, i.e., there are forces (loads) *between* these reactions, regardless of whether one follows around the outside of the truss in one direction or the other. Let the six loads on the top of the truss and the two loads on the bottom be equal. Each reaction will then be equal to four loads. The force polygon is $ABCDEFGHIJA$ — ten-sided — two long sides, GH and JA, each equal to four loads — the other eight sides each equal to one load.

5. *The order in which external forces are drawn in the force polygon for external forces.* External forces should be drawn in the force polygon in succession, in the same order as that in which they are observed as one follows around the outside of the truss. Since one can follow around the truss in two directions, clockwise and counter-clockwise, there are two corresponding orders of succession. Theoretically, it is immaterial in which of these two orders the external forces are laid off in the force polygon (27 : 1), but it is well to adopt one and use it to the exclusion of the other, unless, in some particular case, there is a good reason for not doing so. In this book, as a general rule, the clockwise order will be followed.

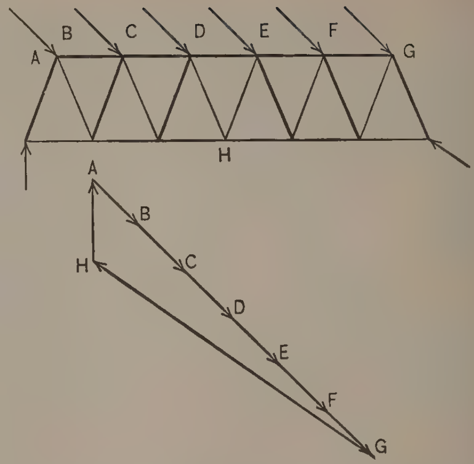


Fig. 232 (c).

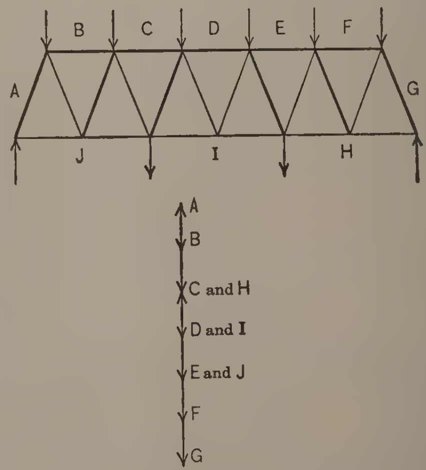


Fig. 232 (d).

1. **The Graphic Method of Successive Joints Applied to a Parallel-chord Truss.** The completion of the force polygon for external forces marks the completion of the first four steps outlined in 230 : 1. The remaining step, that of drawing a force polygon for each joint, is best explained by means of an illustrative example. Assume that the truss diagram shown on page 235 has been drawn to scale, that the panel loads of 1000 lbs. each and the reaction of 2500 lbs. have been indicated, and that the whole diagram has been lettered in accordance with Bow's notation. Assume, moreover, that the force polygon for external forces (diagram marked "(K)") has been drawn, as explained in the preceding article. It is now desired to draw a force polygon for each joint in the left-hand half of the truss. In practice, these polygons are all drawn superimposed one on another to form one **force diagram,** but on page 235 a separate figure is shown for each joint in order to indicate, step by step, the growth of the entire force diagram.

Note: The complete force diagram for any truss is composed of the force polygon for external forces and the force polygons for the various joints of the truss. From these latter polygons the values for the stresses in the truss are determined. For this reason the diagram is frequently called a "stress diagram." This term, however, is often used for a truss diagram on which is written along each member the stress in that member. In this book, the term "force diagram" will be used for the complete diagram which represents both external and internal forces. The term "force polygon for external forces" will be used for that portion of the force diagram which represents only the external forces.

2. The problem at each joint is one in concurrent forces in which the unknowns are two magnitudes. (Case B.) These two unknowns are determined by a force polygon for the forces at the joint — the graphic equivalent for $\Sigma H = 0$ and $\Sigma V = 0$. All the sides of the force polygon for any joint will be drawn *before* that joint is attacked, except the two sides that correspond to the two unknown magnitudes or stresses to be determined at that joint. The graphical work for any one joint will consist, therefore, in merely drawing *two* lines, namely, the closing lines of the force polygon that corresponds to that joint. Before explaining in detail how each polygon on page 235 was drawn, a general method of procedure will be given which makes it easy to decide quickly and surely what two lines must be drawn in order to close the force polygon for any joint in any type of truss.

3. *Note:* Since the force polygon for each joint is superimposed on the polygons already drawn for preceding joints, the force diagram becomes more and more complex as one proceeds from one joint to another. It is not easy to distinguish in this network of lines the force polygon for any particular joint, or to pick out the points from which to draw the two closing lines, unless one adopts some systematic method of procedure. Such a method is explained in the next article. Though the explanation may seem lengthy, because it is given in detail, the method itself is simple and easy to apply, as will be evident in the illustrations of its use that follow. No article in the entire chapter is more important than the next; once the method there explained is understood and followed, there should be little difficulty in drawing the force diagram for any type of truss.

4. *How to determine and how to draw the two closing lines of the force polygon for any joint:*

5. First: As one proceeds from joint to joint, there will be at each joint two members and *only two* for which the stresses have not already been determined. The first step in attacking any joint is to concentrate on these two members; *ignore all other members and all external forces* either at the joint or elsewhere. For purposes of explanation let *either* of these two members be known as the *first member* and the other as the *second member.*

6. Second: Turning to the truss diagram, fix the attention on the *first member* at the joint, ignoring the second. This first member will lie between two letters in the *truss* diagram. *One* of these letters will already have been used to designate a point in the *force* diagram. Find this point, and through it draw a line of indefinite length in the force diagram parallel to the *first member* in the truss diagram.

7. Third: Turning again to the joint in the truss diagram, fix the attention on the *second member* at the joint, ignoring the first. This second member will lie between two letters in the *truss* diagram. *One* of these letters will already have been used to designate a point in the *force* diagram. Find this point, and through it draw a line in the force diagram parallel to the *second member* until it intersects the line drawn in the second step parallel to the *first member.*

8. The third step merely repeats for the second member the method of procedure followed in the second step for the first member. Two new lines have now been drawn in the force diagram, parallel, respectively, to the first and second members at the joint, i.e., the two members for

which stresses are being determined. These two lines will complete the force polygon for the joint, the other sides of the polygon having been drawn previously in proceeding from joint to joint. The letter to be placed at the intersection of these two new lines will be the same as that which, according to Bow's notation, lies *between* the two corresponding members of the truss diagram, i.e., the letter between the *first* and *second members.* The length of each of these two closing lines will represent to scale the magnitude of the stress in the corresponding member.

Though the method as just outlined may seem wholly mechanical, it will cease to be so as soon as the reason for each step is understood. These reasons should become obvious from the illustrations which follow.

1. *Illustrations of the method of drawing the two closing lines for the force polygon at any joint.* In the truss diagram on page 235 the joints are numbered "(1)," "(2)," "(3)," "(4)," and "(5)" in the order in which they will be attacked. This order is determined by the rule already followed in the algebraic method of successive joints, namely, to begin with the joint at a support, because there are only two unknown magnitudes (stresses) at that joint, and to be guided in the selection of successive joints by the principle that there must not be more than two unknown stresses at the joint attacked, otherwise the problem at that joint will be indeterminate.

2. Below the truss diagram are five force diagrams numbered "(1)," "(2)," "(3)," "(4)," and "(5)," to correspond to the five joints. In addition there is the force polygon for external forces lettered "(K)." Throughout the explanation that follows, a number or letter in parentheses means that the corresponding diagrams on page 235 should be consulted.

3. The truss diagram is a space diagram and one should always keep in mind the distinction that a line in the truss diagram represents a truss member whereas the corresponding line in the force diagram represents the stress in that member. This distinction is particularly important when the same letters are used to designate the two lines.

4. Beginning with diagram (1), each force diagram is a reproduction of the preceding force diagram plus two (and only two) additional lines. These two lines in a diagram for any joint are the *closing lines* of the force polygon for that joint; they represent the two stresses that are determined at that joint, and are drawn heavier than the other sides of the polygon in order that they may easily be distinguished. Broken lines represent sides of other polygons previously drawn. Above each diagram are designated by letters the sides of the force polygon for the corresponding joint in the order in which they occur; it is also stated which of these sides are the two closing sides, i.e., the two new lines that are drawn heavy in the polygon itself.

5. It should be borne in mind that all six of these diagrams are drawn ordinarily as one diagram, shown in its completeness in diagram (5), and that they have been drawn here as separate diagrams in order to indicate the growth, two lines at a time, of this final diagram.

6. The work will now be explained step by step. The explanation for each joint is, in a sense, a repetition of that for the preceding joint, but perhaps in no other way can one problem be made to illustrate as clearly a general method of procedure that renders easy the application of the graphic method of successive joints to any type of truss.

Joint (1)

7. Concentrate on the two members at this joint in the truss diagram in which the stresses are unknown, namely, Aa and aB, — ignore the reaction.

8. The first member Aa lies between A and a in the truss diagram. One of these letters must already be on the force diagram (K) for external forces — it is A. Through A (diagram (1)) draw a line of indefinite length parallel to the member Aa.

9. The second member aB lies between a and B in the truss diagram. One of these letters must already be on the force diagram (K) for external forces — it is B. Through B (diagram (1)) draw a line parallel to the member aB until it intersects the line previously drawn through A. The letter to be placed at the point of intersection is that between the members Aa and aB in the truss diagram, namely, a. The lengths of Aa and aB in the force diagram (1) represent to scale the magnitudes, respectively, of the stresses in members Aa and aB.

Joint (2)

10. Concentrate on the two members at this joint in the truss diagram in which the stresses are unknown, namely, ab and bC — ignore the load at the joint and the member Ba.

11. The first member ab lies between a and b in the truss diagram. One of these letters must already be on the force diagram (1), — it is a. Through a (diagram (2)) draw a line of indefinite length parallel to the member ab.

12. The member bC lies between b and C in the truss diagram. One of these letters must already be on the force diagram (1), — it is C. Through C (diagram (2)) draw a line parallel to the member bC until it intersects the line previously drawn through a. The letter to be placed at the point of intersection is that between the members ab and bC in the truss diagram, namely, b. The lengths ab and bC in the force diagram (2) represent to scale the magnitudes, respectively, of the stresses in members ab and bC. Note that the stress in ab (diagram (2)) is equal to the load BC, and the stress in bC is equal to the stress in aB. This is in agreement with the principle in 71 : 5.

Joint (3)

13. Concentrate on the two members at this joint in the truss diagram in which the stresses are unknown, namely, Ac and cb, — ignore the members ba and aA. The member Ac lies between A and c in the truss diagram. One of these letters must already be on the force diagram (2) — it is A. Through A (diagram (3)) draw a line of indefinite length parallel to the member Ac.

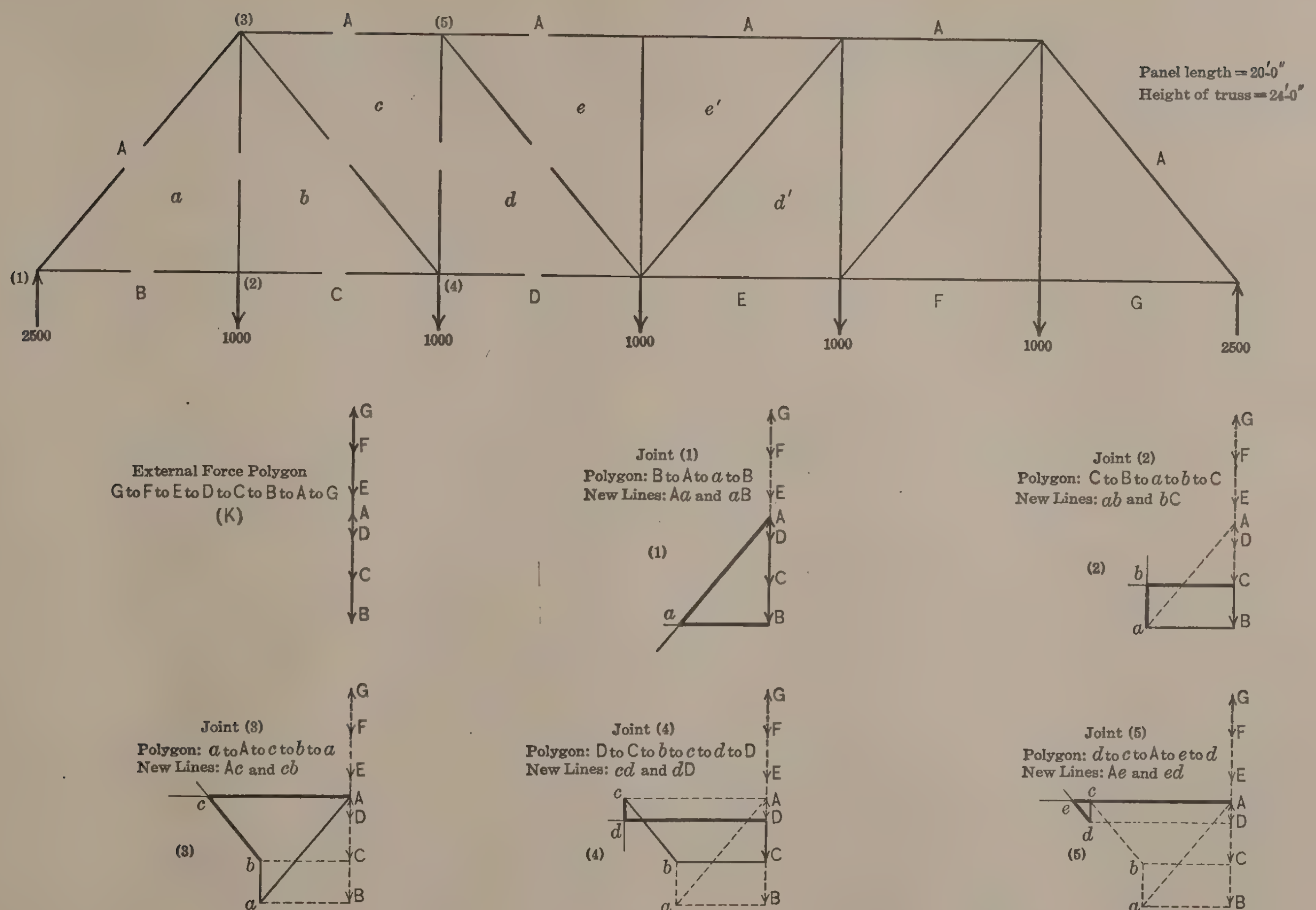

Panel length = 20'-0"
Height of truss = 24'-0"
(1)
(2)
(3)
(4)
(5)
A
B
C
D
E
F
G
a
b
c
d
e
e'
d'
2500
1000
1000
1000
1000
1000
2500
External Force Polygon
G to F to E to D to C to B to A to G
(K)
Joint (1)
Polygon: B to A to a to B
New Lines: Aa and aB
Joint (2)
Polygon: C to B to a to b to C
New Lines: ab and bC
Joint (3)
Polygon: a to A to c to b to a
New Lines: Ac and cb
Joint (4)
Polygon: D to C to b to c to d to D
New Lines: cd and dD
Joint (5)
Polygon: d to c to A to e to d
New Lines: Ae and ed

Fig. 235.

1. The member *cb* lies between *c* and *b* in the truss diagram. One of these letters must already be on the force diagram (2), — it is *b*. Through *b* (diagram (3)) draw a line parallel to member *cb* until it intersects the line drawn through *A*. The letter to be placed at the point of intersection is that between members *Ac* and *cb*, namely, *c*. The lengths *Ac* and *cb* in the force diagram (3) represent to scale the magnitudes, respectively, of the stresses in members *Ac* and *cb*.

Joint (4)

2. The two members for which stresses are unknown: *cd* and *dD*. Letters already on force diagram (3): *c* and *D*. Through *c* and *D* (diagram (4)) draw lines parallel, respectively, to *cd* and *dD* — their intersection is *d*.

Joint (5)

3. The two members for which stresses are unknown: *Ae* and *ed*. Letters already on force diagram (4): *A* and *d*. Through *A* and *d* (diagram (5)) draw lines parallel, respectively, to *Ae* and *ed* — their intersection is *e*.

4. The five steps of the graphic method as outlined in 230 : 1 have now been explained in detail. There remain three more steps, namely, to check the completed force diagram, to scale all stresses from this diagram, and to determine for each of the truss members whether it is in tension or compression.

5. **To Check the Completed Force Diagram.** Calculate by the method of sections the stress in some member near the center of the truss, as, for example, the member *Dd*. Scale the line in the force diagram that corresponds to this member. If the stress thus found is the same as that calculated by the method of sections, it is probable that the entire force diagram is correct. (Why?)

6. *Note:* In addition to the check just explained, it is well to apply certain other checks from time to time as the work progresses, in order to detect any error in the force diagram as soon as possible after it has been made. One method of checking is to observe what stresses are evident from an inspection of the truss diagram. For example, from the truss diagram on page 235, it is evident that the stress in *ab* must equal the panel load *BC*, that the stress in *Dd* must equal the stress in *Ac* (Why?), and that the stress in *dc* must equal the V component of the stress in *de*. Observe also that since the stress in *ee′* is zero, the points *e* and *e′* must coincide in the force diagram.

7. Another method of checking is symmetry of the force diagram. For example, if the force diagram had been extended to the center joint of the lower chord, the portion of the diagram below *Ae*, as far as that portion is drawn, should be symmetrical with the corresponding portion above *Ae*.

8. **To Determine Stresses from the Force Diagram.** After the check explained in 236 : 5 has been satisfactorily applied, *and not before*, scale the lines in the force diagram in order to obtain the stresses in the corresponding members of the truss. Stresses should either be recorded directly on the truss diagram or tabulated elsewhere.

9. *Note:* It is an inexcusable waste of time to scale the stresses from a force diagram that has not been checked when, had the check described in 236 : 5 been applied, the diagram would have been found to be partly or wholly incorrect.

10. **To Determine Whether a Member Is in Tension or Compression.** The nature of the stress in any member of an ordinary truss is usually evident. When it is uncertain whether the stress in a member is tension or compression, it is merely necessary to follow around the force polygon for the joint at *either* end of the member in order to determine whether the member acts away from or toward the joint (as explained in the graphic method for Case B on page 29). The completed force diagram, however, is made up of a number of force polygons superimposed, one on the other, and it is essential, therefore, to have some method of picking out from this network of lines the polygon for any joint under consideration.

11. *To pick out any force polygon in a force diagram.* The method of picking out, in the force diagram for a truss, the force polygon for any joint of the truss is best explained by an illustrative example. The force diagram already determined on page 235 is shown to a considerably larger scale in Fig. 237 (*a*). A similar force diagram for the same truss and loading is shown to the same scale in Fig. 237 (*b*). This second diagram is identical with the first in form but is inverted in position. The results obtained from the two diagrams are identical. The difference in position is due to the difference in the order in which the *external* forces were laid off. In Fig. 237 (*a*) the loads and reactions were laid off clockwise around the truss, i.e., *ED–DC–CB–BA*, whereas in Fig. 237 (*b*) they were laid off in the reverse order (counter-clockwise), i.e., *AB–BC–CD–DE*. In each figure, only that portion of the complete force diagram is shown that includes the polygons for the five joints in the left-hand half of the truss.

12. If the let ers used to designate the forces, external or internal, that act at any joint in the truss diagram are conceived as grouped around that joint, they will correspond in order to the apices of the force polygon

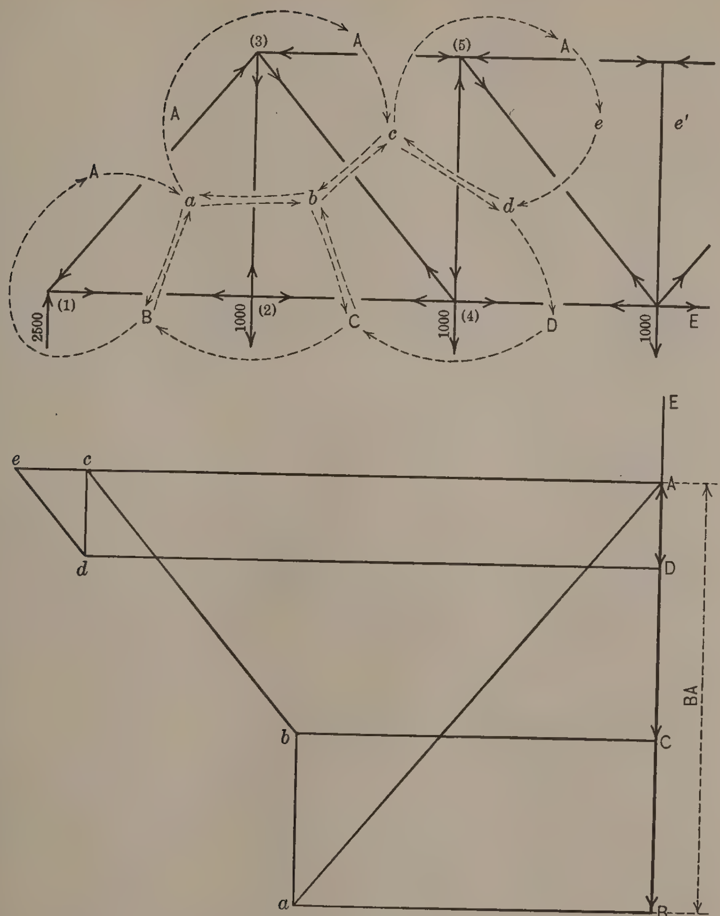


Fig. 237 (*a*).

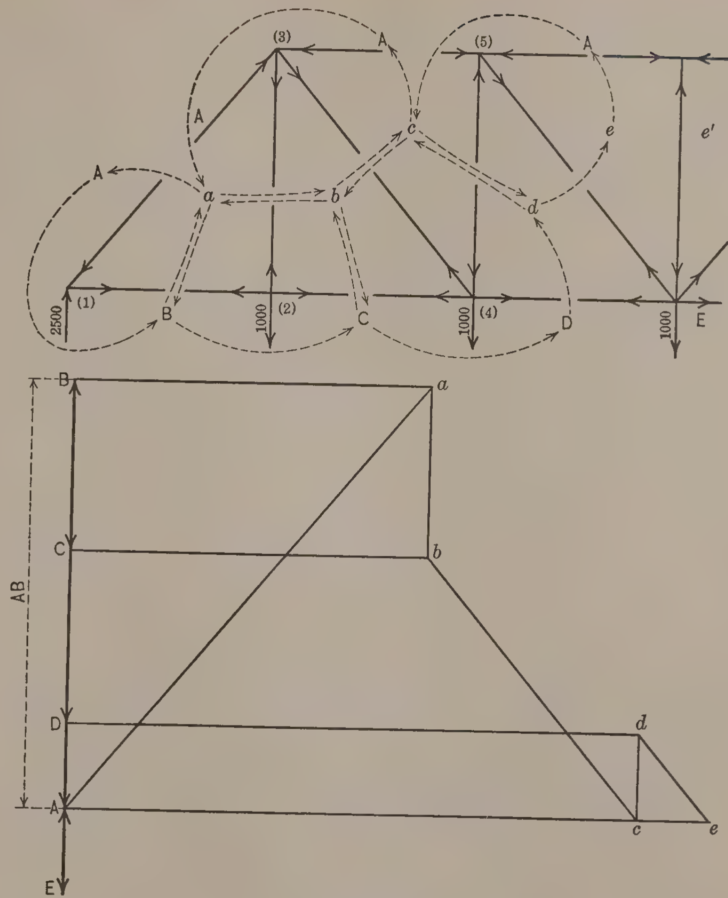


Fig. 237 (*b*).

for that joint. In both truss diagrams the letters around each joint have been connected by lines to indicate this conception. For example, at joint (1) in each of the two truss diagrams, the apices of the force polygon for that joint are indicated as A, B, and a; at joint (2) the apices are indicated as a, B, C, and b; at joint (3) the apices are indicated as a, b, c, and A; and at joint (4) the apices are indicated as b, C, D, d, and c; at joint (5) the apices are indicated as c, d, e, and A. The letters thus grouped at any joint may be conceived as the apices of an *imaginary* polygon. By this simple device, one is able to determine at a glance what letters are used to designate the apices of the force polygon for any joint. Moreover, the apices of the force polygon are lettered in the same order as that in which the letters occur in the imaginary polygon around the joint. If, therefore, one uses the *truss* diagram in this way as a guide, it is easy to pick out the force polygon for any joint. It remains to determine in which direction, clockwise or counter-clockwise, one should follow around a polygon in order to determine whether the corresponding forces in the truss act toward or away from the joint for which the polygon was drawn.

1. *To determine whether to follow around a force polygon in a clockwise or counter-clockwise direction.* The force polygon for the external forces on a truss is drawn by laying off these forces in the order in which they occur as one follows around the outside of the truss in one of two directions, clockwise or counter-clockwise (232 : 5). This order will determine in which direction one must proceed from letter to letter in the imaginary polygon around any joint. If the external forces are laid off in a clockwise direction around the truss, as is usually the case in this book, one must follow in a clockwise direction around the imaginary polygon for each joint, as indicated in Fig. 237 (a) by the arrows on the lines which connect the letters in the truss diagram. For joint (1) the order is from B to A to a to B, and this is the order in which one should follow around the force polygon for joint (1) *regardless of whether in so doing one proceeds around the polygon in a clockwise or counter-clockwise direction.* For joint (2) the order (from the imaginary polygon in the truss diagram) is from C to B to a to b to C; for joint (3) the order is from a to A to c to b to a; for joint (4), from D to C to b to c to d to D; for joint (5) from c to A to e to d to c. (If this same method is applied to the truss diagram in Fig. 237 (b), it will be found that in every case the order is reversed. This is because the external forces in the force polygon for external forces in Fig. 237 (b) were laid off in a counter-clockwise order.)

2. By using the truss diagram as a guide in the manner just explained, one is able, not only to pick out the force polygon for any joint, but also to decide in which direction to follow around that force polygon in determining whether a stress is in tension or compression. It is to be noted that this direction may or may not be clockwise. For example, in Fig. 237 (a), the order in which one proceeds around the imaginary polygon at *each* joint is clockwise, but the order in which one follows around the force polygon for joint (1) is counter-clockwise, the order for the force polygon for joint (2) is clockwise, and the order for the force polygon for joint (3) is counter-clockwise. In other words, the direction, clockwise or counter-clockwise, in which one proceeds around the imaginary polygon at any joint determines the order in which one must proceed from letter to letter (apex to apex) in the force polygon for that joint, regardless of whether in so doing one follows around the force polygon in a clockwise or counter-clockwise direction.

3. It is not well to follow a method of procedure, such as that just outlined, without some means of checking it. The order in which one should proceed around the force polygon for any joint is best checked by observing the sense of some known force at that joint, preferably an *external* force, since the sense of such a force is definitely known. In the following illustrations of the use of this check, the two letters used in designating a force are given in the order that indicates the sense of that force. (For the distinction between AB and BA see 4 : 5.) The check will be applied to joints in Fig. 237 (a).

4. *Joint (1).* The external force at this joint is the reaction *upward.* In the force diagram this force, in order to be upward, must be BA, not AB, i.e., in following upward along the line that represents the reaction BA in the force diagram one must proceed from B to A and not from A to B. This checks the order already determined for the force polygon for joint (1).

5. *Joint (2).* The external force at joint (2) is a panel load downward. In the force diagram this load, in order to be downward, must be CB, not BC, hence one must proceed from C to B. This checks the order already determined for the force polygon for joint (2).

1. *Joint (3).* There is no external force at joint (3). It is necessary, therefore, to use as a check the sense of a stress previously determined for a member at this joint. At joint (1) the end post was found to act from A to a or downward in the force diagram, hence at joint (3) the end post must act in the opposite direction or upward, i.e., from a to A in the force diagram. (6 : 6.) This checks the order already determined for the force diagram for joint (3). (The sense of the other known stress at joint (3), namely, the sense of the stress ba in the hip vertical can be used as a check, instead of the sense of the stress in the end post.)

2. Note that the stress in the end post at joint (1) is Aa, whereas at joint (3) it is aA. This means that in following around the force polygon for the joint at one end of the end post, one proceeds in one direction along the line which represents the stress in the end post, whereas in following around the force polygon for the joint at the other end of the end post one proceeds in the opposite direction along the same line. This reversal of sense holds true for the stress in every truss member. For this reason it is best *not to put arrows on the lines of a force diagram* except on those that represent external forces.

3. *Note:* The method just explained for checking sense in the force polygon for any joint may be used, if preferred, in place of the first method based on the imaginary polygon around the joint. In that case, the first method may be applied as a check almost at a glance. Once the first method is understood, however, it can be applied more easily and quickly than the other method by use of the short cut explained in the next paragraph.

4. *A short-cut method of determining whether a stress is tension or compression.* The following short-cut method of determining the sense of any force in the force polygon for any joint is merely a modification of the method based on the imaginary polygon.

5. First: Note whether the external forces were laid off in the external force polygon clockwise or counter-clockwise. (232 : 5.)

6. Second: Conceive the letters in the truss diagram used to designate the forces at the joint to be the letters at the apices of an imaginary polygon around the joint. (236 : 12.)

7. Third: To determine the sense of the stress in any member, note first the two letters by which that member is designated in the truss diagram; note next which of these letters comes before the other in following around the imaginary polygon in the direction determined in the first step; proceed in the force diagram from this letter to the other along the line that represents the stress in the member. This is the direction in which the member acts at the joint under consideration.

8. *Note:* If, in drawing external force polygons, one habitually lays off the external forces in the same direction, as recommended in 232 : 5, one not only eliminates the first step, but also becomes accustomed to following around the imaginary polygons in the same direction, and is, therefore, less likely to make mistakes. This is one reason why, as a rule, external forces in this book are laid off in the order in which they occur as one proceeds clockwise around the truss.

9. *Illustration:* Let it be required to determine the nature of the stress in any member in the truss diagram shown in Fig. 237 (*a*), as, for example, the member bc.

10. First: Note that the external forces were laid off in clockwise order.

11. Second: Fix the attention on the imaginary polygon in the truss diagram around the joint at one end of the member bc (either end). If joint (3) is taken, the letters in the imaginary polygon are a, A, c, and b.

12. Third: Note first that the two letters used to designate the member are b and c; note next that in following around the imaginary polygon clockwise, c comes before b, hence proceed in the force diagram from c to b, i.e., *downward*. The force cb must therefore act downward *at joint (3)* or away from the joint (tension). (Show by the same method, applied at joint (4), that bc acts upward at that joint.)

13. In using this short-cut method, use also, as often as may be necessary, the check explained in 236 : 5.

14. **Graphic Method of Joints Summarized.** The graphic method may be summarized as follows:

15. *First:* Draw the truss diagram carefully to scale, draw the lines of action of all loads, at each line of action enter the magnitude of the corresponding load, and letter the diagram according to Bow's notation.

16. *Second:* Lay off the force polygon for external forces, including the reactions. If the lines of action or the magnitudes of the reactions are not evident from inspection, they must be determined. For this purpose use the best method, regardless of whether it is an algebraic or a graphic method. (231 : 5 to 232 : 5.)

17. *Third:* Using the external force polygon as a basis, draw the complete force diagram. Begin at a joint of the truss (usually a joint at a support) at which there are not more than two unknowns, and draw for each joint *two and only two* additional lines, parallel, respectively, to the two members at that joint in which the stresses are unknown. When

working at any joint, concentrate on the two members just mentioned and ignore all other members and all external forces. These two members are designated in the truss diagram by three letters. Two of these letters will already be on the force diagram, and it is from these two letters that the two additional lines are drawn in that diagram; the third letter will be placed at the intersection of these two additional lines. Follow for each joint the method of procedure outlined in 233:4 to 8. Check the force diagram from time to time as it is being drawn, either by stresses the magnitudes of which are evident from inspection of the truss diagrams, or by some other method (236 : 6). Put no arrows on the force diagram except on lines that represent external forces. (239 : 2.)

1. *Fourth:* Check the completed force diagram by calculating the stress in a member near the center of the truss and comparing this calculated value with that obtained by scaling the corresponding line in the force diagram. (236 : 5.)

2. *Fifth:* After the completed force diagram has been checked, and not before, scale the lines in that diagram in order to obtain the stresses in the corresponding members of the truss. Record results directly on the truss diagram or tabulate them elsewhere.

3. *Sixth:* Indicate in some way, either by " C " and " T " or by minus and plus signs, which stresses are compression and which are tension. This may be done in most cases from inspection. For example, in an ordinary truss, top-chord members are in compression, and bottom-chord members are in tension. When in doubt concerning any member, apply the short-cut method of 239 : 4.

4. *Suggestion:* In order to gain facility in the use of the graphic method of successive joints, it is well to draw free-hand the truss and force diagrams for different types of trusses before attempting to work to scale. Pay no attention at first to whether members are in tension or compression. When able to draw free-hand force diagrams quickly and easily, practice the short-cut method of determining which members are in tension and which in compression. Blackboard drills of this character will help students to master quickly the method of procedure just summarized, and until it is mastered, it is a waste of time to draw to scale. The student should realize at the start that the graphic method of successive joints is simple, does not differ for different types of trusses, is easily applied, and for certain types of trusses is used almost exclusively.

5. *Exercise:* Draw a series of force polygons for the first three joints of the triangular parallel-chord truss used as a basis of explanation in the preceding chapter. (p. 208.) Follow the general arrangement shown on page 235, and compare the polygon for each joint with the algebraic solution for the corresponding joint as given on page 209.

6. **Practical Suggestions for the Use of the Graphic Method:** The following practical suggestions have to do mainly with the mechanical work of drawing the diagrams:

7. *Instruments and materials:* For the degree of accuracy usually required, ordinary drawing instruments of good quality may be used. The principal requirement is that the edges of the drawing board, the T-square, and the triangles should be *straight.* Ruling edges that are slightly curved are a most common source of error, and all such edges should be tested for straightness. Angles of triangles, particularly right angles, should also be tested.

8. The pencil lead should be of such hardness that it can produce clean, firm hair lines, and it should be sharpened to a long, thin wedge at one end of the pencil for drawing lines and to a fine, conical point at the other end for marking points. Pencil leads that are too soft or are improperly sharpened are also a most common and a most inexcusable source of error.

9. A triangular decimal scale, such as that used in mapping, is the most useful form of scale.

10. Any good drawing paper with a hard, smooth surface that will stand erasing should be used.

11. Metal straight-edges, triangles, T-square, and a metal edge to the drawing board may be used when exceptional accuracy is required, or when a large amount of graphic work is to be done. Much of the work in the graphic method consists in drawing lines in the force diagram parallel to lines in the truss diagram, and any instrument that enables the draftsman to do this quickly and accurately is perhaps the most efficient instrument of all.

12. *Precautions to insure accuracy:* The precautions commonly taken in accurate drawing should be observed. The most important are clear, fine hair lines drawn exactly through the point or points through which they should pass, and clear-cut intersections. Intersections will be more definite if pencil lines are allowed to overrun instead of stopping exactly at the point of intersection. In marking off measurements or in marking a point for any other purpose, make the point as small as a prick point —

just visible. It is not unusual for a beginner to make points so large that the diameter of any one of them will be equivalent to 100 lbs. measured by the scale that he is using.

1. The truss diagram should be drawn accurately, because from it the directions of all lines in the force diagram are determined; slight errors in the inclinations of two intersecting sides of a force polygon may cause errors of considerable magnitude in the corresponding stresses. If an inclined member of a truss is exceptionally short, it may be well to plot the slope of that member to a larger scale than that used for the truss; this is particularly necessary if the short member is near the end of the truss (as it usually is in a roof truss), because then it enters early into the construction of the force diagram, and affects all lines subsequently drawn.

2. In most force diagrams there are several parallel lines. Such lines are frequently the longest lines in the diagram, and consequently errors in their inclinations are likely to be most serious. Partly to insure accuracy and partly to save time, the experienced draftsman will draw these parallel lines, one immediately after the other, with one setting of the instruments. For example, from inspection of the truss diagram in Fig. 237 (*a*), it is seen that there will be in the force diagram four parallel lines, corresponding respectively to the four members *Ba*, *Cb*, *Dd*, and *Ae*. These four lines can be drawn of indefinite lengths, one after the other, as soon as the force polygon for external forces has been completed.

3. If a truss is unsymmetrical or unsymmetrically loaded, it is well to begin at the left-hand end joint and draw about one-half of the force diagram, and then to begin at the right-hand end joint and draw the remainder of the force diagram. By thus working from each end of the truss toward the center, instead of beginning at one end and working from joint to joint throughout the length of the truss, cumulative errors are reduced.

4. *Choice of scale:* The truss diagram may be drawn to any convenient scale, but it should be of sufficient size, if possible, to give accurately the slopes of inclined members. The slope of an exceptionally short member may be drawn, if necessary, to larger scale. With this in mind, draw the truss diagram to such a scale that the shortest member is at least one or two inches long. This will enable slopes to be determined with a fair degree of accuracy, and afford space for recording along each member the numerical value of the stress in that member. The extreme dimensions of the truss and the limits of the drawing paper are other determining factors in the choice of scale.

5. Some decimal scale should be adopted for the force diagram. Decide upon the allowable error in determining the stresses, and use a scale such that the smallest division that can be readily read is well within the allowable error. Nothing is gained, and frequently accuracy is lost, by drawing a force diagram to an unnecessarily large scale. The force diagram for an ordinary truss can usually be drawn with sufficient accuracy to a scale such that the whole diagram will lie within a space eight or ten inches square, or even in a considerably smaller space.

6. *Location of diagrams on the paper:* The truss diagram is usually drawn at the top or at one end of the drawing paper. The first line drawn in the force diagram is the load line, and this should be in such a position that the remainder of the force diagram will not run off the paper. It is not always easy to estimate the space required for this diagram, and it is often advantageous to sketch rapidly, free-hand, some of the longer and consequently limiting lines of the diagram. Such a sketch will often show that the entire force diagram lies to the left or to the right of the load line, and this to a large extent will determine the position of the load line on the paper. From the free-hand sketch an estimate can also be made of the space required for the force diagram drawn to any given scale. If there are several force diagrams to be drawn on the same sheet, a preliminary estimate of the space required is still more necessary.

Checks: In addition to the methods of checking the force diagram given in 236 : 5, one should be on the alert for special checks. For example, if the diagram for an unsymmetrical truss or a truss unsymmetrically loaded is drawn partly from one end and partly from the other, it will be to a large extent self-checking. (Why?) None of these checks, however, will disclose errors in scaling, except in the case of a stress checked algebraically. A rough check on scaling is to observe if the results obtained are reasonable, i.e., in accord with known conditions. For example, do the stresses in a group of members, such as web members or chord members, increase or decrease in a reasonable progression from the end toward the center of the truss or *vice-versa?* Large errors may frequently be detected in this way by inspection of results as a whole.

1. **Advantages and Disadvantages of the Graphic Method of Successive Joints.** The graphic method of successive joints is particularly efficient for determining stresses in roof trusses and other trusses in which the chords are not parallel or in which there are a number of members inclined at different angles. It is a simple and quick method, to a considerable extent self-checking, and involves little or no calculation. To one accustomed to its use, it becomes almost automatic except for the mechanical work of drawing the lines. It will occasionally reveal some fact concerning the stresses in a truss which is not evident at the start and not as quickly discovered by an algebraic method. So important is the graphic method of successive joints that a separate course in Graphic Statics is frequently given, in which the greater part of the work is based on this method.

2. The graphic method of successive joints, like the algebraic method of successive joints, cannot be used to determine the stress in a single member without involving the stresses in other members. It cannot be used to advantage for all types of moving loads, though it will become evident later that even for live loads it is more efficient in some cases than either of the algebraic methods. The graphic method is less efficient than the method of coefficients when the chords of a truss are parallel and panel loads are equal; in fact, in such a case, the stresses can often be obtained algebraically in less time than it would take to get ready the drawing equipment and plot the truss diagram, to say nothing of the rest of the graphic solution.

3. There is, of course, a limit to the degree of precision with which stresses may be determined by the graphic method; but in any case in which this method is the most efficient, the results obtained are usually of sufficient precision for all practical purposes.

CHAPTER XVII

STRESSES IN ROOF TRUSSES

The top chords of a roof truss are usually inclined to conform to the slope of the roof. For determining the stresses in such a truss the graphic method of successive joints is particularly advantageous, and the larger part of this chapter is devoted to the application of that method to various types of roof trusses. Before beginning this chapter, the student should be thoroughly familiar with the general method of procedure for the graphic method summarized in 239 : 14 and with the practical suggestions for the use of that method given in 240 : 6.

1. **Two Types of Supports.** There are in general two types of supports for roof trusses, namely: (1) The single-point type in which the truss may be considered to be connected to the support at a *single* point or joint, as, for example, when a truss merely rests on or is fastened to the top of a wall or column; (2) the multiple-point type in which the truss is connected to the support at *two or more* points or joints, as, for example, when a truss is connected not only to the top of a column, but also, by means of a knee brace, to a second point farther down on the column. Unless otherwise stated, it will be assumed in this chapter that supports are of the first or simple type.

2. **End Conditions.** When supports are of the simple type, an end of a truss may merely rest upon a support and thus be free to move horizontally, as, for example, when the end of a truss rests on expansion rollers, or, on the other hand, an end may be rigidly connected to the support, in which case it is not free to move horizontally. These two types of connections or **end conditions** will be referred to henceforth as, respectively, **free end** and **fixed end.** (135 : 7 and 137 : 2.)

3. **External Forces.** The following facts concerning the external forces which act on roof trusses are of fundamental importance.

4. Loads may be *vertical,* such, for example, as dead loads and snow loads, or they may be *inclined,* such, for example, as wind loads.

5. Loads, both vertical and inclined, are assumed ordinarily to be uniformly distributed, and to be applied at the upper joints of the truss. (Pages 130 and 134.)

6. *Note:* The load at an end joint of a truss is usually half that at an intermediate joint. When loads are vertical this half load goes directly into the support and therefore does not affect the stress in any member of the truss. It may be omitted in determining stresses in the truss but not in determining stresses in the support. When loads are inclined, the half load at the end joint should be included as one of the external forces. (Why?)

7. A reaction due to vertical loads can have no H component, and hence it is immaterial whether the corresponding end of the truss is free to move horizontally on its support or whether it is fixed.

8. A reaction due to an inclined load may or may not have an H component, depending on the corresponding end condition. If that end is free, the reaction can have no H component, but if the end is fixed, the reaction will have an H component. (138 : 7 and 8.) In the latter case, the H component of the reaction may be equal to the whole or to only a part of the H component of the total inclined load.

9. For a given inclined load applied at a given point, the V component of a reaction is always the same regardless of whether the corresponding end of the truss is free or fixed.

Note: For a more complete treatment of external forces on roof trusses, see pages 130 and 133 to 135.

1. **Stresses.** The stress due to any vertical load is always the same for any member of a roof truss regardless of whether either end of the truss is free or fixed. (Why?) The stress due to wind load will depend first upon end conditions and second upon whether the wind is on one side or the other of the roof. For a given wind pressure and given end conditions there will be two wind-load stresses for each member, one when the wind is blowing in one direction, the other when the wind is blowing in the opposite direction.

2. The stresses due to vertical loads may first be determined, regardless of end conditions. The stresses due to wind loads may then be determined separately. Finally the stresses due to vertical loads may be combined with stresses due to wind pressure in order to obtain the *total* stresses.

3. *Combined stresses:* The vertical-load stress in any member, added to the greater of the two wind-loads stresses for that member, will give the total *maximum* stress, provided the vertical-load stress and the wind-load stress are both tension or both compression. If one is tension and the other is compression and the wind-load stress is the greater, there will be reversal of stress. It is sometimes desirable to obtain the *minimum* stress that can occur in a member, whether that stress be simply the dead-load stress or the stress obtained by combining the dead-load stress with one of the two wind-load stresses.

4. If the structure is in a region where the roof must sustain snow, the vertical load will include not only dead load but snow load. The question then arises: What are the probable maximum external forces due to both snow and wind? It is not probable, for example, that the maximum snow load and the maximum wind pressure will exist at the same time. Assumptions for the best combination of snow load and wind pressure vary for roofs of different pitch and for structures in different latitudes. (134: 8.) (See *Assignment (4)* on page 148.)

5. **To Determine Reactions.** *Reactions for vertical loads.* Most roof trusses are symmetrical, hence the vertical loading is symmetrical and the reactions at the two ends of a truss are each equal to half the total load. When a truss is unsymmetrical or the vertical loading is unsymmetrical, the problem of determining reactions is one in parallel forces, Case B′. (Pages 137 to 140.)

6. *Reactions for inclined loads when one end of the truss is free and the other fixed:* The unknowns are the magnitude of the reaction at the free end (its line of action is vertical) and the magnitude and direction of the reaction at the fixed end. (M, M and D, Case 3 (Concurrent Forces).) It is well to change to Case 4 by replacing the reaction at the fixed end by its H and V components. (Pages 143 to 148 and *Problem 5*, page 149.)

7. *Reactions for inclined loads when both ends of the truss are fixed:* The magnitude and direction of each reaction are unknown, hence the problem of determining reactions is statically indeterminate (four unknowns). One of two assumptions is usually made, namely, (1) that both reactions are parallel to the inclined loads and to each other, or (2) that the H components of both reactions are equal. Under the first assumption the unknowns are two magnitudes (Case B′, Parallel Forces); under the second assumption the magnitudes of the two components of each reaction are unknown, but the assumption that the H components are equal provides a fourth equation in addition to the three equations of equilibrium, and thus the four unknown magnitudes may be determined. Note that under the assumption that reactions are parallel, the H components are proportional to the corresponding V components.

8. Methods of determining reactions are given in more detail in Chapter XII (pages 137 to 148.) For vertical loads an algebraic method is recommended (138 : 14). For inclined loads the graphic method of 146 : 3 is recommended.

9. **To Determine Stresses.** *Number of diagrams required.* For determining all stresses in an ordinary roof truss the graphic method of successive joints is recommended. When stresses due to vertical loads and those due to wind loads are determined separately, there will be at least two force diagrams, one for vertical loads and one for wind loads. When the truss is symmetrical and symmetrically loaded with vertical loads, it is necessary to draw the force diagram for vertical loads for one-half of the truss only. Since wind loads are applied to one side of a truss only, a force diagram for wind loads is drawn for the whole truss. When the truss is fixed at both ends, it is necessary to draw only one complete force diagram for wind pressure, since the stress in any member of the left-hand half of the truss when the wind is blowing from the left will be equal to the stress in the corresponding member of the right-hand half of the

truss when the wind is blowing from the right. When one end of the truss is free and the other fixed, it is necessary to draw two complete force diagrams for wind loads. (Why?) The statement just made may be summarized as follows: For a roof truss with both ends fixed, there will be (1) a truss diagram, (2) a force diagram for one-half of the truss for vertical loads, and (3) a force diagram for the whole truss for wind loads. For a truss with one end free, there will be an additional force diagram, i.e., two force diagrams for the whole truss for wind loads instead of one.

1. *Method of Procedure.* First: Determine the stresses due to vertical loads. These vertical loads may be simply the dead loads or, if desired, they may include snow loads. Follow closely the method of procedure summarized in 239 : 14.

2. Second: Determine the stresses due to wind pressure. The truss diagram already drawn for dead loads may be utilized. The method of procedure is essentially the same as that for vertical loads. The truss is unsymmetrically loaded, which makes it a little more difficult to determine reactions, and also makes it necessary to draw a force diagram for the whole truss. The loads are inclined, which makes it necessary to take into account end conditions. These are minor differences which do not essentially change the general method of procedure.

3. *Note:* The student should follow not only the method of procedure summarized in 239 : 14, but also the practical suggestions given in 240 : 6.

4. *Note:* Instead of combining snow loads with dead loads, the snow-load stresses may be obtained directly from dead-load stresses by multiplying by the ratio of snow load to dead load. Instead of determining separately the stresses due to vertical loads and wind loads, it may be best in some cases to combine dead loads, snow loads, and wind loads, and draw the force diagram for this combined loading.

5. *Note:* When panel loads are equal, as they usually are, it is occasionally advantageous to use some convenient unit panel load, such, for example, as 1000 lbs., and then to multiply the stresses thus found by the ratio of actual panel load to unit panel load in order to obtain the true stresses. For the sake of simplicity, unit panel loads are used in most of the illustrative examples, but this does not mean that such loads should be used as a general rule.

6. *Note:* In many cases it is sufficiently accurate to determine the stresses due to wind loads directly from the dead-load stresses without drawing a separate diagram for wind loads, as will be explained later.

7. *Note:* For a given type of roof truss of a given number of panels, a general formula for the stress in any member may be derived in terms of the dimensions of the truss, as, for example, in terms of the ratio of the length of span to the height of truss (the reciprocal of the pitch). From these formulas, tables of coefficients may be prepared for different standard types of trusses and for trusses of the same types but with different numbers of panels. By means of these tables, stresses can then be found more quickly than by the graphic method. When, therefore, stresses are to be determined for a large number of trusses of the same type and the same character of loading, it is well to prepare such tables of coefficients. (See *Assignment* (*2*) at the end of this chapter.)

8. **GRAPHIC METHOD OF SUCCESSIVE JOINTS APPLIED TO A TYPICAL ROOF TRUSS.** Given: The roof truss shown in the truss diagram on page 246. Span = 48 ft.; rise = 12 ft.; vertical loads at the joints in the top chord as indicated. Required: The stresses in the truss due to the vertical loads.

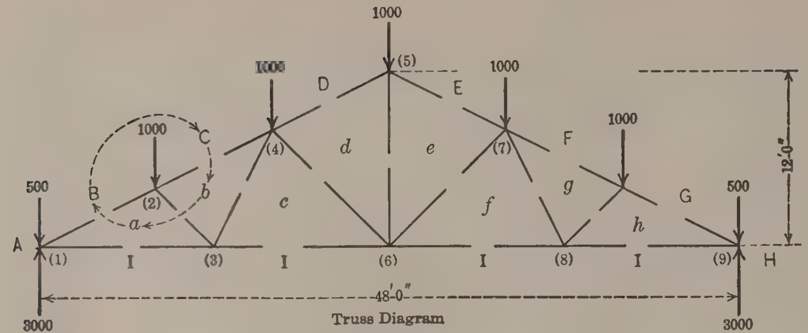
9. The method of procedure summarized in 239 : 14 will be followed. This illustrative problem should be studied and each step in the solution should be verified, not for the purpose of observing how the graphic method is applied to this particular type of truss, but in order to learn how it may be applied most efficiently to any type of roof truss.

10. *First:* Draw the truss diagram carefully to scale, draw the lines of action of all loads, at each line of action enter the magnitude of the corresponding load, and letter the diagram according to Bow's notation.

11. *Second:* Draw the force polygon for the external forces. (Diagram K.) Lay off first the load line from A to H, then lay off the reactions HI and IA, each equal to 2500 lbs., thus making the polygon close at A. (231 : 5.)

12. *Third:* Using the external force polygon as a basis, draw the complete force diagram. (Diagram 5.)

13. *Note:* The growth of this diagram, step by step, is shown by the force polygons for successive joints, exactly as in the illustrative problem on page 235. This device for illustrating the method of procedure is, perhaps, hardly necessary if the student has really mastered CHAPTER XVI. Remember that, in order to complete the force polygon for any joint, it is only necessary to draw *two* closing lines, the other sides of the polygon having been previously drawn; to draw these two lines quickly and correctly, *follow the method of procedure outlined in* 233 : 4. In each of the force diagrams on page 246, the force polygon for the corresponding joint is shown by full lines, the two closing sides are shown by heavy lines, and the other sides, previously drawn, are shown by fine lines. Other lines, previously drawn but not a part of the force polygon for the joint, are indicated by broken lines. It is understood, of course, that all of these diagrams are supposed to be drawn, superimposed on each other, to form the final force diagram.



Truss Diagram

Member	Stress
Ba	5,580# C
Cb	4,850# C
Dd	3,350# C
aI	5,000# T
cI	4,000# T
ab	950# C
bc	750# T
cd	1,410# C
de	2,000# T

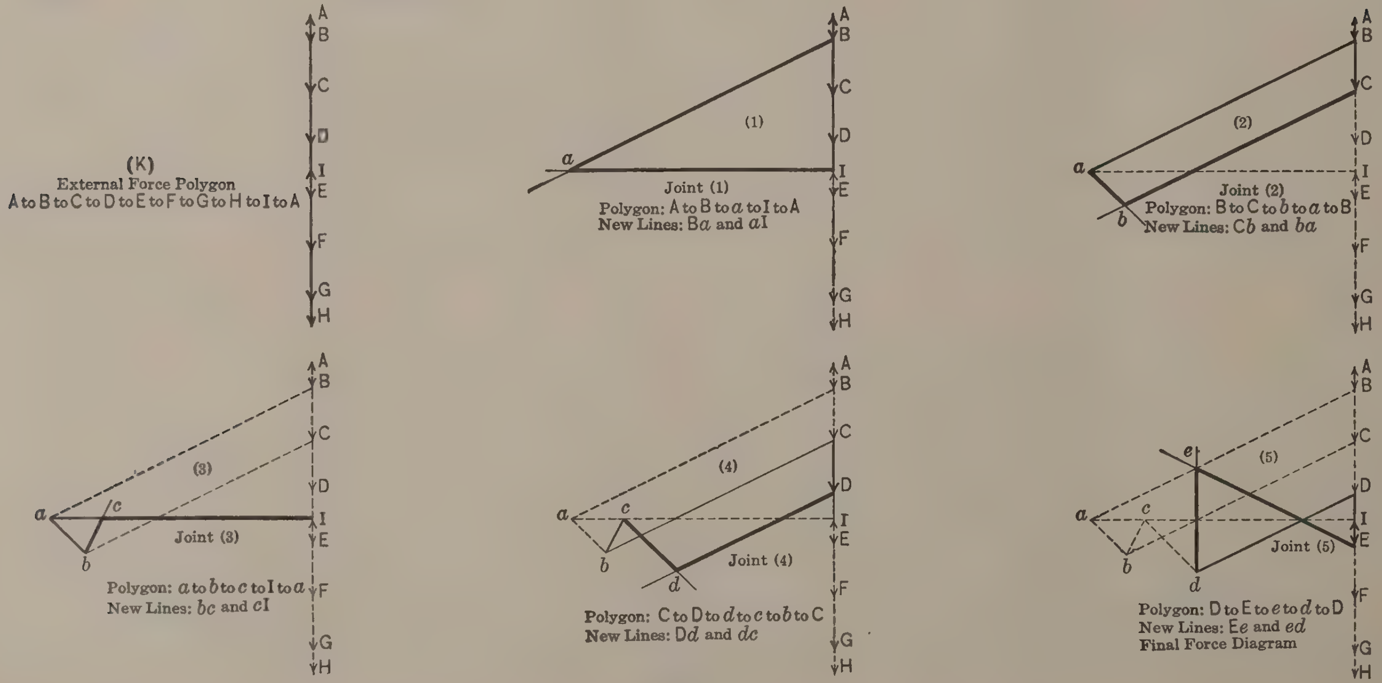


Fig. 246.

1. *Fourth:* Check the stress in some member near the center of the truss by the method of sections. Choose cI. (Why?) (191 : 9 and *Problem 9*, page 205.)

$$\Sigma M \text{ about } (4) = 3000 \times 16 - 500 \times 16 - 1000 \times 8 + cI \times 8 = 0$$
$$cI = -4000 \text{ lbs. tension (check).}$$

The line cI in the force diagram should equal to scale 4000 lbs. As an additional check (from symmetry) the lines Ee and Dd in the force diagram should not only be equal in length but should intersect in a point of the horizontal line through I. (Why?)

2. *Fifth:* When the completed force diagram has been checked, as just explained, *and not before*, scale the lines in that diagram in order to obtain the stresses in the corresponding members of the truss. These may be recorded directly on the truss diagram or tabulated as shown.

3. *Sixth:* Indicate, either by "T" and "C" or by plus and minus signs, which stresses are tension and which are compression. It is evident that the stresses in the top-chord members are compression, and that those in the bottom-chord members are tension. If in doubt concerning any web member, apply the short-cut method. (239 : 4.) For example, assume that one is not sure whether the stress in ab is tension or compression. Note first that in the external force polygon the forces were laid off in the order in which they occur as one proceeds *clockwise* around the outside of the truss. Concentrate on the imaginary polygon around the joint at either end of ab, as, for example, the imaginary polygon indicated by the broken curved lines around joint (2). To follow clockwise around this polygon one must proceed from b to a and not from a to b. To proceed from b to a in the final force diagram, one must follow the line ba upward, therefore at joint (2) ba acts upward or toward the joint, i.e., ba is compression.

4. The entire method of procedure as just outlined for this particular roof truss does not differ in any respect from that outlined for the parallel-chord trusses in the preceding chapter, nor will the method of procedure outlined for any of the other problems in roof trusses in the remainder of this chapter differ, except in minor details, from that given here.

5. *Note:* The suggestion in 240 : 4 is worth repeating here, namely, that the best way in which to gain facility in the use of the graphic method is to sketch free-hand the truss and force diagrams for different types of trusses, and, until one can do this quickly and easily, it is a waste of time to draw to scale. Free-hand work of this character can be used to advantage in connection with typical problems throughout the remainder of this chapter.

6. *Note:* The two loads, AB and GH, of 500 lbs. each, applied at the end joints, were included in the force polygon of external forces and consequently in all of the other figures which indicate the development of the force diagram. Inspection of these figures will show that these two forces do not affect the stresses in any way, and that, except for the purpose of completeness, they might better have been omitted entirely. (243 : 6.) Henceforth, when loads are *vertical*, those at the two supports will not be included in the force diagram unless there is good reason for doing so. In designing the supports, however, they should be considered.

7. *Note:* Since the truss is symmetrical and symmetrically loaded, it is necessary to lay off in the external-force polygon only forces AB, BC, CD, and DE, provided care is taken to represent the reaction IA correctly. It is evident that no use was made of forces EF, FG, GH, and HI. Similarly, it is necessary to draw only one-half, or a little more, of the truss diagram.

8. **Wind-Load Stresses in a Typical Roof Truss. (Both Ends Fixed.)** Given: The truss shown in the truss diagram on page 248. (This is the same truss as that for which vertical-load stresses were determined in the preceding illustrative problem.) Required: To find the stresses due to the wind loads indicated, when the truss is fixed at both ends.

9. The method of procedure is very much the same as that outlined for vertical loads. The chief differences may be summarized as follows:

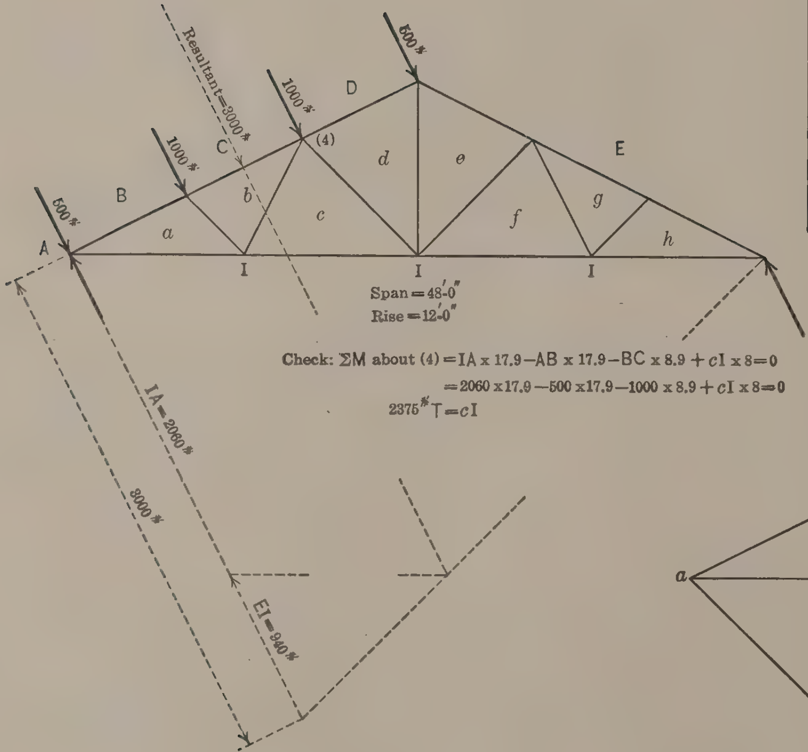
10. In lettering the truss diagram, only one letter (E) is placed on the outside of the right-hand half of the truss.

11. *Note:* When it is desired to designate each number by the same two letters that were used in determining stresses due to vertical loads, a single letter is not used; instead, all letters are allowed to remain on the truss diagram but the letters replaced by E in the truss diagram of Fig. 248 will all fall at the same point (E) of the force diagram.

12. The load line is inclined instead of vertical, and should include the load AB at the support. The reactions are indeterminate without some assumption. The assumption was made that both reactions are parallel to the wind loads. Their magnitudes were then determined by the graphic method explained on page 146 and indicated on the figure of the truss diagram.

13. The general method of drawing the force polygons for the joints is exactly the same as that outlined for vertical loads (two new lines for each joint), but the complete force diagram will not be symmetrical.

Member	Stresses			
	Dead	Wind R	Wind L	Maximum
Ba	5,580 C	1,870 C	3,120 C	8,700 C
Cb	4,850 C	1,870 C	2,790 C	7,640 C
Dd	3,350 C	1,870 C	1,620 C	5,220 C
aI	5,000 T	1,250 T	3,490 T	8,490 T
cI	4,000 T	1,250 T	2,370 T	• 6,370 T
ab	950 C	0	1,050 C	2,000 C
bc	750 T	0	830 T	1,580 T
cd	1,410 C	0	1,580 C	2,990 C
de	2,000 T	1,120 T	1,120 T	3,120 T



Check: ΣM about (4) = IA x 17.9 − AB x 17.9 − BC x 8.9 + cI x 8 = 0
= 2060 x 17.9 − 500 x 17.9 − 1000 x 8.9 + cI x 8 = 0
2375# T = cI

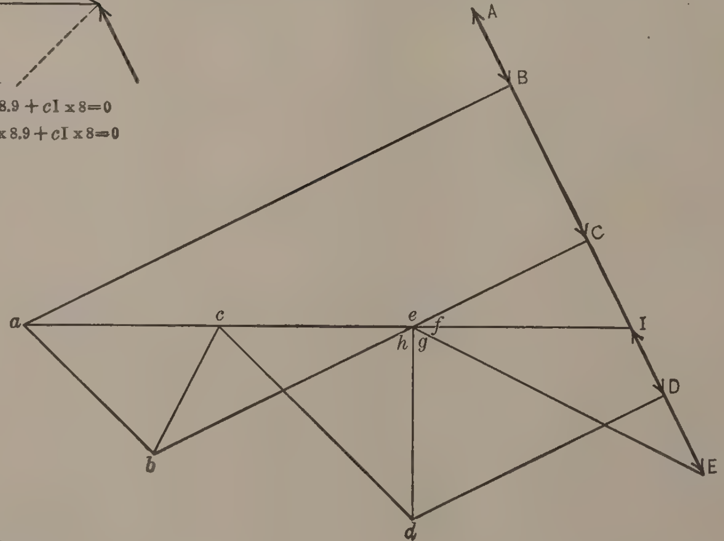


Fig. 248.

1. The general method of checking the stress in cI by the method of sections is also exactly the same, but this check cannot be supplemented by the check for symmetry. There is, however, a similar supplementary check. The stresses in the members ef, fg, and gh are all zero, since these members are inside of the triangle which forms the right-hand half of the truss and no external forces are applied to this triangle except at its apices (72 : 1). The points e, f, g, and h should, therefore, all coincide in the force diagram, and they will coincide if the line through E in the force diagram, drawn parallel to Ee in the truss diagram, intersects de in a point e on the horizontal line through I. (Check.) (The members ef, fg, and gh could have been temporarily removed from the truss diagram, and a single letter substituted for the four letters e, f, g, and h.)

2. The magnitudes of the wind-load stresses, as given in the table, were scaled from the force diagram in the usual manner, but the stress in a member in one half of the truss is not necessarily the same as the stress in the corresponding member in the other half. Note that the stress in any member in one half of the truss for the wind on the left-hand side is equal to the stress in the corresponding member in the other half of the truss when the wind is on the right-hand side. (Why?)

3. The nature of the stress in any member may be determined in the usual manner. Since the stresses in ef, fg, and gh are zero, the only members concerning which there can be any doubt are the web members in the left-hand half of the truss. If uncertain as to the nature of the stress in any of these members, apply the short-cut method. (239 : 4.)

4. The stresses due to vertical loads, determined in the illustrative problem on page 246, are combined with the wind-load stresses in the tabulation.

5. *Note:* Instead of the assumption that the reactions are parallel, the assumption may be made that the H components of the reactions are equal. In this case, the external forces are not all parallel, therefore the force polygon for those forces will not lie in a straight line. (232 : 4.) With this exception, the graphic solution is exactly the same as that just given. It is suggested that the student draw this force diagram in order that he may compare the stresses obtained from one assumption with those obtained from the other.

6. **Wind-Load Stresses in a Typical Roof Truss. (One End Free.)** (See diagrams on page 250.) This problem is the same as the preceding one except that, instead of both ends of the truss being fixed, one is fixed and the other is free. (The left-hand end is free.) The method of procedure is so nearly like that of the preceding problem that only the essential differences between the two methods will be given.

7. The line of action of the reaction at one end is known (vertical at the free end). (138 : 5.) Both reactions are statically determinate and therefore no assumption is necessary. If the limits of the paper permit, reactions may be checked by the principle that the lines of action of the two reactions must intersect the line of action of the resultant wind load in the same point. (71 : 3.)

8. The force polygon for external forces will not lie in a straight line, since two of the external forces (reactions) are not parallel to the others (loads).

9. Since the conditions at the two supports are different — one free, the other fixed — two force diagrams are necessary. For the sake of clearness, a truss diagram is shown above each force diagram, but only one truss diagram is necessary. In fact, the truss diagram drawn for vertical loads may be used for wind loads as well, provided precautions are taken to avoid confusion in the loading and lettering.

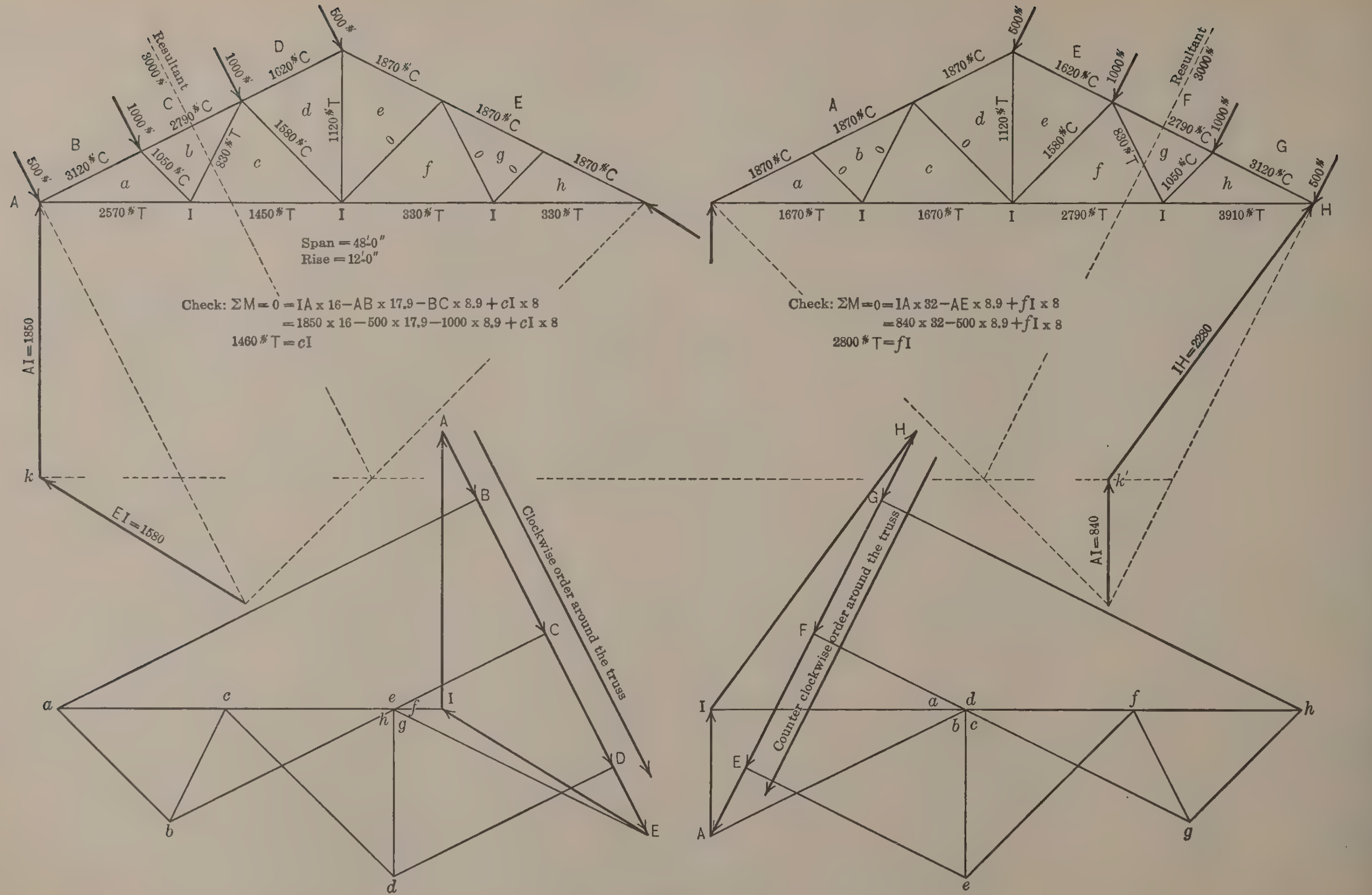
10. The stress in any member in one half of the truss when the wind blows from the left is not necessarily the same as the stress in the corresponding member in the other half when the wind blows from the right.

Note: If the reactions are determined algebraically, the right-hand reaction may be replaced by its H and V components; the problem is then one under Case 4 (see *Problem 5*, page 149). The graphic method of determining reactions indicated in the diagrams is that of 146 : 5. Note that the point k', which determines the reactions AI and IH when the wind is on the right, may be found either by the regular method of construction, or by merely drawing a horizontal line through k until it intersects the vertical in which lies AI. (Why?)

11. *Note:* Instead of changing the wind from one side to the other, one can assume the wind to remain on the same side and interchange the fixed and free ends. It is more logical, however, and less confusing to the beginner, to change the wind, as was done in this illustrative problem.

12. *Note:* When the wind is changed from one side to the other, the V components of the two reactions are merely interchanged. (Why?)

13. *Note:* To avoid any possible confusion, different letters were chosen for the right-hand half of the truss diagram from those used for the left-hand half. It is helpful in tabulating stresses, however, if a member in one half is designated by the same two letters as those used for the corresponding member in the other half.



Resultant 3000#
500#
1000#
1000#
500#
B
C
D
E
A
3120#C
2790#C
1620#C
1870#C
1870#C
1870#C
1050#C
830#T
1580#C
1120#T
2570#T
1450#T
330#T
330#T
I
Span = 48'-0"
Rise = 12'-0"
Check: ΣM = 0 = IA x 16 − AB x 17.9 − BC x 8.9 + cI x 8
= 1850 x 16 − 500 x 17.9 − 1000 x 8.9 + cI x 8
1460#T = cI
AI = 1850
EI = 1580
k
Clockwise order around the truss
F
G
H
1670#T
1670#T
2790#T
3910#T
Check: ΣM = 0 = IA x 32 − AE x 8.9 + fI x 8
= 840 x 32 − 500 x 8.9 + fI x 8
2800#T = fI
IH = 2280
AI = 840
k'
Counter clockwise order around the truss

Fig. 250.

1. *Exercise.* The stresses obtained in this problem are entered directly on the truss diagrams. Assuming that the dead-load stresses are those obtained on page 246, tabulate all dead- and wind-load stresses, including the maximum combined stresses. Use a form similar to that on page 248. Compare the results obtained with the statements in the next article.

2. *Question:* The force diagrams were checked by the method of sections. What additional check is there for each diagram?

3. **General Principles Pertaining to Wind-Load Stresses.** A comparison of the results obtained on the illustrative problem in which both ends of the truss were fixed with the results in the problem in which one end of the truss was free will show that the following statements are true:

4. For the wind blowing in a given direction, right or left, the V component of *either* reaction is constant regardless of whether the corresponding end of the truss is fixed or free.

5. There is no stress in any web member that is *within* the triangle which forms the leeward portion of the truss.

6. For the wind blowing in a given direction, right or left, the stress in any member, except that in a member in the *lower* chord, is the same regardless of whether both ends are fixed or one end is fixed and the other free.

7. The stresses in the *lower-chord* members of the windward half of a truss are greater than the stresses in the corresponding members of the leeward half and are greatest when the end of the windward half is fixed and the end of the leeward half is free.

8. *Exercise.* Prove that the statements given above are general principles which hold true for any standard roof truss in which the bottom chord is straight and in a horizontal line between the two points of support.

9. *Note:* The stresses in the *upper-chord* members of the windward half of a truss may or may not be greater than the stresses in the corresponding members of the leeward half, depending upon the slope of the roof and the type of truss; hence no general principle, such as that stated for lower-chord members, can be given for top-chord members.

10. **Special Types of Roof Trusses.** Different types of roof trusses do not require different methods of determining stresses, and if one thoroughly understands the application of the general method to one type of truss one should be able to apply it to any other type, for the method is always essentially the same. There are, however, several types of roof trusses in common use each of which involves some special minor difficulty in the application of the general method. The difficulty usually arises either (1) because at some joint of the truss there are three unknown magnitudes and it seems impossible to proceed or (2) because the reactions at the ends of the truss seem to be or actually are indeterminate, as in the case of a truss which is fixed at both ends. In the treatment of these special cases, only those steps in the solution will be explained that differ from or are in addition to the general method of procedure. It is hoped in this way to avoid creating the too common impression that each truss is a separate problem in itself, different from other problems. The bulk of the work of determining stresses is the same for all common types of roof trusses.

11. **The Fink Truss.** The Fink truss shown in Fig. 252 is an example of a truss in which the difficulty in determining stresses is due to the fact that at one of the joints there are three unknown magnitudes.

12. The polygon for external forces is laid off in the usual manner, and no difficulty is encountered in applying the general method to the first three joints. This much of the work is shown by full lines in the force diagram. If joint (4) is attacked next, there will be three unknown magnitudes, *cd*, *dg*, and *gI*; and if joint (5) is attacked there will also be three unknown magnitudes, *Ce*, *ed*, and *dc*. The problem apparently is indeterminate because there appear to be too many members in the truss. If, however, the criterion given on page 91 is applied, it will be found that there is no redundant member.

13. If one of the three unknown magnitudes at either joint (4) or joint (5) can be determined by some other method, it will be possible to proceed with the application of the general method. The stress which will thus be determined is that in *gI*. It can be found *algebraically* by the method of sections, and its value laid off on a horizontal line through *I* in the force diagram. The point *g* thus having been located, the force polygon for joint (4) can be drawn. The *graphic* method of determining the stress in *gI* is based on the following principle: The member *de* is the diagonal of a quadrilateral whose sides are *Ce*, *ef*, *dg*, and *cd* — shown by heavy lines in the truss diagram. If *de* is taken out and the other diagonal (shown by a broken line) is put in as a temporary member, the only members in the

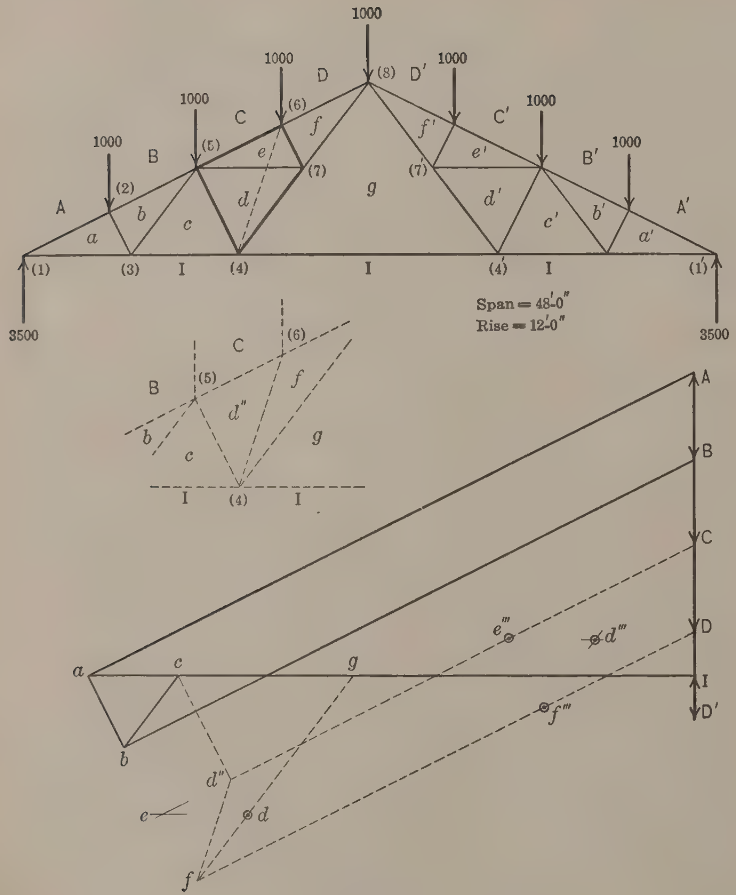


Fig. 252.

whole truss in which the stresses will be changed will be the four members that form the sides of the quadrilateral. (Why?) Let such a substitution be made. There will then be only three members at joint (7), and, since two are in the same line, there can be no stress in *ef*, which is the third, hence *ef* may be taken out. Joint (7) thus disappears and the portion of the truss under consideration is changed to the form shown by broken lines below the truss diagram. The letters *d* and *e* are replaced by *d″*, and this is the only change in notation. The stress in the changed portion of the truss may be found graphically by attacking joint (5), then joint (6), and finally joint (4). The broken lines in the force diagram indicate this work. At joint (4) the stress in *gI* is found, and since this member is not a part of the quadrilateral, the stress thus found is the same as that which exists when the diagonal *de* is in the truss. The point *g* in the force diagram is thus determined. The temporary diagonal *d″f* may now be taken out and the members *de* and *ef* put back. It will be necessary now to redraw the force diagram for joints (4), (5), and (6), attacking them in that order. (The points *d* and *e* on the force diagram were thus determined.) When joint (6) is solved, the point *f* should coincide with *f* as previously found. (Why?) Another check is afforded by the fact that the stresses in *bc* and *de* are equal. (Prove that they are equal.) Still another check is the fact that *a*, *b*, *e*, and *f* in the force diagram should all be in a straight line, provided the panel lengths and loads are equal. The method just explained has been called the **method of the reversal of the diagonal.**

1. *An alternative method* of getting around the difficulty encountered at joint (4) of the Fink truss is outlined in the following construction in Fig. 252: Through *C* and *D* in the force diagrams, draw lines of indefinite length, parallel to members *Ce* and *Df*. On the line through *C* assume any point as, for example, *e‴*. Through *e‴* draw a line parallel to the member *ef* until it intersects the line through *D* at the point *f‴*. Through *e‴* draw a line parallel to the member *ed*, and through *f‴* draw a line parallel to the member *fg*; these two lines intersect at the point *d‴*. Through *d‴* draw a line parallel to *Ce‴* (or *Df‴*), and through *c* draw a line parallel to the member *cd*; these two lines intersect at the point *d*. The stress in the member *cd* is now known, and the force polygon for joint (4) may be completed in the usual manner by drawing two closing lines parallel, respectively, to the members *dg* and *gI*. This method, as just

outlined, is fully as good as the method based on the reversal of the diagonal.

1. *Special characteristics of the stresses in the Fink truss in Fig.* 252 may be summarized as follows:

(*a*) The stress in the members *ab* and *ef* are equal to each other, and each is equal to the component, normal to the roof, of the panel load. (71: 7.)

(*b*) The stresses in *bc* and *de* are equal to each other, and the component of each, normal to the roof, is equal to one-half the stress in *ab* or *ef*. (Why?)

(*c*) The stress in *cd* is equal to twice the stress in *ab* or *ef*. (Why?)

Similar statements can be made concerning members in the right-hand half of the truss. These special characteristics afford additional checks in drawing the force diagram.

2. *Exercise.* Prove that the construction given for the alternative method is correct. (Note that 71 : 7 applies to joints (3), (4), and (7) as well as to joints (2) and (6).)

3. *Short-cut method for a Fink truss with equal panels and equal panel loads.* (Fig. 252.) The method of the reversal of the diagonal and the alternative method are general methods that can be used regardless of whether the panels and panel loads are equal or unequal. When they are equal, the stress in *cd* is equal to twice the stress in *ab*, hence in attacking joint 4 the line *cd* may be drawn in the force diagram equal in length to the line *ab* which has already been drawn. This makes it unnecessary to use either the method of reversal of the diagonal or the alternative method, since the regular method of procedure may be followed in drawing the remainder of the force diagram.

4. *Question:* In calculating stresses in a Fink truss by the algebraic method of sections, a difficulty is encountered similar to that met in the graphic method. For example, let it be required to calculate the stress in the member *dg*. It is impossible to take a section through this member without cutting at least three other members, making four unknowns in all. How best could this difficulty be overcome?

5. *Note:* An additional complication sometimes occurs in connection with the Fink truss, namely, that caused by a load at joint (7) or loads at both joints (7) and (7′). In the case of a special load like this, it is better not to include the load in the regular force diagram for the truss, but, instead, to draw a separate force diagram in which the special load or loads and the corresponding reactions are the only external forces. The stresses thus obtained, due to the special loads, may then be combined with those caused by the regular loads. It is, however, an interesting and comparatively easy problem to work out a graphic method for drawing a force diagram for the regular and special loads combined. Several such methods are to be found explained in engineering periodicals and elsewhere.

6. *The stresses in a Fink truss due to inclined wind loads* may be determined by the usual method of procedure supplemented by the method of the reversal of the diagonal (251 : 13) or by the alternative method (252 : 1). Since, however, there will be no stress in any web member *within* the main triangle that forms the leeward portion of the truss (72 : 1), the following method of procedure may be used to advantage. *First step:* Determine the reactions and draw the force polygon for external forces. *Second step:* Consider the web members in the leeward side of the truss (except the one that forms the main triangle with the top and bottom chords) to be removed. Draw the portion of the force diagram that corresponds to the leeward side of the truss, including the center member of the bottom chord. *Third step:* Draw the portion of the force diagram that corresponds to the windward side of the truss. Since the stress in the center member of the bottom chord was determined in the second step, the third step will involve no special method, such as the reversal of the diagonal or the alternative method.

7. *Illustration:* Assume the wind to blow on the left-hand side of the truss in Fig. 252. *First step:* Determine the reactions and draw the force polygon for the external forces. *Second step:* Consider the members *f′e′*, *e′d′*, *d′c′*, *c′b′*, and *b′a′* temporarily removed. Beginning at joint (1′), draw the force diagram for the triangle (1′) (8) (4′), and thus obtain not only the stresses in the members that form the sides of that triangle, but also the stress in *gI* (from joint (4′)). *Third step:* Begin at joint (1) and draw the remainder of the force diagram, thus obtaining the stresses in the left-hand portion of the truss. When joint (4) is reached, the usual difficulty will not be encountered, since the stress in *gI* was determined in the second step.

8. *When panels and panel loads due to wind pressure are equal* and the wind blows on the left-hand side, the stresses in *ab* and *ef* are each equal to a panel load (normal to the roof) and the stress in *cd* is equal to two panel loads. This suggests still another method of procedure, similar to that of 253 : 3.

9. *A Fink truss with eight panels on each side* is shown in Fig. 254 (*a*). When the fourth joint is reached, the method of the reversal of the diagonal (251: 13) or the alternative method (252: 1) may be used. When joints

b and g are reached, the general method of procedure, corresponding to the method of the reversal of the diagonals, is as follows: Replace all web members in the triangle bgf by the three temporary members indicated by the three broken lines. In substituting these members, care was taken to fulfill the conditions that the truss should be composed of triangles and that there should be a web member at each joint where there is an external load. The usual method of procedure may now be followed until the stress in the member gg' has been determined. This stress is not affected by the substitution of the temporary members. (Why?) The three temporary members may now be replaced by the original web members,

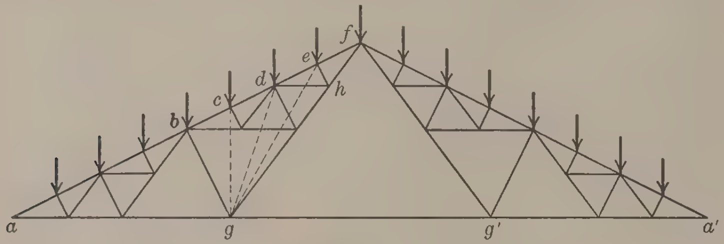


Fig. 254 (*a*).

and the force diagram may be completed by the regular method of procedure. (Which of the stresses that were determined when the three temporary members were acting are true stresses? What checks, corresponding to those at the end of 251 : 13, may be applied to the force diagram?)

1. *Questions: What special characteristics of the stresses in the Fink truss in Fig. 254* (*a*) *are similar to those in the Fink truss in Fig. 252? Why is the stress in* bg *twice that in the strut between the fourth and fifth joint, and four times that in the strut between the second and third joint?*

2. *Short-cut method for the Fink truss in Fig.* 254 (*a*). The general method just explained can be used regardless of whether the panels and panel loads are equal or unequal. When they are equal, the force diagram may be drawn without using any special method, such as the substitution of temporary members, by a method similar to that in 253 : 3. Proceed as follows: When the fourth and fifth joints are reached, draw the line in the force diagram that represents the stress in the strut between those joints twice as long as the line already drawn that represents the stress in the short strut between the second and third joints. When joints b and g are reached, draw the line in the force diagram that represents the stress in bg twice as long as that which represents the stress in the strut between the fourth and fifth joints, or four times as long as that which represents the stress in the strut between the second and third joints. The regular method of procedure may be followed in drawing the remainder of the force diagram.

3. **Cantilever Trusses.** The stresses in all or nearly all of the members of a cantilever truss can often be determined without first finding the reactions, by beginning at the unsupported end. This would be true, for example, of the truss shown in Fig. 254 (*b*), provided the tie ce is omitted and the truss is supported solely by the tie be and a hinged joint at a. Lay off the seven vertical loads. This load line forms only a portion of the force polygon for the external forces since the reactions of the tie at b and of the wall at a are not included. Beginning at the unsupported end, joint d, the force diagram for the truss may be drawn in the usual manner. When joint b is reached the force exerted on the truss by the tie be will be one of the two unknown forces at that joint, since the force in the members bb' is equal to the load A. When joint a is reached, the only unknowns will be the magnitude and direction of the reaction R at that joint. This reaction will close the force polygon for external forces since the other reaction of the tie be on the truss at b has already been drawn in the force diagram. The force diagram for the entire truss may now be checked by calculating the reactions and comparing the values thus obtained with those scaled on the force diagram. When there is an additional tie ce, it is necessary to determine the reaction before drawing the force diagram. (See *Problem 9*, page 152.)

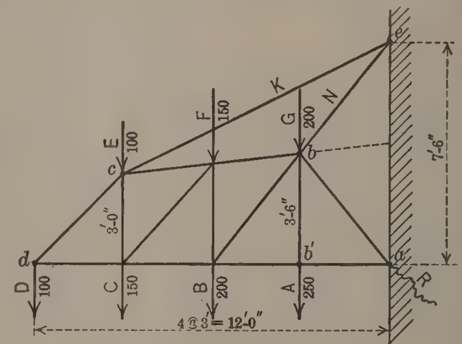


Fig. 254 (*b*).

1. The tower bent shown in Fig. 255 (*a*) is a cantilever truss with the upper end free. After laying off the known horizontal forces in a horizontal load line, one can begin at the joint at which the force D is applied, and draw the force diagram for the truss as in the preceding illustration. (It is assumed that only one set of diagonals, those in tension, are in action.) When joint a is reached, there will be three unknowns, namely, the stress in ab and the magnitude and direction of the reaction R_L. The problem at that joint is therefore indeterminate. If one attempts to calculate the reactions, that problem is also indeterminate unless some element of one reaction is known or assumed. (Why?) If, for example, the reaction at a is known or assumed to be vertical, both reactions can be calculated. (See *Problem 10*, page 152.) Moreover, the problem at joint a then becomes determinate since the only unknowns at that joint are the stress in ab and the magnitude of the reaction R_L. One can therefore proceed to complete the force diagram for the truss, including the determination of the magnitude and direction of the other reaction R_R. (If the assumption is made that the H components of the reactions R_L and R_R are equal, could one proceed at joint a to complete the diagram without first determining the reactions?)

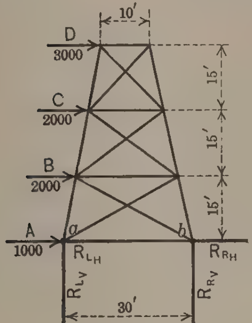


Fig. 255 (*a*).

2. A double cantilever truss, such as that shown in Fig. 152 (*a*), presents no special difficulties. In the force polygon for external forces, however, the two lines that represent the reactions will not be adjacent sides of the polygon.

3. **Roof Truss with Monitor.** A roof truss with a monitor truss is shown in Fig. 255 (*b*). In drawing the diagram for stresses no difficulty is encountered until the joint is reached at which the vertical member Fg of the monitor truss is connected to the main truss. At this joint there will be three unknown magnitudes, namely Fg, gf, and fe. If the stresses in the members of the monitor truss can be determined it will be possible to proceed with the diagram for stresses in the main truss. Beginning at the peak of the monitor truss, the load HH′ is known and the other two forces Hi and $H'i$ can be determined graphically. At the joint at which GH and GF act there are apparently three unknown magnitudes, namely, ih, hg, and gF. The assumption may be made, however, that hg and hg' are tension members, and that only one acts at a time. For the wind as shown, hg' acts, and the stress in hg is zero. This leaves only two unknowns, Fg and ih, at the joint. The stresses in these members and in hg' and $H'g'$ should be determined graphically before proceeding with the work of determining stresses in the main truss.

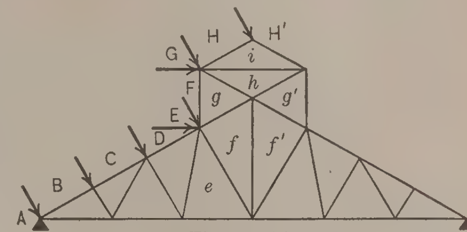


Fig. 255 (*b*).

4. In Fig. 255 (*c*) is shown a monitor with two panels on each side. The wind loads on the monitor should be transmitted to the main truss through members which will carry them most directly. The diagonals fc' and $e'b'$ on the leeward side could carry them only very indirectly, and for this reason the stresses in these diagonals are assumed to be zero. This is equivalent to assuming that there will be no stresses in the right-hand half of the monitor. (Why?) The diagonal cf must then be designed for compression. The problem is still statically indeterminate, and hence some additional assumption must be made, such, for example, as the removal of another member. It is not best to take out the vertical ce since this member carries a gravity load. It is better to consider the diagonal be to be removed, as if the members ab and bc acted like the column and the rafter of a lean-to. Usually the diagonal be is actually in the monitor for the sake of rigidity, but the changes in the stresses due to its action are negligible in designing the truss. With these assumptions, it is now possible to construct the force diagram for the main truss by the ordinary method of procedure. It is best to draw the diagram as far as possible, first for

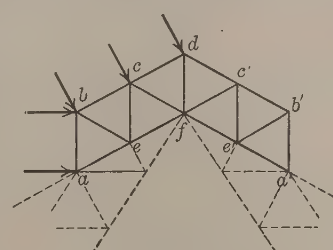


Fig. 255 (*c*).

one side of the truss and then for the other; the connecting lines necessary to complete the diagram may then be obtained by attacking joints at b and d.

1. **Saw-Tooth Roof Truss.** Assuming that the reactions for the saw-tooth roof truss shown in Fig. 256 (a) have been calculated (*Problem 12*, page 153), the force diagram may be constructed as follows: Remove, temporarily, all web members in the leeward half of the main truss. (Why?) (72 : 1.) Replace the three members that are within the saw-tooth portion of the truss by one horizontal member extending from the joint at which the load of 500 lbs. is applied to the peak of the main truss. Draw the force diagram for the external forces. Attacking the top joint first, draw that part of the force diagram that corresponds to the saw-tooth portion of the truss. Beginning at joint d, draw that part of the force diagram that corresponds to the leeward half of the main truss until the stress in the vertical member at the center of the truss has been determined. This stress is not affected by the substitution of the horizontal member for the three members within the saw-tooth portion of the truss. (Why?) Restore the three members. The portions of the force diagram corresponding to the left-hand half of the main truss and to the saw-tooth with the three original members restored may now be drawn by following the usual method of procedure.

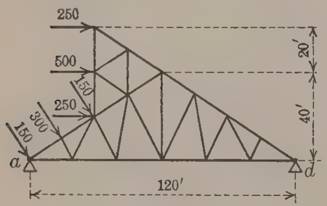


Fig. 256 (a).

2. **The Three-Hinged Arch.** Once the reactions have been determined, the stresses in a three-hinged arch may be found by the graphic method just as for any other truss. The reactions may be determined graphically or algebraically as explained in *Problem 13*, page 154.

3. **Special Types of Trusses Used for Architectural Effect.** In order to secure certain architectural effects, roof trusses are often used which are not as rigid as the roof trusses thus far considered. Two of the most common types of such trusses are the **scissors truss** and the **hammer-beam truss.** Each may be used separately or the two may be combined.

4. *The scissors truss* in its simplest form is shown in Fig. 256 (b). For vertical loads, the reactions are vertical, provided the truss is rigid. The form of truss is such, however, that the elastic deformation of the truss causes a thrust at the supports. The stresses in the truss due to both dead and wind loads may be determined graphically by the usual method of procedure, just as if the truss were absolutely rigid. From the maximum combined stresses the thrust at the supports may then be calculated. (See *Assignment* (*5*) at the end of this chapter.)

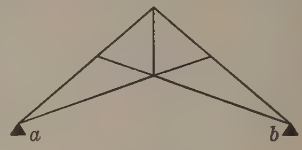


Fig. 256 (b).

5. *Note:* The load on the truss tends to change its shape, and, in particular, to lengthen the span by causing joints a and b to move apart horizontally. This horizontal movement must be provided for, prevented, or reduced to a negligible quantity. The movement can be provided for by making the span a little short, putting a slight camber in the rafters, and allowing one end of the truss to be a free end, so that when the truss is fully loaded it will settle down to its true length. The horizontal movement may be prevented by designing the supports to resist the corresponding horizontal thrust. The movement can be reduced by using excess material in the top and bottom chord members, particularly in the latter. If the conditions are such that the horizontal resistance of the supports is considerable, the stresses must be revised to include the effect of this resistance, since they were originally calculated for reactions due simply to the dead and wind loads.

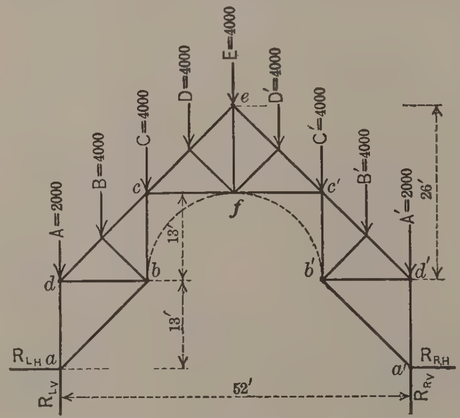


Fig. 256 (c).

6. *The hammer-beam truss* in a simple form is shown in Fig. 256 (c). In order that the stresses may be statically determinate, the joints a, c, c', and a' are assumed to be hinged joints. These assumptions are very nearly in accord with actual conditions. The entire structure is really composed of three trusses, namely, the two lower or supporting **hammer beams** and the upper truss that these hammer beams support. This upper truss may be one of a number of types, such as, for example, as the

simple type shown in the figure or a more complex truss such as a scissors truss.

1. The first step is to determine the H and V components of the reactions. This is best done algebraically. (See *Problem 14*, page 155, for this calculation.) Once these components of the reactions are known, the stresses for the entire structure may be determined graphically by the usual method of procedure.

2. Though the structure is stable for vertical loads it is not stable for inclined loads. It may be made so by the addition of members between joints b and f and between joints b' and f. These members are usually curved, but the stresses in them are first determined as if they were straight; when the member is designed the curvature may be taken into account. If such members are rigidly attached, the truss becomes a scissors truss, and stresses may then be determined by the method already explained for that type of truss. Another method is to assume that when the wind blows from the left the member between b' and f acts, but the member between b and f does not; when the wind blows from the right, the member between b and f acts but the member between b' and f does not. When either of these members acts, it eliminates one of the hinged joints. For example, when the member between b' and f acts, it eliminates the hinged joint at c'; the structure will then be composed of two trusses meeting at the hinged joint at c.

3. **Head Frames.** Head frames for mines vary in design from the simple type in which the stresses are statically determinate, such as that shown in Fig. 257 (a), to the complex type in which stresses are statically indeterminate. The external forces which act on such a structure in addition to its own weight are due to the weight of the loaded skip or cage that is being hoisted, the weight and friction of the rope, the friction of the skip or cage in its guides, and the accelerating force. When these external forces have been determined, the stresses may be found graphically by the usual method of procedure, provided the head frame is of the simple type.

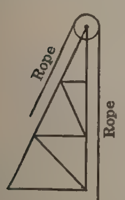


Fig. 257 (a).

4. *Note:* The steel head frame is an example of a structure that is designed to do more than merely carry loads. Among the many structures of this kind may be mentioned coal tipples, hoisting cranes, and ore conveyors. It is not within the scope of this book to discuss the stresses in such structures — indeed, a complete discussion for any one type would form a treatise in itself. It is well to note, however, that in the case of any structure in which stresses are statically determinate, little can be new except whatever may be involved in finding the external forces that act on the structure; once these forces are known, the stresses may be determined, as in the case of the head frame, by one of the old and well-known methods of statics. If one has mastered these methods one can quickly acquire whatever else may be necessary in determining stresses in unusual or special forms of structures.

5. **Roof Truss with Knee Braces.** When a roof truss is connected to the supporting columns by knee braces, the determination of the external forces exerted on the truss by the columns and knee braces is not so simple as the determination of ordinary reactions. Once these external forces are known, however, the stresses may be determined graphically by the usual method of procedure.

6. Let ABC in Fig. 257 (b) represent a roof truss supported by two columns AD and CE with knee braces FH and IG. For *vertical loads* the reactions on the truss are vertical, and their lines of action lie, respectively, in AD and CE. Vertical loads do not cause stresses in the knee braces. The vertical reactions at A and C are therefore the only external forces which act on the truss in addition to the loads. This makes the problem of determining stresses due to vertical loads exactly like typical problems already explained.

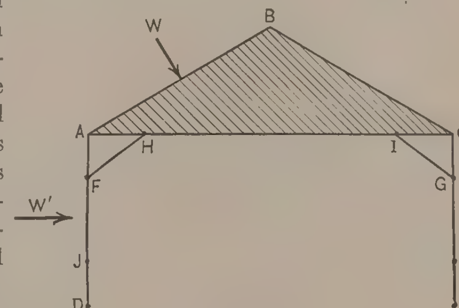


Fig. 257 (b).

7. *Wind pressure* causes the knee braces to act, and thus complicates the problem. Let W represent the resultant of the wind pressure on the roof and W' the resultant of the wind pressure on the side of the building. The roof truss and columns will carry the V component of W, but the truss, columns, and knee braces acting together must resist both the H component of W and the horizontal force W'.

8. The windward column AD will be considered first. The upper portion AF of this column may be considered as part of a rigid structure

since it is part of the triangle AFH; the lower portion FD, however, is merely a vertical beam when horizontal forces alone are considered, and these horizontal forces will cause it to bend to the right or left. The nature of the curve representing the distortion will depend upon the conditions at the ends F and D. If both of these ends were fixed there would be a point of contraflexure half way between them, but if one end is fixed and the other partially fixed, the point of contraflexure will be nearer the partially fixed end. Wherever the point is, it may be considered as a hinged joint at which forces may act in any direction, but for which the moment due to horizontal forces acting on FD is zero.

1. The end F may be considered fixed; though the foot of the column is usually anchored it may be considered as only partially fixed. The point of contraflexure J is therefore nearer D than F, and a good assumption — the first thus far made — is that its distance from the foot of the column is one-third of the distance from the foot of the column to the foot of the knee brace, i.e., $DJ = \frac{1}{3}\ DF$. The corresponding point of contraflexure K on the leeward column is similarly assumed, i.e., $EK = \frac{1}{3}\ EG$. (See *Assignment* (*4*) at the end of this chapter.)

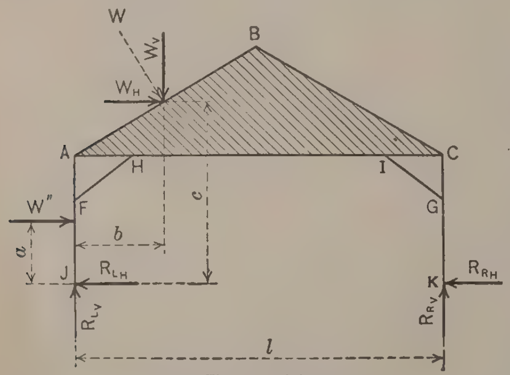


Fig. 258 (*a*).

2. Since there can be no moment for either J or K, these two points of contraflexure may be considered as points of support for the entire structure except the portions DJ and EK of the columns. The structure under these conditions, with DJ and EK omitted, is shown in Fig. 258 (*a*). The wind loads are the same as before except that instead of W' there is a force W'' which is the resultant of the wind pressure on that portion of the side of the building that is *above* the point of contraflexure J.

3. The problem of determining the reactions at J and K may now be analyzed as follows:

Body in equilibrium: Truss, columns, and knee braces considered as one structure.

Known: External forces W and W''.

Unknown: R_{L_V}, R_{L_H}, R_{R_V}, and R_{R_H}. (Four unknowns — problem indeterminate.) Since the problem is indeterminate some assumption must be made. A common assumption is that $R_{L_H} = R_{R_H}$. (See *Assignment* (*4*) at the end of this chapter.) This assumption having been made, the problem of finding the reactions falls under Case 4, non-concurrent forces.

Equations: $\Sigma H = 0 = W_H + W'' + R_{L_H} + R_{R_H}$.

$(R_{L_H} = R_{R_H}$ by assumption.)

$$\Sigma M \text{ about } J = 0 = W'' \times a + W_V \times b + W_H \times c + R_{R_V} \times l. \quad (\text{Solve for } R_{R_V}.)$$

$$\Sigma V = 0 = -W_V + R_{R_V} + R_{L_V}. \quad (\text{Solve for } R_{L_V}.)$$

4. The reactions at J and K were calculated as a preliminary step to determining the forces which the columns exert on the truss, directly at A and C, indirectly through the knee braces at H and I. These four forces are best determined by calculating the corresponding forces that the truss and the knee braces exert on the column. The windward column will be considered first. This column from A to J may be considered as a vertical beam, and the horizontal forces that act on this beam must be in equilibrium. These forces (Fig. 258 (*b*) are W'', the resultant of the wind pressure; A'_H, the H component of the force exerted on the column by the truss; F'_H, the H component of the force exerted on the column by the knee brace; and R_{L_H}, the H component of the force (reaction) exerted on the column at its (point of contraflexure). The problem of determining A'_H and F'_H may now be analyzed as follows:

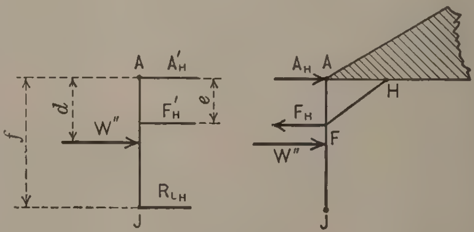


Fig. 258 (*b*).

Body in equilibrium: Beam AJ.
Known: W'', R_{L_H}, A'_H (L) and F'_H (L).
Unknown: A'_H (M) and F'_H (M). (Case B′, parallel forces.)
Equations: $\Sigma M = 0$ about $A = R_{L_H} \times f - W'' \times d + F'_H \times e$.
(Solve for F'_H.)

$$\Sigma H = 0 = -R_{L_H} + W'' + F'_H + A'_H.$$
(Solve for A'_H.)

It will be found that F'_H is positive (acts toward the right) and A'_H is negative (acts toward the left).

1. Let F_H represent the H component of the force which the column exerts on the knee brace and let FH represent the stress in the knee brace. At joint F, F_H must equal F'_H but act toward the left, and, from $\Sigma H = 0$, FH_H must equal F_H but act toward the right. FH is therefore tension, and its magnitude may be determined from FH_H. This tensile stress in the knee brace is the pull which the knee brace exerts on the main truss at H. Let A represent the force which the column exerts on the truss at A. Its H component A_H is equal in magnitude but opposite in sense to A'_H. Its V component must be equal in magnitude to the stress in AF, the portion of the column above F, and this stress AF may be found by applying $\Sigma V = 0$ to joint F.

$$\Sigma V = R_{L_V} + F_V + AF = 0. \quad \text{(Solve for } AF.\text{)}$$

2. (In place of the equation, a force triangle may be drawn to obtain AF.) The forces at I and C which the leeward column and knee brace exert in the truss may be found by exactly the same methods as those just explained. It is to be noted that there is no wind pressure on the leeward column, and it will be found that the stress in the knee brace IG is compression instead of tension.

3. The external forces at A, H, I, and C are now known, and the stresses in the truss may therefore be determined graphically by the usual method of procedure.

4. *Alternative method.* Instead of calculating the stresses in the knee brace FH and in the member AF (upper part of the column), these two members may be considered a part of the truss, in which case the external forces are applied as shown in Fig. 259. The stresses for the entire structure, including those in the knee braces and in the portions of the columns above the foot of the knee braces, may then be determined. This

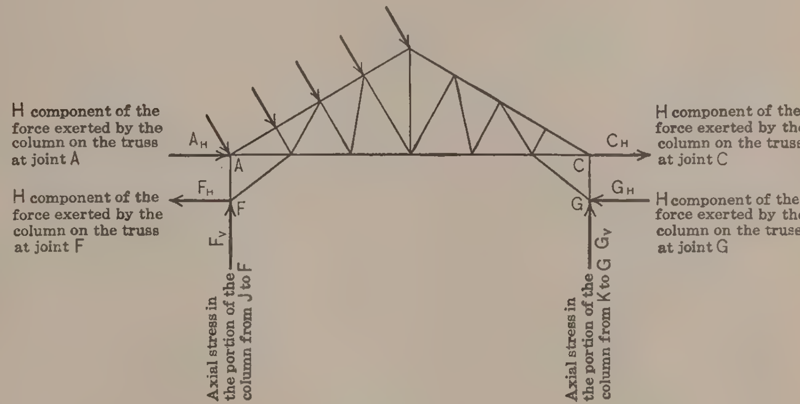


Fig. 259.

modification of the method just explained will be used in the illustrative problem that follows.

5. *Illustrative Problem:* (See page 260.) The wind loads on the roof are those shown on the right-hand truss diagram. (Fig. (*c*).) The wind loads on the side of the building are those shown in the left-hand truss diagram (Fig. (*a*)), obtained from a wind pressure of 280 lbs. per linear foot of vertical surface — measured along the column. The portions of the columns below the points of contraflexure J and K are not involved in the calculations, and are shown by broken lines in the left-hand figure. The complete solution may be outlined as follows:

6. First: Assume the points of contraflexure J and K so that DJ and EK are equal, respectively, to $\frac{1}{3}$ FD and $\frac{1}{3}$ GE. (258 : 1.)

7. Second: Consider the entire structure as supported at J and K (Fig. (*a*)) and calculate J_H, J_V, K_H, and K_V, the H and V components of the reactions of these points. (258 : 3.)

8. Third: Consider the portion AJ of the windward column (Fig. (*b*)) as a body in equilibrium, and calculate A'_H and F'_H, the H components, respectively, of the forces exerted on that body by the truss at A and the knee brace at F. (258 : 4.)

9. Fourth: Consider the portion CK of the leeward column (Fig. (*d*)) to be a body in equilibrium, and calculate C'_H and G'_H, the H components, respectively, of the forces exerted on that body by the truss at A and the knee brace at G.

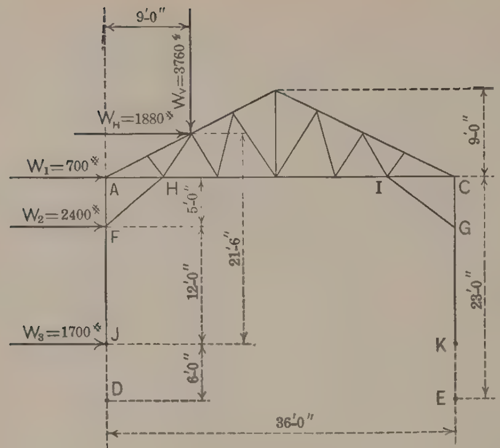


Fig. *a*

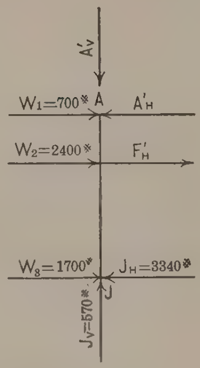


Fig. *b*.

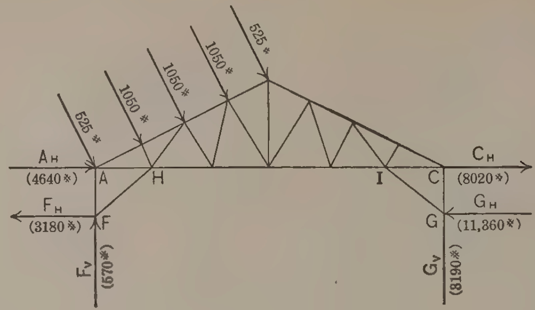


Fig. *c*.

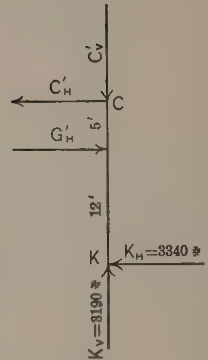


Fig. *d*.

$W = 4200\#$, $W_H = 1880\#$, $W_V = 3760\#$.

$JD = \frac{1}{3}\ FD = KE$. (J and K are points of contraflexure.)

To find components of reactions at J and K (Fig. *a*).

Body in equilibrium: Entire structure above J and K.
Known: W_1, W_2, W_3, W_H, W_V, J (P) and K (P).
Unknown: $J_H(M)$, $J_V(M)$, $K_H(M)$, and $K_V(M)$ (Indeterminate).

Assume $J_H = K_H$. Now determinate, (Case 4).

Equations: $\Sigma M_J = 2400 \times 12 + 700 \times 17 + 1880 \times 21.5 + 3760 \times 9 + K_V \times 36 = 0$

$$3190\# = K_V \uparrow$$
$$\Sigma V = -3760 + 3190 + J_V = 0$$
$$570\# = J_V \uparrow$$
$$\Sigma H = 1700 + 2400 + 700 + 1880 + J_H + K_H = 0$$
$$-3340\# = J_H \leftarrow$$
$$-3340\# = K_H \leftarrow$$

To find A'_H and F'_H (Fig. *b*.)

Body in equilibrium: AJ, the portion of the column above J.
Known: W_1, W_2, W_3, J_V, J_H, $F'_H(L)$, $A'_H(L)$ and $A'_V(L)$.
Unknown: $A'_H(M)$, $F'_H(M)$, and $A'_V(M)$ (Case 4).
Equations: $\Sigma M_A = 3340 \times 17 - 1700 \times 17 - 2400 \times 5 + F'_H \times 5 = 0$

$$-3180\# = F'_H \curvearrowleft \rightarrow$$
$$\Sigma H = -3340 + 1700 + 2400 + 700 + 3180 + A'_H = 0$$
$$-4640\# = A'_H \leftarrow; \quad -570\# = A'_V \downarrow \text{ (from } \Sigma V = 0)$$
$$570\# = F_V = J_V \uparrow; \ F_H = -F'_H = 3180 \leftarrow; \ A_H = A'_H = 4640 \rightarrow$$

To find C'_H and G'_H (Fig. *d*).

Body in equilibrium: CK, the portion of the column above K.
Known: K_V, K_H, $C'_H(L)$, $G'_H(L)$, and $C'_V(L)$.
Unknown: $C'_H(M)$, $G'_H(M)$, and $C'_V(M)$ (Case 4).
Equations: $\Sigma M_C = 3340 \times 17 + G'_H \times 5 = 0$.

$$-11{,}360\# = G'_H \curvearrowleft \rightarrow; \quad -3190 = C'_V \downarrow \text{ (from } \Sigma V = 0)$$
$$\Sigma H = -3340 + 11{,}360 + C'_H = 0$$
$$-8020\# = C'_H \leftarrow$$
$$3190 \uparrow = G_V = K_V; \ G_H = -G'_H = 11{,}360 \leftarrow; \ C_H = -C'_H = 8020 \rightarrow$$

1. Fifth: Consider the truss and knee braces (Fig. (*c*)) as one structure held in equilibrium by eleven external forces, namely, the five wind loads and the forces F_H, G_H, F_V, G_V, A_H, and C_H. The forces F_V and G_V are equal, respectively, to J_V and K_V, and F_H, G_H, A_H, and C_H are equal in magnitude but opposite in sense, respectively, to F'_H, G'_H, A'_H, and C'_H. Draw the force polygon for these external forces, and with this polygon as a basis, following the usual method of procedure, draw the force diagram for the entire structure. This diagram is not shown as it does not differ essentially from the force diagrams already explained.

2. *Alternative method for a roof truss with knee braces.* Given: The truss and wind loads shown on page 260. The method just explained for determining the stresses may be modified as follows:

First: Assume the points of contraflexure J and K just as before, i.e., so that the distance from the foot of the columns to the point of contraflexure is one-third of the distance from the foot of the columns to the foot of the knee brace.

Second: Add a temporary framework on the outside of each column as indicated in Fig. 261 (*a*) by the broken lines AS, SF, and SJ, and by CT,

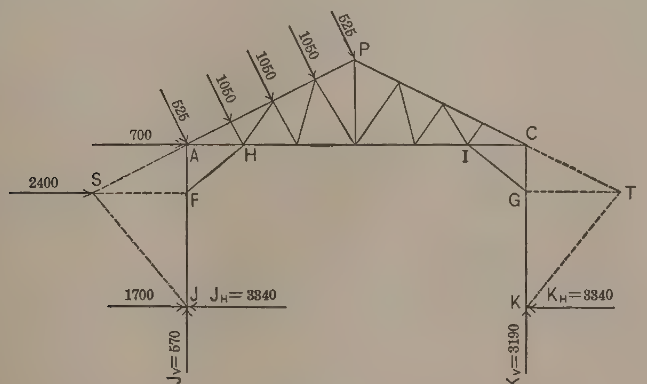


Fig. 261 (*a*).

TG, and TK. The column AJ now acts as a part of the rigid framework $ASJFA$ and the column CK acts as a part of the rigid framework $CTKGC$.

Third: Assume all of the wind loads to act as before, except that the load of 2400 lbs. will act at S instead of at F.

Fourth: Calculate the H and V components of the reactions at J and K, assuming that $J_H = K_H$, just as on page (260).

Fifth: Lay off the force polygon for all external forces on the temporary frameworks and the truss, and draw the force diagram for the entire structure by the usual method. This diagram will be almost the same as that drawn in the fifth step of the first method (261 : 1); in fact, the lines that represent the stresses in the knee braces and in the members of the roof truss will be exactly the same.

Sixth: All stresses obtained from the force diagram will be true stresses except those that are in members of the temporary frameworks. Consider the temporary frameworks removed. The true stress in $JF = J_V =$ 567 lbs., and the true stress in AF may be found from $\Sigma V = 0$ applied to joint F, i.e., $AF = JF + FH_V$. The true stresses in KG and CG may be found in a similar manner.

3. *Note:* The line SF must be horizontal, but it is not necessary that S should be in the line PA produced, or that T should be in PC produced. If, however, these two points are thus located, SA and TC will have the same slopes as AP and CP, respectively — conditions that contribute to both convenience and accuracy in drawing the force diagram. (Why must the line SF be horizontal? Must the line TG be horizontal?)

4. *Note:* A roof truss supported on columns and braced by knee braces forms a **transverse bent** which is, in reality, a two-hinged arch or a fixed arch, according to whether the columns are free to turn at the base or are fixed. The problem of determining the stresses in such a structure is statically indeterminate, and the methods explained in this chapter are only approximate. More exact methods are not within the scope of this book.

5. *A truss in which the top and bottom chords do not meet* may present a problem similar to that of a truss with knee braces. For example, in the truss shown in Fig. 261 (*b*), the columns are extended to points A and C of the top chords. Wind pressure will cause a horizontal force at D, the base of the column AD, and this force will develop horizontal forces at A and F. Similarly, the horizontal force at E will develop horizontal forces at C and G. Points of contraflexure, J and K, may be assumed, and the H and V components

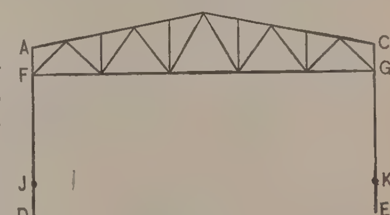


Fig. 261 (*b*).

of the reactions at these points due to wind pressure may be determined. The horizontal forces at A, F, C, and G may then be calculated by the method explained for trusses with knee braces, or a temporary framework may be added on the outside of each column in order to follow the method of procedure explained in the preceding article.

1. **Force Diagrams for Combined Loads.** Throughout this chapter, it has been assumed that the force diagram for the vertical loads and the force diagram for the inclined wind loads are drawn *separately*, as two different diagrams. It is sometimes advantageous to draw a single force diagram for the two types of loading combined. The stresses scaled from this force diagram will then be the *total* stresses due to the combined loads. The force polygon for external forces will be slightly more complex than for vertical loads or for inclined loads alone; but, once this polygon has been constructed, the remainder of the force diagram may be drawn by following the usual method of procedure. For trusses like the Fink truss that present special problems, it is best to draw separate force diagrams for vertical loads and for wind loads.

2. **Equivalent Combined Wind and Snow Load.** The short-cut method of combining wind loads and snow loads, explained in 134 : 9, is based on the assumption of an equivalent *vertical* load which causes stresses in the truss approximately equal to the total stresses which would be obtained by combining stresses due to snow loads with those due to inclined loads. The trend in practice is toward the use of such an equivalent vertical load when the slope of the roof is not more than 45°. The magnitude of the equivalent load depends largely on the slope of the roof. (135 : 1.)

3. The equivalent load having been selected, the total stresses may be determined by one of the following combinations: (*a*) Calculate the total apex loads due to the dead load and the equivalent vertical combined wind and snow load, and from these apex loads determine the total stresses just as if each apex load were composed wholly of dead load. (*b*) Determine the dead-load stresses. Determine the stresses due to the equivalent vertical combined load as follows: Multiply the dead-load stresses by the ratio obtained by dividing the equivalent combined load per apex by the dead load per apex. This method is used when for any reason it is desired to keep the dead-load stresses separate from those due to snow and wind. (*c*) Determine the dead-load stresses. To determine the total stresses, multiply the dead-load stresses by the ratio obtained by dividing the total dead load per apex by the total load per apex (the dead load plus the equivalent vertical combined load).

ASSIGNMENTS

(1) Report on various values for equivalent combined wind and snow loads, and state to what extent one is justified in using the short-cut method of 134 : 9 which involves such combined loads.

(2) Report on the method of determining stresses in symmetrical trusses symmetrically loaded by means of equations or coefficients. Report also on tables of coefficients for roof trusses.

(3) Prove that the stress in a web member of an ordinary roof truss is affected only by the loads between the nearer support and a section cutting that member and two other members.

(4) Report on experiments and discussions concerning the assumption, made in 258 : 1, that the point of contraflexure is a distance from the foot of the column equal to one-third of the distance from the foot of the column to the foot of the knee brace, and also on the assumption, made in 258 : 3, that the H components of the reactions at the points of contraflexure are equal.

(5) Report on the methods of determining the thrust at the supports of a scissors truss.

PART III — STRESSES DUE TO LIVE LOAD

INTRODUCTION TO PART III

The aim in **PART III** is to show how the methods of determining stresses explained in **PART II** are applied when the load is moving instead of static. There can be only one dead-load stress in any member of a structure, but there can be an indefinite number of live-load stresses. As a train moves across a railroad bridge or as vehicles and pedestrians move across a highway bridge, the live-load stress in any member varies in magnitude from zero to a maximum; it may even change in some cases from tension to compression or *vice-versa*.

1. **Live-load Stresses.** The live-load stress in any member of a structure for a given position of the live load is the stress determined *as it would be if the live load were standing still in that position.* The live-load stress required is usually the **maximum live-load stress** because it is necessary to design each member to carry, in addition to the dead-load stress, the greatest live-load stress that can occur in that member. If, for different positions of the live load, the stress in a member can change in character, it may be necessary to determine both the maximum tension and the maximum compression. To determine the maximum live-load stress, it is necessary first to determine the position in which the live load must be placed in order to cause the maximum stress.

2. Although it is true that there can be an indefinite number of live-load stresses in a member, corresponding to different positions of the load, the live-load stress in any member is understood to mean the maximum live-load stress unless otherwise stated. The maximum live-load stress in one member of a truss may occur when the live load is in a position different from that in which it causes the maximum stress in another member, hence *the maximum live-load stresses do not occur simultaneously in all members of a truss* as do the dead-load stresses.

3. Since live-load stresses are determined as if the live load were static, they do not include any increase in stress due to the motion of the live load. It will be explained later that an empirical allowance is made for such increase; for the present the term **impact stress,** or simply **impact,** will be used for the increase in stress due to the motion of the live load on a straight track. (135 : 4.) Impact does not include the effect of centrifugal force (135 : 5) or of tractive forces. (135 : 6.)

4. **Types of Live Loads.** Live loads for railroad bridges may be concentrated loads (locomotive loads) or uniform loads (train loads). (131: 4.) Likewise, live loads for highway bridges may be concentrated loads (road rollers or automobile trucks) or uniform loads (compact crowds of people). (131 : 7.) An equivalent uniform live load is frequently used in place of a given concentrated-load system. (131 : 5.) A locomotive excess load (131 : 6) is sometimes used in combination with a uniform load, and a similar combination of one or two concentrated loads with a uniform load is now often specified for highway bridges.

5. In determining the required live-load shears and bending moments for beams in a floor system, concentrated loads are generally used. (132 : 1.) In determining the stresses in the trusses of a railroad bridge, locomotive concentrated loads followed by a uniform train load are generally used, or else an equivalent uniform live load. In determining the stresses in the trusses of a highway bridge, a uniform live load is used, or else a combination of uniform load and one or two excess loads. In the case of short-span

highway bridges, concentrated live loads are used for the trusses as well as for the floor system. When a highway bridge carries a street railroad, concentrated live loads corresponding to those of a typical street car are used for the portion of the floor system that carries the track, and some combination of these concentrated loads — with live loads that may be on the remaining portion of the roadway — is used for trusses.

1. **How Live Loads Are Applied.** *Stringers.* In determining live-load reactions, shears, and bending moments for stringers, the portion of the live load carried by a stringer may be considered as moving along the top of the stringer as if applied directly, regardless of whether the loads are concentrated or uniform. This is because the live load is distributed more or less uniformly to the stringers by the ties or flooring. Concentrated live loads may be near the middle of the beam, near one end, or distributed from end to end, depending upon the system of loads and upon the results required. Similarly, a uniform live load may cover some portion of the beam or the entire beam.

2. *Floor beams.* In determining live-load reactions, shears, and bending moments for floor beams, the portion of the live load carried by a floor beam may be assumed to act as concentrated loads applied at the points where the stringers are connected to the floor beam, but these concentrated loads will vary in magnitude as the position of the live load changes.

3. *Main girders without floor beams.* In determining live-load reactions, shears, and bending moments for main girders without floor beams, such, for example, as the girders of a deck plate-girder bridge, the portion of the live load carried by one girder may be considered as moving along the top of the girder as if applied directly, regardless of whether the load is uniform or a series of concentrated loads. As in the case of stringers, the live load may be on some portion of the beam or may extend over the entire length.

4. *Girders or trusses with floor beams.* Any live load on a floor system with floor beams is carried to the main girders or trusses by the floor beams and takes effect only at panel joints (97 : 1), consequently the live load is applied to the main girders or trusses as a series of concentrated loads acting at panel points. This is true of uniform as well as of concentrated live loads. As the live load moves on to and across a bridge, the first panel joint is at first the only one to receive any load, then the first two joints, then the first three joints, and so on until there are loads at all panel joints. The bridge is then fully loaded. When some panel joints are loaded and some are not, the bridge is partially loaded. Thus panel loads due to live load may be considered as applied to as many or as few panel joints as may be necessary in order to obtain any desired result, whereas panel loads due to dead load are fixed and extend from end to end of the girder or truss.

5. When the live load is a series of concentrated loads, the corresponding loads at panel points are usually unequal; but when the live load is uniform, the corresponding panel loads are equal except in the case of a partially loaded bridge. This exception is treated in the next paragraph.

6. **The Exact and the Conventional Methods of Uniform Loading.** One of two assumptions is commonly made concerning the distribution of uniform live load to the panel joints of a truss when the load extends from one end of the bridge to some point part way across. Let a main girder or truss support floor beams at joints b, c, d, e, f, and g in Fig. 264, and let the uniform live load extend from the support h to any point n between floor beams at c and d. There will be a full panel load W at each of the joints g, f, and e, but only a partial panel load at d and a still smaller partial panel load at c. In the **exact method** the loads W_c and W_d at c and d respectively are calculated and used in the determination of shear and the corresponding stresses caused by the load in the given position. It is called the "exact method" because the assumption of the distribution of the load is in accord with the actual conditions of loading. In the **conventional method,** the load at c is ignored, and a full panel load is assumed to act at d as well as at e, at f, and at g; this assumption is contrary to fact, but, in the majority of cases, resulting errors are negligible. A more general statement of the difference between the exact and the conventional methods is the following:

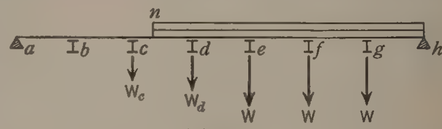


Fig. 264.

7. *When the uniform live load extends from one end of a truss part way into a given panel, the actual partial panel load at each end of that panel is used in the exact method, whereas in the conventional method it is assumed that there is no load at the end of the given panel in advance of the load and a full panel load at the other end of the given panel. This statement holds true regardless of whether the right-hand or the left-hand segment of the bridge is*

loaded. When the uniform live load extends from end to end of bridge there is no difference between the exact and the conventional methods of loading.

1. **POSITION OF THE LIVE LOAD.** *Concentrated live loads.* The position of a system of live loads composed of two or more concentrated wheel loads is fixed when the position of one wheel, called the critical wheel, is determined, since the distances between wheel loads are fixed. This applies both to beams in the floor system and to main girders or trusses.

2. *Uniform live load.* When the uniform load is considered as applied directly to the top of a beam, as in the case of a stringer or a deck plate girder, the position of the load is fixed by the length of the beam which it covers. When only a part of the beam is covered, it is usually a portion which begins at one end of the beam. When the uniform load is distributed to panel joints of a girder or truss, the position of the load on the girder or truss is fixed by the number of panel joints that are considered as loaded. When some but not all are loaded, the loaded joints are consecutive and extend usually to one end or the other of the truss.

3. *Criteria.* Criteria for determining the positions in which live loads will cause maximum reactions, shears, bending moments, and stresses may be expressed in the form of fundamental principles or in algebraic forms. Some of these criteria will be derived, but in many cases a criterion will be given without proof in order that the student may derive it for himself.

4. **METHODS OF DETERMINING REACTIONS, SHEARS, BENDING MOMENTS, AND STRESSES.** When the live load has been placed in the position in which it causes the maximum reaction, shear, bending moment, or stress, as the case may be, it is assumed to be static in that position. The method of determining the required result does not differ, therefore, from the corresponding method for dead load. Throughout **PART III** it will be necessary to explain the application of the methods of determining reactions, shears, bending moments, and stresses already given, but it should be kept in mind that these basic methods are the same as those explained in **PART II.** It is almost literally true that the only fundamentals in **PART III** that have not already been given are those that are used in determining the correct position of the live load.

5. *Formulas and tables.* In engineering practice many formulas are used in the calculation of reactions, shears, bending moments and stresses. Likewise many tables are prepared from which such quantities may be obtained for different lengths of span and different loading. Such formulas and tables, however, are intended for the use of those who thoroughly understand the fundamental principles and methods upon which they are based; they may be found in standard structural-engineers' handbooks.

6. **COMBINATION OF DEAD-LOAD AND LIVE-LOAD STRESSES.** Most members of a bridge are designed to withstand the combined action of dead load and live load, including impact. In discussing any combination of dead-load and live-load stresses it should be understood that the live-load stress and the corresponding impact stress must be taken together, since the live-load stress cannot occur without the impact stress when the live load is moving at the usual speed. The combined stress in any member at any instant is the *algebraic* sum of the dead-load stress (which is constant) and the stress due to the live load in the position which that load occupies at the instant. This combined stress will vary in magnitude as the live load moves across the bridge. The combinations for which the member is designed will now be given.

7. *When the maximum live-load stress is the same in character as the dead-load stress,* the sum of the two stresses, including impact, is the maximum combined stress.

8. *When the maximum live-load stress is not the same in character as the dead-load stress,* the algebraic sum of the two stresses, including impact, is the minimum combined stress. When the dead-load stress is less than the maximum live-load stress plus impact, the minimum combined stress is opposite in character to the dead-load stress. In this case, the member is subject to reversal of stress, and in changing from tension to compression, or *vice-versa,* the stress in the member must pass through zero.

9. **MAXIMUM AND MINIMUM STRESSES.** Certain members of a truss are subject to only one kind of stress. For example, the members in a top chord of an ordinary truss are always in compression and those in the bottom chord are always in tension, regardless of the position of the live load. In such a member the maximum combined stress is the sum of the dead-load stress and the maximum live-load stress plus impact; the minimum stress is the dead-load stress.

10. Certain members, usually web members, may be subject to two kinds of live-load stress but to only one kind of combined stress. In such a member the maximum combined stress is the sum of the dead-load stress and the

maximum live-load stress of the same kind plus impact; the minimum stress is the algebraic sum of the dead-load stress and the live-load stress of the opposite character, including impact, and this minimum stress will be less than the dead-load stress but the same in character. For example, the dead-load stress in a diagonal is 16,000 lbs. tension and the maximum live-load stresses, including impact, are 46,000 lbs. tension and 9000 lbs. compression. The maximum combined stress is $16{,}000 + 46{,}000 = 62{,}000$ lbs. tension, and the minimum stress is $16{,}000 - 9000 = 7000$ lbs. tension, or 9000 lbs. less than the dead-load tension and the same in character.

1. Certain members, usually web members, may be subject to two kinds of combined stress, i.e., reversal of stress. This occurs only when the dead-load stress is less than the maximum live-load stress of the opposite character, including impact. The maximum combined stress in such a member is the sum of the dead-load and maximum live-load stress of the same kind, including impact; the minimum combined stress is the algebraic sum of the dead-load stress and the maximum live-load stress of the opposite character, including impact, and this minimum stress may be greater or less than the dead-load but it will be opposite in character. For example, the dead-load stress in a diagonal is 10,000 lbs. tension, and the maximum live-load stresses, including impact, are 37,000 lbs. tension and 17,000 lbs. compression. The maximum combined stresses are $10{,}000 + 37{,}000 = 47{,}000$ lbs. tension, and the minimum stress is $10{,}000 - 17{,}000 = 7000$ lbs. compression, or 3000 lbs. less than the dead-load tension and opposite in character.

2. In certain types of trusses, such, for example, as the Pratt truss or Baltimore truss, counters may be used to prevent reversal of stress in certain diagonals by acting in the place of these diagonals when the live load is in any position that would cause reversal of stress. It will be shown later that when a counter in any panel is acting, there is no stress, not even dead-load stress, in the main diagonal in that panel. The minimum stress for such a diagonal is therefore zero. The action of counters may also cause minimum stresses of zero in certain vertical members.

3. **Stress Sheet.** The customary method of showing the stresses in a truss is by means of a stress sheet on which are drawn diagrams of the main truss and of the lateral systems. The stresses in each member are entered along the line that represents that member. On the main-truss diagram, the dead-load, live-load, and impact stresses in each member are shown separately. Longitudinal or lateral stresses are shown on the diagrams of the lateral systems. If the bridge is on a curve, stresses due to centrifugal force may also be shown.

4. **Effect of the Reversal of Stress and Range of Stress on Design.** When reversal of stress takes place it is usually necessary to determine the maximum stress of each character, maximum tension and maximum compression; when there is no reversal of stress it may be desirable to determine the minimum as well as the maximum stress of the same character. In either case, the two total stresses in a member are found by adding, algebraically, first the dead-load stress and the *maximum* live-load tension, and then the dead-load stress and the *maximum* live-load compression. When these two combinations result in a maximum and a minimum stress of the same character, it may be desirable to design the member to resist the effect of this range of stress; if the range is great it might result in a gradual weakening of the member, due to "fatigue of metal" or to other causes, unless the members were designed to resist such weakening. When the two combinations of dead-load and maximum live-load stresses result in a reversal of stress, the member should be proportioned for the kind of stress, tension or compression, that requires the larger section. Not only must the member be strong enough as a whole, but each part of the member must be designed to withstand either of the total maximum stresses. A compression member, for example, will easily carry the maximum tension, as a rule, except at some point, such as at the end, where a connection, though adequate for the compression, may be incapable of carrying the tension.

5. When counters are used, the necessity of designing a web member for two kinds of stresses is avoided. For example, the main diagonal of a Pratt truss is normally a tension member and it can be designed as such even when it is subject to reversal of stress, provided a counter is used to prevent this reversal.

6. Special provision is made in many specifications for members in which alternate stresses of tension and compression occur in succession during the passage of the live load. The following is an extract from the specifi-

cations for railroad bridges, page 478 of the *Transactions of the American Society of Engineers, Vol. LXXXVI:*

1. *Alternate Stresses.* Members subject to alternate stresses of tension and compression shall be proportioned for the kind of stress requiring the larger section. If the alternate stresses occur in succession during the passage of one train, as in stiff counters, each stress shall be increased by 50 per cent of the smaller. The connections of such members shall be proportioned in all cases for the sum of the stresses thus increased.

2. If the live-load and dead-load stresses are opposite in character, only two-thirds of the dead-load stress shall be considered as effective in counteracting the live-load stress. This reduction of dead load shall not be made in proportioning members subject to alternate stresses.

3. *An increase in live load* is often specified to be used in determining the stresses in certain members, such, for example, as those in which reversal of stress occurs, or in which the range of stress is great as, for example, in counters. An increase of 50, or even of 100 per cent, in the live load is frequently specified. An increase, however, in the unit stresses, used in designing, is also generally specified.

4. It will be explained later that in many cases the specified increase in live load causes a reversal of stress which the original live load without the increase would not have caused.

5. **Effect of the Motion of the Live Load.** Since the maximum live-load stress in any member of a floor system or of a truss is determined as if the live load were standing still, the stress thus determined is usually much less than the actual maximum stress when the live load is in motion, particularly if the member is one upon which the live load acts more or less suddenly or directly. (135 : 4.) The increase in the stress due to motion cannot be determined theoretically, but an empirical allowance is made for it in one of two ways, namely, (1) by an *impact allowance* (135 : 4) or by the use of a smaller unit stress in designing.

6. *Impact allowance.* As stated in 135 : 4, the term *impact* is somewhat misleading since the allowance for impact is usually intended to provide for the increase in stress due to a variety of causes having their origin in the motion of the load. The allowance for impact or impact stress in any member is usually a certain percentage of the maximum live-load stress in that member; if there are two maximum stresses, tension and compression, there will be two corresponding impact stresses. The percentage used in calculating the impact stress for any member is usually determined from an empirical formula for impact allowance. In the most common type of impact formula, the percentage is determined by the **loaded length**, i.e., the length in feet (L) of that portion of the track or roadway which is loaded when the maximum stress in the member occurs. When there are two maximum stresses, tension and compression, the impact percentage for one is different from that for the other, if, as is usually the case, the loaded length when one stress occurs is different from the loaded length when the other occurs.

7. *General forms of impact formulas.* Let S represent the computed maximum live-load stress, I the impact stress or the dynamic increment to be added to live-load stresses, and L the loaded length. The most common impact formulas for railway bridges are as follows:

$$(1)\quad I = S\frac{300}{300 + L}; \qquad (2)\quad I = S\frac{300}{300 + \dfrac{L^2}{100}};$$

$$(3)\quad I = S\frac{2000 - L}{1600 + 10\,L}; \qquad (4)\quad I = S\frac{400 - \frac{1}{2}L}{400 + L}$$

The increment to be added to any computed stress S depends in any one of these formulas solely upon the loaded length L. With the exception of formula (2), each is a straight-line formula.

8. *Impact percentage.* Any one of the impact formulas may be expressed as a percentage by substituting "100" for "S." Formula (4), for example, may be expressed as:

$$I \text{ (in percentage)} = 100\frac{400 - \frac{1}{2}L}{400 + L}$$

This is in many respects a more convenient form for general use.

9. *The impact formula recommended for railway bridges* is the straight-line formula just given.* The maximum impact percentage from this formula is when $L = 0$, and is equal to 100; when $L = 800$ the percentage is zero. In applying the formula to multiple track bridges, L is the loaded length for *one* track only, and the impact stress is found by multiplying the live-load stress due to the load on *one* track by the impact percentage, as explained in 268 : 4.

* This formula is from the proposed specifications for railway bridges, prepared by a joint committee of the American Society of Civil Engineers and the American Railway Engineering Association.

1. *Note:* As a general rule, the shorter the span the greater the impact percentage should be, but this rule does not hold true in all cases. The impact effect for a very short span may be less than that for a longer span. Moreover, impact percentage does not really vary directly as the length of span, or as the loaded length, therefore a straight-line formula for impact is not strictly correct. Present knowledge of impact, however, does not justify great refinement in impact formulas.

2. *Loaded length.* The impact stress in any member due to the movement of a given live load is largely a function of the time that it has taken the live load to reach the position in which it causes the computed maximum live-load stress in that member. The loaded length L in an impact formula is intended to provide for this time interval. It may be defined as the length of that portion of the structure over which the live load has passed when it has reached the position in which it causes the computed maximum live-load stress. This length may be taken to the nearest foot.

3. When the length of span is short, the live load may pass over it almost instantly, and it is customary, therefore, in the case of a short beam, such as a stringer, to consider the loaded length as equal to the length of the beam; in determining the impact to add to the calculated load brought to a floor beam by two stringers at the same point, one in each of the two adjacent panels, the loaded length may be taken as the sum of the lengths of the two panels. The loaded lengths for trusses will be treated later.

4. In the application of impact formulas to double-track bridges, some specifications require that the aggregate loaded length of both tracks producing the maximum stress in a member shall be used for L in determining the percentage for impact stress in that member; this impact stress is then calculated by multiplying the maximum live-load stress due to loads on *both* tracks by the impact percentage. It is doubtful, however, if a train on the second track would add to the impact calculated for a train on the first track; on the contrary, it is probable that it would destroy synchronism and thus decrease rather than increase the impact effect. The trend in practice, therefore, is to calculate the live-load stress for full live load on both tracks, but to use for L the loaded length for *one* track only; the impact stress is then found by multiplying the maximum live-load stress due to the load on *one* track by the impact percentage, as if there were no impact from live load on the other track. The loads on the third and fourth track for four-track bridges will be train loads only, without engines and without impact. To simplify calculation, the loading on the third and fourth tracks may be assumed as 90 per cent of the engine load.

5. *Causes of impact.* Among the causes of impact in a railway bridge are suddenness of application of the load, unbalanced driving wheels, lurching of the locomotive, and cumulative vibration due to a variety of conditions pertaining to the structure itself. Contrary to the general impression, the effect of speed on impact is small, particularly on spans exceeding 40 ft. in length. Impact should not be confused with the effect of repetition of load, or the effect of reversal of stress, and allowance for impact should not properly provide for such effects. Provision for these effects can be made wholly apart from impact, as, for example, by an increase in live load, as explained in 267 : 3.

6. *Electric traction.* Impact from electric locomotives is much less than that from steam locomotives, probably not more than one-third to one-half as great. This is because impact from steam locomotion is due largely to unbalanced drivers and reciprocating parts, whereas vibration from this source is reduced to a minimum in electric traction.

7. *Impact allowance for highway bridges.* There is a considerable diversity of practice in the impact allowance used for highway bridges. Some formulas provide for one-third of the impact percentage used for railway bridges, others for one-half. A committee of the American Society of Civil Engineers recommends for spans longer than forty feet:

$$I \text{ (in percentage)} = 100 \frac{50}{L + 160}$$

8. *A flat allowance for impact* is often specified. For example, the committee just referred to recommends a flat impact allowance of 25 per cent for floors, floor-beam hangers, and for members of trusses for spans of less than forty feet.

9. *Impact allowances are not usually added* to stresses produced by longitudinal, centrifugal, and lateral or wind forces, or to the forces on supports such as piers and abutments.

10. *Impact stresses are not determined in the illustrative problems* in live-load stresses in this book. To determine them would mean a mere process of multiplication repeated over and over. It should be remembered, however, that in general, for every live-load stress there is a corresponding impact stress, and that in designing any member, the impact stress is included in the total stress that the member must carry.

11. *Note:* The stresses that are usually required in order to design any member of a bridge are those due to dead load, live load, and impact. In designing certain members, stresses due to other forces may also be required, such forces, for example, as wind pressure, centrifugal force, and longitudinal traction.

12. Notwithstanding the numerous careful experiments and investigations upon which empirical impact formulas and allowances for increase in live load are based, results are only approximate. Engineers differ with regard to such formulas and increase in the live loads. The fact that the increase in stress due to the motion of the live load cannot be determined theoretically is one of the most important limitations which prevents the exact determination of the total stress due to live load. It is a limitation which should be considered carefully in deciding what refinements are justified in the determination of stresses due to live load when it is considered as standing still.

CHAPTER XVIII

INFLUENCE LINES

In this chapter the fundamental properties of influence lines are explained. The use of influence lines in determining live-load stresses will be illustrated more fully in succeeding chapters.

1. **Introductory.** The maximum live-load stress in any member of a structure will occur when the live load is in some particular position on the structure. In the case of simple structures with trusses of a standard type, this position may be determined by means of simple guiding principles. In the case of more complex structures, however, these guiding principles may not apply, and it is in such cases that influence lines can be used to the greatest advantage.

2. The whole subject of live-load stresses in simple structures can be treated without any reference whatever to influence lines. Nevertheless, a study of influence lines is an excellent introduction to the study of live-load stresses. This is because an influence line is a graphic representation of the effect of a moving load — a picture of what the load is doing to a structure as it moves from one end to the other. Such a picture is helpful, even in the study of live-load stresses in simple structures. Moreover, the guiding principles for determining the positions of live loads on such structures are, in some cases, most easily derived by means of influence lines. Incidentally, facility in the use of influence lines is more easily gained in connection with simple than with complex structures.

3. It is important to note that an influence line pictures the effect of a *single* concentrated load, not the effect of two or more loads or of a uniform load. One must keep this fact constantly in mind in studying influence lines. Although the concentrated load is a moving load, the effect it produces in any particular position is considered to be the same as that produced by an equivalent static load in the same position. Any allowance for the effect of motion, as, for example, an allowance for impact, is not involved in the construction of influence lines. This is equivalent to saying that in any particular position on the structure a moving load may be considered as stationary.

4. *Principles pertaining to a single concentrated load on a simple beam.* It will be helpful in studying influence lines to keep in mind three simple fundamental principles that were used frequently in the study of dead-load stresses. These principles pertain to a single concentrated load on a simple beam supported at each end.

(*a*) The reaction at either end is equal to a fraction of the load, and that fraction is equal to the distance of the load from the other end divided by the length of span. (139 : 6.)

(*b*) The shear for a given segment when the load is on the other segment is positive and equal to the reaction on the given segment; the shear for a given segment when the load is on that segment is minus and equal in magnitude to the reaction on the other segment (158 : 15, 12, and 10).

(*c*) The bending moment for a given segment when the load is on the other segment is positive, and equal to the reaction on the given segment multiplied by the length of that segment; the bending moment for a given segment when the load is on that segment is still positive, but it is equal in magnitude to the reaction on the other segment multiplied by the length of that segment. (163 : 11.)

5. **The Simplest Form of Influence Line.** The **influence line for a reaction** is the simplest form of influence line. Let a load of 1 lb. move across the beam *ae* shown in Fig. 270. It is desired to picture the effect of the load on the reaction at *a* as the load moves from *e* to *a*. When

the load reaches the quarter point at d, the reaction that it causes at a is $\frac{1}{4}$ lb.; lay off the ordinate dd', equal to $\frac{1}{4}$ lb., to any convenient scale. When the load reaches the center c, the reaction that it causes at a is $\frac{1}{2}$ lb.; lay off cc' equal to $\frac{1}{2}$ lb. When the load reaches the three-quarter point at b, the reaction that it causes at a is $\frac{3}{4}$ lb.; lay off bb' equal to $\frac{3}{4}$ lb. When the load reaches a, the reaction that it causes at a is equal to 1 lb.; lay off aa' equal to 1 lb. The points a', b', c', and d' will lie in the straight line ea'. When the load reaches any point a distance x from a, the reaction that it causes at a is 1 lb. multiplied by $\frac{l-x}{l}$, and therefore the corresponding ordinate $y = \frac{l-x}{l}$. This is the equation of the straight line ea'. This straight line is the **influence line** for the reaction at a, and the diagram bounded by the triangle $aa'e$ is the **influence diagram** for that reaction. The following properties of this diagram should be noted:

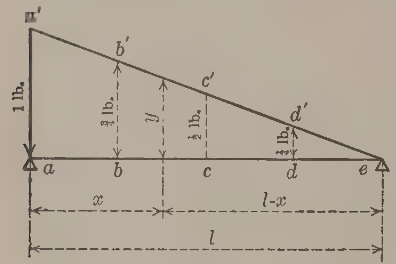


Fig. 270.

(*a*) The diagram pictures the effect of a *single* concentrated load as it moves from one end of the beam to the other.

(*b*) The diagram pictures the effect of this moving load on the reaction at a *fixed* point a.

(*c*) The effect pictured is the *variation* in the magnitude of the reaction at a.

(*d*) The ordinate at any point represents the magnitude of the reaction at a fixed point a when the load is at the point from which the ordinate is drawn — or, briefly, *when the load is at the ordinate.*

1. These four statements may be combined as follows: An influence diagram for a reaction pictures the *variation* in the magnitude of the reaction at a *fixed point* caused by a *single* moving concentrated load; this variation is shown by the variation in the lengths of ordinates from the base line of the diagram to the influence line; the ordinate at any point represents the magnitude of the reaction *when the moving load is at that ordinate.*

2. *Note:* The separate statements and the combined statement have been made in such a way as to emphasize fundamental conceptions or properties that are common to *all* types of influence lines. These fundamentals will be summarized later.

3. *Question:* What would be the influence line for the reaction at e?

4. The interpretation of the particular influence line for reaction, shown in Fig. 270, is exceedingly simple. It is merely this: When the load is at e, the reaction at a is zero. As the load moves along the beam, the reaction at a increases as the ordinates to the influence line increase until, when the load reaches a, the reaction at a is equal to the load itself. If the maximum reaction at a is desired, the load must, therefore, be placed at a. These statements, are, of course, self-evident, but they illustrate the method of interpreting any influence diagram for a reaction, no matter how complex that diagram may be.

5. Although the influence line was drawn for a unit load of 1 lb., it can be used to determine the reaction for any load in any position on the beam. It is merely necessary to scale the ordinate at the point where the load is placed, and then to multiply the value thus obtained by the magnitude of the load. For example, assume that a load of 12,000 lbs. is placed at b. The reaction at a caused by this load is $\frac{3}{4} \times 12{,}000 = 9000$ lbs.

6. From the last two paragraphs it is evident that the influence line for a reaction may be used for two quite different purposes, namely, (1) to picture the variation in the reaction as a unit load moves from one end of the beam to the other, and (2) to determine the magnitude of the reaction for any load at any point of the beam.

7. **Influence Line for Shear.** Let the beam in Fig. 271 be divided into two segments: as, 8 ft. long; and sf, 16 ft. long. Assume a unit load of 1 lb. to move across the beam from f to a. When the load reaches e, the shear for the segment as is $\frac{1}{4}$ lb.; lay off ee' equal to $\frac{1}{4}$ lb. to any convenient scale. When the load reaches d, the shear for the *same* segment as is $\frac{1}{2}$ lb.; lay off dd' equal to $\frac{1}{2}$ lb. When the load reaches s, it is at the section. The shear for the *same* segment as will now be $\frac{2}{3}$ lb. or $-\frac{1}{3}$ lb. according to whether the load is considered on segment sf or on segment as. Plot $ss' = \frac{2}{3}$ lb. and $ss'' = -\frac{1}{3}$ lb. When the load reaches b, the shear for the *same* segment as will be $-\frac{5}{24}$ lb.; plot $bb' = -\frac{5}{24}$ lb. When the load is at either support a or f, the shear for the *same* segment as is zero. The points f, e', d', and s' fall in a straight line and the points s'', b' and a will fall in

another straight line. (What are the equations for the two lines?) If at any point x on segment sf the ordinate to fs' is measured, the result will be the shear for the segment as when the unit load is at x, and this shear will be positive; likewise at any point z on the segment as the ordinate to the line as'', measured to scale, will give the shear for segment as when the load is at z, and this shear will be negative.

1. The irregular line from f to s' to s'' to a is the **influence line for shear** for the segment as, and the two triangles bounded by this irregular line and the base line af form the **influence diagram for shear** for the segment as. The following properties of this diagram parallel closely the properties of the influence diagram for a reaction (269 : 5):

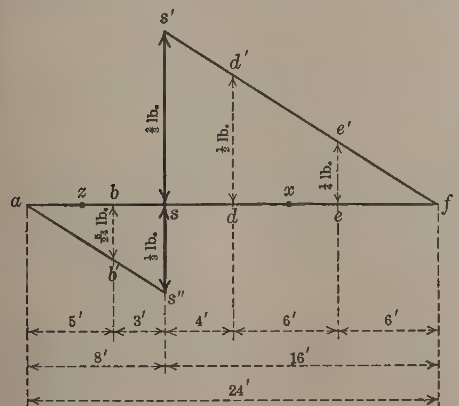


Fig. 271.

(*a*) The diagram pictures the effect of a *single* concentrated load as it moves from one end of the beam to the other.

(*b*) The diagram pictures the effect of this moving load on the shear for a segment of *fixed* length, namely, the segment as.

(*c*) The effect pictured is the *variation* in the magnitude of the shear for the segment as and the change in that shear from positive to negative.

(*d*) The ordinate at any point represents the magnitude and algebraic sign of the shear for the segment **as** *when the load is at that ordinate.*

2. These four statements may be combined as follows: An influence diagram for shear pictures the *variation* in the magnitude and algebraic sign of the shear for a segment of *fixed length* due to a *single* moving concentrated load; this variation is shown by the variation in the ordinates from the base line of the diagram to the influence line; the ordinate at any point represents the magnitude of the shear *when the moving load is at that ordinate*, and also, by its position *above* or *below* the base line, indicates whether that shear is *positive* or *negative*.

3. *Questions:* Why must the lines as'' and $s'f$ in Fig. 271 be parallel? Why must the line fs' coincide with the corresponding portion of the influence line for the reaction at a, assuming that the diagrams are drawn to the same scale? In what respect does the line as'' differ from the corresponding portion of the influence line for the reaction at f? What would be the influence diagram for shear for segment sf?

4. The interpretation of the particular influence line for shear shown in Fig. 271 is as follows: When the unit load is at f, the shear for the segment as is zero. As the load moves from f to s, the shear for segment as is positive and increases as the ordinates to fs' increase until the load reaches s, the point at which the section was taken. As the load passes this point, the shear for segment as changes from the greatest positive value represented by the ordinate ss' to the greatest negative value represented by the ordinate ss''. As the load moves from s to a, the negative shear for segment as decreases as the ordinates to $s''a$ decrease, until when the load reaches a the shear becomes zero. It is evident that if the greatest positive shear for segment as is desired, the load must be placed on segment fs indefinitely close to s; if the maximum negative shear for segment as is desired, the load should be placed on that segment indefinitely close to s. Simple as this interpretation is, it illustrates the method of interpreting the influence diagram for shear, no matter how complex it may be. Notice that throughout the interpretation the shear is for the *same* segment as — it is the position of the load that changes, *not* the segment.

5. The magnitude of the shear for segment as due to any load in any position on the beam may be found by scaling the ordinate at the point where the load is placed, and then multiplying the result thus obtained by the magnitude of the load. For example, the shear for segment as due to a load of 12,000 lbs. at e is $\frac{1}{4} \times 12{,}000 = 3000$ lbs. positive; with the same load at b the shear for segment as is $\frac{5}{24} \times 12{,}000 = 2500$ lbs. negative.

6. The influence diagram for shear for a given segment may be used for two quite different purposes, namely, (1) to picture the variation in the shear for the given segment as a unit load moves from one end of the beam to the other and (2) to determine the magnitude and algebraic sign of the shear for the given segment due to any load placed at any point on the beam.

1. **Influence Line for Bending Moment.** An explanation of the construction of an influence line for bending moment could be given that would closely parallel the explanations in 269 : 5 and 270 : 7 for the construction of influence lines for reaction and shear. The following explanation, however, is more general in its character.

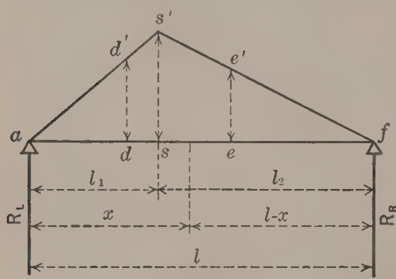


Fig. 272.

Let the beam af in Fig. 272 be divided into two segments as and sf; let the lengths of these segments be, respectively, l_1 and l_2, and let the length of the beam be l.

The bending moment for a segment as for a moving load of 1 lb., when the load is on segment sf a distance x from a, is $R_L \times l_1$, i.e., $M_B = \frac{l-x}{l} \times l_1$. This is the equation of a straight line in which M_B is zero when $x = l$, and equal to $\frac{l-l_1}{l} \times l_1$ when $x = l_1$. When the moving load is on the segment as a distance x' from a, $M_B = -(-R_R \times l_2)$, i.e., $M_B = -\left(-\frac{x'}{l} \times l_2\right)$. This is also an equation of a straight line in which M_B is zero when $x' = 0$, and equal to $\frac{l_1}{l} \times l_2 = \frac{l-l_1}{l} \times l_1$ when $x' = l_1$. Let the ordinate ss' be laid off to any convenient scale to represent the bending moment for the segment as when the unit load is at s. This ordinate will equal $R_L \times l_1 = R_R \times l_2 = \frac{l-l_1}{l} \times l_1$. The equation $M_B = \frac{l-x}{l} \times l_1$ is that of the straight line fs', and the equation $M_B = \frac{x'}{l} \times l_2$ is that of the straight line as', hence the line $fs'a$ is the **influence line for bending moment** for the segment as, and the triangle $fs'a$ is the corresponding **influence diagram for bending moment.** When the unit moving load is at any point on the segment sf, as, for example, at the point e, the ordinate ee' at that point represents the magnitude of the corresponding bending moment for segment as; similarly, when the load is at any point on the segment as, as, for example, at the point d, the ordinate dd' at that point represents the magnitude of the corresponding bending moment for the segment as. If lengths are expressed in feet and the load in pounds, the magnitudes represented by ordinates are expressed in pound-feet. For the segment as all bending moments are positive. This would be true for any left-hand segment of a simple beam supported at each end, but when a beam extends beyond either support, the influence line for any section between supports will show both positive and negative bending moments.

2. The following statement closely parallels similar statements already made for influence diagrams for reaction and shear: An influence line for bending moment pictures the *variation* in the magnitude and algebraic sign of the bending moment for a segment of *fixed length* due to a *single* moving concentrated load; this variation is shown by the variation in ordinates from the base line of the diagram to the influence line; the ordinate at any point represents the magnitude of the bending moment, *when the moving load is at the ordinate*, and also, by its position above or below the base line, indicates whether that bending moment is positive or negative.

3. *Questions:* Would the influence line for bending moment for any left-hand segment of a beam supported at each end ever extend below the base line, if the only upward forces on the beam are the two reactions? In what respects would the influence diagram for bending moment for the segment sf differ from the diagram in Fig. 272?

4. The interpretation of the particular influence line for bending moment shown in Fig. 272 is as follows: When the unit load is at f, the bending moment for the segment as is zero. As the load moves from f to s, the bending moment for segment as increases as the ordinates to fs' increase until the load reaches s, the point at which the section is taken. As the load passes this point and moves from s to a, the bending moment for segment as, though still positive, decreases as the ordinates to $s'a$ decrease, until when the load reaches a the bending moment for as becomes zero. If the greatest bending moment for segment as is desired, the load must, therefore, be placed at s, that is, at the section.

The magnitude of the bending moment for segment as due to any load in any position on the beam may be found from the influence diagram for bending moment for segment as. (How?)

1. The influence line for bending moment for a given segment may be used for two purposes, namely, to picture the variations in the bending moment for the given segment due to a unit moving load, and to determine the magnitude and algebraic sign of the bending moment for the given segment due to any load placed at any point on the beam.

2. **Influence Diagram Defined.** An influence line may be defined as one that shows the *variation in the effect* on a beam or structure produced by a *single moving concentrated load* as it moves from one end to the other. The variation in the effect may be the variation in the magnitude and algebraic sign of the reaction at a *fixed point*, or of the shear for a segment of *fixed length*, or of the bending moment for a segment of *fixed length*, or of any other quantity which can be expressed as a function of a single moving load and its varying position on the beam or structure. The variation in magnitude is shown by the variation in the lengths of ordinates between the base line and the influence line; the variation in algebraic sign is shown by the variations in the position of the influence line above or below the base line. An influence diagram is the diagram formed by an influence line and the corresponding base.

3. *Note:* Influence diagrams may be used to show the variation in the stress in a given member, or the variation in the deflection of a given point of a truss caused by the change in position of a single concentrated load. Reactions, shears, and bending moments are, however, the functions of a moving load most often represented by influence diagrams.

4. The main points to keep in mind in connection with influence lines are those already emphasized in the illustrative examples, namely:

5. An influence diagram shows the variation due to a *single* load in different positions — not to two or more loads. This single moving load may be assumed as 1 lb., 100 lbs., 1000 lbs., 1 ton, or any other convenient unit.

6. If the variation is in the effect on the reaction, it is the reaction at some *one* point of the structure; if the variation is in the effect on the shear or bending moment, it is the shear or bending moment for some segment of *fixed length*. No single influence line can be drawn for reactions at different points, or for shears for different segments, or for bending moments for different segments. Such a line would not be an influence line, but rather a "curve," such, for example, as a curve for shears or a curve for bending moments. The difference will be explained more fully in the next chapter.

7. The ordinate at any point represents the magnitude of the varying function when the unit load is *at that point*. For example, the length of an ordinate in an influence diagram for reaction represents the magnitude of some particular reaction when the line of action of the unit load coincides with the ordinate. Similarly, the length of an ordinate in an influence diagram for shear or for bending moment represents the magnitude of the shear or the bending moment for a given segment when the line of action of the unit load coincides with the ordinate. It is helpful, when considering any particular ordinate of any influence diagram, to conceive the unit load as at the point from which the ordinate is drawn.

8. In an influence diagram for shear or for bending moment the variation in the lengths of ordinates is due solely to the variation in the position of the unit load and not to a change in length of segment. The length of segment does not change in such an influence diagram any more than the point of support changes in an influence diagram for a reaction.

9. An influence line is a straight line or series of straight lines, and the corresponding equation, therefore, is one of the first degree.

10. **Methods of Plotting Influence Lines for Simple Beams.** In the illustrative examples of influence diagrams that have been given thus far, influence lines were plotted by means of several ordinates of known value. More direct methods will now be given. These methods are for simple beams resting on a support at each end; they are, however, fundamental methods that are involved in plotting influence lines for more complex structures. In every case, positive values are plotted above the base line and negative values below.

11. *To plot the influence line for a reaction.* Let it be required to plot the influence line for the reaction at the support a. Lay off the length of the beam ab to any convenient scale. (Fig. 273.) Lay off the ordinate aa' equal to the unit load to any convenient scale and draw the influence line $a'b$. The influence diagram is the cross-lined area.

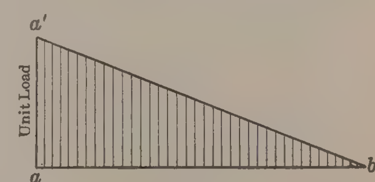


Fig. 273.

12. *To plot the influence line for shear for a given segment.* Let it be required to plot the influence line for shear for the segment as of the beam

ab. (Fig. 274 (*a*).) Lay off the length of beam to any convenient scale, and the length of the segment *as* to the same scale. Lay off *ad* equal to the unit load to any convenient scale. Draw a vertical line through *s* and find the point *s*′ at which this line is intersected by *bd*. Through *a* draw a line parallel to *bd* until it intersects the vertical line through *s* at point *s*″. The influence line is the line *bs*′*s*″*a* and the influence diagram is the cross-lined area.

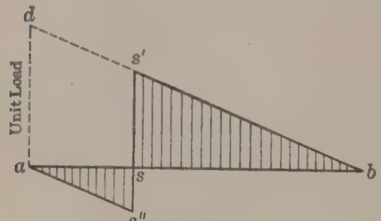


Fig. 274 (*a*).

1. *To plot the influence line for bending moment for a given segment.* Let it be required to plot the influence line for bending moment for the segment *es* of the beam *ef*. (Fig. 274 (*b*).) Lay off the length of the beam to any convenient scale and the length of the segment *es* to the same scale. Lay off *eh* equal (to scale) to the length of the segment *es*. It is not necessary that the scale used in laying off *eh* should be the same as that used in laying off *ef* and *es* — often some other scale is better. Draw a vertical line through *s* and find the point *s*′ at which this line is intersected by the line *fh*. The influence line is the line *fs*′*e*, and the influence diagram is the cross-lined area. The scale used in measuring ordinates is the same as that used in laying off *eh*, but any ordinate represents a quantity equal to the scaled length *multiplied by the unit load*, i.e., *bending moment*. (Force × length.)

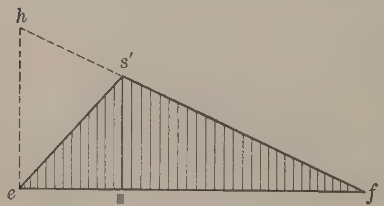


Fig. 274 (*b*).

2. *Exercise.* Prove that each of the three constructions for influence lines just given is correct.

3. *Exercise.* Illustrate by free-hand diagrams general methods for plotting influence lines for: (1) the reaction at *b* in Fig. 273; (2) the shear for segment *sb* in Fig. 274 (*a*); (3) the bending moment for segment *sf* in Fig. 274 (*b*).

4. *Note:* The following three statements hold true, provided it is understood that all beams are simple beams supported at each end:

(*a*) The general form of an influence diagram for a reaction is the same for all simple beams, namely, a right triangle with its base in the base line and its highest apex in the line of action of the reaction; the ordinate to this apex is equal to a unit load. (Fig. 273.)

(*b*) The general form of an influence line for shear is the same for all simple beams, namely, two right triangles, one above and one below the base line. Each triangle has one side in the base line and one side perpendicular to the base line at the section; the two perpendicular sides are ordinates and their combined length represents a magnitude equal to the unit load. The hypotenuse of one triangle is parallel to the hypotenuse of the other. (Fig. 274 (*a*).)

Question: Is the influence diagram for end shear an exception to the description just given?

(*c*) The general form of an influence line for bending moment is the same for all simple beams, namely, a triangle with one base in the base line and a third apex in a vertical line at the section; the ordinate to this apex is equal to the bending moment for the given segment when the unit load is at the section. (Fig. 274 (*b*).)

5. *Exercises.* Draw, free-hand but in approximately the correct proportions, the influence line for each of the following requirements:

(*a*) A simple cantilever beam 10 ft. long is fixed at the right-hand end and unsupported at the left-hand end. Required: The influence lines for shear and for bending moment for a segment 6 ft. long measured from the left-hand end.

(*b*) A beam 40 ft. long rests on two supports 24 ft. apart and has a cantilever or overhanging end at each support 8 ft. in length. Required: The influence lines for reactions at the left-hand support; for the shear and for the bending moment for a left-hand segment 16 ft. long (section 8 ft. to the right of the left-hand support); for the shears and for the bending moments for a left-hand segment 36 ft. long (section 4 ft. to the right of the right-hand support).

Suggestion: Blackboard drills in problems similar to those just given are recommended.

6. **The Uses of Influence Lines.** The two purposes for which influence lines are most used are: (1) To show the position in which a moving load must be placed in order to cause a required reaction, shear, bending moment, or stress; (2) to serve as a graphic or partially graphic means of determining such a reaction, shear, bending moment, or stress. An influence line may also be used in the development of a general criterion for placing a moving load, but such criteria can usually be derived better by other methods.

7. Although an influence line is constructed to show the effect of a single moving concentrated load, it can be used in studying the effect of any number of concentrated loads as they move across a structure; likewise, to show the effect of a moving uniform load. An influence line, therefore, can be used in determining the required positions of a number of

moving concentrated loads, or of a uniform moving load, as well as the required position of a single load.

1. In like manner, influence diagrams may be used to determine graphically the magnitude and algebraic sign of reactions, of shears, of bending moments, or of stresses due to a number of moving loads or to a uniform moving load placed in the required positions.

2. **Uses of Influence Lines Illustrated.** In order to illustrate the uses of influence lines a few simple examples will be given. The first set of examples illustrate the use of influence lines in determining the position of one or more moving loads, and the second set the use of influence lines as a graphic method of determining magnitudes and algebraic signs of reactions, shears, and bending moments. It is to be borne in mind that the results in these simple examples could have been obtained as easily, if not more easily, by other methods, and that in practice influence lines are used mainly in connection with more complex structures.

3. If an influence line is to be used solely to determine the position of the loading, it usually may be sketched free-hand; but if it is to be used to determine magnitudes as well, it must be drawn carefully to scale.

4. *Illustrative examples in determining positions of loads.* In examples that illustrate the use of influence lines in determining the position of a single load, the magnitude of the load is immaterial. If there are two loads, the relative magnitudes rather than the actual magnitudes are involved, i.e., it is usually necessary to consider which load is the greater, and, in some cases, how much greater one is than the other.

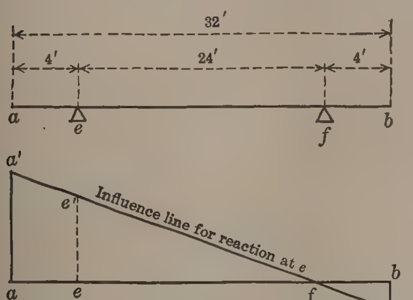


Fig. 275 (*a*).

5. *Illustrative example.* Given: A beam *ab* 32 ft. long resting on supports *e* and *f* 24 ft. apart, as shown in Fig. 275 (*a*). Required: The position of a single concentrated load which will result in the greatest reaction at *e*.

Draw free-hand the influence line for the reaction at *e*. It is evident from inspection that the greatest positive reaction at *e* for a single load will occur when that load is at *a*, and that the greatest negative reaction at *e* will occur when the load is at *b*. Since the ordinate *ee′* is equal to the unit load used for the influence diagram, and since *aa′* is greater than *ee′*, the reaction at *e* due to a single concentrated load is greater than the load itself.

6. *Illustrative example.* Given: A beam *ac* 24 ft. long supported at *a* and *b*, as shown in Fig. 275 (*b*). Two concentrated loads a fixed distance, 8 ft., apart move from one end to the other across the beam. One load is 8000 lbs. and the other is 12,000 lbs. The loads may be turned around, i.e., either may be the left-hand or the right-hand load. Required: The positions of the loads that will result in the greatest positive and negative reactions at *a*.

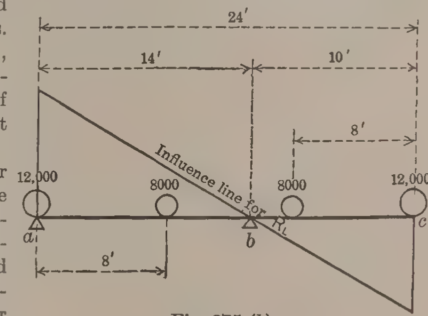


Fig. 275 (*b*).

Draw free-hand the influence line for the reaction at *a*. From the influence diagram it is evident that any load between *a* and *b* will cause a positive reaction at *a*, and that the nearer the load is to *a* the greater will be this reaction. The greatest positive reaction for the two given loads will occur, therefore, when the larger load is at *a* and the smaller load as near to *a* as the fixed distance between the loads will permit, namely, 8 ft. from *a*.

It is also evident that any load between *b* and *c* will cause a negative reaction at *a*, and that the nearer the load is to *c* the greater will be this reaction. The greatest negative reaction at *a* for the two given loads will occur, therefore, when the larger load is at *c* and the smaller load as near to *c* as the fixed distance between the loads will permit, namely, 8 ft. from *c*.

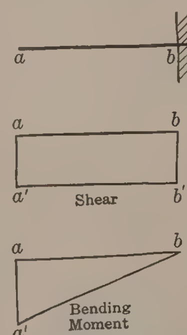


Fig. 275 (*c*).

7. *Illustrative example.* Given: A simple cantilever beam *ab* 8 ft. long, with the unsupported end at *a*. (Fig. 275 (*c*).) Required: The position of a single moving concentrated load that will result in the greatest shear and bending moment for a section at the support *b*.

Draw free-hand the influence line for shear and bending moment for segment *ab*. It is evident from inspection that the greatest shear for segment *ab* will occur for any position of the load on *ab*, and is equal to the load itself, but the greatest bending moment will occur for one position only, namely, when the load is at *a*.

8. *Illustrative example.* Given: A simple beam *ab*, 28 ft. long. (Fig. 276 (*a*).) Required: The positions of a uniform load which will result in the greatest shear and the greatest bending moment for a segment *as*, 7 ft. long.

Draw free-hand the influence lines for shear and bending moment for segment *as*. In the influence diagram for shear any ordinate to the right of *ss'* is positive and any ordinate to the left of *ss'* is negative, hence, for the greatest positive shear for segment *as*, the uniform load should cover segment *sb* from end to end as indicated in the figure, but should not extend on to segment *as*; for greatest negative shear for segment *as*, the uniform load should cover entirely segment *as* but should not extend on to segment *sb*.

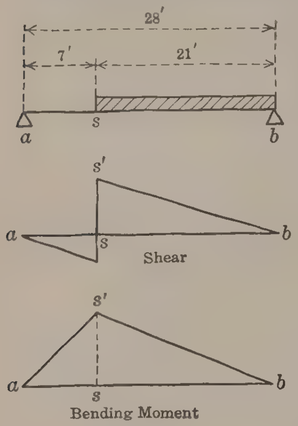


Fig. 276 (*a*).

For the greatest bending moment for segment *as*, the uniform load should cover the entire beam from *a* to *b* since all ordinates between *a* and *b* in the influence diagram for bending moment are positive.

1. *Illustrative example.* Given: A simple beam *ab*, 30 ft. long and two moving concentrated loads of 1500 lbs. each, 8 ft. apart. Required: The position of the loads which will result in the greatest bending moment for a segment *as* 10 ft. long. Draw free-hand the influence line for bending moment for segment *as*. (Fig. 276 (*b*).) The ordinates increase as a unit load moves from *b* to *s*, hence if the loads are on segment *sb*, the greatest bending moment for segment *as* will occur when one load is at *s* and the other is as near to *s* as the fixed distance apart of 8 ft. will permit. The question arises, however, whether or not the bending moment will be increased by moving one or both loads on to segment *as*. This is equivalent to asking whether or not two ordinates 8 ft. apart can be found whose sum is greater than the sum of *ss'* and *ee'*. Since the slope of *as'* is greater than the slope *s'b*, there are no two ordinates 8 ft. apart between *a* and *s* whose sum can equal the sum of *ss'* and *ee'*, nor is there any pair of ordinates 8 ft. apart, one between *a* and *s* and the other between *s* and *b*, whose sum can equal the sum of *ss'* and *ee'*. Hence the bending moment is decreased by moving both loads or one load onto segment *as*.

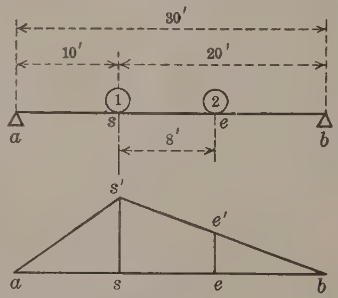


Fig. 276 (*b*).

2. *To determine the position of a load from geometrical properties of an influence line.* In all the illustrative examples just given, the required positions of the load are evident from inspection of the influence lines, and these positions are in accord with common sense or common knowledge. In many cases, however, the required positions are not so evident. In such cases the required positions of the loads may be determined from the geometrical properties of the influence lines. A simple illustration of this is when two moving concentrated loads are unequal and cannot be reversed in order. For example, assume that the two loads in Fig. 276 (*c*) are 8 ft. apart; assume that the right-hand load (2) is larger than the left-hand load (1), and that the order cannot be reversed. The loads are first placed at *s* and at *e*, respectively. It is required to determine whether or not a greater bending moment for segment *as* may be obtained by moving the loads to another position. Let the weight of (1) be represented by W and the weight of (2) by W'. Let the loads be moved from positions at *s* and *e* to the left, any distance x. The ordinate for load (1) has been decreased by an amount equal to $s's''$ and the ordinate for load (2) has been increased by an amount $g'g''$. The total bending moment has been increased or decreased according to whether $W' \times g'g''$ is greater or less than $W \times s's''$. But $g'g'' = \frac{x}{20} \times ss'$, and $s's'' = \frac{x}{10} \times ss'$, hence if $W' \times 10$ is greater than $W \times 20$, the bending moment will be increased by moving the loads to the left; it will be greatest when load (2) is at *s* and load (1) is on the segment *as*, 8 ft. from *s*.

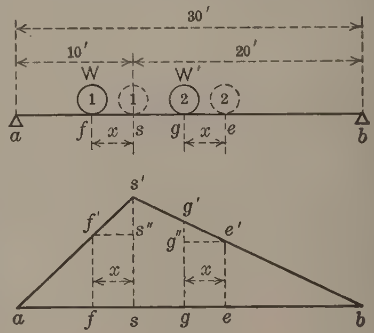


Fig. 276 (*c*).

3. Note that, in general, if the weight of the concentrated load on the right-hand segment multiplied by the length of the left-hand segment is greater than the weight of the concentrated load on the left-hand segment multiplied by the length of the right-hand segment, the bending moment is increased by moving the loads to the left until the right-hand load is at the section.

4. Guiding principles or criteria for placing moving loads, such as that given in the preceding paragraph, may be formulated from the geometrical properties of influence lines, but usually such criteria are more easily derived by algebraic methods. This is particularly true when the criterion is for the purpose of ascertaining the position of a load that will result in

the absolute maximum shear or the maximum bending moment, regardless of where the section is taken. In such a case, it is not always evident where the section should be taken in order to obtain a maximum value, i.e., the length of segment is not known. But influence lines are always drawn for a given segment of known length, and therefore influence lines cannot be used to advantage when the position of the load and the length of segment both vary.

1. *Illustrative examples in determining magnitudes of shears and bending moments.* Examples of the use of influence lines in determining the positions of loads have just been given. In the illustrative examples which

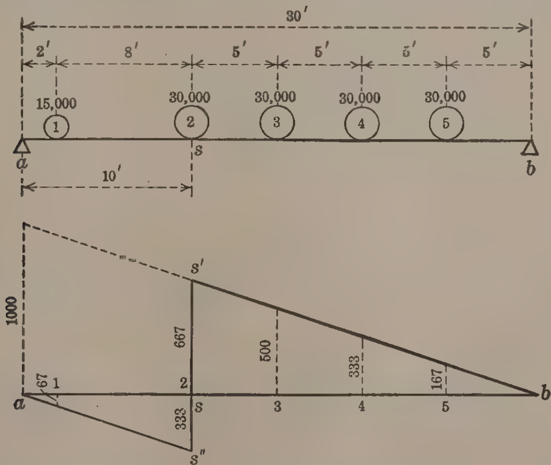


Fig. 277 (*a*).

follow, it is assumed that the position of the load has been determined, that the load has been placed in this position, and that it is required to determine the magnitudes of the shear and bending moment from influence diagrams drawn carefully to scale. The unit load used in plotting influence lines in these particular examples is 1000 lbs.

2. *Illustrative example.* Given: A simple beam *ab*, 30 ft. long. Five moving concentrated loads have been placed on the beam in the position shown in Fig. 277 (*a*). (Consider load (2) on the segment *sb* for positive shear. (Why?) (173 : 13).) Required: The shear for the segment *as*, 10 ft. long.

Lay off the length of the beam to scale and plot the positions of the loads to the same scale. Plot the influence diagram for shear for segment *as* to any convenient scale. (273 : 12.) (Unit load is 1000 lbs.) The influence line is *as″s′b*. Since the ordinate at any point represents the shear for segment *as* when a unit load of 1000 lbs. is at that point, the ordinate at any load multiplied by that load in thousands of pounds will equal the shear for segment *as* due to that load. The total shear will equal the algebraic sum of the five such products that correspond to the five loads.

Scale each of the five ordinates. Assume that the values thus found are those shown in the figure. Then the total shear for segment *as* is:

$$V = -67 \times 15 + 667 \times 30 + 500 \times 30 + 333 \times 30 + 167 \times 30$$
$$V = -67 \times 15 + 1667 \times 30 = 49{,}000 \text{ lbs.}$$

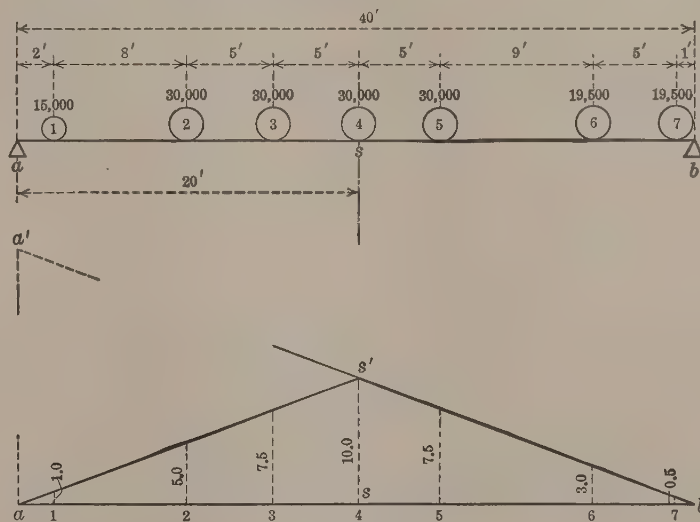


Fig. 277 (*b*).

3. *Illustrative example.* Given: A simple beam *ab*, 40 ft. long. Seven moving concentrated loads have been placed on the beam in the position shown in Fig. 277 (*b*). Required: The bending moment for segment *as*.

Lay off the length of beam to scale and plot the positions of the loads to the same scale. Plot the influence diagram for bending moment for segment *as* to any convenient scale. (274 : 1.) (Unit load 1000 lbs.) The influence line is *as′b*. Since the ordinate at any point represents the bending moment for segment *as* when a unit load of 1000

lbs. is at that point, the ordinate at any load multiplied by that load in thousands of pounds will equal the bending moment in pound-feet for segment *as* due to that load. The total bending moment will equal the algebraic sum of the seven such products that correspond to the seven loads.

Scale each of the seven ordinates. Assume that the values thus found are those shown in the figure. Then the total bending moment will be:

$$M_B = 1 \times 15 + 5 \times 30 + 7.5 \times 30 + 10 \times 30 + 7.5 \times 30 + 3 \times 19.5 + 0.5 \times 19.5$$
$$M_B = 1 \times 15 + 30 \times 30 + 3.5 \times 19.5 = 983250 \text{ lb.-ft.}$$

1. *Note:* In either Fig. 277 (*a*) or Fig. 277 (*b*), the loads on any portion of the beam for which the influence line is straight, as, for example, the loads on the portion *sb*, may be replaced by the resultant of those loads. The magnitude of this resultant multiplied by the value for the ordinate at the line of action of the resultant (at the center of gravity of the loads replaced) will equal the sum of the products obtained by multiplying each individual load by its ordinate.

2. *Exercise.* Assume the loads in Fig. 277 (*b*) to remain in the position shown, but draw an influence diagram for the bending moment for a section at load (3), i.e., for a left-hand segment 15 ft. long. Determine from this diagram the bending moment for the segment.

3. *Exercises.* (*a*) Draw to scale the diagram in Fig. 275 (*a*); determine the greatest positive and negative reactions at *e* due to two moving concentrated loads of 4000 lbs. each, spaced 3 ft. apart.

(*b*) Draw to scale the diagram in Fig. 275 (*b*); determine the greatest positive and negative reactions at *a*.

(*c*) Draw to scale the influence diagrams for shear and for bending moment for the beam shown in Fig. 276 (*b*); determine the greatest shear and greatest bending moment for segment *as* 10 ft. long, caused by two loads of 1500 lbs. each, spaced 8 ft. apart.

4. *Illustrative example.* Given: A simple beam *ab*, 24 ft. long, and a moving uniform load of 100 lbs. per lin. ft. A section is taken at *s*, 8 ft. from the left-hand support, i.e., segment *as* is 8 ft. long. (Fig. 278.) Required: The greatest shear for segment *as*; the shear for *as* when the load covers the entire beam; the greatest bending moment for *as*; the bending moment for *as* when the load covers only the segment *sb*.

The influence lines for shear and for bending moment for the segment *as* are shown in Fig. 278. The ordinates ss' and ss'' are respectively $\frac{2}{3}$ and $\frac{1}{3}$ lb. (Why?) The ordinate ss''' is $5\frac{1}{3}$ lb.-ft. (Why?) The greatest shear for segment *as* will occur when the load extends from end to end of segment *sb*. (Why?) The greatest bending moment for segment *as* will occur when the load extends from end to end of beam. (Why?) The following statements may easily be verified:

The greatest shear for the segment *as*, i.e., the shear when the load extends from end to end of the 16 ft. segment *sb*, is equal to the area of the triangle $ss'b$ multiplied by 100 lbs. or $(\frac{2}{3} \times 16 \times \frac{1}{2}) \times 100 = 533\frac{1}{3}$ lbs.

The shear for the segment *as* when the uniform load extends from end to end of beam is the area of the triangle $ss'b$ minus the area of the triangle $as''s$ multiplied by 100 lbs., or $(\frac{2}{3} \times 16 \times \frac{1}{2} - \frac{1}{3} \times 8 \times \frac{1}{2}) \times 100 = 400$ lbs.

The greatest bending moment for the segment *as*, i.e., the bending moment when the uniform load extends from end to end of beam, is equal to the area of the triangle $as'''b$ multiplied by 100 lb.-ft., or $(24 \times 5\frac{1}{3} \times \frac{1}{2}) \times 100 = 6400$ lb.-ft.

The bending moment for the segment *as* when the uniform load extends from end to end of the segment *sb* is equal to the area of the triangle $ss'''b$ multiplied by 100 lb.-ft., or $(5\frac{1}{3} \times 16 \times \frac{1}{2}) \times 100 = 4266\frac{2}{3}$ lb.-ft.

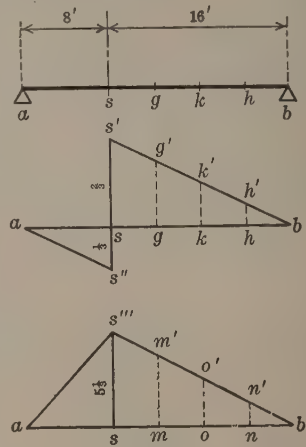


Fig. 278.

The above statements are for a particular segment of a particular beam. Formulate a *general* statement for each of the four statements, and give the corresponding *general* proof.

5. *Exercise:* If a uniform load covers a portion *gh* of the beam (Fig. 278) prove that:

(*a*) The shear for the segment *as* is equal to the distance *gh* multiplied by the ordinate kk' at the middle of that distance multiplied by the load per lin. ft., i.e., the area of the trapezoid $gg'h'h$ multiplied by the load per lin. ft.

(*b*) The bending moment for the segment *as* is equal to the distance $gh = mn$ multiplied by the ordinate oo' at the middle of that distance multiplied by the load per lin. ft., i.e., the area of the trapezoid $mnn'm'$ multiplied by the load per lin. ft.

6. *Exercise:* Draw to scale the influence diagrams shown in Fig. 276 (*a*); determine the greatest shear and the greatest bending moment for the segment *as* 7 ft. long; the uniform load is 600 lbs. per lin. ft.

7. **Influence Line Between Two Successive Panel Points.** In the discussion of influence lines thus far, loads have been treated as if they were applied directly to the beam. In through bridges the loads really take effect at panel points only, i.e., at points where floor beams are connected to the girders or trusses. It is necessary, therefore, to consider the effect of a unit load placed anywhere between two successive panel points.

8. *Influence line for shear for a beam with panels.* In Fig. 279 (*a*) is shown a structure with five equal panels. Let it be required to plot the influence line for shear for the segment *ab*. (The shear for any segment extending from *a* to any point between *b* and *c* will be the same as the shear for segment *ab*, provided the load at *b* is assumed to act on segment *ab*.) (Why?) The influence line for any position of the unit load on the portion

fc is fc' plotted as if the structure were a simple beam according to the method of 273 : 12. (Why?) For any position of the unit load on the portion ab the influence line is ab' drawn parallel to fc'. (Why?) For any position of the unit load W between b and c, the influence line is a *straight* line between b' and c'. This last statement may be proved in two ways.

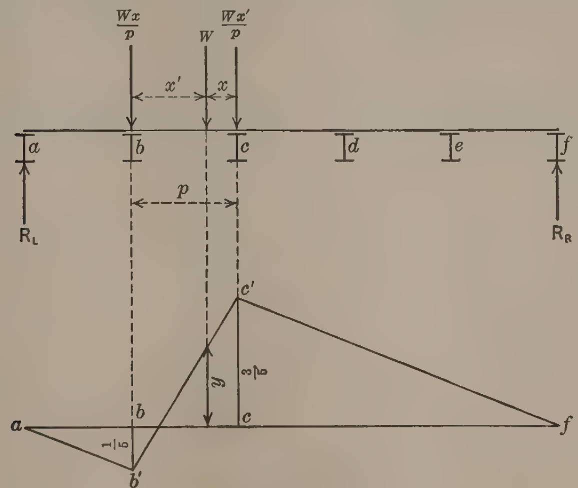


Fig. 279 (*a*).

1. First proof: When the unit load W is between b and c it takes effect at floor-beam joints b and c, and hence it may be replaced by two forces $\frac{Wx}{p}$ and $\frac{Wx'}{p}$ acting at b and c, respectively. The shear for segment ab (or for *any* section between b and c) is:

$$V = R_L - \frac{Wx}{p} = \left(\frac{3}{5}\frac{Wx'}{p} + \frac{4}{5}\frac{Wx}{p}\right) - \frac{Wx}{p}.$$

This is an equation of a straight line. When $x = 0$ and $x' = p$, $V = \frac{3}{5} W$ which is the ordinate cc'; when $x' = 0$ and $x = p$, $V = -\frac{1}{5} W$ which is the ordinate at b. Hence the straight line $b'c'$ is the influence line for a unit load placed anywhere between b and c.

2. Second proof: If $\frac{Wx}{p}$ is the load at b, then $-\frac{Wx}{p} \times bb'$ = shear for segment ab due to that load; similarly $\frac{Wx'}{p} \times cc'$ = shear for ab due to load $\frac{Wx'}{p}$ at c. The total shear for ab due to both loads is $\frac{Wx'}{p} \times cc' - \frac{Wx}{p} \times bb'$. This is an equation of a straight line from b' to c'. Moreover, $W \times y = \frac{Wx'}{p} \times cc' - \frac{Wx}{p} \times bb'$ = shear for segment ab when the unit load W is any distance x from c. This is a general proof regardless of the number of panels, therefore:

3. *An influence line for shear is always straight between panel points (floor-beam joints) of a structure.*

4. *Influence line for bending moment for a beam with panels.* If the center of moments is in a vertical line through a floor-beam joint of the

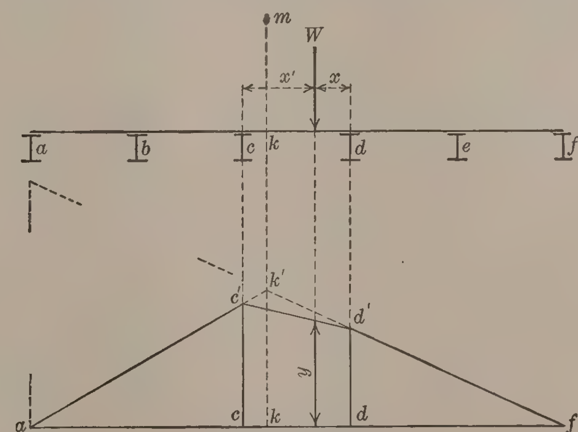


Fig. 279 (*b*).

girder or truss, the structure may be treated as a simple beam (171 : 12), and the other panel points may be ignored in drawing influence lines for bending moment. If the vertical line through the center of moments falls *between* floor-beam joints, the construction is slightly modified as follows: In Fig. 279 (*b*) let m represent a center of moments anywhere in a vertical

line which lies between floor-beam joints c and d. The corresponding segments will be ak and kf. The influence line $ak'f$ is plotted for the segment ak by the method used for a simple beam of length af. (274 : 1.) If the unit load is anywhere on the segment fd, the corresponding influence line is fd', and if it is anywhere on the segment ac, the corresponding influence line is ac'. By a proof exactly like the second proof just given for shear, it may be shown that if the unit load W is anywhere in the panel cd, the corresponding ordinate y is given by the equation:

$$y = \left[\frac{Wx}{p} \times cc' + \frac{Wx'}{p} \times dd'\right] \div W.$$

This is an equation of a straight line from c' to d'. The bending moment for W, a distance x from d, is $W \times y$, or the unit load multiplied by the ordinate to the influence line $c'd'$.

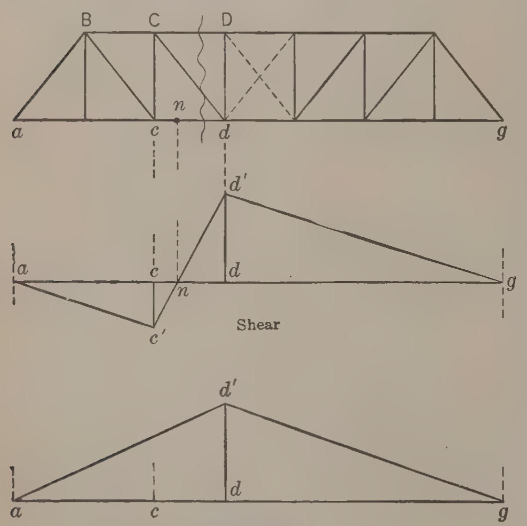


Fig. 280 (a).

1. *Illustrative example.* Given: The seven-panel Pratt truss shown in Fig. 280 (a). Required: To draw the influence lines that will show the positions of a uniform moving load that will result in maximum stresses in members Cd and CD.

The stress in Cd may be calculated directly from the shear for segment $aBCc$ (194 : 5); therefore, in order to cause the maximum stress in Cd, the load should be in such a position as to cause the greatest shear for segment $aBCc$. From the influence diagram for shear in Fig. 280 (a), it is seen that any load between g and n will cause positive shear, but that any load between n and a will cause negative shear. Hence for the greatest positive shear (and for the corresponding maximum tension in Cd) the uniform load should extend from g to the neutral point n. If it is desired to determine this shear from the diagram, it will be equal to the area of the triangle $nd'g$ multiplied by the live load per linear foot. (278 : 4.)

The stress in CD may be calculated directly from the bending moment when d is the center of moments (196 : 3); therefore the position of the load should be such as to cause this bending moment to be as great as possible. From the influence diagram for bending moment in Fig. 280 (a), it is seen that a load anywhere between a and g will cause a positive bending moment; therefore, for the greatest positive bending moment (and for the corresponding maximum stress in CD), the load should extend from g to a, i.e., it should cover the structure from end to end. Note that the influence line for bending moment is exactly the same as that for a segment ad of a simple beam. (What area could be used in determining the bending moment?)

2. *Illustrative example.* Given: The seven-panel Warren truss shown in Fig. 280 (b). Required: To determine from influence lines the positions of a uniform moving load that will result in maximum stresses in members cD and cd.

The influence diagram for shear for segment $aBCc$ is exactly the same in character as the influence diagram for shear in the preceding illustrative example. For the maximum compression in cD, the uniform load should extend from g to n, and for the maximum tension, it should extend from a to n. (Why?)

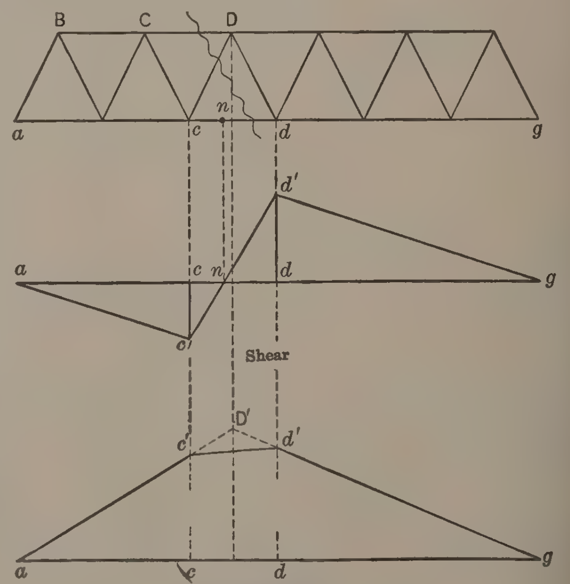


Fig. 280 (b).

The influence diagram $ac'd'g$ in Fig. 280 (b) for bending moment for point D, the center of moments used in finding the stress in cd, differs in character from the influence diagram in Fig. 280 (a) for bending moment for center of moments at d. This is because the point D in Fig. 280 (b) falls *between* panel points whereas the point d in Fig. 280 (a) falls *at* a panel point. Nevertheless, the conclusion is the same, namely, that for the greatest bending moment (and therefore for the maximum stress in cd) the load

should cover the structure from end to end. If it is desired to calculate the bending moment from the influence diagram, it will be equal to the area of the polygon $ac'd'g$ multiplied by the live load per linear foot.

1. *Note:* The position of the load that will result in maximum bending moments, as just determined in the two illustrative problems, namely, that the load should extend from end to end of the structure, is in accord with the general rule for placing uniform moving load in calculating live-load stresses in chord members. The position determined for the greatest shear, namely, that the load should extend from one support to the neutral point, is correct, and when the exact method of loading (264 : 6) is used, this principle is followed in placing uniform live load where it will cause the maximum stress in a web member; when the conventional method is used, the principle is modified slightly.

2. *Exercise.* Draw, superimposed on each other and with a common base line, the six influence diagrams for shears for the six left-hand segments that correspond to sections through the six panels in Fig. 280 (a).

3. **Influence Table.** A single unit live load as it moves from one end to the other of a span may be conceived as applied at each of the panel points in succession. For each of these positions, there will be a corresponding live-load stress in each member of the truss. These stresses may be tabulated in such a way that the table will show the change in the live-load stress in each member as the load moves across the span. Such a table is called an **influence table.** An example of an influence table will be found on page 312.

SUMMARY OF THE CHAPTER

4. An influence diagram is one that shows the *variation* in magnitude and algebraic sign of some function of a unit load as that load moves from one end of a structure to the other. The most common functions of the load for which influence lines are drawn are reactions, shears, and bending moments; they are also sometimes drawn for such functions as the stress in a given truss member or the deflection of a given point of the truss. (273 : 2.)

5. An influence line shows the variation due to a *single* moving unit load — not to two or more loads. The unit load may be 1 lb., 10 lbs., 100 lbs., 1000 lbs., 1 ton, or any convenient unit.

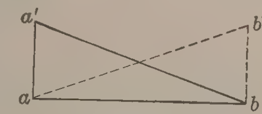
6. An influence line for a reaction is for the reaction at a *fixed point;* an influence line for shear or for bending moment is for the shear or the bending moment for a segment of *fixed length* not for different segments of different lengths. (273 : 6.)

7. The ordinate at any point of an influence diagram represents the magnitude of the varying function when the unit load is at that point. (273 : 7.)

8. The variation in lengths of ordinates is due solely to the variation in the position of the load. (273 : 8.)

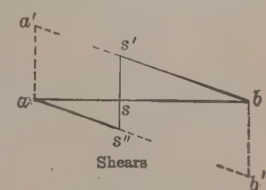
9. An influence line is a straight line or a series of straight lines and the corresponding curve is, therefore, one of the first degree.

10. An influence line for reactions at a given support a of a simple beam ab is a straight line $a'b$ whose ordinate aa' at the given support is equal to the unit load, the ordinate at the other support being zero. Similarly the line ab' is the influence line for reactions at the support b. (273 : 11.)



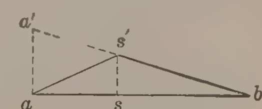
Reactions
Fig. 281 (a).

11. An influence line for shears for a given segment as of a simple beam ab is composed of three straight lines, namely: (1) the line bs' which coincides with a portion of the influence line for reactions at a; (2) the line as'' which is parallel to the line bs'; (3) the line $s's''$ which is a vertical line at the section s where the shear changes from positive to negative or *vice-versa.* (273 : 12.) The sum of the two ordinates ss' and ss'', i.e., the line $s's''$, is equal to the unit load.



Shears
Fig. 281 (b).

12. An influence line for bending moments for a given segment as of a simple beam ab is composed of two lines as' and $s'b$. The point s' is obtained as follows: Lay off aa' equal to as (to any convenient scale, not necessarily to the same scale used in laying off as). The point s' lies at the intersection of $a'b$ and a vertical line at the section s. (274 : 1.)



Bending Moments
Fig. 281 (c).

1. When the loads on a simple beam or truss are applied at panel points only, an influence line for shears for a given left-hand segment which ends anywhere between two given panel points c and d is composed of three lines, namely: (1) bd' which coincides with a portion of the influence line for reactions at a; (2) ac' which is parallel to bd'; (3) the line $c'd'$ which lies between vertical lines through the two given panel points c and d. The point n, at which $c'd'$ crosses the base line, is the **neutral point.** Any load to the right of n cause positive shear; any load to the left causes negative shear. The portions ac' and $d'b$ of the influence line coincide with the corresponding portions of an influence line for shears for a left-hand segment of a simple beam without panels. (278 : 8.)

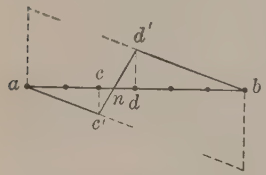


Fig. 282 (a).

2. When the loads on a simple beam or truss are applied at panel points only, an influence line for bending moments for a given left-hand segment may be drawn as follows: (1) When the given segment ends at a panel point, the influence line is identical with that for a corresponding segment of an ordinary beam of the same length as the given beam. (2) When the given segment ends at a section s between two given panel points c and d, temporary lines as' and bs' are drawn first, precisely as if as were the segment of an ordinary beam without panels. The point c' in which as' intersects a vertical line through c and the point d' in which bs' intersects a vertical line through d are then joined. The complete influence line is $ac'd'b$, and the portion $c'd'$ is the only portion that does not coincide with the corresponding influence line for bending moments for a simple beam without panels. (279 : 4.)

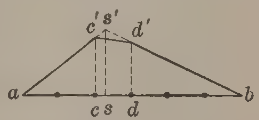


Fig. 282 (b).

3. *The principal use of an influence line* is to show the position in which a moving load, concentrated or uniform, must be placed in order to cause a required reaction, shear, bending moment, or stress. The required position is often evident from inspection of the influence line, but, when it is not, it may be determined from the geometrical properties of the influence line. (276 : 2.) A second but less common use of an influence line is to determine graphically the magnitude of a reaction, shear, bending moment, or stress, once the moving load has been placed in the correct position. When an influence line is to be used solely for determining the position of a load it may be sketched free-hand, but when it is to be used to obtain the magnitude of a function it must be plotted to scale.

4. *The magnitude of the reaction* at a given support for a given concentrated load in any position may be found from the influence line for reactions at that support by multiplying the magnitude of the load by the value of the ordinate which corresponds to the position of the load. If there are two or more loads, there will be two or more such products and the total reaction will be equal to the algebraic sum of these products. For a uniform load, the total reaction will be equal to the load per unit of length multiplied by the area that is included between the portion of the base line covered by the uniform load and the corresponding portion of the influence line.

5. *The magnitude of the shear or bending moment for a given segment and a given concentrated load* in any position may be found from the influence diagram for shear or bending moment for that segment, by multiplying the magnitude of the load by the value of the ordinate which corresponds to the position of the load. If there are two or more loads, there will be two or more such products, and the total shear or bending moment will be equal to the algebraic sum of these products. (277 : 1 to 3.)

6. *The magnitude of the shear or bending moment for a given segment and a given uniform load* in any position may be found from the influence diagram for shear or bending moment for that segment, by multiplying the uniform load per unit of length by the area that is included between that portion of the base line covered by the uniform load and the corresponding portion of the influence line. If there are two such areas, one positive and the other negative, the two corresponding products should be added algebraically. (278 : 4 and 5.)

7. *Influence lines for cantilever beams.* For influence lines for a simple cantilever beam see 275 : 7. For influence lines for a beam on two supports with one cantilever end see Fig. 295 (c).

CHAPTER XIX

REACTIONS, SHEARS, AND BENDING MOMENTS DUE TO MOVING LOADS

Maximum reactions, shears, and bending moments for moving loads occur when those loads are in certain positions, and these positions are determined in accordance with a few simple fundamental principles. These fundamental principles, in so far as they apply to simple types of loading on simple types of beams, are explained in this chapter. The more complex systems of concentrated loads, such for example, as a system of locomotive wheel loads, are treated in CHAPTER XXII.

1. **INTRODUCTORY.** All loads treated in this chapter are understood to be moving loads. Only the simplest form of moving loads are included, namely, a single concentrated load, two concentrated loads a fixed distance apart, and a uniformly distributed load. Unless otherwise specified, shears and bending moments are for *left-hand* segments, and reactions are those at *left-hand* supports. For example, maximum positive shear or maximum negative shear means the maximum positive or maximum negative shear for a left-hand segment. (176: 8 and 9.)

2. The terms "maximum shear" and "maximum bending moment" are ambiguous — they may mean the maximum shear and maximum bending moment for a *given segment* or section, or they may mean the maximum shear and the maximum bending moment that can occur, regardless of where the section is taken. In the first case, only the position of the load must be determined before the shear or bending moment is calculated, but in the second case, both the position of the load and the position of the section must be determined. For example, let a single concentrated load move across a beam *ab*, 20 ft. long. (Fig. 283.) A section is taken at *s*, 6 ft. from *a*. It is required to determine the maximum shear for segment *as*, and also the maximum of all shears. The greatest shear for *as* will occur when the load is on segment *sb* indefinitely close to *s*, but the maximum of all shears will occur when the section is taken indefinitely close to *a* and the load is indefinitely close to that section. Similarly, the greatest bending moment for segment *as* will occur when the load is at *s*, but the maximum of all bending moments will occur for a section at *c*, the center of the beam, when the load is at *c*. This example illustrates the importance of distinguishing between the greatest shear or bending moment for a given section and the maximum of all shears or of all bending moments. The term "maximum of maximums" or the "absolute maximum" is sometimes used in the latter case. To avoid ambiguity, the terms **greatest shear** and **greatest bending moment** will be used throughout this chapter when a given segment is under consideration, and the terms **maximum shear** and **maximum bending moment** will be reserved, respectively, for the maximum of all shears and the maximum of all bending moments due to a given load.

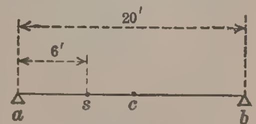


Fig. 283.

3. Once the position of any moving load has been determined and the load has been placed in that position, the load becomes static, and the methods of determining reactions, shears, and bending moments are exactly the same as those explained for dead loads in **PART II,** CHAPTERS XII and XIII. In most cases, therefore, it will be sufficient in this

chapter to show how to determine the position of a moving load in order to obtain a required result without giving the calculations which would follow in getting that result. The aim all through the chapter should be to study guiding principles or **criteria** for placing loads rather than to study methods of calculating reactions, shears, and bending moments once the loads are placed.

1. Influence lines are used throughout the chapter because the criteria for placing loads in order to obtain the greatest reaction, the greatest shear for a given segment, and the greatest bending moment for a given segment may usually be derived from mere inspection of corresponding influence lines. These same criteria may also be derived algebraically.

2. *Note:* The use of influence lines in determining positions of loads was explained in the preceding chapter. (274 : 7.) The assumption that the student has mastered that chapter and understands the illustrative examples on pages 275 and 276 permits many statements of principles to be made in this chapter without further explanation or proof.

3. The criteria for placing loads in order to obtain the maximum of all shears and the maximum of all bending moments are most easily derived, as a rule, by algebraic methods. In many cases, the derivation of these criteria is left to the student as a simple but valuable exercise.

4. *Note:* Instead of attempting to memorize criteria for positions of loads, it is better in every case to seek the reason *why* a load should be placed in a certain position in order to obtain a desired result. In this connection, influence lines, shear curves for moving loads, and bending moment curves for moving loads are helpful. Once the underlying reason for any criterion is understood, it is easy to recall the criterion whenever it is needed.

5. *Notation and algebraic signs.* The notation and the system of algebraic signs explained in 176 : 6 will be used throughout this chapter.

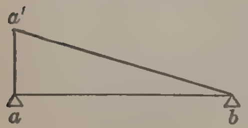


Fig. 284 (*a*).

6. **Maximum Reactions Due to Live Loads.** The influence line for the reaction at the left-hand support of a simple beam *ab* is shown in Fig. 284 (*a*). (281 : 10.) From inspection of this influence line the positions of moving loads which will result in the greatest reactions at *a* will be evident.

7. *A single concentrated live load.* The greatest reaction at *a* will be when the load is at support *a* as shown in Fig. 284 (*b*).

Fig. 284 (*b*).

8. *Two equal concentrated live loads.* The greatest reaction at *a* will be when one load is at support *a* and the other is as near to that support as the fixed distance *d* between the loads will permit as shown in Fig. 284 (*c*).

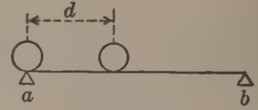


Fig. 284 (*c*).

9. *Two unequal concentrated live loads.* Let load *2* be larger than load *1*. (a) Assuming that the two loads can move on to the beam with either load *1* or load *2* in front, the greatest reaction at *a* will occur when the loads are in the position shown in Fig. 284 (*d*). (b) Assuming that load *2* is always the right-hand load, the greatest reaction at *a* may occur when load *1* is at *a* and load *2* is on the beam as near to *a* as the fixed distance between the loads will permit, or it may occur when load *1* is off the beam to the left and load *2* is at *a*, depending upon the relative magnitudes of the two loads. If load *2*, the larger load, is always a right-hand load, the *maximum* reaction will occur at *b*. (What would be the positions of the loads?)

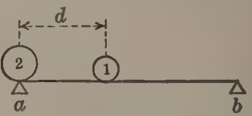


Fig. 284 (*d*).

10. *Note:* Once the concentrated loads have been placed in position, the reaction is calculated by the method explained in 139 : 8.

11. *Uniform live load:* (a) If the length covered by the uniform load is unrestricted, the greatest reaction at *a* will occur when the load covers the beam from end to end. (b) If the uniform load is restricted to a length *d* less than the length of the beam, the greatest reaction at *a* will occur when the load is as near to *a* as possible (Fig. 284 (*e*)).

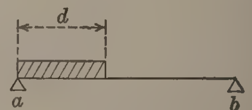


Fig. 284 (*e*).

12. *Note:* The method of calculating a reaction due to a uniform load of restricted length is explained in 141 : 2.

13. *General statement: The greatest reaction at a support of a simple beam will occur when as much of the moving load as possible is on the beam, and the center of gravity of the load on the beam is as near as possible to the support.*

(The case in which the larger of two loads is at the support and the smaller is off the beam is an exception to the statement that as much of the moving load as possible should be on the beam.)

1. **Maximum Shears Due to Live Loads.** The influence line for shear for a given segment *as* of a simple beam *ab* is shown in Fig. 285 (*a*). (281 : 11.) From inspection of this influence line the following statements may be verified:

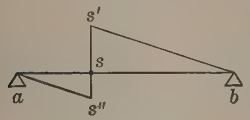


Fig. 285 (*a*).

2. *A single concentrated live load.* The greatest positive shear for segment *as* will occur when the load is on segment *sb* indefinitely close to *s*. For purposes of calculation it is assumed at *s*. (Fig. 285 (*b*).) The greatest negative shear for segment *as* will occur when the load is on segment *as* indefinitely close to *s*. Although in both cases the load is assumed for purposes of calculation to be at *s*, the positive and negative shears for segment *as* are not equal in magnitude. (Why?) (158 : 13.)

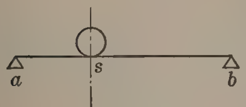


Fig. 285 (*b*).

3. *Note:* The greatest positive shear is equal to the reaction; the greatest negative shear is equal to the reaction minus the load.

4. *Two equal concentrated live loads.* The greatest positive shear for segment *as* will occur when both loads are on segment *sb*, one load indefinitely close to *s* and the other as near to *s* as the fixed distance *d* between loads will permit. (Fig. 285 (*c*).) The greatest negative shear for segment *as* will occur when both loads are on that segment with one load indefinitely close to *s*. When the length of the segment *sb* is less than the distance *d*, the greatest positive shear will be that for one load only, and when the length of the segment *as* is less than the distance *d*, the greatest negative shear will be that for one load only.

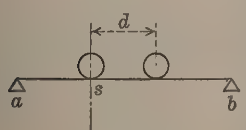


Fig. 285 (*c*).

5. *Note:* With two concentrated loads in the position for greatest positive shear for a given segment, that shear is equal to the corresponding reaction on that segment. With two concentrated loads in the position for greatest negative shear for a given segment, that shear is equal to the corresponding reaction on that segment minus the sum of the two loads. (158 : 8.)

6. *Two unequal concentrated live loads.* The greatest positive shear for segment *as* will occur when both loads are on segment *sb*, the larger load *2* indefinitely close to *s* and the smaller as near to *s* as the fixed distance *d* between loads will permit. (Fig. 285 (*d*).) The greatest negative shear for segment *as* will occur when both loads are on that segment with the larger load *2* indefinitely close to *s*, the loads being thus reversed in order. If the loads cannot be reversed in order and the smaller load *1* is always the right-hand load, the greatest negative shear for *as* may occur for the position shown in Fig. 285 (*d*), assuming, however, that load *2* is on segment *as* instead of on segment *sb*, or it may occur when both loads are on segment *as* with the smaller load indefinitely close to *s*. (If the length of the segment *as* is less than the fixed distance between loads and either load may be a right-hand load, what position of the loads will result in the greatest negative shear for segment *as*?)

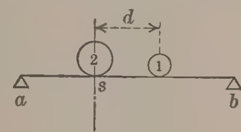


Fig. 285 (*d*).

7. *Note:* The magnitudes of the greatest positive and negative shears are determined as explained in the note for two equal concentrated loads. (285 : 5.)

8. *Uniform live load.* The greatest positive shear for segment *as* in Fig. 285 (*e*) will occur when the uniform load extends from end to end of segment *sb*, and the greatest negative shear for segment *as* will occur when the load extends from end to end of that segment. If the amount of uniform load is restricted so that it extends for a given distance *d*, the greatest positive shear for segment *as* will occur when the load is in the position shown in Fig. 285 (*f*). (What position of the restricted load will result in the greatest negative shear for segment *as*?)

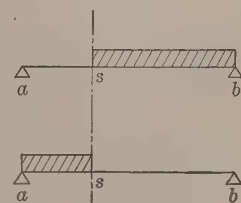


Fig. 285 (*e*).

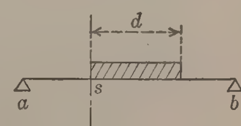


Fig. 285 (*f*).

9. *Note:* With the uniform load in the position for greatest positive shear for a given segment, that shear is equal to the corresponding reaction on that segment. With the uniform load in the position for greatest negative shear for a given segment, that shear is equal to the corresponding reaction on that segment minus the total uniform load on the segment. (160 : 4.)

1. *General statements: The greatest positive shear for a given segment of a simple beam will occur when there is no load on the given segment and as much load as possible is on the other segment, with the center of gravity of that load as near as possible to the section. The greatest negative shear for a given segment will occur when as much load as possible is on that segment, with the center of gravity of the load as near as possible to the section, and no load is on the other segment.*

2. *The greatest shear for a given section will occur when there is as much load as possible on the longer segment, with its center of gravity as near as possible to the section, and there is no load on the shorter segment. If the longer segment is the right-hand segment, the shear for the left-hand segment is positive; if the longer segment is the left-hand segment, the shear for the left-hand segment is negative.* (Why?)

3. *The maximum shear* which can occur is for a section indefinitely close to one of the supports (end shear). This shear will be equal to the reaction which varies as the position of the load changes. Hence the positions of the different types of loading which will result in maximum reactions will also result in maximum shears for simple beams. (284 : 6.) For the moving loads thus far considered, the maximum shear may occur for a section at either end of the beam, except in the case of two unequal loads which cannot be reversed in order; in that case, the maximum shear will occur for a section at the left-hand or right-hand support, according to whether the larger load is to the left or to the right of the smaller. If the shear is for a section at the right-hand support, it is negative for the left-hand segment (equal to the length of the beam).

4. *General statement: The maximum shear for a simple beam and the maximum reaction are equal in magnitude, and the position of a single concentrated load, two concentrated loads, or a moving uniform load that will result in one will result in the other.*

5. **Maximum Bending Moments Due to Live Loads.** In order to calculate the greatest bending moment for a given segment, it is necessary to determine first the position of the loading. In order to calculate the *maximum* of all bending moments for a given beam and loading, it is necessary first to determine at what point the section should be taken, and then to determine the position of the loading. (283 : 2.) In either case, if the loads are concentrated, *one of the loads will be at the section.*

6. For each type of loading treated, the criterion will be given (a) for the greatest bending moment for a given segment and (b) for the maximum of all bending moments.

7. The influence line for bending moment for a given segment *as* is shown in Fig. 286 (*a*). (281 : 12.) From inspection of this influence line the following statements may be verified:

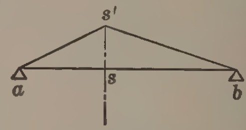


Fig. 286 (*a*).

8. *A single concentrated live load:* (a) For a given segment *as* the *greatest bending moment* will occur when the load is at *s*. (Fig. 286 (*b*).)

(b) The *maximum bending moment* will occur when $as = \frac{1}{2}\,ab$, i.e., when the section and the load are both at the center of the beam. The truth of this statement will become evident from a comparison of the influence line for a section at the center of the beam and an influence line for any other section drawn to the same scale. It is also evident at once from the curve for bending moments for a single concentrated moving load to be shown later.

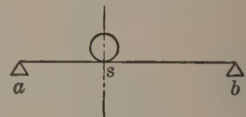


Fig. 286 (*b*).

9. *Note:* With a single concentrated load in the position for the greatest bending moment for a given segment, that bending moment is equal to the corresponding reaction on that segment multiplied by the length of the segment. (163 : 9.)

10. *Two equal concentrated live loads:* (a) For a given segment *as* the *greatest bending moment* will occur when one of the loads is at *s* and the other load is on the longer segment whether it be *as* or *sb*. (Fig. 286 (*c*).)

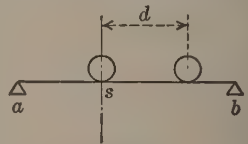


Fig. 286 (*c*).

(b) The *maximum bending moment* will occur when $as = \frac{1}{2}\,ab - \frac{1}{4}\,d$, where *d* is the fixed distance between the loads. (Fig. 286 (*d*).) It will also occur when $as = \frac{1}{2}\,ab + \frac{1}{4}\,d$, provided the right-hand load is at *s*. In general, the maximum bending moment will occur when the section and one of the loads at that section are a distance on one side of the center of the beam

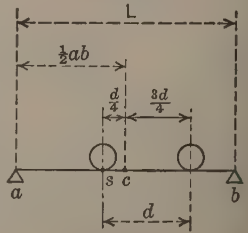


Fig. 286 (*d*).

equal to $\frac{1}{4}$ d, and the other load is on the other side of the center a distance $\frac{3}{4}$ d. (This statement is not evident from inspection of the influence line in Fig. 286 (a).) (Prove the statement algebraically by means of the equation for bending moment for two equal loads a fixed distance apart.)

(c) *Exception:* When the length of span L is less than the distance d between loads multiplied by 1.71, the maximum moment will occur when the section and one of the loads are at the center of the beam. (The other load will be off the beam.) The condition just given may be stated in two ways, namely, when $L < d \times 1.71$, or when $d > 0.586\,L$. (Prove.)

1. *Note:* With two concentrated loads in the position for the greatest bending moment for a given segment, that bending moment is equal in magnitude to the corresponding reaction on the *shorter* segment multiplied by the length of that segment. (163 : 9 and 11.)

2. *Two unequal concentrated live loads:* (a) For a given segment *as* the *greatest bending moment* will occur when the larger load is at *s* and the smaller load is on segment *sb* (Fig. 287 (a)), provided the loads can be placed in that order. (Would this statement be true if *as* were longer than *sb*?) If the left-hand load is the smaller and the order cannot be reversed, the greatest moment may still occur when the larger load is at *s*, even though this brings the smaller load on *as* or off the beam, particularly if one load is much larger than the other. Let one load be considered on one segment and the other load on the other segment. The following principle may then be used: When the weight of the concentrated load on the right-hand segment multiplied by the length of the left-hand segment is greater than the weight of the concentrated load on the left-hand segment multiplied by the length of the right-hand segment, the bending moment will be greatest when the right-hand load is at the section. (276 : 2 and 3.)

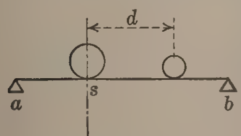


Fig. 287 (a).

(b) The *maximum bending moment* will occur when the length of segment *as* is equal to one-half of the length of the beam minus one-half of the distance of the larger load from the line of action of the resultant of the two loads, i.e., when the section and the larger load are both as far on one side of the center of the beam as the center of gravity of the two loads is on the other side. If in Fig. 287 (b) G represents the resultant of the two loads acting through the center of gravity, and g represents the distance from the larger load to the center of gravity, the maximum bending moment will result for either of the positions shown. Note that in the first position, the segment *as* is less, and in the second position, greater than half the length of the beam. The general statement just given for maximum bending moment holds true for two equal loads as well as for two unequal loads, and is most easily proved algebraically, although it can be proved by means of influence lines. (Prove the statement algebraically from the equation for bending moments for two unequal loads.)

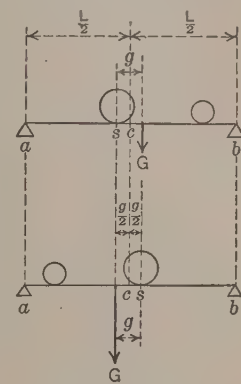


Fig. 287 (b).

(c) *Exception:* Let L = length of span; d the fixed distance between the two loads; r the ratio of the larger load to the sum of the two loads. When $L < d(1 + \sqrt{r})$, the maximum moment will occur when the section and the larger load are at the center of the beam. (Prove.)

3. *Note:* The magnitude of the bending moment due to two unequal concentrated loads is determined as explained in the note for two equal concentrated loads. (287 : 1.)

4. *Uniform live loads:* (a) For any segment *as* the *greatest bending moment* will occur when the uniform load extends from end to end of beam. If the load is restricted to a length d less than the length of the beam L and x is the distance from the left-hand support a to the center of the load, the greatest bending moment for a segment of length *as* will occur when $x = as + \dfrac{d}{2} - \dfrac{d \times as}{L}$. (Prove.) This is equivalent to the statement that the greatest bending moment for *as* will occur when the load on *as* divided by the length *as* is equal to the total load on the beam (length d) divided by the length of the beam L. (Why?)

(b) *Note:* If x represents the distance from the left-hand support to the beginning of the uniform load instead of to the center, the equation just given becomes:

$$x = as - \frac{d \times as}{L}.$$

(c) The *maximum bending moment* will occur for a segment equal in length to one-half the length of the beam, i.e., when the section is at the center of the beam and the uniform load extends from end to end of beam. (Prove.) If the uniform load is restricted so that it covers a distance d, the *maximum moment* will occur for a section at the center of the beam when the load extends equal distances on either side of the center.

1. *Note:* When the uniform load extends from end to end of beam, the bending moment is equal to the product of the lengths of the two segments multiplied by one-half the load per linear foot. (167:1.) When the load is restricted so that it partially covers the beam, the bending moment is determined by the method explained in 167 : 6.

2. *General statement: The greatest bending moment for a given section due to concentrated live loads will occur when a load is at the section. If there are two loads, one should be at the section (the larger load if the loads are unequal) and the other should be on the longer segment. For uniform live load, the greatest bending moment for a given segment will occur when the load covers the entire beam, or, if the load is restricted, when the portion of the load on either segment divided by the length of that segment is equal to the total load on the beam divided by the length of the beam.*

3. *General statement: The maximum bending moment for concentrated live loads will occur for a section at or near the center of the beam; for a single load, the section and load will be at the center; for two loads the section and one load (the larger if the loads are unequal) will be as far on one side of the center of the beam as the center of gravity of the two loads is on the other side. (For exceptions that are likely to occur in the case of short spans see* 286: 10 (c) and 287 : 2 (c).) *For uniform live load the maximum bending moment will occur for a section at the center of the beam when the load covers the entire beam, or, if the load is restricted, when it extends the same distance on each side of the center.*

4. *Note:* Many of the criteria or rules for placing live loads are known from common experience. Assume, for example, that a man walks across a beam. Since the greatest positive shear for any left-hand segment is equal to the reaction on that segment, the *maximum* shear will be for a segment for which the reaction is the greatest possible. Common experience tells us that this greatest possible reaction will occur when the man (a concentrated load) comes indefinitely near to the left-hand support and hence the corresponding end shear is the maximum positive shear. When the number of men is unlimited (uniform load), maximum shear will be the end shear equal to the reaction caused by as many men as it is possible to crowd on to the beam. Similarly, the greatest bending moment due to one man's weight will occur for a given segment when the man is at the corresponding section (the greatest deflection being wherever the man is), and the maximum bending moment will occur when both the man and the section are at the center of the beam. The maximum bending moment due to the weights of an unlimited number of men will occur for a section at the center when as many men as possible are on the beam. Similar analogies could be made in connection with many other criteria.

5. *Illustrative problem.* Given: A simple beam resting on supports 10 ft. apart; a single concentrated live load of 30,000 lbs. Required: (a) the maximum live-load reaction at the left-hand support and the maximum shear; (b) the greatest live-load shear for a left-hand segment 2.5 feet long; (c) the greatest live-load bending moment for the same segment; (d) the maximum live-load bending moment.

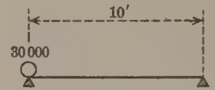
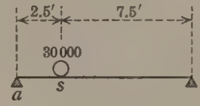


Fig. (*a*).



Figs. (*b*) and (*c*).

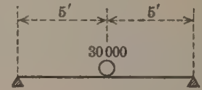


Fig. (*d*).

Fig. (*a*) 30,000 lbs. = Maximum reaction = maximum end shear. (284 : 7 and 286 : 3).
Fig. (*b*) 22,500 lbs. = 30,000 × 7.5 ÷ 10 = R_L = greatest shear for segment *as*. (285 : 2.)
Fig. (*c*) 56,250 lb.-ft. = R_L × 2.5 = 22,500 × 2.5 = greatest bending moment for segment *as*. (286 : 8.)
Fig. (*d*) 75,000 lb.-ft. = R_L×5 = 15,000 × 5 = maximum bending moment. (286 : 8 (b).)

6. *Illustrative problem.* Given: A simple beam resting on supports 12 ft. apart; two equal concentrated live loads of 30,000 lbs. spaced 5 ft. apart. Required: (a) the maximum live-load reaction and end shear; (b) the greatest live-load shear for a left-hand segment 3 ft. long; (c) the greatest live-load bending moment for the same segment; (d) the maximum live-load bending moment.

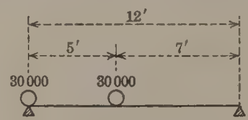
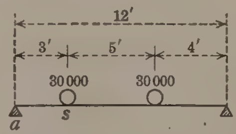


Fig. (*a*).



Figs. (*b*) and (*c*).

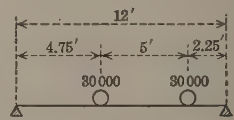


Fig. (*d*).

Fig. (*a*) 47,500 lbs. = R_L = 30,000 + (30,000 × 7) ÷ 12 = maximum reaction = maximum end shear. (284 : 8.)
Fig. (*b*) 32,500 lbs. = R_L = 30,000 × (9 + 4) ÷ 12 = greatest shear for segment *as*.
Fig. (*c*) 97,500 lb.-ft. = R_L × 3 = 32,500 × 3 = greatest bending moment for segment *as*. (286 : 10.)
Fig. (*d*) 112,800 lb.-ft. = R_L × 4.75 = [30,000 × (7.25 + 2.25) ÷ 12] × 4.75 = maximum bending moment. (286 : 10 (b).)

1. *Illustrative problem.* Given: A simple beam resting on supports 8 ft. apart and the two concentrated live loads of the preceding problem. Required: The maximum live-load bending moment.

Since the length of the beam is less than the distance between the two loads multiplied by 1.7, the maximum bending moment will be for a section at the center of the beam when one of the loads is at the section and the other is off the beam. (286 : 10 (c).) This maximum moment is equal to $30{,}000 \div 2 \times 4 = 60{,}000$ lb.-ft.

2. *Illustrative problem.* Given: A simple beam resting on supports 20 ft. apart; two concentrated live loads of 35,000 and 25,000 lbs., respectively, spaced 5 ft. apart. Required: (a) The maximum live-load reaction and end shear; (b) the greatest live-load shear for a left-hand segment 8 ft. long; (c) the greatest live-load bending moment for the same segment; (d) the maximum live-load bending moment.

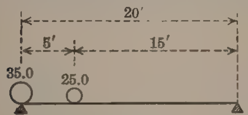
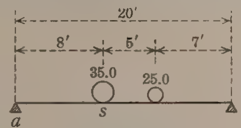


Fig. (*a*).



Figs. (*b*) and (*c*).

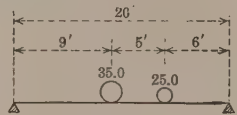


Fig. (*d*).

The forces will be expressed in kips, except in the final results.

Fig. (*a*) 53,750 lbs. $= R_L = 35 + (25 \times 15) \div 20$ = maximum reaction = maximum shear. (284 : 9.)

Fig. (*b*) 29,750 lbs. $= R_L = (35 \times 12 + 25 \times 7) \div 20$ = greatest shear for segment *as*. (285 : 4 and 158 : 6.)

Fig. (*c*) 238,000 lb.-ft. $= R_L \times 8 = 29.75 \times 8$ = greatest bending moment for segment *as*. (287 : 2 and 163 : 7.)

Fig. (*d*) 2.08 ft. = c. of g. from larger wheel $= (25 \times 5) \div (35 + 25)$.

Fig. (*d*) 241,000 lb.-ft. $= R_L \times 9 = [(35 \times 11 + 25 \times 6) \div 20] \times 9$ = maximum moment. (287 : 2 (b).)

3. *Illustrative problem.* Given: A simple beam resting on supports 30 ft. apart; a uniform live load of 6000 lbs. per linear foot. Required: (a) The maximum live-load reaction and shear; (b) the greatest live-load shear for a left-hand segment 10 ft. long; (c) the greatest live-load bending moment for the same segment; (d) the maximum live-load bending moment.

(*a*) 90,000 lbs. $= R_L = 30 \times 6000 \div 2$ = maximum reaction = maximum shear. (284 : 11.)

(*b*) 40,000 lbs. $= R_L = (20 \times 6000 \times 10) \div 30$ = greatest shear for 10-ft. segment. (285 : 8 and 160 : 4.)

(*c*) 600,000 lb.-ft. $= (6000 \div 2) \times 10 \times 20$ = greatest bending moment for 10-ft. segment. (287 : 4 and 167 : 1.)

(*d*) 675,000 lb.-ft. $= (6000 \div 2) \times 15 \times 15$ = maximum bending moment. (287 : 4 (b).)

4. *Exercise.* Determine the greatest negative shear for the given segment *as* in each of the four illustrative problems, beginning with 288 : 5.

5. **Comparison of Different Forms of Diagrams for Shear and Bending Moment.** A diagram for shear or bending moment for a given beam and a given load may be one of three general types depending upon whether the position of the load and the length of segment *both* vary or whether only *one* varies. The three possible combinations or general types of diagrams are:

(a) *Position of the load fixed, length of segment variable.* Shear and bending-moment diagrams for dead load (Chapter XIII) are of this character.

(b) *Position of the load variable, length of segment fixed.* Influence diagrams for shear and bending moment (Chapter XVIII) are of this character.

(c) *Position of the load and length of segment both variable.* Shear and bending-moment diagrams for moving loads are of this character, except the bending-moment diagram for a uniform live load that can cover the entire beam.

6. For a single concentrated load or for uniform load, the equation for shear or for bending moment which corresponds to any one of the three types of diagram is simple and easily derived. It is important, however, to keep clearly in mind the distinctions between the three types. For this reason, the three types of diagrams for the simplest and most common cases of shear and bending moment are shown on pages 290 to 293 in three parallel columns for comparison. In studying these contrasted diagrams, note first of all in each case what is variable and what is not. If the length of segment varies, it is represented by x; if the length of segment is constant, it is represented by a; if the position of a concentrated load varies, the load is shown by a broken circle and its distance from the left-hand support is represented by x'; if the position of the concentrated load is fixed, the distance from the support is represented by b; fixed uniform load is indicated by a cross-hatched rectangle; moving uniform load by broken horizontal lines in the rectangle.

7. *Note:* The equations given in connection with the diagrams on pages 290 to 293 are not intended as formulas though they may be used as such. Each equation should be regarded not only as the equation from which the corresponding diagram is plotted, but also as indicating the steps in the calculation of shear or bending moment as the case may be. Each equation should be checked by the student with this in view. Failure to master the diagrams and principles, simple as they are, is too often the cause of difficulty in mastering the entire subject of live-load stresses.

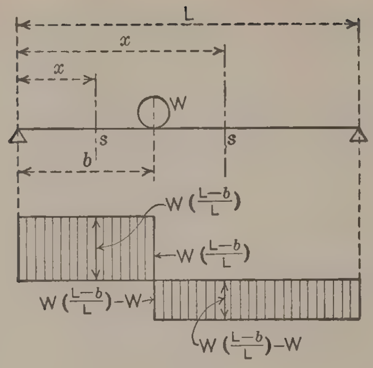


Fig. (*a*).

SHEAR DIAGRAM FOR A SINGLE FIXED LOAD

Fixed: The position of the load $= b$.
Variable: The length of the segment $= x$.

Equations: (1) When $x < b$,

$$V = W\left(\frac{L-b}{L}\right)$$

(2) When $x > b$,

$$V = W\left(\frac{L-b}{L}\right) - W$$

Ordinates: Any ordinate represents the shear for a left-hand segment which ends at that ordinate, when the load is a fixed distance b from the support, i.e., the section is at the ordinate but the load is not.

Maximum positive shear: When x, the length of the segment, is any length less than b, the distance from the support to the load, i.e., positive shear is constant.

Maximum negative shear: When x is any length greater than b, i.e., negative shear is constant.

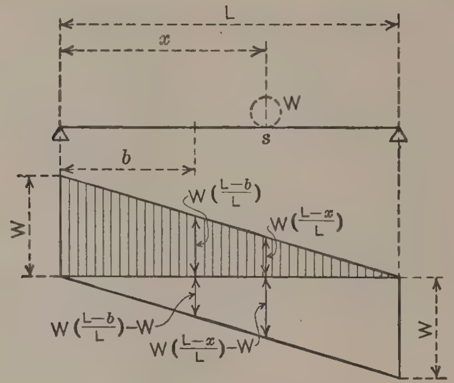


Fig. (*b*).

SHEAR DIAGRAM FOR A SINGLE MOVING LOAD

Variable: The length of the segment and the distance of the load from the support. (Both may be represented by x, since the greatest shear for any segment will occur when the load is practically at the section.)

Equations: When the load is considered on the right-hand segment,

$$V = W\left(\frac{L-x}{L}\right)$$

When the load is considered on the left-hand segment,

$$V = W\left(\frac{L-x}{L}\right) - W$$

Ordinates: Any ordinate represents the greatest shear for the left-hand segment which ends at that ordinate when the load is at the ordinate. If the load is considered as on the right-hand segment, the ordinate is positive (measured above the base line); if the load is on the left-hand segment, the ordinate is negative (measured below the base line).

Maximum positive shear: When $x = o$, i.e., when the section is indefinitely close to the left-hand support, and the load is on the right-hand segment indefinitely close to the section.

Maximum negative shear: When $x = L$, i.e., when the section is indefinitely close to the right-hand support, and the load is on the left-hand segment indefinitely close to the section.

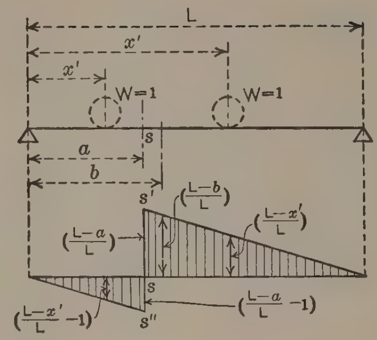


Fig. (*c*).

INFLUENCE DIAGRAM FOR SHEAR FOR A UNIT MOVING LOAD

Fixed: The length of the segment $= a$.
Variable: The position of the load $= x'$.

Equations: (1) When $x' > a$,

$$V = \left(\frac{L-x'}{L}\right)$$

(2) When $x' < a$,

$$V = \left(\frac{L-x'}{L}\right) - 1$$

Ordinates: Any ordinate represents the shear for the left-hand segment of length a when the unit load is at the ordinate.

Greatest positive shear for segment a: When x', the distance from the support to the load, equals a, the length of the segment.

Greatest negative shear for segment a: Also when $x' = a$. As the load passes from the right-hand to the left-hand segment, the shear, changing from positive to negative, passes through zero.

Comment: Assuming that all three diagrams are for the same beam and load, the shear scaled from an ordinate a distance b from the support in all three diagrams will be the same; namely, $W\left(\frac{L-b}{L}\right)$, but in the first two diagrams it is the shear for a segment of length b, while in the influence diagram it is the shear for a segment of length a instead of b.

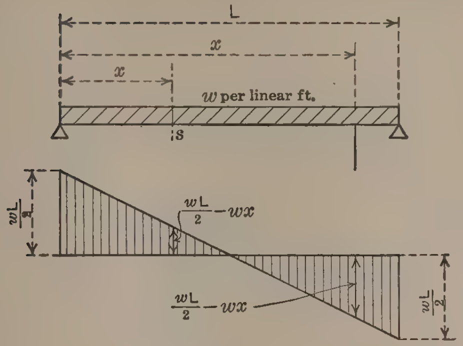


Fig. (*a*).

SHEAR DIAGRAM FOR FIXED UNIFORM LOAD

Fixed: The load over the length L.
Variable: The length of the segment $= x$.

Equation: $V = \frac{wL}{2} - wx$ (straight line), regardless of where the section is taken.

Ordinates: Any ordinate represents the shear for a left-hand segment that ends at that ordinate when there is a fixed uniform load covering the entire beam.

Maximum positive shear: When $x = o$, i.e., the end shear at the left-hand support. $V = \frac{wL}{2}$.

Maximum negative shear: When $x = L$, i.e., the end shear at the right-hand support. $V = -\frac{wL}{2}$.

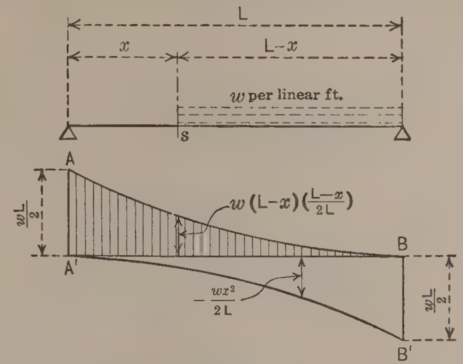


Fig. (*b*).

SHEAR DIAGRAM FOR MOVING UNIFORM LOAD

Variable: The position of the load and the length of the segment both may be represented by x since the length covered by the load should equal the length of one or the other of the segments in order to cause the greatest shear for the segment.

Equations: $V = w(L - x)\left(\frac{L - x}{2L}\right)$. (Equation of a parabola with vertex at the right end. AB is a portion of this parabola.) When the load is on segment x instead of $L - x$:

$$V = wx\left(L - \frac{x}{2}\right) \div L - wx = -\frac{wx^2}{2L}$$

(Equation of a parabola with vertex at left end. $A'B'$ is a portion of this parabola.)

Ordinates: Any ordinate to AB represents the greatest positive shear for the left-hand segment which ends at that ordinate, the load covering only the corresponding right-hand segment.

Maximum positive shear: When x, the length of the left-hand segment is zero and the load covers the beam, which in that case is the right-hand segment. $V = \frac{wL}{2}$

Maximum negative shear: When x, the length of the left-hand segment is equal to L and the load covers the beam, which in that case is the left-hand segment. $V = -\frac{wL}{2}$.

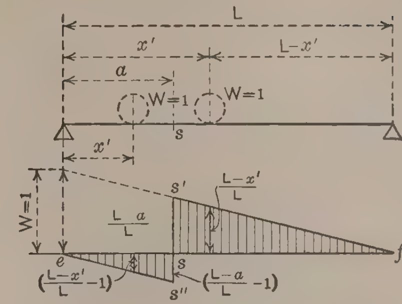


Fig. (*c*).

INFLUENCE LINE FOR SHEAR (UNIT MOVING LOAD)

Fixed: The length of the segment $= a$.
Variable: The position of the unit load $= x'$.

Equations: When $x' > a$,

$$V = \frac{L - x'}{L}$$

When $x' < a$,

$$V = \frac{L - x'}{L} - 1$$

Ordinates: Any ordinate represents the shear for the left-hand segment of length a when the load is at the ordinate.

Greatest positive shear for segment a: When $x' = a$. For the unit load this shear is represented by the ordinate $ss' = \frac{L - a}{L}$. For a uniform load it is equal to the area of the triangle $ss'f$ multiplied by the uniform load per unit of length.

Greatest negative shear for segment a: When $x' = a$. For a unit load this shear is represented by $ss'' = \frac{L - a}{L} - 1$. For a uniform load it is equal to the area of the triangle $ss''e$ multiplied by the uniform load per linear foot.

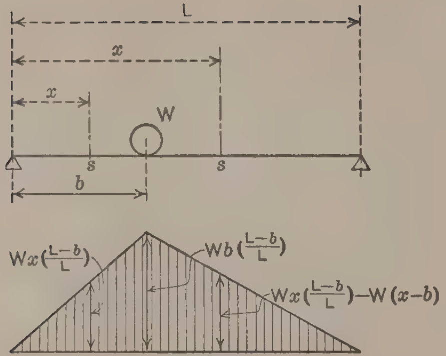


Fig. (*a*).

BENDING-MOMENT DIAGRAM FOR A SINGLE FIXED LOAD

Fixed: The position of the load $= b$.

Variable: The length of the segment $= x$.

Equations: (1) When $x < b$,

$$M_B = Wx\left(\frac{L-b}{L}\right)$$

(2) When $x > b$,

$$M_B = Wx\left(\frac{L-b}{L}\right) - W(x-b)$$

Ordinates: Any ordinate represents the bending moment for a left-hand segment which ends at that ordinate when the load is a fixed distance b from the support.

Maximum bending moment: When $x = b$, i.e., when the section is at the load. This moment is positive; there can be no negative bending moment for a left-hand segment under the given conditions.

$$M_B = Wb\left(\frac{L-b}{L}\right)$$

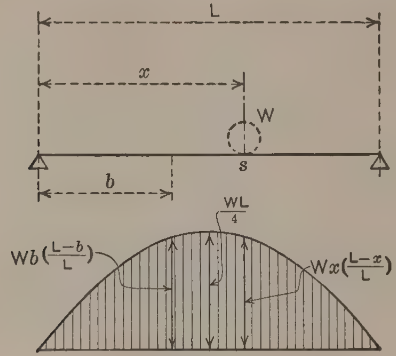


Fig. (*b*).

BENDING-MOMENT DIAGRAM FOR A SINGLE MOVING LOAD

Variable: The length of the segment and the distance of the load from the support; both may be represented by x, since the greatest bending moment for any segment will occur when the load is at the section.

Equation: $M_B = Wx\left(\frac{L-x}{L}\right)$, an equation of a parabola which passes through the points of support and has its vertex above the center of the beam.

Ordinates: Any ordinate represents the greatest bending moment for a left-hand segment which ends at that ordinate when the load is at the ordinate.

Maximum bending moment: When $x = \frac{1}{2} L$, i.e., when the section and the load are both at the center of the beam. $M_B = \frac{WL}{4}$. This moment is positive; there can be no negative bending moment for a left-hand segment under the given conditions.

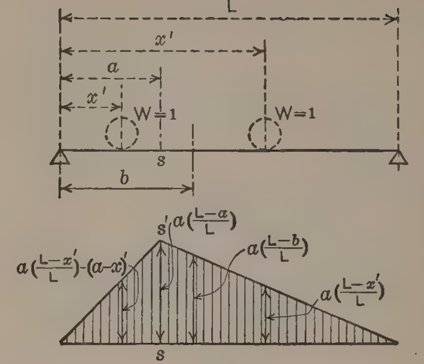


Fig. (*c*).

INFLUENCE DIAGRAM FOR BENDING MOMENT (UNIT MOVING LOAD)

Fixed: The length of the segment $= a$.

Variable: The position of the load $= x'$.

Equations: (1) When $x' > a$,

$$M_B = a\left(\frac{L-x'}{L}\right)$$

(2) When $x' < a$,

$$M_B = a\left(\frac{L-x'}{L}\right) - (a - x')$$

Ordinates: Any ordinate represents the bending moment for the left-hand segment of fixed length a when the unit load is at the ordinate.

Greatest bending moment for segment a: When $x' = a$, i.e., when the load is at the section.

$$M_B = a\left(\frac{L-a}{L}\right)$$

Comments: (1) The bending-moment diagram for a single *fixed* load is similar to the influence diagram for a single load, and if the distance b from the support to the load in the former were equal to the length a of the segment in the latter, the two diagrams would be exactly alike in outline, but the ordinates would have an entirely different significance.

(2) Assuming that all three diagrams are for the same beam and load, the bending moment scaled from an ordinate a distance b from the support will be the same for the first two; namely, $Wb\left(\frac{L-b}{L}\right)$, but not for the third. (Why?)

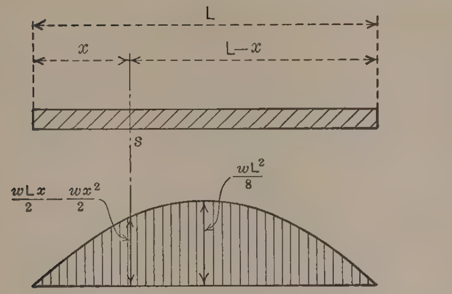


Fig. (*a*).

BENDING-MOMENT DIAGRAM FOR FIXED UNIFORM LOAD

Fixed: The load over length L.

Variable: The length of the segment $= x$.

Equation: $M_B = \frac{wLx}{2} - \frac{wx^2}{2}$ (equation of a parabola with vertex in a vertical line through the center of the beam).

Ordinates: Any ordinate represents the bending moment for the left-hand segment which ends at that ordinate when there is a fixed uniform load covering the entire beam.

Maximum bending moment: When $x = \frac{L}{2}$, i.e., when the section is at the center of the beam. $M_B = \frac{wL^2}{8}$. This moment is positive; there can be no negative bending moment for a left-hand segment under the given conditions.

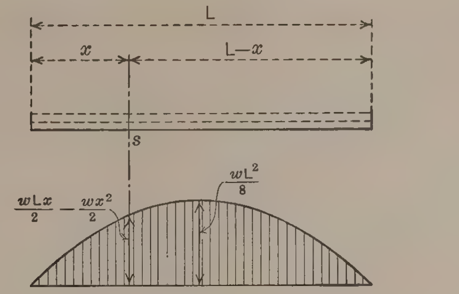


Fig. (*b*).

BENDING-MOMENT DIAGRAM FOR MOVING UNIFORM LOAD

Variable: The position of the load and the length of the segment. The greatest bending moment for any segment will occur when the load extends from end to end of the beam, hence although the position of the load can vary it actually does not. The varying length of segment is therefore represented by x and the curve is exactly the same as for fixed uniform load.

Equation: $M_B = \frac{wLx}{2} - \frac{wx^2}{2}$ (equation of a parabola with vertex in a vertical line through the center of the beam).

Ordinates: Any ordinate represents the greatest bending moment for the left-hand segment which ends at that ordinate. The moving uniform load will cover the entire beam, regardless of the length of segment.

Maximum bending moment: When $x = \frac{L}{2}$, just as for fixed uniform load. $M_B = \frac{wL^2}{8}$.

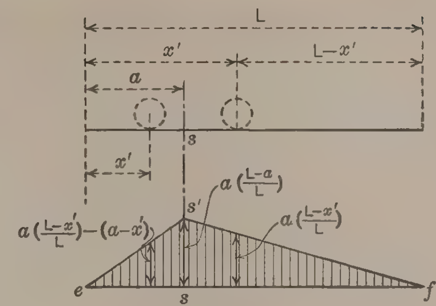


Fig. (*c*).

INFLUENCE LINE FOR BENDING MOMENT (UNIT MOVING LOAD)

Fixed: The length of the segment $= a$.

Variable: The position of the unit load $= x'$.

Equations: When $x' > a$, $M_B = a\left(\frac{L - x'}{L}\right)$

When $x' < a$, $M_B = a\left(\frac{L - x'}{L}\right) - (a - x')$

Ordinates: Any ordinate represents the bending moment for the left-hand segment of length a, when the unit load is at the ordinate.

Greatest bending moment: When $x' = a$. For the unit load this bending moment is represented by the ordinate $ss' = a\left(\frac{L - a}{L}\right)$. For a uniform load it is equal to the area of the triangle $es'f$ multiplied by the load per unit of length. This moment is positive; there can be no negative bending moment for a segment a under the given conditions.

1. **Simple Cantilever Beam — Shears Due to Live Loads.** The influence line for shear for a given segment *as* of a simple cantilever beam *ab* is shown in Fig. 294 (*a*). (275 : 7.) From inspection of this influence line the following statements may be verified:

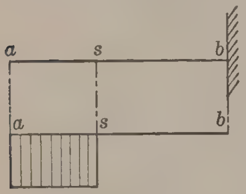


Fig. 294 (*a*).

2. *For a single concentrated live load* the shear for the segment *as* is constant, negative, and equal to the load as long as the load is anywhere on the segment *as*; when the load moves on to the segment *sb*, the shear for segment *as* becomes zero. (169 : 4.)

3. *For two concentrated live loads,* equal or unequal, the shear for the segment *as* is constant, negative, and equal to the sum of the two loads as long as the two loads are anywhere on the segment *as*; when one load moves on to the segment *sb*, the shear for segment *as* is decreased by an amount equal to that load, and when both loads move on to segment *sb* the shear for segment *as* becomes zero. (169 : 4.)

4. *For a uniform live load* the greatest shear for segment *as* will occur when the load covers the entire segment and this shear is equal to the total load on the segment; any uniform load on segment *sb* does not affect the shear for segment *as*. (169 : 5.)

5. *General statement: The greatest shear for any segment* **as** *extending from the unsupported end of a cantilever beam will occur when there is as much live load as possible on that segment, regardless of whether or not the load is concentrated or uniform, regardless of what the position of the load on the segment may be, and regardless of any load that may be on the other segment* **sb**. *The maximum shear will be when segment* **as** *is of sufficient length to carry all of the load or loads; for a single concentrated live load this may be any length between zero and the length of the beam* **ab**, *but for a uniform live load the maximum shear will occur when* **as** = **ab**, *i.e., when the section is indefinitely close to the support* **b**. *In any case, if the shear for a section indefinitely close to the support is not greater than that for any other section it is at least as great, and therefore when the maximum shear is required the section may always be taken at the support.*

6. **Simple Cantilever Beam — Bending Moments Due to Live Loads.** The influence line for bending moment for the segment *as* is shown in Fig. 294 (*b*). From inspection of this influence line the following statements may be verified:

7. *For a single live concentrated load* the greatest bending moment for the segment *as* will occur when the load is at *a* and the bending moment for segment *as* becomes zero when the load moves on to segment *sb*.

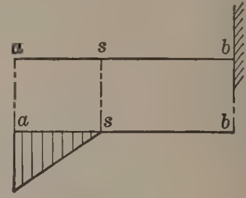


Fig. 294 (*b*).

8. *For two concentrated live loads* the greatest bending moment for the segment *as* will be when one load is at *a* and the other load is as near *a* as the fixed distance between the loads will permit. If the loads are unequal, the larger load should be at *a*. The bending moment for segment *as* decreases as the loads move toward the right and becomes zero when both loads are on segment *sb*.

9. *For a uniform live load* the greatest bending moment for the segment *as* will occur when the load covers the entire segment; any uniform load on segment *sb* does not affect the bending moment for segment *as*.

10. *Note:* The method of determining bending moments for simple cantilever beams for either concentrated loads or uniform load is explained in 169 : 4 to 8.

11. *General statement: The greatest bending moment for any segment* **as** *extending from the unsupported end of a cantilever beam will occur when as much live load as possible is on that segment, regardless of any load that may be on the other segment* **sb**; *if the live load or loads are concentrated, the center of gravity should be as near to the unsupported end as possible. The maximum bending moment will be for a section at* **b**, *i.e., when* **as** = **ab** *(just as for maximum shear), regardless of whether the live load is concentrated or uniform.* If *ab* were the overhanging or cantilever end of a beam that is supported at *b* and anchored to a second support to the right of *b*, the general statement just given would apply to the cantilever end *ab*.

12. *Illustrative problem.* Given: A simple cantilever beam 12 ft. long; two concentrated live loads of 2000 lbs. and 1000 lbs. respectively, spaced 4 ft. apart — either may be the left-hand load; a uniform load of 300 lbs. per linear foot. The concentrated loads and the uniform load cannot be on the beam at the same time. Required: (a) The maximum shear; (b) the greatest shear for a segment 3 ft. long extending from the unsupported end; (c) the greatest bending moment for the same segment; (d) the maximum bending moment.

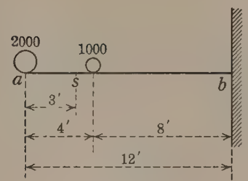


Fig. 295 (*a*).

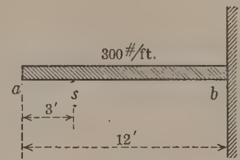


Fig. 295 (*b*).

Concentrated loads (Fig. 295 (*a*).)

(a) 3000 lbs. = 2000 + 1000 = maximum shear (section at *b*). (294 : 5 and 169 : 7.)
(b) 2000 lbs. = greatest shear for segment *as*. (294 : 3 and 169 : 6.)
(c) 6000 lb.-ft. = 2000 × 3 = greatest bending moment for segment *as*. (294 : 8.)
(d) 32,000 lb.-ft. = 2000 × 12 + 1000 × 8 = maximum bending moment (section at *b*). (294 : 11 and 176 : 3.)

Uniform load (Fig. 295 (*b*).)

(a) 3600 lbs. = 12 × 300 = maximum shear (section at *b*). (294 : 5 and 169 : 7.)
(b) 900 lbs. = 3 × 300 = greatest shear for section *as*. (294 : 4 and 169 : 6.)
(c) 1350 lb.-ft. = 3 × 300 × 1½ = greatest bending moment for section *as*. (294 : 9 and 169 : 5.)
(d) 21,600 lb.-ft. = 12 × 300 × 6 = maximum bending moment (section at *b*). (294 : 11.)

From a comparison of the results obtained from the concentrated loads with those obtained from the uniform load, it is evident that the maximum shear is caused by the uniform load and is equal to 3600 lbs., but that the other three required quantities are obtained from the concentrated loads. These three quantities are: Greatest shear for segment *as* = 2000 lbs.; greatest bending moment for segment *as* = 6000 lb.-ft.; maximum bending moment = 32,000 lb.-ft.

1. **Beam on Two Supports with One Cantilever End — Reactions, Shears, and Bending Moments Due to Live Loads.** A beam *ad* is supported at *a* and *b* and has an overhanging or cantilever end *bd*. From the influence lines in Fig. 295 (*c*), the following statements may be verified:

2. *For a single concentrated live load the greatest positive reaction at* a will occur when the load is at *a*, and the *greatest negative reaction at* a will occur when the load is at *d*. (If there are two concentrated loads, where should they be placed in order to cause the greatest positive reaction at *a*? The greatest negative reaction at *a*?)

3. *For uniform live load the greatest positive reaction at* a will occur when the load extends from *a* to *b*, and the *greatest negative reaction at* a will occur when the load extends from *b* to *d*.

4. *For a single concentrated live load the greatest positive shear for the segment* as will occur when the load is on *sb* indefinitely close to *s*, and the *greatest negative shear for the segment* as will occur when the load is either on *as* indefinitely close to *s* or is at *d* depending upon whether *as* is longer or shorter than *bd*.

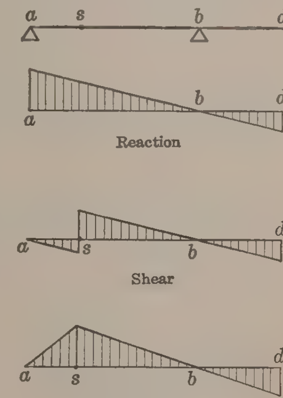


Fig. 295 (*c*).

5. *For uniform live load the greatest positive shear for segment* as will occur when the load extends from *s* to *b*, and the *greatest negative shear for segment* as will occur when the load extends from *a* to *s* and also from *b* to *d*.

6. *For a single concentrated live load the greatest positive bending moment for the segment* as will occur when the load is at *s*, and the *greatest negative bending moment for segment* as will occur when the load is at *d*. (Explain in what positions two concentrated loads a fixed distance apart should be placed in order to cause the greatest positive and the greatest negative bending moments for segment *as*.)

7. *For uniform live load the greatest positive bending moment for segment* as will occur when the load extends from *a* to *b*, and the *greatest negative bending moment for segment* as will occur when the load extends from *b* to *d*.

8. *The maximum positive shear* will occur when *as* equals zero (the section indefinitely close to *a*) and when the single concentrated live load is at *a*, or, if the load is uniform, when the load extends from *a* to *b*.

9. *The maximum positive bending moment* will occur when $as = \frac{1}{2}\,ab$ (when the section is half way between supports) and when the single concentrated live load is at the section, or, if the live load is uniform, when the load extends from *a* to *b*. The *maximum negative bending moment* will occur when $as = ab$ (section at *b*) and when the single concentrated live load is at *d*, or, if the live load is uniform, when the load extends from *b* to *d*. (These statements concerning maximum bending moments may be verified, not from influence lines, but from bending-moment diagrams.)

1. *Note:* The methods of calculating shears and bending moments for beams with overhanging ends was explained in 171 : 1 to 11.

2. *Exercise:* Draw the influence line for the reaction at b. Determine, both for a single concentrated live load and for a uniform live load, what position of the load will cause the maximum reaction at b, and show that this reaction for a single concentrated load is greater than the load itself.

3. *Exercise:* Assume s the section at any point between b and d, draw influence lines for shear and bending moment for the left-hand segment as, and from inspection of these influence lines formulate statements corresponding to those given in 295 : 2 to 295 : 9.

4. *Exercise:* A floor beam for a highway bridge is represented in Fig. 296 (a). There is a roadway 32 ft. wide and two sidewalks each 6 ft. wide. The beam is supported by the trusses at a and b, and it is extended as a cantilever on each side to support the sidewalks. Assume the live load to be a uniform load. Verify the following statements either by means of influence diagrams or by curves for reaction, shear, and bending moment, (sketched free-hand); use for any particular statement whatever diagram is most suitable:

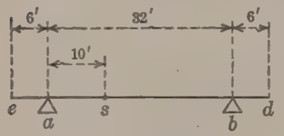


Fig. 296 (a)

(a) The maximum positive shear will be for a section at a when the roadway ab and the sidewalk ea are fully loaded but no load is on the sidewalk bd.

(b) The maximum positive bending moment will be for a section half way between a and b when the roadway is fully loaded but no load is on either sidewalk.

(c) The maximum negative bending moment will be for a section at a when the sidewalk ea is fully loaded, regardless of whether or not any load is on the roadway or on the other sidewalk. (Is there any other section for which the negative bending moment is equally great?)

5. *Exercise:* Given: The beam in Fig. 296 (a). Required: To draw the influence lines for (1) the reaction at a, (2) the shear for segment es, and (3) the bending moment for segment es, and from inspection of these influence lines, to formulate statements corresponding to those made for a beam with one overhanging end in 295 : 2 to 295 : 9.

6. *Exercise:* Explain how to determine the maximum positive shear, the maximum positive bending moment, and the maximum negative bending moment for the floor beam in Fig. 296 (a), once the uniform live load has been placed, in each case, in the correct position.

7. **Reactions, Shears, and Bending Moments When Live Loads Take Effect at Panel Points.** Even though the live load on a bridge may be uniformly distributed over the floor, it is transmitted to the floor beams in the form of concentrated loads at stringer connections, and finally reaches the truss in the form of concentrated loads at panel points. It becomes necessary, therefore, to consider in what respects the calculations for reactions, shears, and bending moments due to concentrated live loads thus applied at fixed points differ from corresponding calculations when live loads are applied directly to any and all points of a beam.

8. **Reaction at an End of a Beam Due to a Single Concentrated Live Load Between Panel Points.** A single concentrated load W between panel points c and d is distributed by the floor system to those panel points. (Fig. 296 (b).) The magnitudes of W_c and W_d, the fractional parts of W that are transmitted respectively to c and d, are the same as if W were applied to a simple beam supported at c and d. In determining reactions at a and g, it is immaterial whether these reactions are calculated from W as if it were applied *directly* to the beam ag, or from W_c and W_d. (Why?) (142 : 7.)

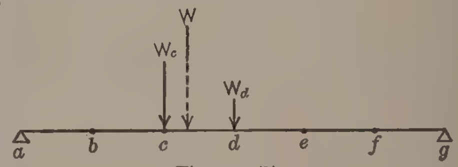


Fig. 296 (b).

9. *If a concentrated live load is considered as applied between panel points directly to a beam or truss, and the reactions at the ends of the beam or truss are calculated from the load on this assumption, the results will be the same as if the load is considered to be distributed proportionately to the two panel points, and the reactions are then calculated from these two fractional parts of the load.*

10. *Note:* It is evident from the preceding statement that in calculating reactions due to any number of concentrated loads, the method of procedure is the same for a beam or truss with panel points as it is for a beam without panel points, i.e., panel points may be ignored, just as if the concentrated loads were applied directly instead of through floor beams. The method of determining the *greatest* reaction at a support is, therefore, the same for a beam or truss with panel points as it is for a beam without panel points. In making these statements it is understood that there are end floor beams. (When there is no end floor beam, the stringers resting directly on the support, how should the statements be modified?)

11. **Reaction at an End of a Beam or Truss Due to Uniform Live Load Carried by a Floor System to Panel Points.** Let ag, Fig. 297 (a), represent a girder which supports floor beams at points a, b, c, d, e, f, and g. In determining a reaction for any position of the uniform

live load, it is immaterial whether the load is considered as applied directly to the beam or at panel points, provided the exact system of loading is used. For example, let a uniform load of 2000 lbs. per lin. ft. extend 72 ft. from the right-hand end. The exact loads in tons at panels point are those indicated. (Check these panel loads.) The reaction at a determined first as if the load were applied directly to the beam, and then from the actual concentrated loads is the same, namely, 21.6 tons, as is evident from the following equations:

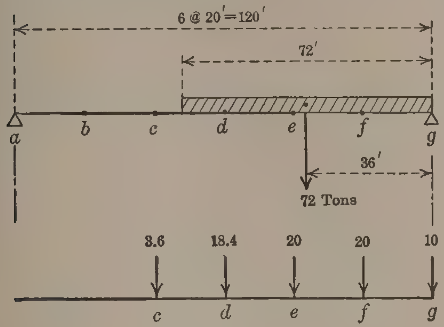


Fig. 297 (*a*).

$$R_L = 21.6 \text{ tons} = (72 \times 36) \div 120$$

$$R_L = 21.6 \text{ tons} = \frac{4}{6} \times 3.6 + \frac{3}{6} \times 18.4 + \frac{2}{6} \times 20 + \frac{1}{6} \times 20.$$

This result was to be expected since the force of 72 tons 36 feet from g is the resultant not only of the uniform load, but also of the actual concentrated loads brought by the floor beams to the panel points at c, d, e, f, and g. (Why?) If, however, *full* panel loads be assumed to be at d, e, f, and a half panel load at g, in accordance with the conventional method (264: 6) the reaction will be:

$$R_L = 20 \text{ tons} = 20\left(\frac{1}{6} + \frac{2}{6} + \frac{3}{6}\right)$$

The reaction is smaller from the conventional loads than from the exact loads, and it would appear that the conventional loads are *not* on the side of safety. Neither reaction, however, would be used in designing the support or any part of the structure, since neither reaction is the greatest that can occur at the support. The only use made of either reaction would be in calculating the shear from which the maximum stress in a web member is determined, and it will be shown later that this shear is greater for the conventional than for the actual loading. The *maximum reaction* at either support will, of course, occur when the live load extends end to end of beam in which case there is a full panel load at each of the intermediate panel points.

1. *Note:* Notice that the half panel load at a support does not affect the stresses in the members of a truss and may, therefore, be ignored in determining such stresses. (127 : 5.) Could it be ignored in designing the shoe or the support if there are end floor beams?

2. In all problems in this book in which the live load on a truss is a uniformly distributed live load, the conventional system of full panel loads will be used, and when the load covers only a portion of the span, the reactions on the unsymmetrically loaded truss may be determined by the method already explained in 142 : 3.

3. **Combined Reaction at Adjacent Ends of Two Successive Spans Due to Live Loads.** It frequently happens that it is necessary to determine the maximum combined reaction at the common support of adjacent ends of two successive spans. For example, let ab and cd in Fig. 297 (*b*) represent two stringers connected at b and c respectively, to the same floor beam. For all practical purposes, b and c coincide. It is desired to determine the maximum combined reaction R equal to the sum of the reactions R_R for the span ab and R_L for the span cd. This maximum combined reaction is equal in magnitude to the maximum concentrated load that the two stringers bring to the floor beam at that point — a load that may be needed in designing the floor beam. The combined reaction R will be a maximum for the following positions of live loads:

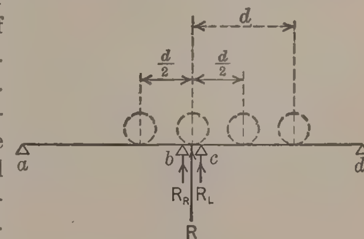


Fig. 297 (*b*).

(a) *Uniform live load.* When both spans are loaded from end to end.

(b) *Two equal concentrated live loads.* When one load is at the common point of support and the other load is on either span as near to that point as the fixed distance d between loads will permit; also when the two loads are equal distances from the support, one on one span, the other on the other. (Why?)

(c) *Two unequal concentrated live loads.* When the larger load is at the support and the other load is on either span; also when the two loads are so placed, one on one span and the other on the other, that the center of gravity of the loads is in a vertical line through the support. (Why?)

1. *Note:* The method of determining the maximum combined reaction when there are more than two concentrated live loads will be explained in CHAPTER XXII.

2. When, in any given case, live loads have been placed in the position that will result in the maximum combined reaction, the value of R_R may be calculated for the span *ab* from the load on that span, and the value of R_L for the span *cd* from the load on that span by the usual method (as if there were two separate problems), and the sum of the results will equal *R*. A load at the support *bc* may be considered as on neither span, in which case it should be added to the combined reaction due to other loads.

3. **Greatest Shear for a Given Segment of a Beam or Truss when Live Loads are Applied at Panel Points.** When a section is taken anywhere between two consecutive panel points of a beam or truss that is supported at each end, one segment extends from the left-hand panel point to the left-hand support, and the other segment from the right-hand panel point to the right-hand support. (184 : 7.)

4. Let it be required to determine the greatest shear due to uniform live load for a section taken anywhere between panel points *c* and *d* in Fig. 298. From the influence line for shear shown in the figure, it is evident that the greatest positive shear will occur when the load extends from *g* to the neutral point *n*, and that the greatest negative shear will occur when the load extends from *a* to *n*. Let l_1 represent the distance from *d* the right-hand end of the panel to the neutral point *n* and l_1' the distance from *c* to *n*; l_2 the length *gd* of the right-hand segment; l_2' the length *ac* of the left-hand segment; *p* the length of the panel *cd*.

$$l_1 = \frac{l_2}{l_2 + l_2'} p \quad \text{and} \quad l_1' = \frac{l_2'}{l_2 + l_2'} p.$$

When panel lengths are equal, the expressions for l_1 and l_1' become:

$$l_1 = \frac{n'}{n-1} p = \frac{l_2}{n-1} \quad \text{and} \quad l_1' = \frac{n''}{n-1} p = \frac{l_2'}{n-1}$$

in which n' is the number of panels in the length l_2, i.e., in the right-hand segment of the truss, n'' the number of panels in l_2', and n the number of panels in the entire truss. The expressions for l_1 and l_1' are general and may be used for determining the distance of the neutral point in any panel from an end of that panel.

5. *Note:* The point *n* may be determined graphically as follows: Draw a line through joints *a* and *C* and a line through joints *g* and *D*. Through the point of intersection of these two lines draw a vertical line; it will intersect *ag* at *n*.

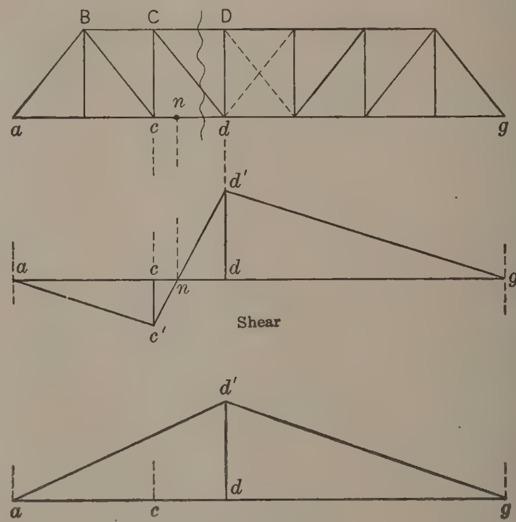


Fig. 298.

6. *To determine uniform-live-load shear by the exact method.* For the greatest positive shear for a section anywhere between *c* and *d* in Fig. 298, the uniform load will extend from *g* to *d* and will be distributed to the truss as follows: a full panel load at each of the intermediate panel points to the right of *d*, less than a panel load at *d*, and still less at *c*. The shear for the segment *aBCc* is equal to the reaction at *a* (determined as if there were no panel points) *minus the partial panel load at c*. Similarly, when the uniform load extends from *a* to *n*, the position for the greatest negative shear for the segment *aBCc*, there is a full panel load at the panel point to the left of *c*, but less than a panel load at *c*, and still less at *d*. The shear for the segment *aBCc* is equal to the reaction at *a* minus the sum of the partial panel load at *c* and the full panel load at the panel point to the left of *c*. (It is also equal in magnitude to the reaction at *g* minus the partial panel load at *d*.) The positive and negative shears

as just determined are the actual shears and the method used is called the **exact method.**

1. *To determine the uniform-live load shear by the conventional method.* For the greatest positive shear for the left-hand segment $aBCc$ in Fig. 298, it is assumed that there is a full panel load at each of the intermediate panel points of the right-hand segment including the point d, but that no load whatever is on segment $aBCc$. The shear is then equal to the reaction at a. This assumption is contrary to fact because there cannot be a full panel load at d unless there is a half panel load at c. (Why?) For the greatest negative shear for segment $aBCc$, it is assumed that there is a full panel load at each of the intermediate panel points of that segment including the point c, but that there are no loads whatever on the other segment. This also is contrary to fact. (Why?) The shear for segment $aBCc$ is equal to the reaction at a minus the sum of the panel loads on that segment. (It is also equal in magnitude to the reaction at g.)

2. *Methods of loading compared.* For the greatest positive shear, the loading is the same for both methods except that in the exact method there is a partial load at each of the points d and c, whereas in the conventional method there is a full panel load at d and no load at c. Similarly, for the greatest negative shear, the loading is the same for both methods, except that in the exact method there is a partial panel load at each of the points c and d, whereas in the conventional method there is a full load at c and no load at d.

3. *Note:* Some engineers regard the conventional method as unscientific and as conducive to slipshod analysis and computation. It will be shown later that the stresses obtained by this method are greater than those obtained by the exact method and therefore the relatively small discrepancies are on the side of safety.

4. *The live-load shear usually required* for a beam or truss with panel points is that due to a single concentrated load, or to two equal concentrated loads, or to a conventional uniform load, or to a series of locomotive concentrated loads. Locomotive loads will be treated in a separate chapter. A summary of the rules for placing loads in positions for shear will now be given.

5. Let ag in Fig. 299 represent a beam or truss with six panels. Let it be required to determine the greatest shear for a section anywhere between c and d, i.e., for a segment ac. From inspection of the influence line in Fig. 280 (a) and from the explanation of the conventional method given in 299 : 1, the following statements concerning the positions of live loads for the greatest shear for segment ac may be verified:

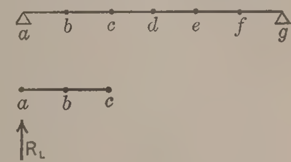


Fig. 299.

6. *Single concentrated live load.* Greatest positive shear when the load is at d, (shear equal to the corresponding reaction at a); greatest negative shear when the load is at c (shear equal to the corresponding reaction at g, but opposite in sense).

7. *Two equal concentrated loads.* The preceding statement holds true except that there will be a second load on the same segment with the first load as near to the first as the fixed distance between the loads will permit.

8. *General statement for greatest shear due to a single concentrated load: When a beam or truss supported at each end is divided into segments by a section anywhere between two consecutive panel points, the greatest positive shear (for the left-hand segment) due to a single concentrated live load will occur when that load is at the right-hand end of the panel cut by the section. (This shear will be equal to the reaction at the left-hand support.) The greatest negative shear (for the left-hand segment) will occur when the load is at the left-hand end of the panel. (This shear will be equal to the reaction at the right-hand support, but opposite in sense.)*

9. *Two equal concentrated loads.* The statements just given may be modified to apply to two equal concentrated loads by considering a second load to be on the same segment as the single concentrated load, as near to that load as the fixed distance between loads will permit.

10. *Conventional uniform live load (conventional panel loads).* Greatest positive shear when panel loads are at d, e, and f, but no loads at b or c; greatest negative shear when panel loads are at b and c, but no loads at d, e, and f. (The shear in each case is equal in magnitude to the reaction that acts on the unloaded segment.)

11. *General statement for greatest shear due to conventional uniform live load: When a beam or truss, supported at each end, is divided into two segments by a section anywhere between two consecutive panel points, the greatest positive shear (for the left-hand segment) that can be caused by conventional*

uniform live load will occur when there is a panel load at each of the intermediate panel points of the right-hand segment, but no loads on the left-hand segment. (This shear will be equal to the reaction at the left-hand support.) The greatest negative shear (for the left-hand segment) will occur when there is a panel load at each of the intermediate panel points of the left-hand segment, but no loads on the right-hand segment. (This shear will be equal to the reaction at the right-hand support, but opposite in sense.)

1. *Note:* The general rule for placing panel loads given in the last statement is sometimes stated less exactly as follows: *The greatest shear for any section between two consecutive panel points will occur when full panel loads are assumed to act at all panel points to the right or to the left of the section.*

2. *Note:* It is well to remember that when there are loads on one segment and no loads on the other, the magnitude of the shear is equal to the reaction that acts on the unloaded segment. (Why?)

3. *Note:* As explained in 299 : 1, it is impossible for uniform live load to be in such a position that there will be a panel load at each of the intermediate panel points of one segment but no loads whatever on the other segment. The conventional method gives results therefore that are only approximately the same as those obtained by the exact method. A comparison will now be made of the greatest shear for a given segment as obtained first by the exact method and then by the conventional method of loading. In Fig. 297 (*a*) the uniform load extends 12 feet into the panel *cd*. This is the position of the load that will cause the greatest shear for segment *ac* if the exact method of loading is followed. (Why?) (298 : 4.) The corresponding reaction R_L is 21.6 tons, and the corresponding shear for *cd* is $21.6 - 3.6 = 18.0$ tons. (Note that the shear cannot be calculated as if *ag* were a beam without panels.) When the conventional panel loads are used, the reaction R_L is 20 tons, and this is the shear for segment *ac*. The shear due to conventional loads is, therefore, greater than the shear due to the exact loading, and the corresponding stress in a web member is greater than any that actually occurs — it is therefore on the side of safety. Show that this holds true as a general proposition.

4. *Maximum shear.* The criteria thus far given for shear due to panel loads are for the *greatest* shear for a *given segment*. The *maximum* shear will occur when the section is taken through an end panel, and corresponds to end shear for a simple beam. The left-hand segment then becomes merely the end point of the beam or the end joint of the truss. Following the general rule for greatest shear for a segment, there should be a panel load at each panel point of the other segment, and this means that *all* intermediate panel points should be loaded, i.e., the live load extends from end to end of beam or truss. The half panel load at the end joint may be ignored in calculating the reaction and the shear, unless the reaction or the shear is to be used in determining the stress in some part or member that transmits the half load to a support, such, for example, as the vertical member at the end joint of a deck truss. (127 : 5.)

5. **Greatest Bending Moment for a Given Segment of a Beam or Truss When Live Loads Are Applied at Panel Points.** In Fig. 300 is shown a Warren truss without verticals; also a simple beam $a'e'$ of the same length of span but *without panels*. Let W be a single concentrated live load that rests on the floor of the bridge *between* the two floor beams at c and d, and let W_c and W_d be the two corresponding fractional parts of W that these two floor beams bring to the truss at joints c and d. Let c' and d' be points (but not panel points) on the beam, and W' a load equal to W. Assume that $a'c' = ac$, $c'd' = cd$, and $x' = x$. The following statement will then be true:

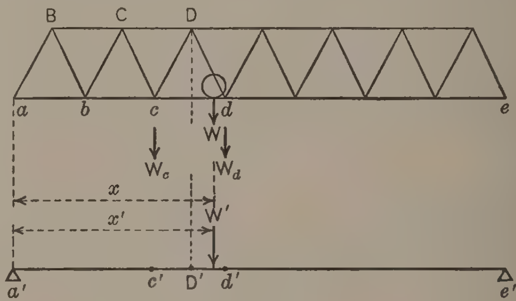


Fig. 300.

6. The bending moment for a center of moments *at* any panel point of the truss, as, for example, a section at d, caused by the two loads W_c and W_d will be the same as the bending moment for a section at the corresponding point d' on the beam $a'e'$ caused by the load W', but the bending moment for a center of moments *between* panel points, as, for example, the joint D, caused by the loads W_c and W_d is *not* the same as the bending moment for a section at the corresponding point D' on the beam $a'e'$ caused by the load W'. (Prove.) (See influence line, Fig. 280 (*b*).)

7. *Note:* The reaction at a may be calculated as if the load W were on a simple beam ae without panels, or it may be calculated from W_c and W_d — it is immaterial which method is used. (296 : 9.)

8. *General statement: When the center of moments is at a panel point of a beam or truss supported at each end, the bending moment due to a single concentrated load is calculated as if the load were on a simple beam without panels;* the same method of calculation may also be used when the center

of moments is between panel points, provided the load is not in the panel cut by the section, but when the load is in the panel, the floor-beam concentration (on the segment at the end of the panel) due to the load is used in place of the load in the equation for bending moment. In any case, the reaction at the end of the beam or truss due to the concentrated load may be calculated as if the load were on a beam without panels.

1. *General statement: The greatest bending moment for a given segment of a beam or truss due to a single concentrated live load will occur when the load is in a vertical line through the center of moments, regardless of whether that center is at a panel point or between panel points.*

2. *When there are two equal concentrated loads, one should be placed in a vertical line through the center of moments, and the other on the longer segment as near to the first load as the distance between loads will permit — just as for a beam without panels.* (286 : 10.)

3. *General statement: The greatest bending moment for a given segment of a beam or truss due to a uniform live load applied at panel points will occur when the load extends from end to end of beam or truss, i.e., when there is a full panel load at each intermediate panel point.*

4. *General statement: The maximum bending moment for a beam or truss due to a uniform live load applied at panel points will occur for a section exactly at the center when there is a full panel load at each panel point. If there is an odd number of panels, the bending moment is a maximum for a section anywhere between the two central panel points.*

5. *Exercise:* Prove each of the three general statements just made.

6. *The exact and the conventional methods of loading for bending moments.* For the purpose of calculating bending moments, there is no difference between the exact and the conventional method of distributing uniform live load to the panel points of a beam or truss. This is because the uniform live load, in the position to cause either the greatest bending moment for a given segment or the maximum bending moment, covers the beam or truss from end to end, and therefore there is a *full* panel load at each intermediate panel point.

7. *Note:* Shears and bending moments for beams and trusses with panels will be treated more fully in succeeding chapters, both for conventional uniform live load and for two or more concentrated live loads.

8. **Impact.** Thus far in this chapter, the live load has been assumed to be static in the position in which it causes the maximum reaction, shear, or bending moment, as the case may be. To the quantity thus determined, should be added, in most cases, a certain percentage of that quantity in order to provide for the effect of the live load in motion. This dynamic increment may be determined by substituting the *loaded length* for "L" in an impact formula. (267: 6.) The loaded length, in the case of a beam, is the length covered by the live load when it is in the position to produce the maximum reaction, shear, or bending moment to which the increment is added. For example, the greatest positive shear for a given segment due to uniform live load occurs when the longer segment is fully loaded, and the greatest bending moment when the whole beam is loaded, consequently in determining the increment for the shear the loaded length is the length of the longer segment, but for bending moment, the loaded length is the length of the beam. In some specifications a flat percentage is specified. For example, in certain specifications for highway bridges, it is stated that the dynamic increment for floor beams and stringers shall be 30 per cent, provided this percentage is not less than that that would be obtained from the impact formula given in the specifications.

9. **Combined Shears or Bending Moments Due to Dead and Live Loads.** The greatest total shear or bending moment *for a given segment* due to both dead and live loads is best determined by calculating the dead-load shear or bending moment and the greatest live-load shear or bending moment separately, and then adding the dead-load shear to the greatest live-load shear, or the dead-load bending moment to the greatest live-load bending moment. Any allowance for impact required by the specifications should be added to the live-load shear and to the live-load bending moment, and this cannot be done unless they are determined separately from the dead-load shear and the dead-load bending moment. There are frequently other reasons pertaining to design why live-load results should be determined separately from dead-load results.

10. To combine results obtained from dead load with those obtained from live load in order to ascertain the total *maximum* shear or *maximum* bending moment is more difficult if, as sometimes happens, the maximum shear or bending moment due to dead load occurs for a section different from that for which the maximum shear or bending moment due to live

load occurs. For example, the maximum moment for a simple beam occurs for a uniform dead load for a section at the center of the beam. If the live load consists of two concentrated loads, the maximum live-load bending moment will occur for a section near the center but not at the center. These two maximum bending moments (dead and live) should not, theoretically, be added together since they are not both for the same section. Such addition, however, would be on the safe side. (Why?) If the error involved is not large, the result thus obtained would be correct for all practical purposes. No general rule can be given which will cover all such cases. In any given case, it may be helpful to sketch free-hand the shear curves or the bending moment curves for dead load and live load separately, and then to combine corresponding dead-load and live-load curves in order to observe for what section the combined dead-load and live-load shears or bending moments are a maximum. Another method is to determine algebraically the combined maximum. (172 : 6 to 9.)

1. *Typical method of procedure.* One of the most typical problems in combining results obtained from dead load with those obtained from live load is in the calculation of shears and bending moments for stringers. It is required, for example, to determine the maximum shear and the maximum bending moment for a given highway-bridge stringer. Assume that the live load is one of the typical truck loadings given in 369 : 2. The following method of procedure is suggested:

(1) Determine the proportion of live load carried by each stringer, i.e., the magnitude of each of the two concentrated live loads that are 14 ft. apart.

(2) Determine the total dead load per linear foot carried by each stringer, including its own weight. (128 : 2.)

(3) Calculate the maximum shear due to live load (286 : 3), and the corresponding dynamic increment. (301 : 8.)

(4) Calculate the maximum shear due to dead load. (175 : 9.)

(5) Calculate the maximum bending moment due to live load (288 : 3), and the corresponding dynamic increment.

(6) Calculate the dead-load bending moment for the section used in calculating the maximum live-load bending moment. Use the short-cut method. (167:1.)

This order of procedure will serve to suggest similar systematic methods for use in different types of problems. Some systematic arrangement of work, similar to that on page 183, in which each step is clearly indicated, should also be used.

Summary of the Chapter

2. In the summary of this chapter which follows, some of the general statements and most of the special illustrations are restricted to *left-hand reactions*, and to shears and bending moments for *left-hand segments*, but such statements and illustrations may easily be modified to apply to right-hand reactions and segments. The term "greatest" is used in connection with a given segment while the term "maximum" is used in connection with that particular segment for which the shear or the bending moment is the greatest possible.

SIMPLE BEAMS — GENERAL STATEMENTS

3. *Reactions:* The greatest reaction at a support will occur when as much of the moving load as possible is on the beam and the center of gravity of the load on the beam is as near as possible to the support. (284 : 6 to 284 : 13.)

4. *Shears:* (a) The greatest positive shear for a given segment will occur when there is no load on the given segment and as much load as possible on the other segment, with the center of gravity of that load as near as possible to the section. The greatest negative shear will occur when there is as much load as possible on the given segment with the center of gravity of that load as near as possible to the section, and there is no load on the other segment. (285 : 1 to 286 : 2.)

(b) The maximum shear is for a section indefinitely close to a support and this end shear will be equal to the corresponding reaction at that support. Hence the position of any load which will cause a maximum reaction will also cause maximum shear. (286 : 3.)

5. *Bending moments:* (a) For concentrated live loads, the greatest bending moment for a given segment will occur when a load is at the section. If there are two loads, one should be at the section (the larger if the loads are unequal) and the other should be on the longer segment. For uniform live load, the greatest bending moment for *any* segment will occur when the load extends from end to end of beam. If the uniform load is restricted in length, the position should be such that the portion of the load which is on either segment divided by the length of that segment will equal the load on the beam divided by the length of the beam. (286 : 5 to 289 : 3.)

(b) The maximum bending moment for concentrated live loads will occur for a section at or near the center of the beam; for a single load the

section and load will be at the center; for two loads, the section and one load (the larger if the loads are unequal) will be as far on one side of the center as the center of gravity of the two loads is on the other side. (For exceptions in the case of short spans see 286 : 10 (c) and 287 : 2 (c).) For uniform live load, the maximum bending moment will occur for a section at the center of the beam when the load covers the entire beam, or, if the load is restricted in length, when it extends the same distance on each side of the center. (286 : 8 (b), 286 : 10 (b), 287 : 2 (b), and 287 : 5 (c).)

1. *Comparison of diagrams.* (1) Curves for dead-load shear or bending moment: Position of the load fixed, length of segment variable. (2) Influence lines: Position of the *unit* load variable, length of segment fixed. (3) Curves for live-load shear or bending moment: Position of the load and length of segment both variable. (Pages 290 to 293.)

SIMPLE BEAMS — CRITICAL POSITIONS OF LIVE LOADS SHOWN GRAPHICALLY

2. The critical positions of concentrated and uniform live loads are the positions in which the loads will cause the greatest reactions or greatest shears or the greatest bending moments for given segments, or the *maximum* shears or the *maximum* bending moments. These positions will now be given in a concise summary illustrated by figures. The following notation will be used throughout the summary: Left-hand and right-hand reactions respectively = R_L and R_R; section = s; left-hand segment = as; right-hand segment = sb; concentrated load = W or W'; distance between two concentrated loads = d; uniform load per unit of length = w; length of restricted uniform load = d; length of beam = L; center of beam = c; shear = V; bending moment = M_B.

3. Values of reactions, shears, and bending moments are expressed in the form of equations. The equations are not given as formulas to be memorized or even to be used; they are intended merely to indicate the steps taken in calculating the reaction, the shear, or the bending moment as the case may be. The student will find it good practice to check each equation by inspection to see if the steps indicated by the equation are correct:

4. *A single concentrated live load.*

(a) Greatest reaction at a in Fig. 303 (a): When W is at a. $R_L = W$. (284 : 7.)

Fig. 303 (a).

(b) Greatest positive shear for segment as in Fig. 303 (b): When W is on the segment sb indefinitely close to s. (285 : 2.) $V = R_L = W \times sb \div L$. Greatest negative shear for segment as: When W is on segment as indefinitely close to s. $V = R_L - W = (W \times sb) \div L - W$, or $V = -R_R = -W \times as \div L$.

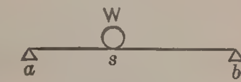


Fig. 303 (b).

(c) Greatest bending moment for segment as in Fig. 303 (c): When W is at s. (286 : 8.) $M_B = R_L \times as = [(W \times sb) \div L] \times as$.

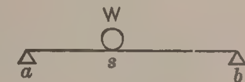


Fig. 303 (c).

(d) Maximum shear: When the section s and W are both indefinitely close to a. (Fig. 303 (d).) (286 : 3.) $V = W$. (End shear.)

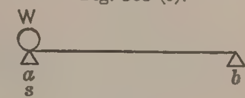


Fig. 303 (d).

(e) Maximum bending moment: When the section s and W are both at c the center of the beam. (Fig. 303 (e).) (286 : 8 (b).)

$$M_B \text{ for } ac = R_L \times ac = \frac{W}{2} \times ac.$$

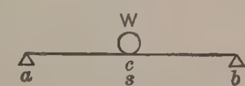


Fig. 303 (e).

5. *Two concentrated live loads.*

(a) Greatest reaction at a in Fig. 303 (f): When the larger load W' is at a and the other load W is on the beam as near to a as the fixed distance d between the loads will permit. (284 : 9.) $R_L = W' + W \times (L - d) \div L$. When the two loads are equal either may be at a, provided the other is on the beam.

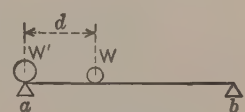


Fig. 303 (f).

(b) Greatest positive shear for segment as in Fig. 303 (g): When both loads are on segment sb, the larger W' indefinitely close to s. (285 : 4.)

$$V = R_L = [W' \times sb + W \times (sb - d)] \div L.$$

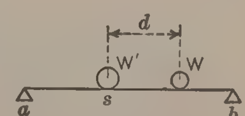


Fig. 303 (g).

Greatest negative shear for segment *as* in Fig. 304 (*a*): When both loads are on segment *as*, the larger indefinitely close to *s*. $V = R_L - W' - W = [W' \times bs + W \times (bs + d)] \div L - W' - W$. V is also equal to $-R_R$. When the two loads are equal, the left-hand load will be indefinitely close to *s* for greatest positive shear, and the right-hand load indefinitely close to *s* for greatest negative shear.

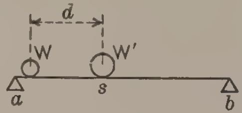


Fig. 304 (*a*).

(c) Greatest bending moment for segment *as* in Fig. 304 (*b*): When the larger load W' is at *s* and the other load *W* is on the longer segment whether it be *as* or *sb*. (287 : 2.) M_B for $as = R_L \times as$ when the smaller load is on *sb*, and $R_L \times as - W \times d$ when the smaller load is on *as*.

$$M_B = R_L \times as = [[W' \times sb + W \times (sb - d)] \div L] \times as$$

$$M_B = R_L \times as - W \times d = [[W' \times sb + W \times (sb + d)] \div L] \times as - W \times d$$

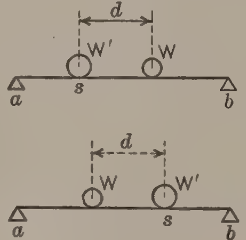


Fig. 304 (*b*).

When the smaller load is on *as*, M_B is also equal to $R_R \times sb$. When the two loads are equal, either load may be at *s*, provided the other load is on the longer segment.

(d) Maximum shear: When the section *s* and the larger load are both indefinitely close to *a*, and the smaller load is on the beam as near to *a* as the fixed distance *d* between loads will permit. (Fig. 304 (*c*).) (Same position as for greatest reaction at *a*.) (286 : 3.)

$$V = R_L = W' + W \times (l - d) \div l$$

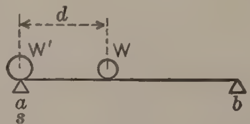


Fig. 304 (*c*).

When the two loads are equal, either may be at *a* provided the other is on the beam.

(e) Maximum bending moment: When the larger load W' and the section *s* are both as far on one side of the center *c* of the beam as the center of gravity *g* of the two loads is on the other side of the center. (Fig. 304 (*d*).) (287 : 2 (b).) (For exceptions likely to occur in the case of short spans see 287 : 2 (c).)

$M_B = R_L \times as$ when *s* is to the left of *c*, or
$M_B = R_L \times as - W \times d$ when *s* is to the right of *c*.

$$M_B = R_L \times as = [[W' \times sb + W \times (sb - d)] \div L] \times as$$

$$M_B = R_L \times as - W \times d = [[W' \times sb + W \times (sb + d)] \div L] \times as - W \times d$$

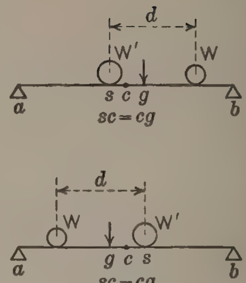


Fig. 304 (*d*).

When *s* is to the right of *c*, M_B is also equal to $R_R \times sb$. When the two loads are equal, *s* will be $\frac{1}{4}$ *d* from *c* (either side of *c*) and either load may be at *s* provided the other load is on the other side of *c* (a distance $\frac{3}{4}$ *d* from *c*). (For exceptions likely to occur in the case of short spans see 286 : 10 (c).)

1. *Uniform live load.*

(a) Greatest reaction at *a* in Fig. 304 (*e*): When the load extends end to end of beam. (284 : 11.) $R_L = wL \div 2$. If the load is restricted to a length *d* less than *L*, the load will cover the beam for a distance *d* from *a*, and $R_L = [wd \times (L - \frac{1}{2} d)] \div L$.

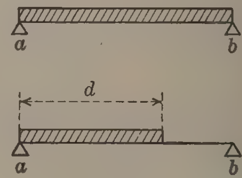


Fig. 304 (*e*).

(b) Greatest positive shear for segment *as* in Fig. 304 (*f*): When there is no load on segment *as* and the load extends from end to end on segment *sb*. (285 : 8.)

$$V = R_L = [(w \times sb) \times \tfrac{1}{2} sb] \div L$$

Greatest negative shear for segment *as*: When there is a load from end to end on *as* but no load on *sb*.

$$V = R_L - w \times as = [(w \times as) \times (sb + \tfrac{1}{2} as)] \div L - w \times as$$

If the load is restricted to a length *d* less than the length of a segment it should extend on that

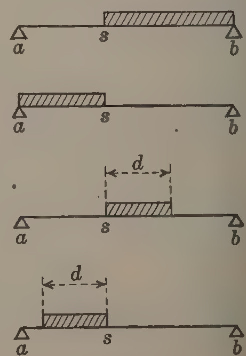


Fig. 304 (*f*).

segment a distance d from the section — on segment bs for positive shear and on segment as for negative shear.

$$\text{Positive V} = [wd \times (sb - \tfrac{1}{2}d)] \div L$$
$$\text{Negative V} = [wd \times (sb + \tfrac{1}{2}d)] \div L - wd = -R_R$$

(c) Greatest bending moment for segment as in Fig. 305 (a): When the load extends from end to end of beam. (287 : 4.)

$$M_B = R_L \times as - (w \times as) \times \tfrac{1}{2}as = \tfrac{1}{2}wL \times as - (w \times as) \times \tfrac{1}{2}as; \text{ also } \tfrac{1}{2}w \times as \times sb$$

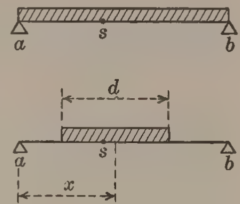


Fig. 305 (a).

If the load is restricted to a length d and x is the distance from a to the center of the load, the greatest bending moment for segment as will occur when $x = as + \frac{d}{2} - d \times \frac{as}{L}$, i.e., when the load on as divided by the length as equals the total load (length d) divided by the length of the beam. (287 : 4.) (The load on sb divided by the length sb also equals the total load divided by l.) When x represents the distance from a to the *beginning* of the uniform load, the equation becomes: $x = as - \frac{d \times as}{L}$.

(d) Maximum shear: When the section is indefinitely close to a and the load extends from end to end of the beam. (Fig. 305 (b).) (286 : 4.) $V = R_L = \frac{1}{2}wL$. If the load is restricted to a length d less than l, it will cover the beam for a distance d from a. (Maximum shear = greatest reaction at a.)

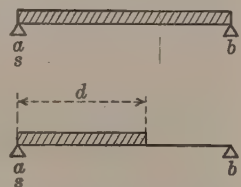


Fig. 305 (b).

(e) Maximum bending moment: When the section s is at c the center of the beam and the load extends from end to end of beam. (Fig. 305 (c).) (287 : 4 (c).)

$$M_B = \frac{w}{2} \times \frac{L}{2} \times \frac{L}{2} = \frac{wL^2}{8} \quad (288 : 1.)$$

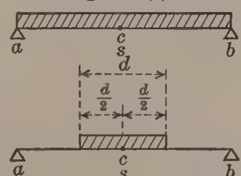


Fig. 305 (c).

If the load is restricted to a length d less than L it will cover the beam for a distance $\frac{d}{2}$ each side of c.

$$M_B = R_L \times ac - \frac{wd}{2} \times \frac{d}{4} = \frac{wdL}{4} - \frac{wd^2}{8}$$

SIMPLE CANTILEVER BEAM — GENERAL STATEMENTS

1. When a simple cantilever beam is divided by a section into two segments, the shear or bending moment for the segment toward the unsupported end is not affected by any load which may be on the other segment. Wherever the term "unsupported segment" is used in this summary in connection with cantilever beams, it is understood that it is the segment on which no support acts.

2. *Shears.* (a) The greatest shear for any unsupported segment of a cantilever beam will occur when there is as much load as possible on that segment regardless of whether the load is concentrated or uniform, regardless of what the position of the load on the segment may be, and regardless of any load that may be on the supported segment. (294 : 1 to 5.)

(b) The maximum shear will occur when the unsupported segment is of sufficient length to carry *all* of the load or loads; for a single concentrated load this might be any length between zero and the length of the beam, but for a uniform load the length of the unsupported segment should usually be equal to the length of the beam. (Section indefinitely close to the support at the fixed end.) In any case, if the shear for a section at the support is not greater than the shear for any other section it is at least as great.

3. *Bending moments.* (a) The greatest bending moment for any unsupported segment of a cantilever beam will occur when as much load as possible is on that segment, regardless of any load that may be on the other segment; when there is a single concentrated load, it should be at the unsupported end; when there are two concentrated loads or a uniform load restricted in length, the center of gravity should be as near the unsupported end as possible. (294 : 6 to 11.)

(b) The maximum bending moment will be when the section is at the support (just as for maximum shear).

SIMPLE CANTILEVER BEAMS — CRITICAL POSITIONS OF LIVE LOADS SHOWN GRAPHICALLY

1. *Greatest shear for a given segment as in* Fig. 306 (*a*): For a concentrated load W, when W is anywhere on *as*. For two concentrated loads W' and W, when both loads, if possible, are anywhere on *as* regardless of whether the larger or the smaller is the nearer to the unsupported end; if only one of the loads can be placed on the segment it should be the larger. For a uniform load, the load should entirely cover segment *as* regardless of whether or not any load is on segment *sb*. In any case the shear equals the total load on the segment. (294 : 1 to 4.)

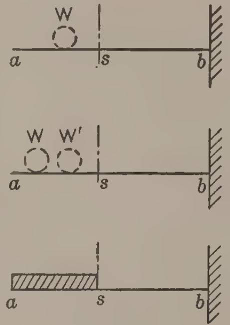


Fig. 306 (*a*).

2. *Maximum shear.* When the segment $as = ab$, i.e., when the section is indefinitely close to b, and as much load as possible is on ab regardless of what its position on ab may be. The shear equals the total load. (294 : 5.)

3. *Greatest bending moment for a given segment as* in Fig. 306 (*b*): For a single concentrated load W, when W is at the unsupported end a. $M_B = -W \times as$. For two concentrated loads when the larger, W', is at a, regardless of whether the smaller comes on *as* or on *sb*. When the smaller is also on *as*, $M_B = -W' \times as - W \times (as - d)$. For a uniform load, when the load entirely covers segment *as* regardless of whether or not any load is on segment *sb*. $M_B = -w \times as \times \frac{1}{2}\, as$. (294: 6 to 10.)

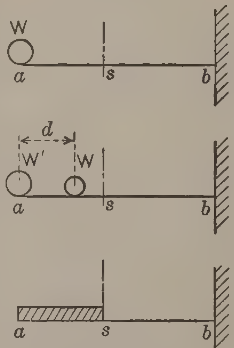


Fig. 306 (*b*).

4. *Maximum bending moment.* Same conditions of loading as those just given for the greatest bending moment for segment *as*, but the length of segment *as* will be equal to the length of the beam *ab*, i.e., the section will be at b. (294 : 1.)

BEAM WITH TWO SUPPORTS AND ONE CANTILEVER END

5. For a summary that gives the critical positions of live loads for a beam with two supports and one cantilever end see 295 : 1 to 296 : 1.

BEAMS WITH TWO SUPPORTS AND TWO CANTILEVER ENDS — CRITICAL POSITIONS OF UNIFORM LIVE LOAD (296: 4.)

6. *Reactions.* Greatest positive reaction at a in Fig. 306 (*c*): When the load extends from b to d. Greatest negative reaction at a: When the load extends from e to b.

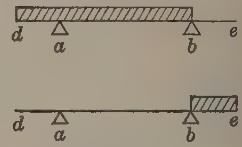


Fig. 306 (*c*).

7. *Shears.* Greatest positive shear for segment *ds* in Fig. 306 (*d*): When the load extends from b to s and at the same time from a to d. Greatest negative shear for segment *ds:* When the load extends from e to b and at the same time from s to a.

8. *Maximum positive shear* for a left-hand segment: When the section is indefinitely close to a and to the right of a. The position of the load will be the same as for the greatest positive reaction at a.

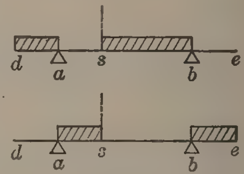


Fig. 306 (*d*).

9. *Bending moments.* Greatest positive bending moment for segment *as* in Fig. 306 (*e*): When the load extends from b to a. Greatest negative bending moment for segment *as:* When the load extends from e to b and at the same time from a to d.

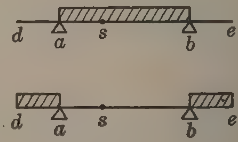


Fig. 306 (*e*).

10. *The maximum positive bending moment* will occur when the section is at the center of the beam and the load extends from one support to the other.

11. *The maximum negative bending moment* will occur when a section is taken anywhere from a to b and the load covers both overhanging ends — no load between supports.

1. *Note:* When the section is taken to the left of support a, the critical positions of the load for greatest shear and greatest bending moment will be the same as for a simple cantilever unsupported at the left-hand end.

BEAMS WITH PANELS — GENERAL STATEMENTS

2. *When floor beams may be ignored.* In certain cases floor beams may be ignored, and moving loads may be treated as if they were applied directly to the top of the main girders that support the floor beams. These cases are:

(a) Calculation of the reaction at either end of the main girder. (142 : 7.)

(b) Calculation of the shear for a section exactly at a panel point.

(c) Calculation of the bending moment for a center anywhere in a vertical line through a panel point. (300 : 8.)

The greatest magnitude in any one of the three cases will be obtained for the same conditions of loading as those given for a beam without panels, regardless of whether the load is concentrated or uniform.

3. *When floor beams may not be ignored.* There are two cases in which floor beams cannot be ignored, both of which occur when the section is taken *between* panel points. These two cases are:

(a) Calculation of the shear for any section between two successive panel points, provided the load is in the panel cut by the section.

(b) Calculation of the bending moment for any center of moments between two successive panel points, provided the load is between these two panel points.

Any load between two successive panel points causes a floor-beam concentration at each end of the panel and this concentration acts as a downward force on the corresponding segment of the beam. This downward force is used in the equation for the shear for a section between the two panel points and in the equation for bending moment for a center of moments between the two panel points. If part of the loading is not between the two panel points, that part may be treated as if it rested on a girder without floor beams, even though the section or center of moments is between the two panel points.

4. *Reactions.* (a) The greatest reaction at a support will occur for the same conditions of loading as those given for a beam without panels, regardless of whether the load is concentrated or uniform.

(b) The greatest reaction at a point on a floor beam where the stringer in one panel meets the stringer in an adjacent panel will occur for uniform load when both panels are fully loaded; for two concentrated loads, when one is at the floor beam (the larger load if the two are unequal) and the other load is in either panel as near to the floor beam as the fixed distance between loads will permit. (Also when the center of gravity of both loads is at the floor beam.) (297 : 3.)

5. *Shears.* (a) The greatest positive shear for a given left-hand segment which ends anywhere between two given panel points will occur for the conventional method of uniform loading when no loads are on that segment and full panel loads are assumed to act at all panel points on the other segment; the greatest negative shear will occur when full panel loads are assumed to act at all panel points on the given segment and there are no loads on the other segment. (299 : 1.)

(b) The maximum shear will occur when the section is taken anywhere between a support and the nearest panel point and all panel points are fully loaded.

6. *Bending moments.* (a) The greatest bending moment for any segment will occur for uniform load when all panel points are fully loaded. (301 : 3.)

(b) The maximum bending moment will occur for uniform live load when all panel points are fully loaded, and the section is taken at the middle panel point if there are an even number of panels or at a panel point adjacent to the center if there are an odd number of panels.

ASSIGNMENTS

(1) Given a simple beam supported at each end and two equal, moving, concentrated loads a fixed distance apart. Report on a method of determining and plotting a curve for the total bending moment due to the two loads, assuming that the moment is always for a section under the left-hand load as the loads move across the beam.

(2) Report on methods of plotting combined shear diagrams representing the total shear due to dead and live loads, and also on methods of plotting similar combined bending-moment diagrams.

(3) Report on a comparison between shears and bending moments for a simple beam supported at each end and without floor beams, with shears and bending moments for a similar beam of the same length but with floor beams. Illustrate by influence lines and curves for shear and bending moment.

(4) Same as the preceding assignment, except one beam is a simple cantilever without floor beam and the other beam is a similar cantilever with floor beams.

CHAPTER XX

STRESSES DUE TO UNIFORM LIVE LOAD
TRUSSES WITH PARALLEL CHORDS

In CHAPTERS XVIII and XIX, guiding principles or criteria were developed for the critical positions of uniform live load on beams or trusses with panels. This chapter explains how these criteria are used in placing uniform live load on parallel-chord trusses in positions that will cause maximum live-load stresses, and how these live-load stresses may be determined by any one of the three methods already explained for finding dead-load stresses. These three methods are the method of sections (CHAPTER XIV), the algebraic method of successive joints (CHAPTER XV), and the graphic method of successive joints (CHAPTER XVI).

1. **UNIFORM LOAD — EXACT AND CONVENTIONAL METHODS.** The difference between the exact and the conventional methods of distributing uniform live load to girders or trusses was explained in 264 : 6. In determining the live-load stress in any member in which the maximum stress occurs when the bridge is fully loaded, no distinction is made between the exact and the conventional methods. (264 : 7.) Since live-load stresses in chord members and end posts are maximum when the bridge is fully loaded, only stresses in web members, as a general rule, are affected by the choice between the exact and the conventional methods of loading.

2. The stress in any web member is a function of the shear for a section through the panel that contains that member. (194 : 5 and 194 : 9.) In the exact method there is a *partial* panel load at the panel point at either end of the panel (unless it is an end panel), whereas in the conventional method there is a *full* panel load at one end and no load at the other, an assumption contrary to fact. (299 : 1.) In no other respect do the two methods of loading differ.

3. The shear is the same for a section through an end panel regardless of whether the exact or the conventional method of loading is used, but for any intermediate panel, the shear obtained by the exact method is less than that obtained by the conventional method, and, therefore, the web stresses determined from the latter method of loading are greater than the actual stresses — a discrepancy on the side of safety. (300 : 3.) Such discrepancies increase in successive panels from an end of a truss toward the center, but since the web stresses themselves decrease in the same order, the absolute difference between the stress in any web member as obtained from the two methods of loading is not usually great enough to materially affect the design of the member.

4. Some engineers prefer the exact method because it is most nearly in accord with the actual conditions of loading. Other engineers regard it as an unnecessary refinement, particularly in view of the fact that to the live-load stresses are added impact stresses that are at best only approximately correct. (268 : 12.) The method of determining live-load stresses from the conventional form of loading is somewhat simpler to explain, and, once one understands it, one can easily change to the exact method should it be desirable to do so. For this reason, the conventional method of loading will be used almost exclusively in this book when the live load is uniformly distributed.

5. **REQUIRED MAXIMUM LIVE-LOAD STRESSES.** In certain members of a truss, live load can cause stress of one character only, tension or

compression. For such a member, only one maximum live-load stress is usually required, though in some cases the maximum and minimum stress may be required. (266 : 4.) In other members, the live load in different positions will cause first tension then compression or *vice versa.* For such a member, two maximum live-load stresses may be required, one the maximum tension, the other the maximum compression. (266 : 4.) When impact is taken into account, each maximum live-load stress of whatever character is increased by the percentage of that stress prescribed by the allowance for impact. (267 : 6.)

1. **Positions of Live Load for Maximum Stresses.** Before the maximum live-load stress can be determined for any member of a truss, it is necessary to know in what position the live load must be placed in order to cause that stress. For parallel-chord trusses and conventional panel loads, two guiding principles for placing live loads will be given, one for the maximum stress in a chord member, and one for the maximum stress in a web member.

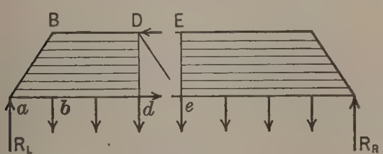


Fig. 309 (*a*).

2. *Position of the live load for maximum stress in a chord member.* Let DE and de in Fig. 309 (a) be the top and bottom chord members, respectively, of a parallel-chord truss that has been divided into two segments by a section through these members.

3. The stress in either DE or de may be determined by dividing the bending moment for either segment by the height of the truss, provided that for DE the center of moments be taken at e and for de the center of moments be taken at D. (196 : 3.) The maximum stress for either member will occur, therefore, when the corresponding bending moment is a maximum. But the greatest bending moment for either segment, *or for any segment,* will occur when there is a full panel load at each intermediate panel point. (301 : 3.) Hence the maximum stress for either DE or de or any other chord member, top or bottom, will occur when the uniform live load extends from end to end of the span.

4. *Position of the live load for maximum stress in a web member.* Let De in Fig. 309 (b) be the web member cut by the section shown. The stress in De is equal to the shear for either segment multiplied by the secant of the angle that De makes with the vertical. (194 : 5.) The maximum stress in De will occur, therefore, when the shear is greatest for either segment. The greatest *positive* shear for the left-hand segment will occur when there is a panel load at each panel point of the right-hand segment but no loads on the left-hand segment, as shown in Fig. 309 (b). (299 : 11.) This shear is equal to R_L, and tends to move the left-hand segment upward; De must, therefore, act downward on that segment, and hence is in tension, i.e., the load in the position shown in Fig. 309 (b) will cause maximum live-load tension in De.

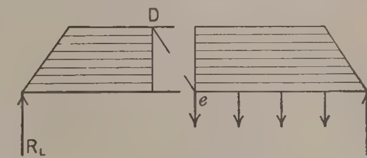


Fig. 309 (*b*).

5. The greatest *negative* shear for the left-hand segment will occur when there is a panel load at each panel point of that segment, but no loads on the other segment as shown in Fig. 309 (c). (299 : 11.) This shear is equal to $-R_R$, and, being negative for the left-hand segment, tends to move that segment downward; De must, therefore, act upward, and hence is in compression, i.e., the load in the position shown in Fig. 309 (c) will cause maximum live-load compression in De.

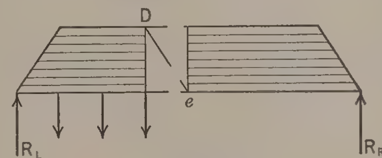


Fig. 309 (*c*).

6. *Note:* Whenever the term "segment" is used in connection with any web member, it refers to one of the two segments of the truss formed by a section taken through that web member in accordance with the method of sections. Such a section will usually cut only two other members, both of which are chord members, but in some types of trusses, as, for example, in the K-truss, such a section will cut four members two of which are web members.

7. *Note: Use of the term "shear."* It is often convenient to speak of the shear for a web member, as, for example, in Fig. 309 (b) the shear for De. It is understood that this means the shear from which the stress in the member is determined. Similarly, "shear for a panel" means shear for the section through the panel, as, for example, shear for the panel de (loaded chord) is the shear for the section through that panel. Unless otherwise specified, shear in this book will mean *shear for a left-hand segment.*

8. The general statement may now be made that the *maximum live-load stress in a chord member will occur when the truss is fully loaded, but*

that the maximum live-load stress in a web member will occur when the truss is partially loaded, namely, when one of the corresponding segments is fully loaded and the other segment is entirely free from loads.

1. *Note:* In any general statement concerning the maximum live-load stress in a web member, it is understood that the statement does not necessarily apply to a vertical web member when it is the only web member at a joint of truss, such, for example, as a hanger in a through bridge truss. The live-load stress in such a member is equal to one panel of live load, and this stress occurs whenever the position of the live load is such that the floor beam brings a full panel load to the panel point where the vertical acts.

2. *Note:* It may be of interest to verify the statement just made without reference to bending moment or shear. Let it be required to calculate by the method of sections the live-load stresses in the three members DD, de, and De. (Fig. 310.) The unknowns are three magnitudes, regardless of how many panel points are loaded, and the combination of equations for this case is $\Sigma M = 0$, $\Sigma M = 0$, and $\Sigma V = 0$. The stress DD is found from a moment equation with e as a center of moments. Any load W on the right-hand segment causes a reaction R_L on the left-hand segment that tends to revolve that segment clockwise about e. As DD is the only force to resist this tendency (de passing through e), DD must act toward the left-hand segment (compression); hence a load W on the right-hand segment causes compression in DD. Similarly, if the load W is moved to the left-hand segment, it will cause a reaction R_R that tends to revolve the right-hand segment counter-clockwise. To resist this tendency DD must act toward the right-hand segment (compression); hence a load W on the left-hand segment also causes compression in DD. The maximum compression in DD will occur, therefore, when there are as many panel loads as possible on each segment, i.e., when the truss is fully loaded. The stress de is found from a moment equation with D as a center of moments. Following a similar method of reasoning, it may be shown that the maximum tension in de will also occur when the truss is fully loaded.

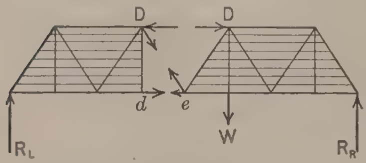


Fig. 310.

3. A load W on the right-hand segment causes a reaction R_L that tends to move the left-hand segment upward, and as De is the only other force with a V component, it must act downward on the left-hand segment (tension) ($\Sigma V = 0$); hence a load W on the right-hand segment causes *tension* in De. Similarly, a load on the left-hand segment causes a reaction R_R that tends to move the right-hand segment upward. To resist this tendency, De must act downward on the right-hand segment (compression), hence a load on the left-hand segment causes *compression* in De. It follows that the maximum tension in De will occur when as many panel loads as possible are on the right-hand segment and the left-hand segment is free from loads; the maximum compression will occur when there are as many panel loads as possible on the left-hand segment and the right-hand segment is free from loads.

4. A load on a right-hand segment always tends to cause a web member to act downward on a left-hand segment, provided the truss is supported at each end, (Why?) but the member will be in tension or compression according to whether it acts away from or toward the segment. A web member that acts downward on a left-hand segment will generally be in tension if it acts at an upper joint, and in compression if it acts at a lower joint. (Why?) As a general rule, therefore, a load on a right-hand segment tends to cause tension in a web member if that member acts at the upper joint of the left-hand segment — compression if it acts at a lower joint; a load on the left-hand segment tends to cause compression in a web member if it acts at an upper joint of a left-hand segment — tension if it acts at a lower joint.

5. *Live-load stress in an end post.* Both criteria for placing live load, the criterion for web stresses and the criterion for chord stresses, hold good for the live-load stress in an end post. If, for example the criterion for web stresses be applied in determining the live-load stress in the end post aB (Fig. 309 (a)), the section will cut only aB and ab, and leave only the joint a as the left-hand segment of the truss. Following the general rule, the other segment should be fully loaded in order to obtain the maximum compression in aB, i.e., *all* panel points of the truss should be loaded. This is the same condition of loading as that which causes maximum chord stresses. The same conclusion may be reached by considering joint a as a particle in equilibrium. From $\Sigma V = 0$, the V component of aB must be equal and opposite to R_L, regardless of the number of panel points loaded, and this component (and the corresponding stress in aB) will be greatest when R_L is greatest, i.e., when all panel points are loaded.

6. *Note:* In a through Warren truss without verticals, both criteria apply not only to an end post but also to a diagonal that meets an end post at an upper joint. (Why?)

7. *To determine the live-load stress in any member of the perimeter of a truss from the dead-load stress.* The top and bottom chord members and the two end posts form the perimeter of a truss. The maximum live-load stress in any member in this perimeter will occur when a full panel of live load is applied at each panel point. When the dead load is considered as applied wholly to one chord instead of being distributed to the top and bottom chords, the dead-load stress in any member in the perimeter (or any other member) will occur for a full panel of dead load applied at each panel point. The conditions of loading for the dead-load and live-load stresses in any member of the perimeter are, therefore, identical, except

that the live load per panel may be greater or less than the dead load per panel. *When, therefore, the dead-load stress is known for any member in the perimeter of a truss, the live-load stress in that member due to conventional panel loads may be obtained by simply multiplying the dead-load stress by the ratio of the live load per panel to the dead load per panel.* For example, if the live load per panel is 30,000 lbs. and the dead load per panel is 12,000 lbs., the maximum live-load stress in any member in the perimeter of the truss will be $2\frac{1}{2}$ times the dead-load stress in that member.

1. *Note:* When part of the dead load is considered as taking effect at joints of the upper chord and part at joints of the lower chord (127 : 4), the dead-load stresses in certain chord members may not be the same as the dead-load stresses in the same members when the dead load is assumed to act at the joints of one chord only. In such a case, it may be necessary to determine the live-load stresses independently from the dead-load stresses. The Warren truss without verticals is one of the few common types of parallel-chord trusses in which these exceptional conditions occur. (199 : 15).

2. The maximum live-load stresses in all members of the perimeter of a truss occur *simultaneously*, i.e., when the live load extends over the entire span, but the maximum live-load stresses in the web members do not occur simultaneously but for different positions of the live load. It is possible for maximum live-load stresses to be in several web members at the same time but not in all. It is also to be noted that, as a general rule, when the live-load stress in a web member is a maximum, the live-load stresses in the chord members are not maximum. This last statement is not true if the maximum stress in a web member occurs when the truss is fully loaded, as, for example, the live-load stress in a hanger or in the diagonal in the end panel of a Warren truss without verticals.

3. *Note:* Live-load stresses are entered along the different members of a truss diagram as if they all occurred simultaneously — it is important to remember that they do not.

4. **General Method of Procedure for Determining Stresses in Parallel-Chord Trusses.**

First: Determine the dead-load stresses in all members of the truss.

Second: Determine the live-load stresses in all members in the perimeter of the truss (chord members and end posts) by multiplying the dead-load stress in each member by a constant obtained by dividing the live load per panel by the dead load per panel. (An exception is noted in 311 : 1.)

Third: Determine the live-load stresses in the web members. The maximum tension and the maximum compression in any web member each occurs when every panel point of one of the corresponding segments is loaded and there are no loads on the other segment, but the segment that is loaded when one maximum stress occurs will be the unloaded segment when the maximum stress of the opposite sense occurs. (An exception is noted in 310 : 1.)

5. *Note:* In a member that acts at an *upper* joint of a segment, maximum compression will occur when that segment is fully loaded, and maximum tension when the other segment is fully loaded; in a member that acts at a *lower* joint of a segment, the reverse is true, maximum tension will occur when that segment is fully loaded and maximum compression when the other segment is fully loaded. To recall or verify these statements at any time, remember that any load on a given segment tends to move that segment *downward* (negative shear), and therefore the web member, to hold it in equilibrium, must act upward regardless of whether it is at an upper or a lower joint of the segment, whereas a load on the other segment tends to move the given segment *upward*, and therefore the web member must act downward on that segment regardless of whether it is at an upper or a lower apex.

6. The criterion for maximum web stresses may be stated differently as follows: When two maximum live-load stresses are possible in a web member (one tension, the other compression), one of these stresses will occur when the positive shear is a maximum, the other when the negative shear is a maximum. The maximum positive shear for either segment, left-hand or right-hand, will occur when there are no loads on that segment and the other segment is fully loaded; the maximum negative shear for either segment will occur when that segment is fully loaded and there are no loads on the other segment. (299 : 11.)

7. The dead-load stresses in the first step and the live-load stresses in the third step may be determined by any one of the three methods explained in **PART II.** As a general rule the method of coefficients (based on the algebraic method of successive joints) is more efficient than the method of sections or the graphic method of successive joints.

8. *Note:* The general method of procedure just given involves little that is new. The criteria for placing the live loads are the same as those already explained in Chapters XVIII and XIX, and the methods of determining live-load stresses do not differ in principle from those used for determining dead-load stresses. The remainder of the chapter is devoted to the application of the general method, and to the consideration

of certain special cases, such for example, as the calculation of stresses in counters or in other supplementary web members. It is important to note, however, that even these special cases involve no new methods — merely special conditions that must be taken into account in applying old methods.

1. A STUDY OF THE STRESSES CAUSED BY A SINGLE LOAD AS IT MOVES ACROSS A TRUSS. Before continuing the study of the methods of determining live-load stresses, it will be helpful to study the variation in the stresses that occur in each member of a typical truss as a single load is applied to each panel point in succession. The panel length of the truss in Fig. 312 is 20 ft., and the height of truss is 24 ft. Assume a unit load of 1000 lbs. to move across the span from left to right, and to take effect only at panel points. On each member of the truss are entered the five stresses that occur in that member at five different times, namely, when the load is at f, at e, at d, at c, and at b. The stresses are also tabulated below the figure. The following statements may be verified from a study of the stresses:

(a) For any member in the perimeter, the five stresses are either all compression or all tension, therefore the maximum stress will occur when there are five loads on the truss, one at each panel point. This is in accord with the criterion for members in the perimeter. (310 : 7.)

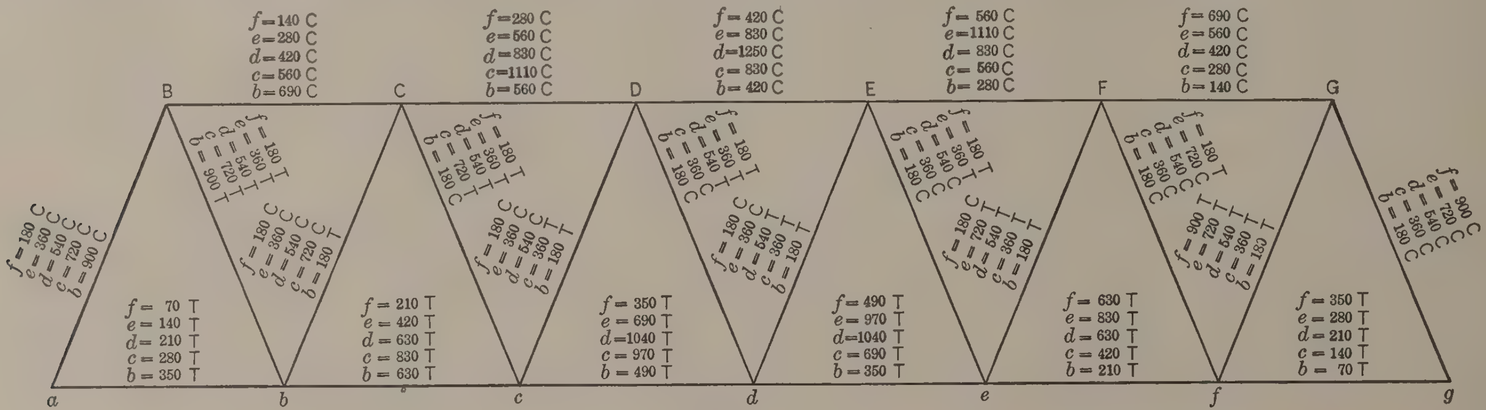


Fig. 312.

INFLUENCE TABLE

Load at	aB Comp.	Bb Tension	bC Comp.	bC Tension	Cc Comp.	Cc Tension	cD Comp.	cD Tension	Dd Comp.	Dd Tension	BC Comp.	CD Comp.	DE Comp.	ab Tension	bc Tension	cd Tension	Load at
f	180	180	180			180	180			180	140	280	420	70	210	350	f
e	360	360	360			360	360			360	280	560	830	140	420	690	e
d	540	540	540			540	540			540	420	830	1250	210	630	1040	d
c	720	720	720			720		360	360		560	1110	830	280	830	970	c
b	900	900		180	180			180	180		690	560	420	350	630	490	b
Totals	2700	2700	1800	180	180	1800	1080	540	540	1080	2090	3340	3750	1050	2720	3540	Totals

(*b*) The stresses in any web member, with the exception of *Bb* and the corresponding member *fG*, are not all tension or all compression, consequently the maximum stress will not occur when there is a load at each of the five panel points.

(*c*) It will be found that if any web member is cut by a section, the compression in that member will occur when the load is at any panel point on one segment, and that tension will occur when the load is at any panel point on the other segment. For example, if *cD* is cut by a section, compression occurs when the load is at any panel point of the right-hand segment, namely, at *f*, at *e*, or at *d*, whereas tension occurs when the load is at any panel point of the left-hand segment, namely, at *b* or at *c*. Similarly, if *Cc* is cut, tension occurs when the load is at any panel point on the right, namely at *f*, at *e*, at *d*, or at *c*, whereas compression occurs when the load is at any panel point on the left, namely, at *b*. The maximum compression and the maximum tension in any web member will therefore both occur when there is a load at every panel point of one segment and no load on the other segment, but the segment that is loaded when one maximum stress occurs will be the unloaded segment when the maximum stress of the opposite sense occurs. This is the criterion for web members given in 311 : 4. (Show that it holds good for member *Bb* and *fG*.)

1. Note that a load at any panel point causes stresses of the same magnitude in all web members to the left or to the right of the panel point, but that it causes varying stresses in each chord. For example, a load at *e* causes a stress of 360 lbs. in each web member to the left, and a stress of 720 lbs. in each web member to the right, but that the stresses that it causes in the top chord members vary from 280 lbs. to 1110 lbs., and in the bottom chord members from 140 lbs. to 970 lbs. Note also that the maximum stress that a load at a given panel point causes in the top chord is in that member for which the panel point is a center of moments (method of sections). For example, the maximum stress that a load at *e* causes in the top chord is in *EF*, the member for which, in the method of sections, *e* is the center of moments. (What statement can be made concerning the maximum stress in the bottom chord caused by a load at a given panel point?) The statements made in this paragraph are in accord with the following principles: (*a*) For a single concentrated load in a given position on a beam, the shear is constant for any section between the load and a support (p. 290), but the bending moment varies for different sections as the ordinates to a sloping line. (p. 292.) (*b*) The maximum bending moment for such a load occurs for a section at the load. (p. 292.)

Influence table. The table of stresses on page 312 is an **influence table** because, like an influence line, it shows the effect of a single concentrated load as it moves from one end of the truss to the other. It shows more than any one influence line could, since it makes clear the effect not merely on the stress in one member but on the stresses in all members. It is thus equivalent to a number of influence lines. The column for *aB*, for example, corresponds to the influence line for the stress in *aB*, and the two columns for *bC* correspond to a different influence line, namely, that for the stress in *bC*.

A similar influence table may be prepared for any type of truss, and from it may be determined the joints that should be loaded to produce the maximum stress in any member. For this purpose it is, perhaps, the surest method. It is not used, however, for trusses of ordinary types, because simple criteria can be developed by means of which the position of live load may be determined. An influence table, nevertheless, may be very helpful to the student in comprehending the actual effect of a concentrated live load as it is placed at different panel points of a truss, and the static effect of uniform live load as it covers more and more of the bridge.

2. **Fundamentals Used in Determining Live-Load Stresses.** Three methods of determining live-load stresses will be explained by means of illustrative problems. In studying these illustrative problems the following fundamentals should be kept in mind:

(*a*) Once the live load has been placed in the critical position for the maximum live-load stress in any member, that stress is determined exactly as if the load were a dead load, i.e., the moving load has become static.

(*b*) Live-load stresses in chord members and end posts are determined directly from the corresponding dead-load stresses. (310 : 7.) (See 311 : 1 for exceptions.)

(*c*) The stress in any web member is determined from the shear for a corresponding segment. This is equivalent to applying $\Sigma V = 0$ to the segment. (193 : 10.)

(*d*) For a given section, the shear for one segment is numerically equal to the shear for the other segment. (158 : 10.)

(*e*) For the maximum stress in a web member, one of the corresponding

segments is free from any external force except the reaction, hence the easiest method of determining the shear for any section through a web member is *to determine the reaction on the unloaded segment.* This means that instead of always considering the left-hand segment as the body in equilibrium, it will be more convenient to consider the unloaded segment as that body, whichever segment it may be.

(*f*) If a web member is vertical, the stress in that member is equal to the corresponding shear; if a web member is inclined, the stress is equal to the shear multiplied by the secant of the truss angle. (194 : 5.)

1. **The Calculation of Live-Load Stresses by the Method of Sections.** Given: The truss shown in Fig. 314. Required: To calculate the live-load stresses by the method of sections for unit panel loads of 1000 lbs. each. Height of truss = 24 ft. The dead-load stresses in this truss for a unit panel load of 1000 lbs. were determined in typical members by the method of sections in 186 : 5. Since the panel loads for live and dead load are the same, the live-load stress in any member of the perimeter of the truss will be the same as the dead-load stress in that member, consequently it will be necessary to calculate here only the live-load stresses in the web members.

2. *Note*: The actual dead and live load per panel, would, of course, be greater than 1000 lbs., and for small trusses like this, the live load would be greater than the dead load. For purposes of explanation, however, a unit panel load of 1000 lbs. is assumed for both dead and live loads. Problems will be solved later in which the panel loads will be actual and not fictitious.

3. The stress in cD is equal to the stress in Dd but opposite in sense, whatever the position of the live load may be. (From $\Sigma V = 0$ applied to joint D; also from fundamental principle 71 : 6.) Consequently, the maximum compression in cD is equal to the maximum tension in Dd, and the maximum tension in cD is equal to the maximum compression in Dd. A corresponding statement may be made concerning any two web members that meet at a joint of the unloaded chord of a Warren truss without verticals. From this principle, the stresses in bC may be determined from the stresses in Cc, and the stress in Bb from the stress in aB. (There is only one maximum stress in Bb, and it is equal to the dead-load stress in Bb when, as in this problem, the panel load for dead load and for live load is the same.) (Why?) Only stresses in Dd and Cc will be calculated.

4. The secant for the truss angle is 1.083. The reaction on the unloaded segment is calculated for each position of the load by the method explained in 142 : 3; the shear in each case is equal to the reaction, and the corresponding stress is equal to the shear multiplied by the secant of the truss angle.

Stress in Dd, segment dEGg loaded.

Segment in equilibrium: $aBDc$

$R_L = \text{shear} = 1000 \times (\frac{1}{6} + \frac{2}{6} + \frac{3}{6}) = 1000$

$\text{Stress} = 1000 \times 1.083 = 1080\ \text{T}$

Stress in Dd, segment aBDc loaded.

Segment in equilibrium: $dEGg$.

$R_R = \text{shear} = 1000 \times (\frac{1}{6} + \frac{2}{6}) = 500$

$\text{Stress} = 500 \times 1.083 = 540\ \text{C}$

Stress in Cc, segment cDGg loaded.

Segment in equilibrium: $aBCb$.

$R_L = \text{shear} = 1000 \times (\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6}) = 1666$

$\text{Stress} = 1666 \times 1.083 = 1800\ \text{T}$

Stress in Cc, segment aBCb loaded.

Segment in equilibrium: $cDGg$.

$R_R = \text{shear} = 1000 \times \frac{1}{6} = 166$

$\text{Stress} = 166 \times 1.083 = 180\ \text{C}$

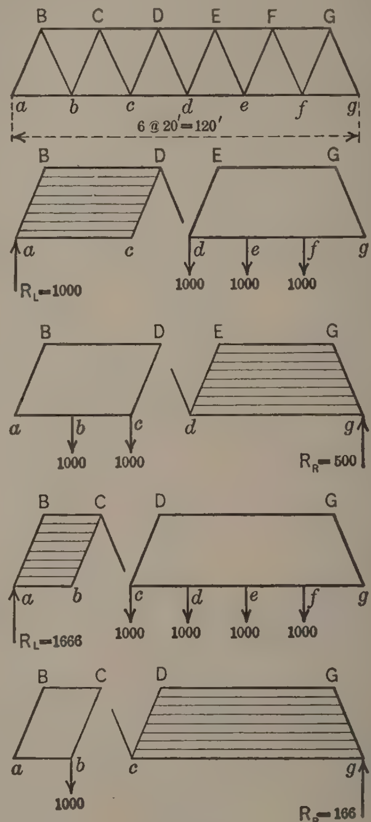


Fig. 314.

5. *Alternative method of sections.* In the illustrative problem just solved, two maximum stresses (maximum tension and maximum compression) were determined for each web member in the left-hand half of the truss except for the member Bb. Throughout the problem the *unloaded*

segment was taken as the body in equilibrium, but this unloaded segment was alternately a left-hand segment and a right-hand segment. The general method of sections used in the problem will now be applied in a slightly different way. Throughout this alternative method, the unloaded segment is taken as the body in equilibrium just as before, but instead of this segment being alternately a left-hand segment and a right-hand segment, the sections are so taken that *the unloaded segment is always a left-hand segment.* The only external force on this segment is the left-hand reaction, consequently, *the shear for each position of the load will be the left-hand reaction.*

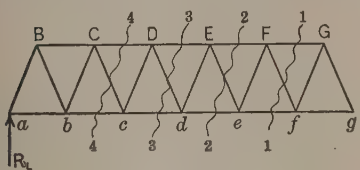


Fig. 315 (*a*).

1. Assuming the same truss and live load as before, the problem will be solved by taking in succession the four sections shown in Fig. 315 (*a*), namely, sections 1–1, 2–2, 3–3, and 4–4.

Section 1–1

Place a load at f. The segment in equilibrium is $aBFe$ and the reaction R_L on this segment is $\frac{1}{6} \times 1000 = 166$ lbs. The stress in Ff is $166 \times 1.083 = 180$ lbs. tension, and the corresponding maximum stress in Fe is 180 lbs. *compression.* This maximum compression in eF is the same as the maximum compression found for Cc in 314 : 4 as, of course, it should be.

Section 2–2

Place a load at f and also at e. The segment in equilibrium is $aBEd$, the reaction R_L is $1000 \times (\frac{1}{6} + \frac{2}{6}) = 500$ lbs., and the stress in Ee is $500 \times 1.083 = 540$ lbs. tension; the corresponding stress in Ed is 540 lbs. *compression.* This maximum compression in Ed is the same as the maximum compression found for Dd in 314 : 4.

Sections 3–3 and 4–4 are identical with the sections taken in calculating respectively the maximum tension in Dd and the maximum tension in Cc in 314 : 4, and need, therefore, no further consideration.

The maximum tensile stress and the maximum compressive stress have now been determined for each web member, just as in the first method. The difference in the two methods is merely one in choosing sections. There is, perhaps, less likelihood of making mistakes if one always works with the left-hand segment as the body in equilibrium, and for this reason the alternative method just explained may be preferable. It will now be applied to a problem in determining the live-load stresses in a Pratt truss.

2. *Method of sections, second illustrative problem.* Given: The Pratt truss shown in Fig. 315 (*b*). Dead load per panel = 1000 lbs.; live load per panel = 1000 lbs. Height of truss = 24 ft. Length of panel = 20 ft. Required: The dead-load and live-load stresses. The dead-load stresses were determined in typical members by the method of sections in 187 : 4. Since the panel loads for live and dead loads are the same, the live-load stress in any member of the perimeter of the truss will be the same as the dead-load stress in that member, consequently, it will be necessary to calculate here only the live-load stresses in the web members.

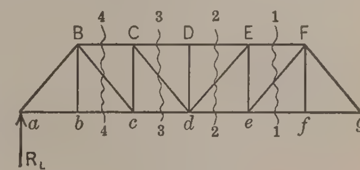


Fig. 315 (*b*).

3. The secant for the truss angle is 1.301. The live-load stresses in the inclined web members are found from the sections 1–1, 2–2, 3–3, and 4–4. The segment in equilibrium for each section is the *unloaded* segment; this is the *left-hand* segment and the corresponding reaction R_L is the shear for that segment. The stress in each diagonal is equal to the corresponding shear multiplied by the secant of the truss angle.

Member	*Section*	*Segment in equilibrium*	*Loads at*	*Shear (Reaction)*	*Stress*
$Fe = Bc$	1–1	$aBEe$	f	$\frac{1}{6} \times 1000$	220 C
$Ed = Cd$	2–2	$aBDd$	f and e	$\frac{3}{6} \times 1000$	650 C
$Ee = Cc$				$\frac{3}{6} \times 1000$	500 T
$Cd = Ed$	3–3	$aBCc$	f, e, and d	$\frac{6}{6} \times 1000$	1300 T
$Cc = Ee$				$\frac{6}{6} \times 1000$	1000 C
$Bc = eF$	4–4	aBb	f, e, d, and c	$\frac{10}{6} \times 1000$	2180 T
$Bb = Ff$					1000 T

4. Note that the maximum live-load stress in each of the hip verticals Bb and Ff is one panel load of 1000 lbs. (from fundamental principle 71 : 5),

and that the maximum live-load stress in each of the verticals *Cc* and *Ee* is the V component of the diagonal that meets the vertical at an upper chord joint. (From $\Sigma V = 0$ applied to the upper chord joint; also from 71 : 6.) The stress in *Dd* is zero. (71 : 4.)

1. *Note:* Note that a load at *f* causes compression in *eF*, likewise a load at *b* causes compression in *Bc*; loads at *e* and *f* cause compression in *dE*, likewise loads at *b* and *c* cause compression in *Cd*. The dead-load stresses in these diagonals (or in the diagonals of any Pratt truss) are always tension. There is, therefore, a possibility of reversal of stress in each of the diagonals. This reversal of stress will be discussed later.

2. **Trusses in Deck Bridges.** The general methods of determining live-load stresses in through trusses hold good for deck trusses. It is to be noted, however, that if a deck truss and a through truss of the same type have the same general dimensions, the stresses in some of the web members of the deck truss are not the same for a given live load as the stresses in the corresponding members of a through truss. For example, the live-load stress in an intermediate post of a deck Pratt truss is greater than the live-load stress in the corresponding post of the through truss, because for a section through the post there will be one more loaded joint on the loaded segment of the deck truss than on the loaded segment of the through truss. (What is the difference in stresses, if any, in a given diagonal?) (Compare the live-load stress in a diagonal of a deck Warren truss without verticals with the live-load stress in the corresponding diagonal of a through Warren truss without verticals, assuming both trusses to have the same general dimensions.)

3. **Comparison of Fundamentals Used in Calculating Live-Load Stresses with Those Used in Calculating Dead-Load Stresses.** Although the methods of determining live-load stresses are primarily the same as those used in calculating dead-load stresses, there are fundamental conceptions and principles pertaining to live-load stresses that, in some cases, differ from similar fundamentals pertaining to dead-load stresses. A comparison of these fundamentals before proceding further may be helpful. From the following arrangement in parallel columns, it is obvious in what respects a statement pertaining to live load differs from the corresponding statement pertaining to dead load.

DEAD LOAD	CONVENTIONAL LIVE LOAD
(*a*) The truss is always fully loaded, and maximum stresses in all members occur simultaneously. The stress in any particular member is always the same.	(*a*) The truss may be fully loaded or only partially loaded. The stress in any member at any given instant depends upon how much of the truss is loaded and varies from zero to a maximum — in some members it may change from tension to compression or *vice-versa* as the live load changes its position. The maximum stresses in all members do *not* occur simultaneously.
(*b*) In any member in the *upper* or *lower chord*, the maximum stress occurs when the truss is fully loaded, since the truss is always fully loaded. The stress may be obtained from the bending moment.	(*b*) In any member of the *upper* or *lower chord*, the maximum stress occurs when the truss is fully loaded, and it may be obtained by multiplying the dead-load stress in the same member by the ratio of a panel of live load to a panel of dead load.
(*c*) In any main *web member*, the maximum stress occurs when the truss is fully loaded, since the truss is always fully loaded, and the stress is always the same in character — either always tension or always compression. The stress may be obtained from the shear.	(*c*) In any main *web member*, the maximum stress occurs, as a general rule, for one position of the load only. The maximum tension will occur when one segment is loaded and the other segment is empty. The maximum compression will occur when the conditions of loading are reversed — the second segment loaded and the first empty. In either case, the stress may be obtained from the corresponding shear.

(*d*) The magnitude of the shear is equal to the algebraic sum of the reaction on either segment and the dead load on that segment. The reaction is always the same, and for a symmetrical truss, symmetrically loaded and resting on two supports equally distant from the center, the reaction is equal to one-half of the total dead load on the truss.	(*d*) The shear is equal in magnitude to the reaction on the unloaded segment. The reaction depends upon how much of the truss is loaded, and must be recalculated for each new position of the loading. If it is desired to obtain both the maximum tension and the maximum compression in a web member, the two shears corresponding to the two positions of the load are used.
(*e*) When a vertical web member is the only web member at a joint, the stress in the member is equal to the dead load concentrated at that joint. This may or may not be a full panel load, according to whether or not all dead load is assumed to take effect at the joints of one chord.	(*e*) When a vertical member is the only web member at a joint, the stress in the member is equal either to a panel load or to zero, according to whether the joint is or is not a joint of the loaded chord
(*f*) The maximum stress in an inclined end post occurs when the bridge is fully loaded, since the bridge is always fully loaded, and may be obtained from the reaction.	(*f*) The maximum stress in an inclined end post occurs when the bridge is fully loaded, and may be obtained directly from the dead-load stress in the same post by the same ratio of live load to dead load that was used in calculating live-load stresses in chord members.

1. **The Calculation of Live-Load Stresses by the Method of Coefficients.** For parallel-chord trusses, the method of coefficients, explained in Chapter XV, is probably the most efficient method of calculating both dead-load and live-load stresses. As a general rule, the live-load stresses in members that lie in the perimeter of the truss may be calculated directly from the dead-load stresses. (See 311 : 1 for exceptions.) When they cannot be thus calculated, the live-load coefficients for these members will be those for a fully loaded truss, and will therefore be the same as the dead-load coefficients in the same members when the dead loads are applied only at joints of the loaded chord. It is only necessary, therefore, to explain the method of determining coefficients for live-load stresses in web members.

2. *The live-load coefficient for any web member is the shear (expressed in terms of panel loads) that causes the maximum live-load stress in that member.* If the unloaded segment is always taken as the segment in equilibrium (as it was in the method of sections), the shear for that segment is the corresponding reaction. This reaction was found by summing up the fractions of the loads that are transmitted to the corresponding support. The fraction that is the sum of these fractions is the live-load coefficient. For example, in Fig. 317 are shown the Warren truss and the Pratt truss already used in illustrating the calculation of live-load stresses by the method of sections. The live-load coefficient for Dd in the Warren truss is equal to the fraction used in calculating the reaction on the unloaded segment, namely, $\frac{6}{6}$, obtained by adding $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$. Similarly, the live-load coefficient for dE in the Pratt truss is $\frac{3}{6}$, obtained by adding $\frac{1}{6}$ and $\frac{2}{6}$. Each of these coefficients is merely the shear (equal to the reaction) expressed in terms of panel loads. The dead-load coefficient for any web member is also the shear expressed in terms of panel loads as explained in Chapter XV. (215 : 7.) The difference is that the live-load coefficient is equal to the coefficient for one of the reactions — that on the unloaded segment, whereas the dead-load coefficient, determined for a fully loaded truss, is, as a rule, not equal to the coefficient for either reaction, because the shear is not equal to the reaction.

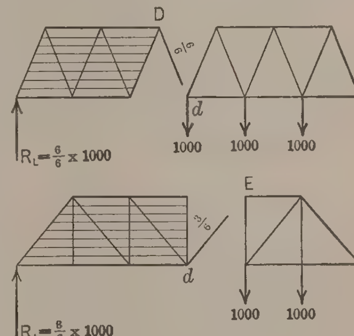


Fig. 317.

3. The method of coefficients used for calculating live-load stresses in web members does not differ essentially from the method of sections. In the method of coefficients, the coefficient used in finding the reaction

on an unloaded segment is entered on the web member cut by the section; in the method of sections this same coefficient is used in finding the reaction (equal to the shear) on the unloaded segment. For example in Fig. 318 are shown the live-load coefficients for all the web members of the Warren truss and the Pratt truss previously used to illustrate the method of sections. Note that the coefficients for the Pratt truss are identical with those in

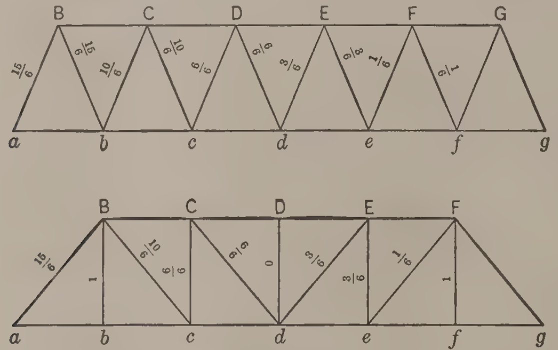


Fig. 318.

the fifth column of the calculation in 315 : 3, i.e., the coefficients used in determining reactions (equal to shears) on unloaded segments. A similar statement is true concerning the coefficients for the Warren truss and the coefficients for reactions (equal to shears) used in the calculations in 314 : 4.

1. *Note:* Compare the coefficients for live-load stresses in the web members shown in Fig. 318 with the coefficients for dead-load stresses in the same members shown in Fig. 215 (*b*) and Fig. 226 (*d*).

2. The coefficients for the live-load stresses in web members may be placed only on the web members in one-half of the truss. In this case nearly every web member in that half will have two coefficients, one for maximum tension and one for maximum compression. This method of placing the coefficients corresponds to the first method of sections explained in 314 : 4. The method that will generally be used in this book is that shown in Fig. 318, and this corresponds to the alternative method of sections explained in 314 : 5, in which only the left-hand segment for each section is considered.

3. In finding the reaction on the unloaded left-hand segment of a partially loaded truss, the numerators of the fractions added are "1," "2," "3," "4," "5," and so on, according to the number of panels loaded. The numerator of the coefficient of the reaction is always "1," or "3" = (1 + 2), or "6" = (3 + 3), or "10" = (6 + 4), or "15" = (10 + 5), or "21" = (15 + 6), or "28" = (21 + 7), or "36" = (28 + 8), or "45" = (36 + 9), and so on. The numerator of the coefficients for the live-load stress in a web member will, therefore, be one of the numbers "1," "3," "6," "10," "15," "21," "28," "36," or "45," unless the number of panels is large enough to require still larger coefficients. These numbers are easily remembered. They should be used not thoughtlessly but as a check. Hangers and similar members are exceptions to the rules. The coefficient for such a member is usually "1," sometimes "2" as in the case of the end vertical of a Baltimore truss, and it is well to enter these unity coefficients on the corresponding web members before placing the coefficients on other web members.

4. When *only two* web members meet at a joint of the unloaded chord, the live-load coefficients are the same for both members. (From $\Sigma V = 0$ applied to the joint.) For example, for any pair of diagonals that meet at a joint of the upper chord of the Warren truss in Fig. 318, the coefficients are the same; for a diagonal and an intermediate post that meet at a joint of the upper chord of the Pratt truss in Fig. 318, the coefficients are the same. In deck bridges, the rule may be applied wherever two and only two web members meet at a *lower* joint. Note that if there were verticals from the upper joints of the Warren truss in Fig. 318, the rule would not hold because there would be *three* web members at each upper joint.

5. *Checking coefficients.* The live-load coefficient for any member may be checked at any time by the shear on the *loaded* segment as well as by the shear on the unloaded segment. For example, in Fig. 318 the coefficient for cD is $\frac{6}{6}$. This coefficient is for loads at d, e, and f. The coefficient of the corresponding reaction at g is $\frac{12}{6}$ (from $\frac{5}{6} + \frac{4}{6} + \frac{3}{6}$), and the shear for the loaded segment is $\frac{12}{6}$ panel loads minus three panel loads or $\frac{12}{6} - \frac{18}{6} = -\frac{6}{6}$. (Check.)

6. The coefficients for nearly all of the web members may be checked to a certain degree simultaneously as follows: In a Warren truss without verticals, the coefficient of the last diagonal to the left should be equal to

the coefficient for the end post found from a fully loaded bridge. (See Fig. 318.) For trusses with verticals, there will usually be one joint of the loaded chord to the left of the section through the diagonal nearest to the end post. To the coefficient for this diagonal add the fraction that represents the proportion of a panel load that would be transmitted to the nearest support if the panel load is assumed at the joint just mentioned. The result should be equal to the coefficients for the end post (which is the same as the coefficient for the corresponding reaction) for a fully loaded truss. For example, in Fig. 318 the live-load and dead-load coefficients for the end post aB are each $\frac{15}{6}$, i.e., the coefficient for a fully loaded truss. The coefficient for the last diagonal Bc is $\frac{10}{6}$; if to this is added $\frac{5}{6}$ which is the fraction of a load at b that would be transmitted to the support at a, the result is $\frac{15}{6}$. (Check.)

1. *Caution:* The maximum live-load stress in a web member does not usually occur simultaneously with the maximum live-load stress in a chord member, consequently no check should be used for the coefficient for a web member that involves the coefficient for a chord member. For example, the check for dead-load coefficients explained in 221 : 9, in which $\Sigma H = 0$ is applied to three members cut by a section, cannot generally be used for live-load coefficients.

2. **Summary of the Method of Coefficients.**

3. *First:* Draw a truss diagram and enter on this diagram the dead-load coefficients as determined by the method explained in Chapter XV. (Apply the checks explained in 221 : 4.)

4. *Second:* Determine the live-load stress for each member in the perimeter of the truss directly from the dead-load stress in that member. (310 : 7.) No live-load coefficients for such members are necessary. (See 311 : 1 for exceptions.)

5. *Third:* Place coefficients for live-load stresses on the web members of the truss diagram in the following order: (1) On all members for which the coefficients are obviously unity, such members, for example, as the hip verticals in a Pratt truss, the hangers at the panel points of a Warren truss with verticals, and the hangers in a Baltimore truss. (2) On all diagonals. Follow the method explained in 317 : 2, in which only left-hand segments are considered. (3) On all of the remaining members, such for example as the posts in a Pratt truss. (Apply $\Sigma V = 0$ to joints of the unloaded chord.) (318 : 4.)

6. *Fourth:* Calculate the live-load stresses in the web members from the live-load coefficients. When a member is vertical, the coefficient is multiplied by the panel load; when a member is inclined, the coefficient is multiplied by the product obtained by multiplying the panel load by the secant of the truss angle. (This product is found once for all in a given problem, and is then used as a constant that is to be multiplied by the coefficient of every inclined member.)

7. *Suggestions:* Before placing coefficients on the web members of the truss diagram, number the panel points 1, 2, 3, 4, 5 and so on, beginning at the first intermediate panel point to the left of the right-hand support. In placing coefficients on diagonals begin with the diagonal in the panel to the left of the panel point numbered "1," and work toward the left. The numerator of the coefficient for any diagonal is the sum of the digits at all panel points on the right-hand segment. (318 : 3.) (An exception occurs when the section cuts two diagonals in the same panel, as, for example, a section through the main diagonal and the half-diagonal of a Baltimore truss.) The numerator of the coefficient (except in the case just noted) will always be one of the numbers 1, 3, 6, 10, 15, 21, 28, 36, 45 and so on. (318 : 3.) The denominator of the coefficient will be the number of panels in the truss.

8. After placing coefficients on the diagonals and before placing them on the remaining members, check the coefficients on the diagonals by the coefficient for the end post. (318 : 6.)

9. It is well to place only the *numerators* of live-load coefficients on the web members, provided some method is used of distinguishing live-load from dead-load coefficients. The student may find it best at first to draw a separate truss diagram upon which to place live-load coefficients for web members.

10. The coefficients for web members in the right-hand half of the truss may be transferred, if desired, to the corresponding members of the left-hand half. Each of the members in the left-hand half will then usually have two coefficients, one for tension and one for compression.

11. The experienced structural engineer knows which members of any standard type of truss are in tension and which are in compression for any position of the live load, but until one has this knowledge, it is well to designate in some way which coefficients are for tension and which are for compression. (Determine the sense of the stress by the method explained in 310 : 4.)

12. *Exercise:* Following the suggestions made in 319 : 7 to 319 : 11, check the live-load coefficients for all web members in each of the trusses shown in Fig. 320 (a). The coefficient for the end post in each case is for a fully loaded truss (equal to the coefficient for the reaction for a fully loaded truss), and should be used as a final check. (318 : 6.) Indicate for each truss which coefficients are for tension and which for compression. (310 : 4.) Note that *only the numerators* of coefficients have been entered on the truss diagram except in the case of the coefficients for the hip verticals of the through Pratt

truss, and of the verticals of the Warren trusses; each of these coefficients is obviously unity, and each has been enclosed in a circle to indicate that it represents a whole panel load instead of a fraction. Note that in the deck Pratt truss, the coefficients for the

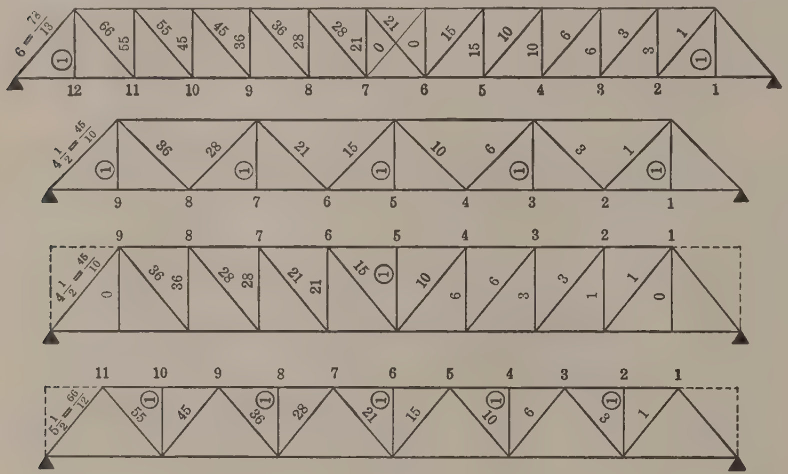


Fig. 320 (*a*).

intermediate posts are found, as usual, by applying $\Sigma V = 0$ to the joints of the *unloaded* chord (318 : 4), but that these joints are *lower* joints. The action of the counters in the center panel of the first truss will be explained later.

1. **Tables of Coefficients.** For any type of parallel-chord truss a table of dead-load and live-load coefficients may be prepared from which can be read the coefficient for any member of a truss of the same type, regardless of the number of panels in the truss. The table on page 321 is for through Pratt trusses of not less than four panels or more than twelve. Only such coefficients for live-load compression in diagonals are given as are likely to be needed in determining reversal of stress, minimum stresses, or counter stresses. The table may be used for deck trusses by merely modifying the coefficients for the verticals to provide for the dead load and the live load that are applied at each upper joint. Tables of this character are easily prepared for standard types of parallel-chord trusses and are very useful.

2. **Range of Stress and Reversal of Stress.** The method of determining the range of stress or the reversal of stress, as the case may be, in a member of a truss, will be explained by an illustrative example.

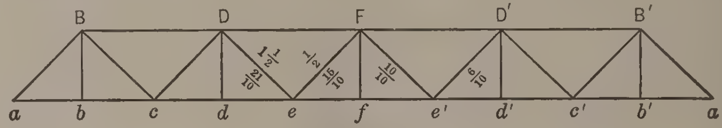


Fig. 320 (*b*).

Given: The Warren truss with verticals in Fig. 320 (*b*). Panel loads: Dead load = 37,500 lbs.; live load (including impact) = 41,250 lbs. Inclination of diagonals = 45°. Required: The range of stress or the reversal of stress in the diagonals *eF* and *De*.

Secant of 45° = 1.414.

Dead-load coefficients: $eF = \frac{1}{2}$; $De = 1\frac{1}{2}$.

Live-load coefficients: $eF = \frac{15}{10}$ and $\frac{10}{10}$; $De = \frac{21}{10}$ and $\frac{6}{10}$ (Fig. 320 (*a*).)

3. The stresses in *eF* are as follows:

(All forces except the final results are expressed in kips.)

Dead-load compression $= \frac{1}{2} \times 37.5 \times 1.414 = -26{,}500$ lbs.

Live-load compression $= \frac{15}{10} \times 41.25 \times 1.414 = -87{,}500$ lbs.

Live-load tension $= \frac{10}{10} \times 41.25 \times 1.414 = +58{,}350$ lbs.

Total compression $= -26.5 - 87.5 = -114{,}000$ lbs.

Total tension $= -26.5 + 58.35 = +31{,}850$ lbs.

It is seen that the live load causes a *reversal of stress* in *eF*, and that the maximum *range of stress* due to dead and live loads is from 114,000 lbs. compression to 31,850 lbs. tension.

4. The stresses in *De* are as follows:

Dead-load tension $= 1\frac{1}{2} \times 37.5 \times 1.414 = +79{,}500$ lbs.

Live-load tension $= \frac{21}{10} \times 41.25 \times 1.414 = +122{,}500$ lbs.

Live-load compression $= \frac{6}{10} \times 41.25 \times 1.414 = -35{,}000$ lbs.

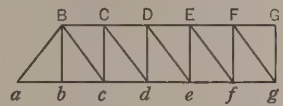
Maximum tension $= +79.5 + 122.5 = +202{,}000$ lbs.

Minimum tension $= +79.5 - 35.0 = +44{,}500$ lbs.

It is seen that the live load does not cause a reversal of stress in *De*, but reduces the tension to a minimum of 44,500 lbs. which is 35,000 lbs. less

COEFFICIENTS FOR THROUGH PRATT TRUSSES

All coefficients are for conventional panel loads.
The dead load is assumed as concentrated at lower joints.
Coefficients in bold face type are for live-load compression in diagonals.
All coefficients except those for verticals hold true for deck trusses.



D represents the dead load per panel.
L represents the live load per panel.
A represents the angle that a diagonal makes with a vertical.

Member	12-Panel Truss	11-Panel Truss	10-Panel Truss	9-Panel Truss	8-Panel Truss	7-Panel Truss	6-Panel Truss	5-Panel Truss	4-Panel Truss	Multiply by
Chords										
abc	5.5 D + 5.5 L	5 D + 5 L	4.5 D + 4.5 L	4 D + 4 L	3.5 D + 3.5 L	3 D + 3 L	2.5 D + 2.5 L	2 D + 2 L	1.5 D + 1.5 L	Panel length
BC, cd	10.0 “ + 10.0 “	9 “ + 9 “	8.0 “ + 8.0 “	7 “ + 7 “	6.0 “ + 6.0 “	5 “ + 5 “	4.0 “ + 4.0 “	3 “ + 3 “	2.0 “ + 2.0 “	divided by
CD, de	13.5 “ + 13.5 “	12 “ + 12 “	10.5 “ + 10.5 “	9 “ + 9 “	7.5 “ + 7.5 “	6 “ + 6 “	4.5 “ + 4.5 “			the height of
DE, ef	16.0 “ + 16.0 “	14 “ + 14 “	12.0 “ + 12.0 “	10 “ + 10 “	8.0 “ + 8.0 “					truss = tan A
EF, fg	17.5 “ + 17.5 “	15 “ + 15 “	12.5 “ + 12.5 “							
FG	18.0 “ + 18.0 “	15 “ + 15 “								
Diagonals										
aB	5.5 D + 5.5 L	5 D + 5 L	4.5 D + 4.5 L	4 D + 4 L	3.5 D + 3.5 L	3 D + 3 L	2.5 D + 2.5 L	2 D + 2 L	1.5 D + 1.5 L	Length of the
Bc	4.5 “ + $\frac{55}{12}$ “	4 “ + $\frac{45}{11}$ “	3.5 “ + $\frac{36}{10}$ “	3 “ + $\frac{28}{9}$ “	2.5 “ + $\frac{21}{8}$ “	2 “ + $\frac{15}{7}$ “	1.5 “ + $\frac{10}{6}$ “	1 “ + $\frac{6}{5}$ “	0.5 “ + $\frac{3}{4}$ “	diagonal di-
Cd	3.5 “ + $\frac{45}{12}$ “	3 “ + $\frac{36}{11}$ “	2.5 “ + $\frac{28}{10}$ “	2 “ + $\frac{21}{9}$ “	1.5 “ + $\frac{15}{8}$ “	1 “ + $\frac{10}{7}$ “	0.5 “ + $\frac{6}{6}$ “	0 “ + $\frac{3}{5}$ “		vided by the
Cd	3.5 “ − $\mathbf{\frac{3}{12}}$ “	3 “ − $\mathbf{\frac{3}{11}}$ “	2.5 “ − $\mathbf{\frac{3}{10}}$ “	2 “ − $\mathbf{\frac{3}{9}}$ “	1.5 “ − $\mathbf{\frac{3}{8}}$ “	1 “ − $\mathbf{\frac{3}{7}}$ “	0.5 “ − $\mathbf{\frac{3}{6}}$ “			height of truss
De	2.5 “ + $\frac{36}{12}$ “	2 “ + $\frac{28}{11}$ “	1.5 “ + $\frac{21}{10}$ “	1 “ + $\frac{15}{9}$ “	0.5 “ + $\frac{10}{8}$ “	0 “ + $\frac{6}{7}$ “				= sec A
De	2.5 “ − $\mathbf{\frac{6}{12}}$ “	2 “ − $\mathbf{\frac{6}{11}}$ “	1.5 “ − $\mathbf{\frac{6}{10}}$ “	1 “ − $\mathbf{\frac{6}{9}}$ “	0.5 “ − $\mathbf{\frac{6}{8}}$ “					
Ef	1.5 “ + $\frac{28}{12}$ “	1 “ + $\frac{21}{11}$ “	0.5 “ + $\frac{15}{10}$ “	0 “ + $\frac{10}{9}$ “						
Ef	1.5 “ − $\mathbf{\frac{10}{12}}$ “	1 “ − $\mathbf{\frac{10}{11}}$ “	0.5 “ − $\mathbf{\frac{10}{10}}$ “							
Fg	0.5 “ + $\frac{21}{12}$ “	0 “ + $\frac{15}{11}$ “								
Fg	0.5 “ − $\mathbf{\frac{15}{12}}$ “									
Verticals										
Cc	3.5 D + $\frac{45}{12}$ L	3 D + $\frac{36}{11}$ L	2.5 D + $\frac{28}{10}$ L	2 D + $\frac{21}{9}$ L	1.5 D + $\frac{15}{8}$ L	1 D + $\frac{10}{7}$ L	0.5 D + $\frac{6}{6}$ L	0 D + $\frac{3}{5}$ L	0 D + $\frac{1}{4}$ L	Unity
Dd	2.5 “ + $\frac{36}{12}$ “	2 “ + $\frac{28}{11}$ “	1.5 “ + $\frac{21}{10}$ “	1 “ + $\frac{15}{9}$ “	0.5 “ + $\frac{10}{8}$ “	0 “ + $\frac{6}{7}$ “	0.0 “ + $\frac{3}{6}$ “			
Ee	1.5 “ + $\frac{28}{12}$ “	1 “ + $\frac{21}{11}$ “	0.5 “ + $\frac{15}{10}$ “	0 “ + $\frac{10}{9}$ “	0.0 “ + $\frac{6}{8}$ “					
Ff	0.5 “ + $\frac{21}{12}$ “	0 “ + $\frac{15}{11}$ “	0.0 “ + $\frac{10}{10}$ “							
Gg	0.0 “ + $\frac{15}{12}$ “									

than the dead-load tension. The maximum range of stress in *De* is from 202,000 lbs. tension to 44,500 lbs. tension.

1. *Note:* For the sake of simplicity of explanation, the allowance for impact was included in the live load per panel. When allowance for impact varies for different members, as it frequently does, it is more usual in determining all of the stresses in a truss to calculate the actual live-load stresses first, and then from the live-load stress in each member to determine the impact stress for that member. (267 : 6.)

2. It is to be noted that in some specifications the allowance for impact used in calculating reversal of stress is greater than that used in calculating maximum and minimum stresses of the same character. (268 : 5.) (Examples of this will be given later.) In some specifications, only a fraction of the dead-load stress, as, for example, two-thirds, is considered as effective in counteracting the live-load stress. (267 : 2.) In any case, however, the impact stress that may be determined for a member must be added to the live-load stress in calculating range of stress or reversal of stress.

3. When there is a reversal of stress or a wide range of stress in a member, special consideration is usually given to this variation of stress when designing the member. (266 : 4.)

4. **Counters.** The live load, moving from one position to another, may cause alternate stresses of tension and compression in a web member, as illustrated in the preceding article. Either the web member must be so designed that it can take this reversal of stress, or the reversal of stress must be prevented by the introduction of a supplementary web member called a counter. (93 : 6.) It has been assumed thus far in this chapter, that a web member is designed to take either tension or compression should reversal of stress occur. There are, however, certain types of trusses, such, for example as the Pratt truss, in which it is often desirable to design the main diagonals to take tension only, and to prevent compression from occurring in such diagonals by the use of counters. The method of calculating stresses in counters will now be explained.

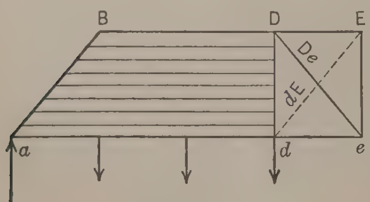


Fig. 322.

5. Let *aBDd* in Fig. 322 be a fully loaded segment of a Pratt truss (less than one-half of the span in length), and let the other segment be entirely free from live loads. This is the condition for maximum *negative* shear. Consider, for the present, the effect on the segment of the live loads only, and assume that the main diagonal *De* is incapable of taking compression. The shear, being negative, tends to move the segment downward, and if *De* were the only diagonal acting on the segment, it would act upward to resist the shear, i.e., it would be in compression. Since, by assumption, *De* is incapable of taking compression, it cannot act upward; the only other way in which the segment may be kept from moving downward is to connect some member to it that can act upward. Such a member *dE* is indicated by a broken line, and this member is a counter. The instant there is negative shear on the segment tending to move it downward, this counter comes into action to prevent such a movement, and at the same time to keep the main diagonal *De* from buckling. Assuming the live load to come on from the left and still leaving the dead load out of consideration, the counter *dE* would begin to act the moment any live load comes on the left-hand segment *aBDd* and it would continue to act until enough live load passes onto the corresponding right-hand segment to change the shear for the left-hand segment from negative to positive. The moment the shear becomes positive, the segment *aBDd* tends to move upward. Since the counter is incapable of taking compression, it cannot act downward to resist this shear, but the main diagonal *De* can and does, i.e., the moment the shear becomes positive the counter ceases to act and the main diagonal resists the shear. Note that the main diagonal acts only when the shear tends to move the segment *aBDd* upward, and that the counter acts only when the shear tends to move it downward. The shear cannot move the segment downward and upward at the same time, hence *the main diagonal and the counter cannot act at the same time* — when there is stress in one there is no stress in the other.

6. It now remains to take into account the dead-load stress in the main diagonal. This dead-load stress is always tension, always the same in amount, and is always to be taken into account whatever the position of the live load may be. (The dead load, as considered in this discussion, does not include any load, such as snow load, which is not always acting on the truss.) It will be shown that the dead load tends to prevent stress in a counter, whereas the live load, in certain positions, tends to cause stress in a counter; also, that in a given panel, a counter will be needed when the negative live-load shear is greater than the positive dead-load shear. In comparing the two shears, the *allowance for impact or for*

increase in the live load (267 : 3) should be included in the live-load shear. Hence in the following explanation, the term "live-load shear" should be understood as including the shear due to live load plus the increase in shear due to the allowance for impact or increased live load. Let V_L represent the maximum negative live-load shear for the segment *aBDd*, and V_D the positive dead-load shear for the same segment. The V component of the dead-load stress in the main diagonal is equal to V_D. Assuming the live load to be in the position to cause maximum negative shear as shown in Fig. 322, the combined dead-load and live-load shear will be:

1. Combined dead-load and live-load shear $= V_D - V_L$

When V_D is greater than V_L, the combined or resultant shear is *positive*, and that diagonal must act that is capable of acting downward, namely, the main diagonal *De*; the corresponding stress in the main diagonal will be the minimum tension that can ever occur in that member — less than the dead-load tension — namely, $(V_D - V_L) \times$ *secant of the truss angle.* When, on the other hand, V_L is greater than V_D, the combined dead-load and live-load shear is *negative*, and that diagonal must act that is capable of acting upward, namely, the counter *dE*; the stress in this counter will be $(V_D - V_L) \times$ *secant of the truss angle.* Note that when the left-hand segment is fully loaded and there is no live load on the other segment, the magnitude of the *combined* dead-load and live-load shear for that segment is a fixed quantity regardless of which diagonal is in action, — it is merely a question of whether it is a plus or minus shear in deciding whether the main diagonal or the counter is acting.

2. From what has preceded, it may be inferred that a counter is not necessary in every panel and this is true. It is only when the live-load *negative* shear, including allowance for impact, exceeds the dead-load positive shear for a left-hand segment, that a counter is needed in the corresponding panel. The dead-load shear increases from panel to panel as one proceeds from the center of the truss toward an end, whereas the reverse is true of live-load negative shear — it increases from the left end toward the center. The live-load negative shear is, therefore, more likely to be greater than the dead-load shear for sections through panels near the center of the truss, consequently counters are more likely to be needed in such panels than in panels near the ends of the truss.

3. *Note:* It is to be noted that the minimum stress in any main diagonal in a panel in which there is a counter occurs when the counter is in action, and that this minimum stress is zero.

4. In determining what counters are needed, begin with a section through a panel at the center and continue to take sections in successive panels toward the left end until a section is made for which the dead-load shear exceeds the live-load negative shear including impact. There is no need of proceeding beyond the panel through which this last section was taken.

5. It may be more convenient in some cases to work with the actual stresses in the main diagonal and counter than with the corresponding dead-load and negative live-load shears. If so, proceed as follows: Consider the main diagonal to be removed and a counter to be connected instead to the left-hand segment. The dead-load stress in this counter will be equal in magnitude to the dead-load stress in the main diagonal, but will be compression instead of tension. (Why?) From the maximum negative live-load shear, calculate the maximum live-load tension in the counter. Add the allowance for impact or increase in live load. If this total live-load tension is less than the dead-load compression in the counter, the counter is not needed (theoretically), but if the live-load tension is greater, the counter is needed, and the stress in it is the live-load tension minus the dead-load compression.

6. *Summary of principles pertaining to stresses in counters in parallel chord trusses:*

7. Stresses cannot occur simultaneously in a main diagonal and in the corresponding counter — when one is acting the other is not.

8. Dead load or any form of symmetrical loading causes no stresses in counters.

9. There is no tendency for a counter to act unless the live load is in a position to cause tension in the counter.

10. A counter will not begin to act until the live-load tension in it is greater than the dead-load tension in the corresponding main diagonal.

11. When the maximum live-load tension (including allowance for impact or increase in live load) in a counter is less than the dead-load tension in the corresponding main diagonal, the counter theoretically, is not needed.

1. The maximum stress in a counter may be obtained by one of three methods:

First method: Add algebraically the dead-load shear and the maximum *negative* live-load shear (including allowance for impact or increase in live load). If the result is positive, no counter is needed, but if the result is negative, multiply this negative result by the secant of the angle that the counter makes with the vertical.

Second method: Subtract from the maximum live-load tension in the counter (including allowance for impact or increase in live load) the dead-load tension in the corresponding main diagonal.

Third method: Subtract from the maximum live-load tension in the counter (including allowance for impact or increase in live load) the dead-load stress in the counter, assuming the main diagonal to have been removed.

2. *Note:* In parallel-chord trusses the first method enables one to tell almost at a glance whether or not a counter is needed, and if it is not, the work of obtaining the live-load stress in the counter from the maximum live-load negative shear is saved. Moreover the combined or resultant shear found in this method is used in determining the effect of counters on verticals as will be explained later. Hence the first method is the best for trusses with parallel chords. On the other hand, the second method is more like that used for trusses with inclined chords. The third method, though based on a slightly different conception than that which underlies the second method, involves identically the same calculations. Whichever method is used, it is advantageous to determine the maximum negative live-load shear and the corresponding live-load stress in the counter in a panel in the left-hand half of the truss by considering the corresponding panel in the right-hand half. (Alternative method (314 : 5 and 319 : 7).)

3. *Note:* The calculation of the stress in a counter of a parallel-chord truss is nothing more or less than the calculation of the reversal of stress in the corresponding diagonal. (320 : 2.)

4. *Illustrative example in calculating stresses in counters:* Given: A Pratt truss of thirteen panels. (Fig. 324.) Panel length = 21 ft.; height of truss = 28 ft. Panel loads: Dead = 16,000 lbs.; live = 21,000 lbs. Allowance for increase in live load for counters = 100%. Required: Stresses in all counters.

$$\text{Secant of truss angle} = \sqrt{21^2 + 28^2} \div 28 = \tfrac{5}{4}.$$

The coefficients on the diagonals in the left-hand half of the truss are those for dead load. (See Fig. 220 (*a*).) In the right-hand half of the truss the main diagonals have been removed and replaced by counters. The coefficients on these counters are for live load, and are the numerators of fractions each of which has 13 for its denominator. (See Fig. 320 (*a*).) The live-load coefficient for a counter in any panel is the same as the live-load coefficient for the main diagonal in the same panel for the same position of the live load. (Why?) The dead-load coefficient for a counter is the same as the dead-load coefficient for the corresponding main diagonal. A coefficient on any diagonal or counter represents the magnitude of the shear (in terms of panel loads) that is to be used in calculating the corresponding dead-load or live-load stress. (215 : 7 and 317 : 2.)

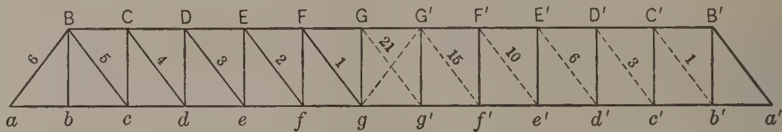


Fig. 324.

5. Beginning with the center panel, the stresses in the counters in the successive panels toward the left-hand end will be calculated until a panel is reached in which no counter is needed. (323 : 4.) It is to be noted that since this truss has an odd number of panels, the dead-load shear for the center panel (and therefore the dead-load stress in either Gg' or $G'g$) is zero. In the calculations indicated in the tabular form that follows, the allowance for increase in the live load is 100%, hence the live-load shear is multiplied by 2 before adding algebraically the dead-load shear.

Counter	*Shear*		*Stress*
	Dead-load	*Live-load*	
$Gg'=gG'$	0	$\left(-21\times\frac{21.0}{13}\right)\times2=-67.8$	$-67.8\times\frac{5}{4}=84.8$ T
$fG=f'G'$	16.0	$\left(-15\times\frac{21.0}{13}\right)\times2=-48.2$	$(16.0-48.2)\times\frac{5}{4}=40.5$ T
$eF=e'F'$	$2\times16.0=32.0$	$\left(-10\times\frac{21.0}{13}\right)\times2=-32.4$	$(32.0-32.4)\times\frac{5}{4}=$ 0.5 T
$dE=d'E'$	$3\times16.0=48.0$	$\left(-6\times\frac{21.0}{13}\right)\times2=-19.4$	$(48.0-19.4)$ 0

6. For the fourth panel the maximum live-load negative shear is not great enough to overcome the dead-load shear, hence the stress in the

counter dE (or $d'E'$) would be zero, i.e., no counter dE is needed. It follows that no other counter is needed in that portion of the truss between e and a or between e' and a'. The dead-load stress in De is 48,000 $\times \frac{5}{4}$ = 60,000 lbs. When the left-hand segment (joints b, c, and d) is fully loaded with live load and there is no live load on the corresponding right-hand segment (position for maximum negative live-load shear), the tension in De is actually less than when no live load is on the truss, i.e., the tension in De is (48,000 − 19,400) $\times \frac{5}{4}$ = 37,500 lbs. as compared with 60,000 lbs. dead-load tension. The stress of 37,500 lbs. is the *minimum* stress that can occur in De.

1. *Counters in the center panel of a truss with an odd number of panels.* When a truss has an odd number of panels and counters are used, the two diagonals in the center panel will both be counters. (Fig. 324.) The dead-load stress in each will be zero, just as in any counter. (The dead-load shear for the center panel will be zero.) It is assumed that when there is live-load stress in one counter there is no live-load stress in the other, that neither counter will take compression, and that for any given position of the live load, that counter will act which by exerting a tensile force will resist the live-load shear for the center panel, i.e., whichever counter in the center panel is in tension will be the one in action. The maximum live-load stress in this counter will be the maximum stress it will ever be required to take, since there is no dead-load stress with which the live-load stress must be combined, as is the case in most counters. The maximum live-load stress in one counter in the center panel is determined therefore just as if it were a main diagonal and the other counter in the panel did not exist. For example, the maximum stress in Gg' in Fig. 324 will occur when the live load is at joints b', c', d', e', f', and g', and will be equal to $\frac{21}{13}$ × 21,000 lbs. $\times \frac{5}{4}$ plus the allowance for impact or increase in live load. (See tabular form for the stress in Gg'.) An equal stress will be in gG' when the live loads are at joints b, c, d, e, f, and g.

2. *Note:* The case in which the two diagonals in a center panel are designed to act simultaneously is treated in 329 : 5.

3. **Effect of Counters on Vertical Members.** In determining the minimum as well as the maximum stresses in a vertical member of a truss with counters, it is important to ascertain whether in *each* of the panels on either side of the vertical it is the main diagonal or the counter that acts when the live load is in a certain position. A counter does not affect the *maximum* live-load stress in a vertical unless it is a vertical in the center of the truss, but it may reduce the stress in a vertical to less than the dead-load stress or even to zero, and thus cause a *minimum* stress. This is best explained by means of typical problems that illustrate the effect of counters on stresses in verticals.

4. *Effect of counters on the stresses in the center post of a through Pratt truss with an even number of panels.* Let Ee in Fig. 325 (a) and Fig. 325 (b) be the center post in *any* through Pratt truss with an even number of panels. When there is no live load on the bridge, the counters dE and Ed' are not in action, and the only members that are in action in the two center panels are those shown by full lines in Fig. 325 (a). Under these conditions, the post Ee is the only web member at joint E, and hence the dead-load stress in Ee is zero, or, in case there is a partial dead load at E, it is equal to this partial load. This dead-load stress is the *minimum* stress that can occur in Ee.

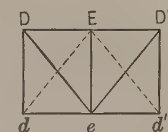


Fig. 325 (a).

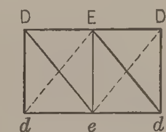


Fig. 325 (b).

5. When the live load is in a position to cause maximum live-load stress in the counter Ed', it will extend from the right-hand end of the truss up to and including joint d'. For ordinary ratios of live load to dead load, the counter Ed' will always be needed when the live load is in this position, and the V component of the stress in the counter will be the algebraic sum of the dead-load and live-load shears for the panel ed'. (324 : 1.) Under these conditions, the main diagonal eD' is not in action, and the only members that are in action in the two center panels are those shown by full lines in Fig. 325 (b). From $\Sigma V = 0$ applied at joint E, the stress in Ee is equal to the V component of the stress in the counter Ed' plus the partial dead load at E if there is any.

6. The live-load shear for panel ed' without allowance for impact may be less than the dead-load shear but greater when impact is added. In such a case, the counter Ed' would not act were it not for impact. This is immaterial, however, because in determining the combined dead-load and live-load shear, allowance for impact should be added to the live-

load shear. This allowance should be that for the post Ee and not, necessarily, the increase in live load or the allowance for impact used in determining the maximum stress in the counter. The corresponding stress in the post Ee, equal to the combined shear for panel ed' will not be, strictly speaking, a live-load stress but a stress due to both live load and impact.

1. *Effect of counters on any post in a through Pratt truss other than a center post.* The maximum live-load stress in any post of a through Pratt truss other than a center post is brought to it by a main diagonal, consequently the counter is not considered in determining such a stress. When, however, it is desired to obtain the minimum stress in a post and there is a counter in the panel on either side of the post, this counter must be considered; if there are two counters, one on each side of the post, both must be considered. A counter may, under certain conditions caused by the changing position of the live load, reduce the stress in a vertical until it is less than the dead-load stress, or even until it is zero.

2. Let Oo in Figs. 326 (a) and 326 (b) represent any post in the left-hand half of a through Pratt truss. Assume that when the live load extends from the left-hand end of the truss up to and including joint o, the counter oP is in action. Under these conditions the main diagonal Op cannot act, but the counter nO may or may not be in action.

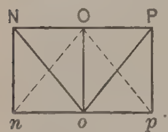


Fig. 326 (a).

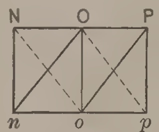


Fig. 326 (b).

3. If the counter nO is not in action and the main diagonal Op cannot act, the diagonals that are in action are those represented by full lines in Fig. 326 (a), namely, the diagonal No and the counter oP. No web member is in action at joint O, hence from the $\Sigma V = 0$ applied to that joint, the stress in Oo is zero, or, in case there is a partial dead load applied at O, it is equal to this partial load. In either case, it is less than the regular dead-load stress, and is the minimum stress that can occur in Oo.

4. *Note:* The diagonal No will act when the dead-load shear for the panel no is greater than the live-load shear for the same panel. The counter oP will act when the live-load shear is greater for the panel op than the dead-load shear for the same panel. Both of these conditions must be fulfilled in order to reduce the stress in Oo to zero or to a partial dead load as the case may be. If they both are fulfilled without allowance for impact, such allowance may be disregarded. (The minimum stress might occur when the live load extends from the left-hand end to joint o but is not in motion.) If, however, the two conditions are fulfilled only when allowance for impact is added to the live-load shear, such allowance should be made.

5. The second condition to be considered is that in which the counter nO acts when the counter oP acts, as represented by the full inclined lines in Fig. 326 (b). Under this condition also the diagonal Op cannot act, hence the only inclined web member in action at joint O is the counter nO. The stress in Oo will now be equal to the V component of the stress in the counter nO, or, if there is a partial dead load at O, it will be equal to the V component plus this partial load. If this stress is less than the regular dead-load stress in Oo, it is the minimum stress that can occur in that post, otherwise the dead-load stress is the minimum. The V component of the stress in the counter nO is equal to the combined shear for panel no. (323 : 1.)

6. *Note:* The counter nO will act when the live-load shear for panel no is greater than the dead-load shear for the same panel. The counter oP will act when the live-load shear for panel op is greater than the dead-load shear for the same panel. Both of these conditions must be fulfilled in order to reduce the stress in Oo to less than the dead-load stress, if that is possible. If they both can be fulfilled without allowance for impact, the V component of nO and the equal live-load stress in Oo will be less than if impact were added, and since it is the minimum stress that is desired, impact would not be added. If, however, it is necessary to add impact in order to bring both counters, nO and oP, into action, this should be done.

7. *Note:* It is conceivable that both counters, nO and oP, would be in action when the live load extends from the left-hand end of the truss up to and including joint n instead of joint o, but the V component of the counter nO would be greater than when the live load extends to joint o (Why?), hence the minimum stress in the post Oo could not occur with the load extending only to joint n. This is equivalent to stating that the minimum stress in the post Oo does not occur simultaneously with the maximum stress in counter nO. It is also conceivable (but not probable) that both counters nO and oP would be in action when the live load extends from the left-hand end of the truss up to and including joint p. In that case, the stress in the post Oo would be the minimum stress since it would be less than when the live load extends to joint o. (Why?)

8. *Illustrative example of the effect of counters on posts of a through Pratt truss.* Assume the panel loads for the truss shown in Fig. 327 (a) to be: Dead = 9000 lbs., one third of which is applied at the upper joint; live = 16,000 lbs. Required: To determine the effect of counters on the posts Ee and Dd.

Dead-load stresses: $Ee = \frac{1}{3} \times 9000 = 3000$ lbs.; $Dd = (\frac{1}{2} + \frac{1}{3}) \times 9000 = 7500$ lbs.

Shears for panel ed': Dead $= -\frac{1}{2} \times 9000 = -4500$ lbs.; live $= \frac{6}{8} \times 16{,}000 = 12{,}000$ lbs.

1. The combined shear for panel ed' is -4500 (dead) $+ 12{,}000$ (live) $= 7500$ lbs. Hence when the live load extends from the right-hand end up to and including joint d', the counter Ed' is acting instead of the diagonal eD', the stress in the counter is a maximum, the V component of this stress is 7500 lbs., and the total maximum stress in the post Ee is 3000 (dead) $+ 7500$ (live) plus the impact stress.

2. *Note:* Assume that the live-load shear for panel ed' is 4000 lbs. instead of 12,000 lbs. Since this is less than the dead-load shear, the counter Ed' will not act unless allowance for impact is added. Assume that the allowance for impact for the post Ee determined from the specified impact formula is 30%. The live-load shear including impact is $4000 + 4000 \times 0.30 = 5200$ lbs. The combined shear is $5200 - 4500 = 700$ lbs. and $3000 + 700$ is the stress in the post Ee including impact stress.

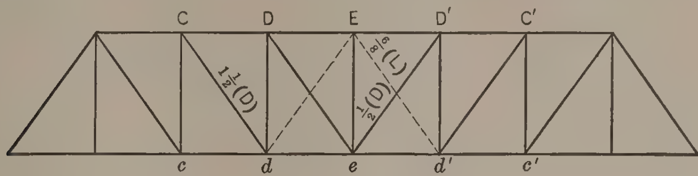


Fig. 327 (*a*).

3. When the live load is in the position to cause the maximum negative shear in panel de, namely, when it extends from the left-hand end of the truss up to and including joint d, the combined shear for panel de is 7500 lbs., i.e., it is the same as that obtained for panel ed' except that it is negative instead of positive, and the counter dE is in action. For this same position of the loading, the live-load shear for panel cd is: $(\frac{18}{8} - \frac{16}{8}) \times 16{,}000 = 4000$ lbs. Since this shear, as well as the dead-load shear for the panel is positive, the diagonal Cd is in action, and since the counter dE is also in action, the post Dd is the only web member that is acting at joint D. The stress in the post under these conditions is merely the dead load applied at D, namely, 3000 lbs. and this is the minimum stress for Dd; it is less than half as great as the stress in Dd when dead load only is acting. (7500 lbs.) If all of the dead load were considered as applied at lower joints, this minimum stress would be zero.

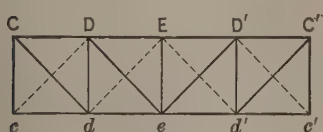


Fig. 327 (*b*).

4. *Effect of counters on verticals in a deck Pratt truss.* The same general principles used in determining the effect of counters on verticals of a *through* Pratt truss hold true in determining the effect of counters on verticals of a *deck* Pratt truss. Let Ee in Fig. 327 (*b*) be the center post of any deck Pratt truss with an even number of panels.

5. *Stresses in the center post Ee of a deck Pratt truss.* When dead load only is on the truss, counters dE and Ed' are not acting, and the stress in Ee is equal to the dead load that is applied at E. (This will be a whole panel load or a partial panel load according to whether the dead load is applied wholly at upper joints or a portion is considered as applied at lower joints.) The stress thus determined is the *minimum* stress that can occur in the center post Ee. When the live load extends from the right-hand end of the truss up to and including joint E, either the counter Ed' or the main diagonal eD' will act, according to whether the *resultant* shear in panel ED' from dead load and live load is positive or negative. (The counter dE will not act. Why?)

6. If the counter Ed' acts when the live load extends from the right-hand end of the truss up to and including joint E, the live-load stress in Ee (from $\Sigma V = 0$ at E) will equal the V component of the stress in Ed' plus the panel live load at E. In deck Pratt trusses with an even number of panels exceeding six, the stress obtained as just explained will under ordinary ratios of live to dead load, be the *maximum* live-load stress in the center post. (Why?)

7. If the main diagonal eD' acts when the live load extends from the right-hand end of the truss up to and including joint E, neither the counters Ed' or Ed will act, consequently Ee will be the only web member at E, and the live-load stress in Ee will be equal to the live load at E. In deck Pratt trusses with an even number of panels less than eight, the stress obtained as just explained will be the maximum live-load stress in the center post. (Why?)

8. *Note:* The determination of the live-load stress in the center post of a deck Pratt truss differs from the determination of the stress in the center post of a through Pratt truss in two respects, (1) the live load extends one panel farther (to the top of the post), and (2) this additional panel load is added to the V component of the stress in the counter.

9. *Note:* In determining the resultant or combined shear from dead load and live load, allowance for impact should be added to the live-load shear; this allowance should be that for the post Ee and not, necessarily, that for the counter Ed'. That portion of the live-load stress in Ee that is equal to this combined shear is then due to both live load and impact. (325 : 6.)

10. *Stresses in any post of a deck Pratt truss other than the center post.* The *maximum* stress in any post other than the center post of a deck Pratt truss is determined without regard to counters by following the general

method for determining stresses in web members. The *minimum* stress will occur when the live load is in the position to cause maximum negative shear for a section taken through the post. If under this condition of loading, the post is the only web member in action at the corresponding joint in the upper chord, the minimum stress in it will be simply the dead load that is applied at that joint; otherwise the minimum stress is determined from the shear in the usual manner. For example, in Fig. 328 (*a*) let *Dd* be any post other than a center post. The position of the live load for maximum negative shear for a section *ss* through *Dd* is from the left-hand end of the truss up to and including joint *C*. If, under this condition of loading, the diagonal *Cd* and the counter *dE* are both in action, the counter *cD* and the diagonal *De* are not, consequently the post *Dd* is the only web member in action at *D*, and the minimum stress in it is merely that due to the dead load at *D*. If, on the other hand, under the same condition of loading as before, both diagonals *Cd* and *De* are in action, neither counter can be, and the minimum stress will be determined from the combined dead-load and live-load shear for the section *ss*, just as if there were no counters in the trusses.

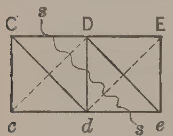


Fig. 328 (*a*).

1. *Illustrative example of the effect of counters on posts of a deck Pratt truss.* Assume the panel loads for the truss shown in Fig. 328 (*b*) to be: Dead = 9000 lbs., one-third

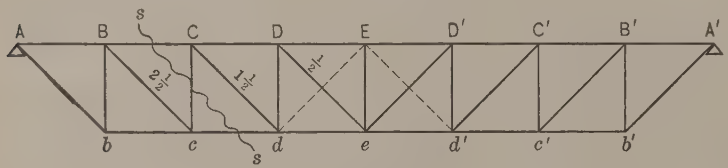


Fig. 328 (*b*).

of which is applied at the lower joints; live = 16,000 lbs. Required: To determine the effect of counters on posts *Ee* and *Dd*.

Dead-load stresses: $Ee = \frac{2}{3} \times 9000 = 6000$ lbs.; $Dd = (\frac{1}{2} + \frac{2}{3}) \times 9000 = 10{,}500$ lbs.

2. Shears for counter *Ed′*: Dead-load shear $= -\frac{1}{2} \times 9000 = -4500$ lbs.; live-load shear (when joints *B′*, *C′*, *D′*, and *E* are loaded) $= (\frac{10}{8} - \frac{6}{8}) \times 16{,}000 = 4000$ lbs. Unless impact is added, the dead-load shear is greater than the live-load shear, and there is no stress in the counter *Ed′*. Assume that the impact allowance for the post *Ee* as determined from the specified impact formula is 30%. The live-load shear, including this allowance, will then be $4000 + 4000 \times 0.30 = 5200$ lbs. The combined dead-load and live-load shear will be $-4500 + 5200 = 700$ lbs. and this is the V component of the stress in the counter *Ed′*. The live-load stress in *Ee* (from $\Sigma V = 0$ at *E*) is $700 + 16{,}000 + 16{,}000 \times 0.30 = 21{,}500$ lbs. including impact stress. Had the counter *Ed′* not been in action, the post *Ee* would have been the only web member acting at *E*, and the maximum live-load stress in it would have been equal to the live load at *E*, namely 16,000 lbs., and the impact stress would be $16{,}000 \times 0.30 = 4800$ lbs. In any case the dead-load stress in *Ee* is 6000 lbs.; and this is the minimum stress.

3. The maximum stress in the post *Dd* is found by the usual method. To determine the minimum stress, assume the live load to be applied at joints *B* and *C* only, and calculate the combined shears for each of the panels *CD* and *DE*.

Shears for panel $CD = 1\frac{1}{2} \times 9000 = 13{,}500$ lbs. (dead) and $(\frac{13}{8} - \frac{16}{8}) \times 16{,}000 = -6000$ lbs. (live). The dead-load shear for panel *CD* is greater than the live-load shear even when any reasonable allowance for impact has been made, hence the diagonal *Cd* is in action.

Shears for panel $DE = \frac{1}{2} \times 9000 = 4500$ lbs. (dead) and -6000 lbs. (live). (For loads at *B* and *C* only, the live-load shear for panel *DE* is the same as that for panel *CD*.) The live-load shear for panel *DE* is greater than the dead-load shear, even without allowance for impact, hence the counter *dE* is in action. Since the diagonal *Cd* is in action the counter *cD* is not, and since the counter *dE* is in action the diagonal *De* is not. This leaves the post *Dd* as the only web member acting at *D*, and the stress in *Dd* is the dead load applied at *D* which is 6000 lbs. This is the minimum stress in *Dd* and is 4500 lbs. less than the stress in *Dd* when only dead load is on the truss.

4. In a similar manner the minimum stress in *Cc* may be determined, the live load in this case extending from the left-hand end of the truss up to and including joint *B*.

Shears for panel $BC = 2\frac{1}{2} \times 9000 = 22{,}500$ lbs. (dead) and $-\frac{1}{8} \times 16{,}000 = -2000$ lbs. (live). The dead-load shear for panel *BC* is greater than the live-load shear including any reasonable allowance for impact, hence the diagonal *Bc* is in action. This might have been inferred from the fact that the diagonal *Cd* was in action when the live load extended to joint *C*, as previously determined.

Shear for panel $CD = 1\frac{1}{2} \times 9000 = 13{,}500$ lbs. (dead) and -2000 lbs. (live). (Live load at joint *B* only.) The dead-load shear for panel *CD* is also greater than the live-load shear, hence the diagonal *Cd* is also in action. The live-load stress in *Cc* is (from $\Sigma V = 0$ at *C*) the V component of *Cd* equal to the live-load shear for panel *CD* equal to 2000 lbs. tension. Assume that the impact allowance has been determined as 35%. The impact stress is 700 lbs. tension. The dead-load stress in *Cc* is $1\frac{1}{2} \times 9000 + 6000 = 19{,}500$ lbs. compression. The combined dead-load and live-load stress (including impact) is $19{,}500 - 2000 - 700 = 16{,}800$ lbs. compression, and this is the minimum stress for the post *Cc*. Note that this minimum stress is equal to the combined dead-load and live-load shear (including impact) for a section *ss* through *Cc* as, of course, it should be.

1. *Practical conditions that affect counters.* Conceive a rectangular frame composed of elastic material (as all structural material is). Let there be two diagonals, each a single wire and therefore incapable of taking compression. Assume two external forces applied at diagonally opposite corners of the frame to act toward each other, thus tending to produce compression in one wire. In order to cause compression, the two corresponding corners of the rectangle must be moved toward each other, but if the sides of the rectangular frame remain unchanged in length, the other two diagonally opposite corners would be moved away from each other. The corresponding diagonal would act in tension to prevent this movement. In other words, any tendency to produce compression in one diagonal wire would be resisted by the other wire acting in tension. This is the theory of the counter. In order that the action just described may be perfect, the forces must be exerted under ideal conditions which never exist in any structure. Among these conditions, the most important is that each diagonal wire should be exactly equal in length to the theoretical length of the diagonal of the rectangle in order that either wire may begin to act in tension the instant the other begins to buckle from compression. The impossibility of making diagonals of trusses of the exact theoretical length together with other practical reasons led to the use of **adjustable counters.** Such a counter usually consists of a rod or bar in two parts united by an open turnbuckle; a counter member of a truss may consist of two or more such counters side by side. Each counter can be adjusted by means of the turnbuckle until it is tight, i.e., until it reaches the theoretical length of the diagonal. If the turnbuckle is screwed up still more, the diagonal will begin to act in tension. This tension is caused, not by loads external to the truss, but, primarily, by the forces exerted by the turnbuckle, and is called **initial tension.**

2. *Initial tension in counters.* To insure the proper action of counters under the most unfavorable conditions, the more important of which have to do with lengths of members, connections of members at joints, and suddenly applied loading, an adjustable counter is usually screwed up until there is initial tension in the counter. This tends to cause an equal initial tensile stress in the main diagonal. (Why?) If the initial stress in the counter is moderate as it should be, it will have either no effect or little effect on the resultant stress in either the counter or the main diagonal. It can affect only members of the quadrilateral of which it is a diagonal and by an amount that can be ignored, as a rule, in designing those members. An exception is the case of a counter in which the initial tension is likely to be greater than the regular counter stress. Some specifications state that adjustable counters shall be designed for an initial stress of 10,000 lbs., and shall have open turnbuckles. The latter provision makes it possible to inspect the distance between the ends of the counter within the turnbuckle, and thus, to guard against excessive screwing up and corresponding excessive initial tension. There are certain advantages in non-adjustable or "stiff" counters when properly made, and some specifications state that when counters are used such rigid counters are preferred.

3. *Compression counters.* In wooden trusses of the Howe type, diagonals are normally in compression, and the connections of these diagonals at the joints of the truss are often such as to require that these diagonals should always act in compression. To prevent tension from occurring in the diagonals under movement of live load, **compression counters** are sometimes used. Such counters act in compression, but the method of determining stresses is essentially the same as for tension counters.

4. *Advantages and disadvantages of counters.* One of the advantages of the Pratt truss is that its longest members (diagonals) are normally in tension, and therefore lend themselves to economy of design and construction, particularly in pin-connected trusses. Counters make possible the use of bars for main diagonals — a simple and economic type of construction. On the other hand, trusses with counters are considered less rigid than trusses without counters. For this and other practical reasons, the trend seems to be toward trusses without counters for bridges of short span, even when the type used is the Pratt truss.

5. **Simultaneous Stresses in Two Diagonals in the Same Panel.** When two diagonals in the same panel are designed as "stiff" members to take either tension or compression and to act simultaneously, the stresses in these two diagonals are statically indeterminate, as explained in 200 : 5. In such a case, it is usually sufficiently accurate to assume that the greatest live-load shear for the panel is resisted equally by the two diagonals and may, therefore, be divided equally between them. (200 : 5 and 6.)

1. The only stresses that can be affected by the assumption that two diagonals in a panel act simultaneously (in addition to the stresses in the diagonals) are those in the four members that form the quadrilateral that contains the two diagonals. For example, if the two diagonals in the center panel of a Pratt truss with an odd number of panels are designed to act simultaneously, the only stresses that can be affected are those in the top and bottom chord members of the center panel and in the two verticals that form the sides of the panel. The live-load stress in each chord member may be obtained from the dead-load stress in that member, and does not differ from the live-load stress as determined on the assumption that only one diagonal in the panel acts at a time. (Why?) The maximum live-load stress in each of the two verticals, however, will be only half as great as it would be if only one diagonal acted at a time. (Why?) In a panel in which the shear is not zero when the maximum stress in a chord member occurs, the stress in each chord member of the panel is affected by the assumption that the two diagonals act simultaneously; it may be determined by the method explained in 200 : 6.

2. **Illustrative Problem — Dead-Load and Live-Load Stresses in a Pratt Truss.** The following problem will illustrate a general method of procedure in the calculation of all of the stresses, dead-load and live-load, in a Pratt truss with counters. It is assumed that the dead and live loads per linear foot of bridge are given. (Load per truss will equal one-half of the load for the bridge.) Impact stresses will not be calculated. The various steps in the work are numbered in order. This order of procedure is only one of several that are equally good. Likewise the arrangement of work is only one of several that are equally convenient.

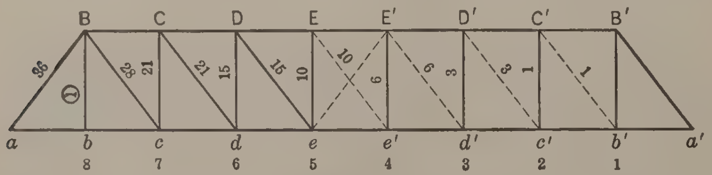


Fig. 330 (*a*).

3. Given: A through Pratt-truss bridge with counters. Span = 198 ft.; length of panel = 22 ft.; height of truss = 28 ft. Dead load per lin. ft. of bridge = 900 lbs. all of which is applied at lower joints; live load per lin. ft. of bridge = 1500 lbs. Required: Dead-load and live-load stresses.

4. *First step:* Draw free-hand a truss diagram, and place on this diagram the numerators of live-load coefficients for web members. Check coefficients for Bc. (Fig. 330 (*a*).)

Check for Bc : $\frac{28}{9} + \frac{8}{9} = \frac{36}{9} = 4$ panel loads $= R_L$. (318 : 6.)

5. *Second step:* Draw free-hand a truss diagram for one-half of the truss, and place on this diagram the dead-load coefficients. Check the coefficients for aB and EE'. (Fig. 330 (*b*).)

Check for $aB : 4 = R_L$. Check for $EE' : 4 + 3 + 2 + 1 + 0 = 10$. (223 : 5.)

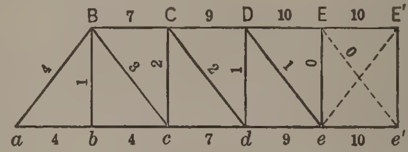


Fig. 330 (*b*).

6. *Third step:* Make the necessary preliminary calculations.

$9900 = 900 \times 22 \div 2 = W_D =$ dead load per panel per truss.

$16{,}500 = 1500 \times 22 \div 2 = W_L =$ live load per panel per truss.

$\frac{5}{3} =$ ratio of live load to dead load.

$0.786 = 22 \div 28 =$ *tangent* of truss angle A.

$1.272 = \sqrt{22^2 + 28^2} \div 28 =$ *secant* of A.

$7780 = 9900 \times 0.786 = W_D \times \tan A$.

$12{,}600 = 9900 \times 1.272 = W_D \times \sec A$

$21{,}000 = 12{,}600 \times \frac{5}{3} = W_L \times \sec A$.

$2333 = W_L \times \sec A \div 9$.

$1833 = W_L \div 9$.

In the fourth, fifth, and sixth steps, the work of calculation will be merely indicated. It is well to indicate the work in some such complete form as this *before* any of the stresses are calculated, leaving blank spaces, as shown here, in which to enter the stresses as they are determined, one after the other.

7. *Fourth step:* Calculate stresses in chord members.

	Dead Load			Live Load	
Member	*Stress*	$=$ *Coef.* $\times$	$W_D \times \tan A$	*Stress*	$=$*D.L. Stress*$\times \frac{5}{3}$
$BC = cd$		$=$ 7 $\times$	7780		
$CD = de$		$=$ 9 $\times$	"		
$DE = EE' = ee'$		$=$ 10 $\times$	"		

1. *Fifth step:* Calculate stresses in diagonals.

	Dead Load			Live Load		
Member	*Stress*	*= Coef.* ×	$W_D \times \sec A$	*Stress*	*= Coef.* ×	$(W_L \times \sec A) \div 9$
Bc		= 3 ×	12,600		= 28 ×	2333
Cd		= 2 ×	"		= 21 ×	"
De		= 1 ×	"		= 15 ×	"
Ee′	zero				= 10 ×	"
aB		= 4 ×	"		= D.L. Stress × $\frac{5}{3}$	

2. *Sixth step:* Calculate stresses in verticals.

	Dead Load			Live Load		
Member	*Stress*	*= Coef.* ×	W_D	*Stress*	*= Coef.* ×	$W_L \div 9$
Bb		= 1 ×	9900	16,500		
Cc		= 2 ×	"		= 21 ×	1833
Dd		= 1 ×	"		= 15 ×	"
Ee	zero				= 10 ×	"

3. *Seventh step:* Calculate stresses in counters assuming the allowance for increase in live load as 100%.

	Shear			
Panel	*Dead*	*Live*	*Increase*	*Combined Shear*
de	9900 = 1 × 9900	$-11{,}000 = 6 \times \frac{16{,}500}{9}$	−11,000	−12,100
cd	19,800 = 2 × 9900	$-\ 5500 = 3 \times \frac{16{,}500}{9}$	−5500	+ 8800

Stress in dE = 15,400 = −12,100 × 1.272.

4. In order actually to determine whether or not a counter is needed, the allowance for increase in the live load for counters, must be taken into account; for this reason such an allowance has been included. (322 : 6.) Since the resultant or combined shear for panel *cd* is positive, no counter is needed in that panel. Note, however, that this results in no stress — not even dead-load stress — in the post *Dd*, since, for the same condition of loading (joints *b* and *c* loaded), the counter *dE* is acting. (Why?) This zero stress is the minimum stress in the post; it means a range of stress in that post from zero to the maximum compression due to the dead-load and live-load stresses (including impact stress).

5. *Note:* The coefficients for both dead and live loads used in the fourth, fifth, sixth, and seventh steps could have been taken from the coefficients for a 9-panel Pratt truss in the table of coefficients on page 321.

6. **General Method of Procedure for the Calculation of Dead-Load and Live-Load Stresses in Parallel-Chord Trusses.**

7. *First step:* Draw a truss diagram.

8. *Note:* A small diagram, drawn free-hand, on which to enter coefficients will suffice. It may be well to draw two diagrams, one of the whole truss for live-load coefficients, and one of half of the truss for dead-load coefficients.

9. *Second step:* Make the necessary preliminary calculations.

10. *Note:* These calculations are usually: Panel loads (live and dead) per truss and the ratio of live to dead; length of diagonal; ratio of panel length to height of truss (tangent of truss angle); ratio of length of diagonal to height of truss (secant of truss angle); the dead load per panel multiplied by the tangent and by the secant of the truss angle; the live load per panel multiplied by the secant of the truss angle, and this product divided by the number of panels; if the truss has vertical posts, the live load per panel divided by the number of panels.

11. *Third step:* Enter dead-load and live-load coefficients on the truss diagram, and check these coefficients.

12. *Note:* Live-load coefficients will be needed for web members only, and these may be entered on all such members beginning at the right-hand end of the truss and working toward the left in accordance with the suggestions of 319 : 7. The checks for dead-load coefficients were explained in 221 : 4; for live-load coefficients in 318 : 5. If a portion of the dead load is considered as applied at joints of one chord and a portion at joints of the other chord, corresponding corrections should be made in the dead-load coefficients. (Usually in only those for verticals.) (218 : 3.)

13. *Fourth step:* Write in tabular form the expressions for all stresses, dead-load and live-load, in terms of the panel loads and functions of the truss angle. Check the expressions.

14. *Note:* Expressions for dead-load stresses should not be combined with those for live-load stresses. Expressions should be grouped, one group for chord members, one for diagonals, and one for verticals, but it is immaterial in what order the groups are arranged. Calculations for reversal of stress or for stresses in counters may be left until later. A vacant column is left in the tabular form in which to enter the stresses when calculated. Assuming that the preliminary calculations and the coefficients are correct, mistakes may still occur in writing the expressions. Such mistakes will usually become evident from a careful inspection of the tabulation, if this is arranged in some such systematic form as that shown in the illustrative example at the end of this chapter. This checking by inspection should be done before calculations for stresses are made.

1. *Fifth step:* When all expressions have been written and checked by inspection, and not before, carry out the calculations indicated by the expressions, and enter the results (stresses) thus determined in the columns left vacant for that purpose.

2. *Note:* The grouping of expressions suggested in the fourth step facilitates computation, particularly when the slide rule is used, since, in that case, the computations in each group can be made from one setting of the slide rule.

3. *Sixth step:* Tabulate the dead-load shear and the maximum negative live-load shear for each panel in the left-hand half of the truss for which combined shear may be needed.

4. *Note:* The coefficients for live-load shear are those for the diagonals in the *right-hand* half of the truss, transferred to the corresponding panels in the *left-hand* half. (319 : 10.) (See the seventh step in the illustrative problem 331 : 3.) The dead-load shear for any panel may be combined with the negative live-load shear for that panel for the purpose of calculating the stress in a counter or the reversal of stress or the minimum stress in a member, according to the type of truss and the stresses required. In combining dead-load and live-load shears, it is necessary to add to the latter any increase (due to impact or increase in live load) that may be required by the specifications, unless it is obvious that the dead-load shear is greater than the live-load shear (including impact).

5. *Note:* Instead of tabulating shears as a final step, it may be desirable to include in the fourth and fifth steps the determination of two live-load stresses, maximum tension and maximum compression, for each diagonal, as was done in the illustrative problem at the end of the chapter. In designing the members, such combination of stresses can then be made as may be necessary or desirable.

6. *Note:* It may be desirable to include in the tabular forms, columns for percentages allowed for impact and the impact stresses as determined from these percentages.

7. *Note:* As a final step a "stress sheet" may be prepared. Such a sheet may include a truss diagram drawn to a scale sufficiently large to permit three stresses to be entered on each member, namely, dead-load, live-load, and impact stresses. Principal dimensions of the truss should be shown. Data used in calculating stresses, may be given on the sheet; such data would include dead load, live load, and the allowance made for impact — the latter usually in the form of an empirical formula.

8. **The K-Truss.** The method of determining live-load stresses in a K-truss with parallel chords, such as that shown in Fig. 332, does not differ essentially from the general method used for the Pratt truss and other parallel-chord trusses. The live-load stress in any chord member may be determined directly from the dead-load stress in that member. (For the method of calculating chord stresses see 200 : 7.) The stresses in two diagonal web members in the same panel are equal but opposite in character, (200 : 7); the V component of each is equal to one-half of the shear for the panel, and will be a maximum when the shear is a maximum. For example, for the panel cd the positive live-load shear will be a maximum when the load extends from the right up to and including d, and the maximum live-load stresses in $c'D$ and $c'd$ will be each equal to one-half of this shear multiplied by the secant of the angle which each of these members makes with the vertical; under this condition of loading the live-load stress in $c'D$ will be compression — in $c'd$, tension. (Why?) The same position of the loading will result in the maximum live-load tension in Dd' — equal to the V component of the maximum compression in $c'D$. (From $\Sigma V = 0$ at D.) Maximum compression in dd' will occur when the load extends from the right up to e. (Why?) In calculating for reversal of stress in any web member referred to above, only those joints should be loaded which were not loaded when calculating the maximum stress in that member. All of this is in accord with the general method of calculating live-load stresses.

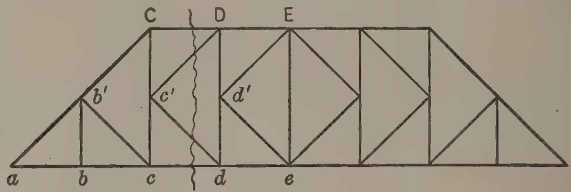


Fig. 332.

9. *Exercise:* In Fig. 227 (*c*) is shown one-half of a ten-panel K-truss. Assume that the joints of the lower chord of the other half are lettered from left to right g, h, i, j, and k. The live-load coefficients for the end post and for all chord members are the same as the dead-load coefficients for the corresponding members shown in Fig. 227 (*c*). Prove that the following tabulation of live-load coefficients, character of stress, and loaded joints is correct. (See 224 : 2.)

Member	*Coefficient*	*Character of Stress*	*Joints Loaded*
$d'E$	$\frac{21}{20}$	Compression	$e, f, g, h, i,$ and j.
$d'E$	$\frac{6}{20}$	Tension	$b, c,$ and d.
$d'e$	$\frac{21}{20}$	Tension	$e, f, g, h, i,$ and j.
$d'e$	$\frac{6}{20}$	Compression	$b, c,$ and d.
Dd'	$\frac{28}{20}$	Tension	$d, e, f, g, h, i,$ and j.
Dd'	$\frac{3}{20}$	Compression	b and c.
$d'd$	$\frac{21}{20}$	Compression	$e, f, g, h, i,$ and j.
$d'd$	$\frac{16}{20}$	Tension	$b, c,$ and d.
Ff	$\frac{10}{20}$	Tension	$f, g, h, i,$ and j.

1. *Suggestion:* In determining the live-load coefficient for either the upper or the lower half of a vertical, place the load in the position to cause the required maximum stress in the half of the vertical under consideration, and then determine first the corresponding coefficient for the diagonal that meets that half of the vertical at a joint of a chord, or, if there are two such diagonals, the coefficient for each.

2. *Question:* What other positions of the live load will give a coefficient of $\frac{10}{20}$ in Ff besides the position when joints f, g, h, i, and j are loaded?

3. **Special Problems in Determining Live-Load Stresses.** The criteria for placing live load, namely, at all panel points of the loaded chord for the maximum live-load stress in a chord member, and at all panel points of one segment only for the maximum stress in a web member, hold true for all types of parallel-chord trusses supported at each end. Likewise the general methods of calculating live-load stresses, namely, to determine the live-load stress in a chord member directly from the dead-load stress in that member, and to determine the live-load stresses in a web member from the maximum live-load shears (positive and negative) for a section through that member, hold good for *all* standard types of parallel-chord trusses. Any idea that each type is a different problem involving a different method should be discarded, for it is a real handicap in the study of stresses. The general method of procedure just outlined, or one similar to it, can be used for any common type of parallel-chord truss. There are, nevertheless, certain special problems that occur, that at first seem to be exceptions to the general rule. Few, if any, of these are involved in the calculation of stresses in chord members. More special problems are due to a secondary system of web members, such for example as that in a Baltimore truss, and are caused by the fact that any section through the primary web member also cuts a secondary web member as well as two chord members. The four unknown stresses corresponding to the four members cut by the section, seemingly make the problem of determining these stresses indeterminate. (69 : 1 (a).) It should be noted that *any member of a truss that can be cut by a section which intersects only two other members needs no special consideration — the live-load stress in that member is determined by the usual method.* The most common cases in which any section through a primary web member cuts also another web member will now be considered. These cases occur in different forms of the Baltimore truss.

4. **Special Problems Involved in the Determination of Stresses in a Baltimore Truss.** *Through Baltimore truss with sub-struts.* In Fig. 333 is shown a typical double panel $eEGg$ of a through Baltimore truss with *sub-struts.* The following general statements concerning the determination of live-load stresses may easily be verified:

5. The live-load stress in any member in the perimeter of the truss such as the chords EG and eg and the end post $ab' - b'C$, may be found directly from the dead-load stress in that member, just as in a Pratt truss.

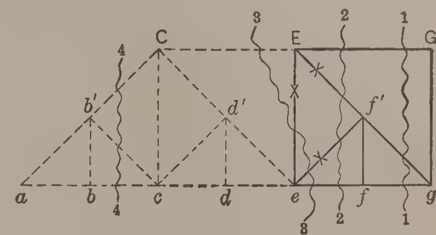


Fig. 333.

6. The stress in the lower half of a main diagonal will occur when all the joints of one segment only are loaded, exactly as in the case of a Pratt truss. (Only three members are cut by the section through the corresponding panel.) For example, in the case of $f'g$, the section is 1–1, and only the joints from the right up to and including g should be loaded for maximum positive shear; for maximum negative shear, only the joints from the left up to and including f.

7. The maximum stress in a sub-vertical such, for example, as ff' is equal to a panel load.

8. The stress in ef is equal to that in fg.

This leaves only three members in the panel to be considered, namely ef', Ef', and Ee. These three members marked " x " (and corresponding members in other panels) are the only members of the truss in which the stresses cannot be obtained exactly as in a Pratt truss. Note that any section through any one of these three members cuts three other members, or four members in all.

9. The V component of the stress in the half-diagonal ef' is equal to one-half of the stress in the sub-vertical $f'f$, equal to one-half the panel load at f. (201 : 5.) If there is no panel load at f, the sub-vertical $f'f$ may be considered removed, the removal of the half-diagonal ef' will follow as a consequence, and only the diagonals Ef' and $f'g$ will be left to act as one main diagonal.

1. The maximum positive live-load shear for Ef' will occur (in accordance with the usual criterion) when only the joints of the right-hand segment (section 2–2) up to and including f are loaded; the maximum negative shear when only the joints from the left up to and including e are loaded.

2. A portion of the maximum positive live-load shear for section 2–2 will be carried by the half-diagonal ef', namely, an amount equal to one-half a panel of live load, hence this amount must be subtracted from the positive shear in order to determine the V component of Ef', i.e., $Ef'_V = R_L - \frac{1}{2} W_L$ in which R_L is the live-load reaction at the left-hand support, and W_L is a panel of live load. Note that once the shear is known, this subtraction of the V component of ef' is the only difference between the calculation of the stress in the diagonal Ef' and in the calculation of the stress in the diagonal of a Pratt truss.

3. From $\Sigma V = 0$ applied at E, the maximum stress in the post Ee must be equal to the V component of the maximum stress in the diagonal Ef'. Once the maximum stress in the diagonal is known, the calculation of the stress in the post Ee differs in no respect from the calculation of the post in a Pratt truss. Note that the stress in the post may be obtained directly from the shear for the section 3–3 by taking into account the V component of the half-diagonal ef', and that this shear is the same as for the section 2–2.

4. The live-load stress in the hip vertical Cc is equal to two panel loads as in the case of dead load. (Fig. 220 (d).)

5. The stress in a counter $f'G$, if one is needed, is found from the combined dead-load and live-load shears for the panel fg, the live load extending from the left up to and including joint f. For the counter to act, the live-load negative shear, including allowance for increased live load or impact, must exceed the dead-load positive shear, or, to state it differently, the live-load stress in $f'G$ must exceed the dead-load stress in $f'g$ as these two members cannot act simultaneously. The method of determining the stress in the counter $f'G$ does not differ essentially from that used for determining stresses in the counters of a Pratt truss.

6. *Note:* When there is a counter $f'G$ in action, the sub-strut ef', normally a compression member, also acts as a counter thus making the whole diagonal eG a counter. Since there is a live load at f, there must also be a live-load stress due to this single panel load either in Ef' or $f'g$. (Why?) If it were in $f'g$ it would be compression, but the object of the counter is to prevent compression in $f'g$; the stress is therefore in Ef', and its V component is equal to one-half of the live load at f. (Why?) The stress in ef' when the live load extends from the left up to and including f, is not the same as the maximum stress which occurs in the counter $f'G$ for this position of the load, (Why?) nor is it the maximum counter stress that can occur in ef'.

7. The maximum stress in ef' acting as a counter, will occur when the live load extends from the left up to and including the joint e. It is found from the combined dead-load and live-load shear (including allowance for increased live load or impact) just as for any counter, and unless this shear is negative, ef' will not be required to act as a counter. (The combined shear is for section 2–2 in Fig. 333.) Note that for the position of the load just stated, there is no live load at f, consequently there will be no live-load stress in either ff' or Ef'.

8. *Note:* Summarized, the principal difference — almost the only difference — between the determination of stresses in a through Baltimore truss with sub-struts and in a Pratt truss, is in the calculation of stresses in the upper portions of the main diagonals. Even here, the criterion for placing the live load is the same. The difference consists merely in subtracting from the shear the portion carried by the half-diagonal, and this is the same difference that exists between the methods of determining dead-load stresses in the two types of trusses. (201 : 2.) The statements made in this note must be modified, however, when sub-ties are used in a Baltimore truss instead of sub-struts, as will now be explained.

9. *Through Baltimore truss with sub-ties.* In Fig. 334 is shown a typical double panel of a Baltimore truss with sub-ties. The method of determining live-load stresses in a truss of this type is, in general, the same as that for a Baltimore truss with sub-struts, but there are several important differences. These differences are due to two changes in conditions, namely: (1) The half-diagonal ($f'G$ or $d'E$) acts at a joint of the *unloaded* chord instead of at a joint of the loaded chord, hence at the top of a post there may be *two* inclined members in action instead of one, as, for example,

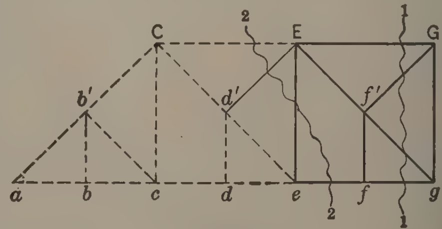


Fig. 334.

$d'E$ and Ef'. (2) The half-diagonal is in *tension* instead of compression, hence it exerts a force *upward* instead of downward on a left-hand segment. For example, for the section 1–1, the half-diagonal $f'G$ acts upward on the left-hand segment. This means that if the shear is positive, the half-diagonal does not act to resist this shear but, on the contrary, its V component is added to the shear.

1. Let W_L represent a panel of live load. If W_L is placed at f, it will cause a V component in $f'G$ equal to $\frac{1}{2} W_L$, and at the same time it will decrease the positive shear for section 1–1 by an amount *less* than $\frac{1}{2} W_L$. (Why?) The net result is that it has added to the resultant force that tends to move the left-hand segment upward, which is equivalent to increasing the positive shear due to any live load that may be on the right-hand segment. A similar statement may be made concerning a live load at the foot of any sub-vertical in the left-hand half of the truss, regardless of the number of panels in the truss. Assume, for example, that there are 12 panels. A load W_L at f decreases the positive shear by $\frac{5}{12} W_L$ (Why?) but adds at the same time an upward force on the left-hand segment (due to the V component of $f'G$) of $\frac{6}{12} W_L$, and the net result is equivalent to increasing the positive shear by $\frac{1}{12} W_L$. This leads to one of the few exceptions to the general criterion for the position of live load, as will now be explained.

2. The maximum live-load stress in the diagonal $f'g$ will occur when all joints of the right-hand segment (section 1–1) are loaded, and one joint of the left-hand segment is also loaded, namely, joint f. Of the four members cut by the section 1–1, the diagonal $f'g$ is the only one that can act downward on the left-hand segment, hence the stress in $f'g$ will be a maximum when there is the greatest resultant force upward on that segment. This occurs when the live load is in the position just stated, and, not, as the general criterion would require, when only the joints of the right-hand segment are loaded. The V component of the maximum live-load stress in $f'g$ is equal to the live-load shear (taking into account the load at f) plus the V component of the live-load stress in $f'G$ (equal to $\frac{1}{2} W_L$). In a truss of fourteen panels, for example, $f'g_V = \frac{21}{14} W_L + \frac{7}{14} W_L = \frac{28}{14} W_L$. If the loads extend from the right up to and including joint g, in accordance with the general criterion, the member $f'G$ would not be in action (since there would be no load at f), and the V component of the stress in $f'g$ would be equal to the live-load shear, or $\frac{26}{14} W_L$. Thus it is seen that a load at f adds $\frac{2}{14} W_L$ to the V component of $f'g$ when the number of panels is fourteen.

3. The maximum live-load stress in Ee will occur for one of two positions of the load, either when the right-hand segment formed by the section 2–2 is wholly loaded and there are no loads on the left-hand segment, or when, in addition to the loads on the right-hand segment, there are loads at the two joints e and d, depending upon whether, for the latter position, the loads at e and d decrease the positive shear or add to it. Assume that there are sixteen panels. Loads of W_L at e and d decrease the positive shear by $\frac{3}{16} W_L + \frac{4}{16} W_L = \frac{7}{16} W_L$, but the load at d adds $\frac{8}{16} W_L$ to the upward forces on the left-hand segment, due to the V component of the stress it causes in $d'E$, hence the net result is equivalent to increasing the positive shear by $\frac{1}{16} W_L$. For a truss of sixteen panels, therefore, the maximum stress in Ee will occur when the load extends from the right to d. Assume now the truss to have fourteen panels. Loads at e and d decrease the positive shear by $\frac{3}{14} W_L + \frac{4}{14} W_L = \frac{7}{14} W_L$, and the load at d adds $\frac{7}{14}$ to the upward forces on the segment, hence for a truss of fourteen panels, the maximum stress in Ee will occur for both positions, when the load extends from the right to f, and when it extends to d. Assume twelve panels. The net result of placing additional loads at e and d will be $-\frac{3}{12} W_L - \frac{4}{12} W_L + \frac{6}{12} W_L = -\frac{1}{12} W_L$, i.e., these loads do not add to the upward forces on the left-hand segment, but decrease them by $\frac{1}{12} W_L$, hence for the maximum stress in Ee, the load should extend from the right up to and including f. Similar reasoning may be applied in the case of any other post to determine which position of the live load will cause the maximum live-load stress in the post. It is evident that the nearer a post is to the center of the span the more likely it is that the maximum stress will occur when the right-hand segment only is loaded, as required by the usual criterion.

4. *Deck Baltimore truss.* In Fig. 336 (a) is shown a typical double panel of a deck Baltimore truss. The only difference between this type of truss and a through Baltimore truss with sub-ties is that the sub-vertical $f'F$ extends from f' to a joint of the upper chord instead of to a joint of the lower chord, and is in compression instead of in tension.

1. In general, live-load stresses are determined as for a through Baltimore truss with sub-ties. The most important exceptions to be noted are: From $\Sigma V = 0$ applied at e, the maximum live-load stress in Ee is equal to the V component of the maximum live-load stress in $d'e$, and hence it may be calculated directly from the stress in $d'e$. It will occur when the load extends from the right up to and including D. (An exception to the usual criterion.) It cannot occur when the stress in Ef' is a maximum, as it may in the corresponding through truss.

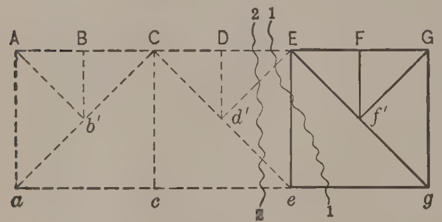


Fig. 336 (*a*).

2. The maximum stress in $b'C$ in the upper half of the *inclined* end post will occur when the load extends from the right up to and including C. A load at B will decrease the compression in $b'C$. (Why?)

3. The maximum live-load stress in the center vertical Gg of a twelve-panel truss will be equal to the full load at G plus one-half of the load at F plus one-half of the load at H, the panel joint immediately to the right of G. The stress is therefore equal to two panel loads. (Compare with the stress in Cc of a through Baltimore truss with sub-struts.) When there are more than twelve panels, an equal or greater stress may be obtained in the center post by bringing the load from the right up to and including the first joint to the left of the post. Under these conditions, the sub-tie at the top of the post and the right-hand diagonal at the bottom of the post are not in action, and the stress in the post is equal to the V component of the stress in the left-hand diagonal at the foot of the post. It is evident that the methods of determining the stress in the center post are similar to those used for any other post.

4. In the explanation of the special cases which arise in the determination of stresses in the three types of Baltimore trusses, it has been assumed that counters would be used. The methods of calculation may readily be modified to apply to trusses without counters. If minimum stresses are required, the methods explained in connection with the Pratt truss may be used, with few exceptions, for determining such stresses. In certain members, such as counters or members which are not in stress when counters are in action, the minimum stress is obviously zero. In other members, such as sub-verticals and sub-diagonals, the minimum stress is obviously the dead-load stress. For the great majority of the remaining web members, the minimum stress in any particular member will occur when only those joints are loaded that were not loaded for the maximum stress in that member. The few exceptions usually occur in web members that are in the same panel with a counter, and minimum stresses in such members may be determined by considering the effect of a counter on members in the same panel. (325 : 3.) In calculations for reversal of stress or for stresses in counters, allowance should be made for any increase in the live load or any impact allowance required by the specifications. When a portion of the dead load is applied at joints of the unloaded chord, it affects dead-load stresses in certain verticals (199 : 14), and this fact should be kept in mind when combining dead-load and live-load shears for such members.

5. **Graphic Method for Live-Load Stresses.** Let it be required to find by a graphic method the maximum live-load stress in the member De in the truss shown in Fig. 336 (*b*). Let the live load per panel be 8000 lbs. The right-hand segment will be loaded, and the reaction at the end of the unloaded left-hand segment will be 10,000 lbs. The graphic method of successive joints, explained in Chapter XVI, will be used.

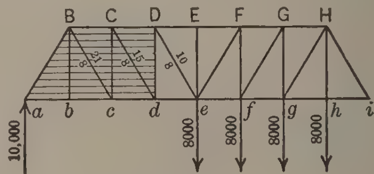


Fig. 336 (*b*).

6. The forces aB, ab, and the reaction (10,000 lbs.) at joint a being in equilibrium will form a force triangle, and this is the beginning of a force diagram *for the segment aBDd* which may be continued from joint to joint until at joint D the force De (the maximum live-load stress for the member De) is found. Note that the only force external to the truss which acts on the segment is the reaction, and that stresses for all members in the segment as scaled from the force diagram, though not maximum stresses, are correct for that reaction of 10,000 lbs. Hence the stress in either Cd or Bc, though not a maximum stress (and therefore of no practical use) is, nevertheless, the stress due to a reaction of 10,000 lbs.

1. Let it now be required to find the maximum stress for *Cd*. The segment is now *aBCc*, loads are placed at *h*, *g*, *f*, *e*, and *d*, and the reaction becomes 15,000 lbs. instead of 10,000 lbs. A new force diagram could be drawn for this reaction, beginning at joint *a*, but the stresses in all members in the new *segment aBCc* found from this new diagram would be 1.5 times as great as the stresses found in the same members from the original force diagram drawn for segment *aBDd*. (Why?) Hence instead of drawing a new force diagram it is better simply to multiply the stress found in *Cd* from the original force diagram by 1.5.

2. In a similar manner the stress found for *Bc* from the original force diagram multiplied by the proper ratio will give the maximum stress for *Bc*. This ratio is the reaction (21,000 lbs.) for loads on the right-hand segment *CHic* divided by 10,000 or 2.1. A general rule for the determination of maximum live-load stresses by the graphic method of successive joints may now be stated as follows:

3. *For members which lie in the perimeter of the truss: Draw a force diagram for a fully loaded truss. Dead-load stresses and maximum live-load stresses may all be determined from this diagram.*

4. *Note:* The diagram may be drawn for the actual panel loads (dead), in which case the dead-load stresses are found directly from the diagram and the live-load stresses by multiplying the dead-load stresses by the ratio of a panel live load to a panel dead load; or the diagram may be drawn for a unit panel load such as 1000 lbs. or 10,000 lbs., in which case the stresses thus found must be multiplied by the ratio of dead panel load to unit panel load to obtain dead-load stresses, and by the ratio of live panel load to unit panel load to obtain live-load stresses.

5. *For web members: Assume a convenient unit reaction such as 10,000 lbs. or 50,000 lbs. or 100,000 lbs. Draw a force diagram for a segment equal to at least one half of the truss, assuming that the unit reaction is the only force external to the truss which is acting on that segment. From this diagram scale the stresses for all of the web members that lie in the segment. The maximum stress in any web member will then equal the stress obtained for that member from the force diagram multiplied by the ratio of the actual reaction (when the truss is loaded to produce the maximum stress in the member) to the assumed reaction.*

6. *Note:* The ratios to be used in finding the maximum stresses in web members may be obtained algebraically as follows: Divide the actual live panel load by the assumed reaction thus obtaining a constant. The ratio for any web member will then be this constant multiplied by the live load coefficient for that member. For example, suppose that in Fig. 336 (*b*) the assumed reaction is 10,000 lbs. and the actual panel live load is 12,000 lbs. The constant is 1.2. The stresses in *De*, *Cd*, and *Bb*, as found from the force diagram, must be multiplied respectively by the ratios $1.2 \times \frac{10}{8}$, $1.2 \times \frac{15}{8}$, and $1.2 \times \frac{21}{8}$, in order to obtain the actual maximum stresses.

7. *Note:* For trusses with equal panels and equal panel loads, the reaction may be expressed in terms of a panel load multiplied by a fraction, the numerator and denominator of which are both expressed in terms of panel length. Such conditions permit a modification of the general method just explained. Instead of assuming any convenient reaction, a reaction is assumed that is equal to a panel load (live). The stress in any member determined graphically from this assumed reaction should then be multiplied by the fraction used in getting the true reaction for the live load in the position that causes the maximum stress in that member. For example, assume the panel load due to live load to be 12,000 lbs. The reaction used for the force diagram will then be 12,000 lbs. The stress in *De*, as determined by scaling the corresponding line in the force diagram, should be multiplied by $\frac{10}{8}$, the fraction that would be used in calculating the true reaction when the live load is in the position to cause the maximum stress in *De*. The result will be the required stress in *De*.

8. *Note:* To determine reverse stresses or counter stresses, the original force diagram may be continued for an *unloaded* left-hand segment of any length until that segment includes, if necessary, all but the extreme right-hand panel of the truss. This corresponds to the alternative algebraic method. (314 : 5.)

9. *Caution:* The rule given for web members does not apply in a few exceptional cases in which the maximum stress in a member occurs when some of the live load acts on the left-hand segment in addition to the reaction on that segment.

10. Maximum stresses in hip verticals, hangers, and similar members are determined by the usual method of analyzing a joint from inspection.

11. The graphic method of determining live-load stresses is not as efficient for parallel-chord trusses as the algebraic method of coefficients. It is useful, however, in determining live-load stresses in trusses with inclined chords, as explained in the next chapter.

12. **The Exact Method of Determining Stresses Due to Uniform Live Load.** If the maximum live-load stress in a member occurs when the live load extends from end to end of the bridge, there is no difference between the conventional and the exact methods of determining the live-load stress in that member. This applies, as a general rule, to all chord members and inclined end posts. It also applies to a web member, such as a hanger, in which the stress is determined from one or more full panel loads. The difference in the two methods exists, there-

fore, only in the case of a web member which acts to resist shear. The maximum live-load tension or compression in such a member may be determined from the corresponding maximum live-load shear. In the conventional method this shear is equal to the reaction on the unloaded segment. (299 : 11.) In the exact method, the live load is assumed to extend to the neutral point in the panel cut by the section through the web member. This neutral point is determined by the method explained in 298 : 4. The shear is equal to the algebraic sum of the reaction and the partial panel load on the otherwise unloaded segment, as explained in 298 : 6. (It may also be determined by an equivalent method to be explained in the next article.) From this shear the stress in the member may be determined by the usual method. To one who understands the conventional method of determining stresses in web members, the exact method is merely a simple modification of the conventional method and *vice-versa*.

1. **The Use of the Segments of the Base of an Influence Triangle in the Determination of Stresses.** *Chord members.* The maximum live-load stress in a chord member due to uniform live load occurs when the load extends from end to end of the bridge. This live-load stress may be determined from the bending moment for a center of moments somewhere in one of the chords. (196 : 3.) The center of moments for any chord member of any of the most common types of trusses will usually be either at a panel point or in a vertical line through a panel point. In either case, the bending moment may be determined as if the truss were a beam without panels, hence the short-cut method of 167 : 1 may be used, i.e., the bending moment is equal to the product of the two segments multiplied by one-half the load per linear foot. In the case of a truss, however, the segments are not segments of the truss as defined in 184 : 7, but they are the two horizontal distances from the center of moments (or a vertical through that center) to the two supports at the ends of the truss. These two horizontal distances are equal, respectively, to l_1 and l_2, the two segments of the base of the corresponding influence triangle for bending moments, one segment on either side of the maximum ordinate of that triangle. For example, in Fig. 280 (a), the center of moments for the stress in the chord member CD is d, the influence triangle for the corresponding bending moment is $ad'g$, and the two segments of the base of this triangle are $l_1 = ad$ and $l_2 = dg$. If w represents the uniform live load per linear foot of truss, the bending moment is: $M_B = \frac{1}{2} w \times ad \times dg$. The general expression for bending moment is:

$$M_B = \tfrac{1}{2} w \times l_1 \times l_2.$$

2. When the center of moments for the stress in a chord member is at a panel point or in a vertical line through a panel point, the corresponding influence diagram for bending moment is a triangle, and the method just given may be used; when, however, the center of moments is neither at a panel point nor in a vertical line through a panel point, the corresponding influence diagram is not a triangle, and the method cannot be used. The latter condition occurs in only a few types of bridge trusses. The Warren truss without verticals is one of these exceptional types. In this truss, the center of moments for any chord member of the loaded chord is in the other chord, in a vertical line half-way *between* panel points. For example, in Fig. 280 (b), the center of moments for the lower chord member cd is D in a vertical line half-way between panel points c and d, the corresponding influence diagram is not a triangle but the polygon $ac'd'g$, and the short-cut method cannot be used.

3. *Exercise:* Prove that the short-cut method $M_B = \frac{1}{2} w \times l_1 \times l_2$ can be used for determining the greatest bending moment due to uniform live load when the center of moments is at a panel point or in a vertical line through a panel point, but that it cannot be used when the center of moments is not so situated.

4. *Web members.* When the maximum live-load stress in a web member is determined by the exact method, the uniform live load extends from one of the supports to the neutral point in the panel. (280 : 1 and 2.) The corresponding influence diagram for shear is a triangle, and the base of this triangle is divided into two segments by the maximum ordinate. The length of one of these segments l_2 is the distance from the support at the loaded end of the truss to the nearer end of the panel through which the section is taken; the length of the other segment l_1 is the distance that the uniform load extends into the panel, i.e., the distance from the end of the panel to the neutral point. (298 : 4.) The greatest shear for any panel *for the exact method of loading* may be determined from the corresponding values of l_1 and l_2 as follows:

$$V = (\tfrac{1}{2} w \times l_1 \times l_2) \div p$$

w represents the live load per linear foot of truss, and p the panel length. As an illustration of the method, the truss in Fig. 298 may be used. The maximum live-load tension in the diagonal Cd due to uniform live load will occur when the load extends from g to n. The corresponding influence diagram for positive shear is the triangle $gd'n$, and the two segments of the base of this triangle are $l_1 = nd$ and $l_2 = dg$. Hence $V = (\frac{1}{2} w \times nd \times dg) \div cd$. The maximum live-load compression in the diagonal Cd will occur when the uniform live load extends from a to n. The corresponding influence triangle for negative shear is $ac'n$, and the two segments of the base are $l_1' = cn$ and $l_2' = ac$. Hence the greatest negative shear is $V = (\frac{1}{2} w \times cn \times ac) \div cd$.

1. *Note:* It should be remembered that l_1 for bending moment and l_1 for shear represent quite different distances. Although in each case l_1 represents a segment of a base of an influence triangle, for bending moment the base is equal to the length of span, whereas for shear, the base is less than the length of span, except possibly for shear in an end panel.

2. *Exercise:* Prove that the greatest shear for any panel, determined by the exact method, is $V = (\frac{1}{2} w \times l_1 \times l_2) \div p$.

3. The methods of determining bending moments and shears and the corresponding live-load stresses by means of the segments l_1 and l_2 are particularly useful in connection with equivalent uniformly distributed live load. Tables or diagrams may be prepared that give for different values of l_1 and l_2 the equivalent uniform live load for any system of concentrated loads, such as the loads on the wheels of a locomotive. By means of such tables or diagrams and the methods of determining bending moments and shears just explained, any live-load stress for which the influence diagram is a triangle may readily be calculated.

4. Impact Allowance — Uniform Live Load. The impact stress in any member is a percentage of the maximum live-load stress in that member; the percentage depends upon the "loaded length," i.e., the distance from the beginning to the end of the live load that is on the bridge when the maximum live-load stress in the member occurs. (268 : 2.) The percentage is determined by substituting the loaded length for L in the impact formula. (267 : 8 and 268 : 7.)

5. *The loaded length for impact stress in any chord member* of a parallel-chord truss is the length end to end of the truss, since the maximum live-load stress in such a member occurs when the live load extends from end to end of the bridge.

6. *The loaded length for impact stress in a web member* is the length of the portion of the bridge that is loaded when the maximum stress in the member occurs. This length, in the case of a main web member, is measured from the *neutral point* (in the panel cut by the section through the member) to the end of the bridge. The distance l_1, from the loaded end of the panel to the neutral point may be determined as explained in 298 : 4. If l_2 is the distance from the same end of the panel to the other end of the loaded segment of the truss, the loaded length is equal to $l_1 + l_2$. For the impact stress of the opposite character in the same member, the loaded length is the length of that portion of the bridge that was not loaded when the first maximum stress occurred; it is equal to $l_1' + l_2'$. (298 : 4.) The lengths l_1 and l_2 are usually used to designate the segments of the base of the influence triangle for positive shear, and l_1' and l_2' for the segments of the base of the influence triangle for negative shear. (298 : 4.)

7. For the impact stress in a web member in which the maximum live-load stress may result from a sudden application of the live load, the allowance for impact is frequently specified as a definite percentage of the live-load stress that is at least as great as, and usually greater than, the percentage that would be obtained from the use of the impact formula. This is particularly applicable to hangers that support floor beams.

8. *Note:* Some engineers consider that when the conventional system of loading is used, it is sufficiently accurate to assume as the loaded length for the impact stress in any web member, the length l_2 (or l_2') of the loaded segment to the right or left, as the case may be, of the panel cut by the section through that member, thus disregarding the distance l_1 (or l_1') that the load extends into the panel. The difference in percentages obtained from an impact formula for highway bridges, first by using $l_1 + l_2$ as the loaded length and then by using l_2 only, does not often exceed 2% for any web member in spans of ordinary length. When the panel cut by the section is to the left of the center of the truss and the right-hand segment is the one that is loaded, the difference in percentages will be less if, instead of l_2 for the loaded length, l_2 plus the panel length p is used, because $l_2 + p$ is more nearly equal to $l_2 + l_1$ than is l_2. The percentage for impact obtained by the conventional method (using l_2 only) is greater than that obtained by the exact method (using $l_1 + l_2$); the stresses in web members determined for the conventional method of loading are also greater than those determined for the exact method of loading. (308 : 3.) Consequently the discrepancy between the impact stress in a web member computed by the conventional method and that computed by

the exact method is considerably greater than the discrepancy between the two impact percentages as determined by the two methods. In determining whether or not this discrepancy between impact stresses is enough to require the use of the exact loaded length for L in an impact formula for highway bridges, it is well to consider the uncertainty that exists concerning impact allowance in general. (268 : 12.) For railroad bridges, uniform live load, when used, is generally an "equivalent uniform live load." (131 : 5.) The equivalent uniform live load for any member is best determined by the two segments l_1 and l_2 (or l_1' and l_2') of the influence triangle for the stress in that member, and the loaded length L in the impact formula is equal to $l_1 + l_2$ (or $l_1' + l_2'$).

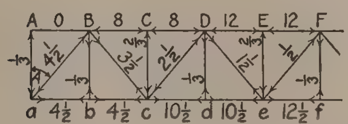
1. **Uniform Live Load with One or Two Concentrated Excess Loads.** In the more recent specifications for highway bridges, the live load for trusses is specified as a uniform load plus one (sometimes two) concentrated excess loads, as, for example, " 450 lbs. per lin. ft. and 21,000 lbs. concentrated, the concentrated load to be placed so as to cause maximum effect." In such a case it is best to determine the live-load stresses due to uniform live load separately, as if there were no concentrated load. The additional live-load stress in each member due to the concentrated load may be determined by placing the concentrated load in the position to cause the maximum stress in that member, and then calculating that stress from the bending moment if it is a chord member, or from the shear if it is a main web member. The methods of determining the positions of the concentrated load for greatest shear or greatest bending moment have been given in Chapter XIX and the application of these methods to a truss will be explained in Chapter XXIII.

2. **Systematic Method of Procedure and Arrangement of Work — Illustrative Example.** It is important to follow some good method of procedure and arrangement of work in the calculation of stresses. One such method is illustrated on the next page. In order that the method may be easily followed throughout by the student, the arrangement is somewhat different than that used by the experienced engineer. It will serve, however, to illustrate the essential features of any good method of procedure and arrangement of work, among which are: (1) Grouping of like operations in logical order. (2) Final results clearly shown in one column, and the quantities used in calculating each result given in the same horizontal line with the result. (In the illustrative example, final results are on the left of the quantities.) The arrangement should be such that the quantities and methods used in obtaining final results may be written first, and when the entire problem has been clearly outlined in this manner, the final results may all be computed, one after another. The arrangement should also be such that one may be able to detect any obviously incorrect variation in the stresses in members of the same general character.

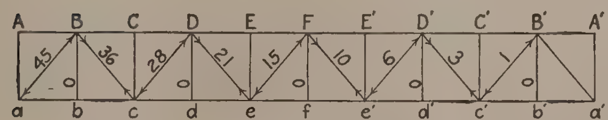
3. The method of procedure shown in the illustrative example is that given for coefficients in 331 : 6. The truss is similar to that shown in the photograph on page 105 except that there are ten panels instead of eight. Impact stresses are not determined in the problem; it is suggested that the student calculate these stresses from the live-load stresses by the method explained in 267 : 5 to 268 : 4 and in 339 : 4 to 339 : 8.

ILLUSTRATIVE PROBLEM IN THE CALCULATION OF DEAD- AND LIVE-LOAD STRESSES IN A DECK WARREN TRUSS WITH VERTICALS BY THE METHOD OF COEFFICIENTS.

Given: A deck bridge; type of truss: Warren with verticals. Span = 200 ft. length of panel = 20 ft.; depth of truss = 24 ft. Dead load per lin. ft. of bridge = 3000 lbs., one third of which is applied at lower joints; live load per lin. ft. of bridge = 6900 lbs. Required: Dead- and live-load stresses.



Dead Load Coefficients



Live Load Coefficients.
Numerators only.

Preliminary Calculations

30.0 = 3000 × 20 ÷ 2 = W_D = dead load per panel per truss
69.0 = 6900 × 20 ÷ 2 = W_L = live " " " "
2.3 = ratio of live to dead load.
0.833 = 20 ÷ 24 = tangent of truss angle A
1.30 = $\sqrt{(20)^2+(24)^2} \div 24$ = secant of A.
25.0 = 30.0 × 0.833 = W_D × tan A
39.0 = 30.0 × 1.30 = W_D × sec A
89.7 = 39.0 × 2.3 = W_L × sec A
8.97 = (W_L × sec A) ÷ 10

Checks for Coefficients (from inspection)

Dead Load

aB = $4\frac{1}{2}$ = R_L (Check)
ef = $+4\frac{1}{2}+3\frac{1}{2}+2\frac{1}{2}+1\frac{1}{2}+\frac{1}{2}=12\frac{1}{2}$ (Check)
$-EF-eF+ef=0=-12-\frac{1}{2}+12\frac{1}{2}$ (Check)

Live Load

aB = 45 ÷ 10 = $4\frac{1}{2}$ = R_L (Check)

Stresses in Chord Members

Member	Dead Load			Live Load		
	Stress	Coefficient	W_D × tan A	Live-Load Stress	Ratio	Dead Load Stress
AB	0.0					
BCD	200.0 (C)	8	25.0	460.0 (C)	2.3	× 200.0
DEF	300.0 (C)	12	"	690.0 (C)	"	× 300.0
abc	112.5 (T)	$4\frac{1}{2}$	"	258.8 (T)	"	× 112.5
cde	262.5 (T)	$10\frac{1}{2}$	"	603.8 (T)	"	× 262.5
ef	312.5 (T)	$12\frac{1}{2}$	"	718.8 (T)	"	× 312.5

Stresses in Diagonal Members

Member	Dead Load		Live Load			
	Stress	Coef × (W_D × sec A)	Compression	Coef × (W_L × sec A) ÷ 10	Tension	Coef × (W_L × sec A) ÷ 10
aB	175.5 (C)	$4\frac{1}{2}$ × 39.0	403.7	45 × 8.97		
Bc	136.5 (T)	$3\frac{1}{2}$ × "	9.0	1 × "	322.9	36 × 8.97
cD	97.5 (C)	$2\frac{1}{2}$ × "	251.2	28 × "	26.9	3 × "
De	58.5 (T)	$1\frac{1}{2}$ × "	53.8	6 × "	1884	21 × "
eF	19.5 (C)	$\frac{1}{2}$ × "	134.6	15 × "	89.7	10 × "

Stresses in Vertical Members

Member	Dead Load			Live Load
	Stress	Coefficient	Panel Load	Stress
Aa	10.0 (C)	$\frac{1}{3}$	30.0	34.5 (C) = Half panel load
Bb = Dd = Ff	10.0 (T)	$\frac{1}{3}$	"	0.0
Cc = Ee	20.0 (C)	$\frac{2}{3}$	"	69.0 (C) = Full panel load

ASSIGNMENTS

(1) Prepare a table of coefficients for a deck Warren truss with verticals similar to the table on page 321.

(2) Report on the effect of initial stress in a counter.

(3) Report on the "exact method" of determining stresses due to uniform live load, and upon differences in stresses as determined by this method and as determined by the conventional method used in this chapter.

(4) Report on the effect of counters on the verticals of a through Baltimore truss with sub-struts.

(5) Report on several different methods of combining dead-load and live-load stresses in proportioning counters and members subject to reversal of stress.

(6) Report on the article in the Engineering News-Record, Vol. 88, p. 95, concerning reversal of stress and repetitive stress.

(7) Report on general formulas that may be used in the calculation of dead-load and live-load stresses in standard types of parallel-chord trusses.

(8) Report on experiments that have been made to obtain data upon which impact formulas for highway bridges may be based, and on different impact formulas in use for highway bridges. Report also on increases in live load and definite percentages for impact specified for use in calculating stresses in counters and in other members subject to sudden loading for both highway and railroad bridges.

CHAPTER XXI

STRESSES DUE TO UNIFORM LIVE LOAD TRUSSES WITH INCLINED CHORD MEMBERS

The methods of determining stresses in trusses with inclined chords are primarily the same as those used for determining stresses in trusses with parallel chords. The algebraic method of sections, the algebraic method of coefficients, and the graphic method of successive joints, one or all, can be used to determine the stresses in a truss with inclined chords, but certain conditions are encountered in such a truss that do not exist in a parallel-chord truss. These conditions make it necessary to include in the application of either of the algebraic methods certain steps which are not necessary when using the same method for parallel-chord trusses.

1. Method of Sections Applied to a Truss with Inclined Chord Members. In Fig. 343 are shown a segment of a six-panel Pratt truss with parallel chords and a corresponding segment of a six-panel Pratt truss with a polygonal top chord, commonly called a Parker truss. (See

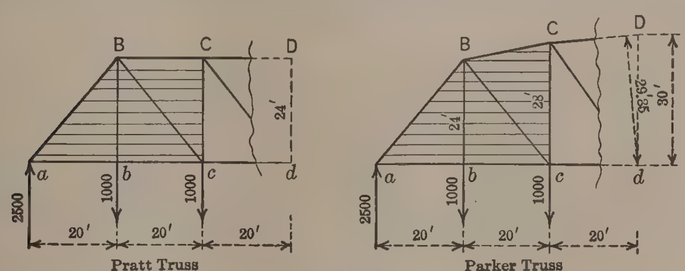


Fig. 343.

photograph page 102.) Each segment includes two panels, the section being taken through the third panel. The essential difference between the two is that in the Parker truss the top chord member CD cut by the section is inclined, whereas the corresponding member CD in the Pratt truss is horizontal. How this difference affects the application of the method of sections in the calculation of *dead-load* stresses in the members cd, CD, and Cd will now be explained by comparing the equilibrium equations used for the Parker truss with those used for the Pratt truss. The problem in each truss falls under Case 4 (70 : 4), and the combination of equations used is the same for each truss, namely:

Determine cd from $\Sigma M_C = 0$
Determine CD from $\Sigma M_d = 0$
Determine Cd from $\Sigma V = 0$

2. *To determine the stress in the lower-chord member cd.*

Pratt truss: $\Sigma M_C = 0 = 2500 \times 40 - 1000 \times 20 + cd \times 24$
Parker truss: $\Sigma M_C = 0 = 2500 \times 40 - 1000 \times 20 + cd \times 28$

These equations are identical except that the lever arm of cd in the Parker truss is four feet longer than the lever arm of cd in the Pratt truss, but each of these lever arms is the height of the corresponding truss at C the center of moments. There is, therefore, no essential difference in the calculation of the stress in a horizontal chord member, and this holds true whether the member is in the top or bottom chord. *The stress is equal to the bending moment divided by the height of truss at the center of moments.* (196 : 3.)

1. *To determine the stress in the upper-chord member CD.*

Pratt truss: $\Sigma M_d = 0 = 2500 \times 60 - 1000\,(20 + 40) + CD \times 24$

Parker truss: $\Sigma M_d = 0 = 2500 \times 60 - 1000\,(20 + 40) + CD \times 29.85$

These equations are identical except that the lever arm of CD in the Pratt truss is equal to the height of truss at the center of moments, whereas the lever arm of CD in the Parker truss is not the height of truss but the perpendicular distance from d to CD. *In each case the stress in CD is equal to the bending moment divided by the lever arm of CD.*

2. *To determine the stress in the diagonal web member Cd.*

Pratt truss: $\Sigma V = 0 = 2500 - 1000 - 1000 + Cd_V = 0$

Parker truss: $\Sigma V = 0 = 2500 - 1000 - 1000 + Cd_V - CD_V = 0$

These equations are identical except that the equation for the Parker truss includes CD_V whereas the other equation does not. This is because the inclined chord CD in the Parker truss has a vertical component, whereas the horizontal chord CD in the Pratt truss has none. The V component of the stress in the diagonal Cd in the Pratt truss is equal to the shear (194 : 4), but *the V component of the stress in the diagonal Cd in the Parker truss is equal to only a portion of the shear, since the shear is resisted in the Parker truss by both the diagonal web member Cd and the inclined chord member CD.* In a Parker truss, therefore, it is impossible to calculate the stress in a web member from the shear alone if the section through that member cuts an inclined chord member — the V component of the inclined chord member must be taken into account.

3. **Calculation of Dead-Load Stresses by the Method of Sections.** The differences between the equilibrium equations for determining stresses in a truss with inclined chords and the corresponding equations for determining stresses in a truss with parallel chords were shown in the preceding article to be slight, but these differences lead to a variety of methods of applying equilibrium equations to a truss with inclined chords that are not commonly used in calculating stresses in a parallel-chord truss. The more important of these methods will now be explained.

4. **To Determine the Stress in an Inclined Chord Member.** There are three methods of determining stresses in inclined chord members by the general method of sections.

5. *First method.* When the web member cut by the section is vertical as in Fig. 344 (a), the H component of the inclined chord member is equal in magnitude to the stress in the horizontal chord member cut by the section. (From $\Sigma H = 0$.) For example, $\Sigma H = 0 = CD_H + de$, hence $CD_H = -de$. (Compare with the first combination of equations for Case 4 (70 : 4) and with the Pratt truss (191 : 2 and 6).) As explained in the preceding article, the stress in de is determined exactly as in a parallel-chord truss from:

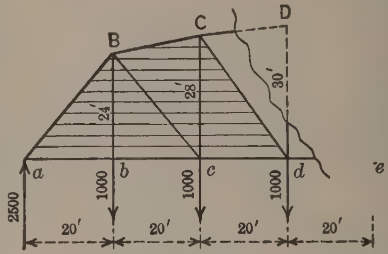


Fig. 344 (a).

$$\Sigma M_D = 0 = 2500 \times 60 - 1000\,(20 + 40) + de \times 30$$

or $\quad de = CD_H =$ Bending moment about D as a center divided by 30.

When this method is used, the stresses in the horizontal chords are calculated first, then from these stresses the H components of the stresses in the inclined chords are determined, and, finally, the stress in each inclined chord is calculated from its H component, usually by the geometric method. (65 : 9.)

6. *Second method.* When the web member cut by the section is inclined as in Fig. 344 (b), the unknown stress in CD may be replaced by CD_H and CD_V applied at D; then CD_H may be calculated from $\Sigma M_d = 0$, and finally the stress CD determined from CD_H.

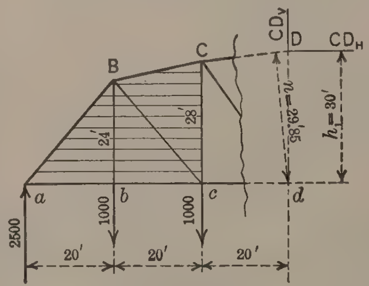


Fig. 344 (b).

$$\Sigma M_d = 0 = 2500 \times 60 - 1000\,(20 + 40) + CD_H \times 30.$$

This equation for determining CD_H is identical with that used in the first method for determining de. When this second method is used, the H components of the stresses in the inclined chords are calculated first, and from these components the stresses in the horizontal chords are determined,

provided the truss is of such a type that a section may be taken through each inclined chord member and a horizontal chord member that will cut a *vertical* web member but no inclined web member.

1. *Third method.* Another method that may be used when the web member cut by the section is inclined, has already been given in the preceding article, namely, to determine the stress in the inclined chord directly from a moment equation of equilibrium. This involves, first of all, the calculation of the lever arm of the inclined chord member CD. Let n represent the length of this lever arm, h_1 the height of truss at the upper end of the inclined chord member, p the panel length, and i the length of the inclined chord member, then:

$$n = h_1 \frac{p}{i} = 30 \frac{20}{20.1} = 29.85$$

The stress in CD may now be determined from the equation:

$$\Sigma M_d = 0 = 2500 \times 60 - 2000 \times 30 + CD \times 29.85$$

This method involves the calculation of the length of the lever arm of the stress in the inclined chord member CD, but this calculation is exactly equivalent to that required in the second method in determining the stress in CD from its H component. There is, therefore, little choice between the second and third methods, unless the H component of the stress in CD is desired for some other purpose. If, for example, the stresses in the horizontal chord members are to be determined from the H components of the stresses in the inclined chord members, as explained in the second method, the second method would be used in preference to the third.

2. *Exercise:* If h_1 represents the height of the truss at d, p the panel length, i the length of the inclined member CD, and M_B the bending moment for the segment $aBCc$, prove that the work done in calculating the stress in CD is indicated by $CD = M_B \div \left(h_1 \times \frac{p}{i}\right)$, regardless of whether the first, second, or third method is used.

3. *Comparison of the three methods.* When the only dead-load stresses required in a Parker truss are those in the inclined members of the top chord, it is immaterial which of the three methods is used, but when, as is usually the case, the stresses in the horizontal bottom chord members are also required, either the first or second method should be used. (Why?) The third method can be used to advantage for trusses which have a triangular web system. (Can the first method be used for trusses of this type?)

4. **To Determine the Dead-Load Stress in a Vertical Web Member.** There are two methods of determining stresses in vertical web members by the general method of sections.

5. *First method.* The first method consists merely in the application of the equilibrium equation $\Sigma V = 0$. For example, the stress in the member Cc in Fig. 345 is determined from the equation:

$$\Sigma V = 0 = (2500 - 1000 - 1000) - BC_V + Cc$$

The dead-load stress in BC having previously been determined, its V component may be calculated and substituted in the equation, which may

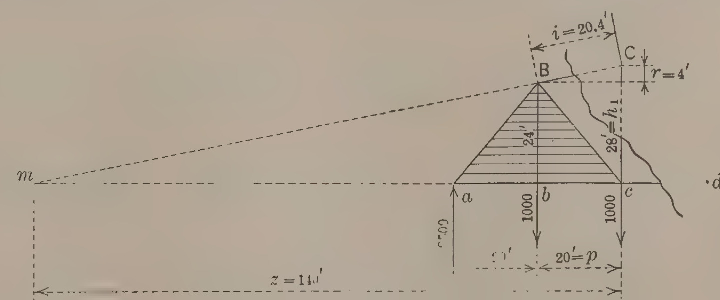


Fig. 345.

then be solved for Cc. Note that if i is the length of BC and r the rise of BC, the V component of the stress in BC is $BC_V = BC \times \frac{r}{i}$.

6. *Note:* The portion of the equation within the parentheses is the shear. For dead load, this shear for any segment in the left-hand half of the truss is positive. The V component of the stress in the inclined chord member BC may be less or greater than the shear. If BC_V is less than the shear, the segment aBc will move upward unless the vertical Cc acts downward (compression); if BC_V is greater than the shear, the segment aBc will move downward unless Cc acts upward (tension). Hence the vertical Cc may be in compression or in tension, depending upon the relative magnitudes of the shear and the V component of the inclined chord. In this respect the Parker truss differs from the Pratt truss, since an intermediate post in the latter is never in tension from dead load.

1. *Note:* If the dead-load stress in Bc or its V component is known, the dead-load stress in Cc may be determined from the application of $\Sigma V = 0$ to joint c, and will be equal to $Bc_V - 1000$.

2. *Second method.* The second method consists merely in the application of the equilibrium equation $\Sigma M = 0$ with m, the intersection of the two chord members BC and cd that are cut by the section, as the center of moments. This involves the calculation of the lever arm z. If $r =$ the rise of the inclined chord BC, $h_1 =$ the height of the truss at c, and $p =$ the panel length, from similar triangles $z = p\frac{h_1}{r}$. This is a general expression for z, and in this case gives 140 feet as the value for z. From z, the lever arms of the reaction and the panel loads may be calculated, and the stress in Cc may then be determined from the equation:

$$\Sigma M_m = 0 = -2500 \times 100 + 1000\,(120 + 140) + Cc \times 140$$

3. *Note:* If the stresses in the chords BC and cd have also been determined by moment equations, the third combination for Case 4 of three moment equations (70 : 4) has been used.

4. *Comparison of the two methods.* For dead-load stresses, the first method is as good, if not better, than the second, but it involves the dead-load stress in the inclined chord member cut by the section; for this reason, as will appear later, the second method, which does not involve any stress previously determined, is better for calculating live-load stresses in verticals.

5. **To Determine the Dead-Load Stress in an Inclined Web Member.** There are three methods of determining stresses in inclined web members by the general method of sections. (Fig. 346.)

6. *First method.* The first method is the same as the first method for vertical web members. Calculate the V component of the stress in the inclined chord member cut by the section; subtract this V component from the shear; the result is the V component of the required stress in the diagonal — from this component determine the stress. Expressed algebraically:

$$\Sigma V = 0 = (2500 - 1000) - BC_V + Bc_V$$

$$Bc_V = -(\text{shear} - BC_V)$$

$$Bc = Bc_V \times \frac{\text{length of } Bc}{\text{length of } Bb} = Bc_V \times \frac{k}{h} = \frac{31.24}{24}$$

7. *Note:* BC_V is the portion of the dead-load shear carried by the inclined chord member. (344 : 2.) Since this shear is positive, the diagonal member Bc must act downward and is, therefore, in tension.

8. *Question:* It is suggested that the student investigate later the question whether or not the V component of the dead-load stress in the inclined chord member cut by a section through a diagonal web member of a Parker truss is likely to be greater than the dead-load shear and thus cause dead-load compression in the diagonal, as may happen in the case of the vertical (345 : 6). (See Assignment (8) at the end of this chapter.)

9. *Note:* The ratio of k to h is the secant of the angle that the inclined web member makes with the vertical. Another statement, therefore, of the first method is: *To determine the stress in a diagonal web member subtract from the shear the portion carried by the inclined top chord member in the same panel, and multiply the result (remaining shear) by the secant of the angle between the diagonal and the vertical.* (How does this compare with the corresponding method of calculating the stress in a diagonal of a Pratt truss with parallel chords?)

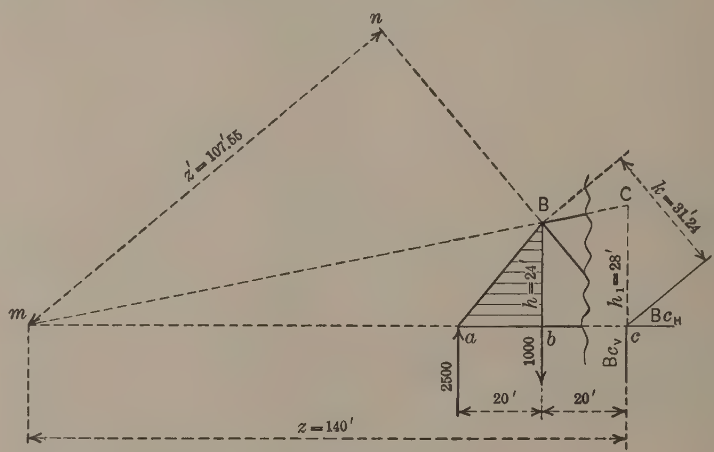


Fig. 346.

10. *Note:* If the dead-load stress in the post Cc is known, the dead-load stress in Bc may be determined from the application of $\Sigma V = 0$ to joint c, i.e., the V component of the stress in Bc is equal to the stress in Cc plus a panel load, or, in this case, $Cc +$ 1000 lbs.

1. *Second method.* Replace Bc by its H and V components applied at c. (Fig. 346.) Take m as the center of moments and determine the length z, the lever arm of Bc_V, as explained in the second method for the post Cc (346 : 2). This length is the same as for the lever arm of Cc, namely, 140 feet. The V component of Bc may then be determined from the equation:

$$\Sigma M_m = 0 = -2500 \times 100 + 1000 \times 120 + Bc_V \times 140$$

This equation is identical with that for the stress in Cc (346 : 2) except the latter has one more term, namely, — 1000×140. Hence the result obtained for Bc_V will be 1000 lbs. greater than that obtained for Cc as it should be. (See note 346 : 10.) Note that $Bc = Bc_V \times \frac{k}{h} = Bc_V \times \frac{31.24}{24}$.

2. *Third method.* This method consists in finding the stress in the inclined web member directly from a moment equation with m (Fig. 346) as a center of moments. This involves the calculation of the length z', the lever arm of the inclined web member, as well as the length z used in the second method. Let k represent the length of the inclined web member Bc. From the similar triangles mnc and Bbc, the length $z' = z\frac{h}{k} = 140 \times \frac{24}{31.24} = 107.55$. The stress in Bc may now be determined from:

$$\Sigma M_m = 0 = -2500 \times 100 + 1000 \times 120 + Bc \times 107.55$$

3. *Relation of the lever arms of two diagonals in the same panel.* Let k' represent the length of the diagonal bC in Fig. 346 and z'' its lever arm with respect to m. Then $\frac{z'}{z''} = \frac{k'}{k}$, that is, *the lever arms of the two diagonal web members in the same panel are in the inverse ratio of the lengths of those diagonal members.* (Prove this statement.) (Does it follow that the stresses in the two diagonals are in the same inverse ratio?)

4. *Comparison of the three methods.* The amount of work involved in determining the dead-load stress in a diagonal is about the same for all three methods. The second method does not involve the determination of the lever arm z', but this apparent gain, as compared with the third method, is offset by the calculation required in the second method in obtaining the stress in the diagonal from its V component, a calculation not required in the third method. The second method has this advantage, that the V component of the diagonal obtained from the moment equation can be used to determine the stress in the vertical that meets that diagonal at a joint of the horizontal chord — in the usual type of truss at a joint of the lower chord. (346 : 1.) By either the second or the third methods, the stress in the diagonal is determined without reference to the stress in the inclined chord or in any other member, and this is an advantage, particularly in calculating live-load stresses, as will appear later.

5. Calculation of Live-Load Stresses by the Method of Sections. The calculation of the live-load stress in any member of a truss with polygonal chord involves first the placing of the live load in the position for maximum stress, and second the use of an equilibrium equation to determine that stress. For simple types of trusses with inclined chords, the criteria for placing the live load are the same as those for trusses with parallel chords. For the calculation of stresses, one of the methods already explained for dead-load stresses may be used. There are certain exceptions or special cases, but these special cases are quite similar to corresponding cases in parallel-chord trusses. Throughout the explanations that follow, therefore, it is important to keep in mind the parallelism that exists between the methods of determining stresses in trusses with parallel chords and in trusses with inclined chords.

6. *Live-load stresses in chord members.* The maximum stress in any member in the perimeter of a truss (a chord member or an end post) will occur when the bridge is fully loaded. The live-load stresses in such members may be calculated, therefore, directly from the dead-load stresses by multiplying the latter by the ratio of live load per panel to dead load per panel, just as in parallel-chord trusses. (310 : 7.) If, for any reason, it is desired to calculate the live-load stress in a chord member independently from the dead-load stress, one of the methods explained in 344 : 4 may be used.

7. *Live-load stresses in verticals.* When a vertical is the only web member at a joint of the loaded chord, the maximum stress in it is obviously equal to one panel load. The criterion for placing the live load for the maximum stress in any other vertical is, in general, the same as for a vertical of a parallel-chord truss, namely, one segment formed by the section through

the vertical should be fully loaded, with no load on the other segment. The exceptions to this general rule will be explained later. When the load has been placed in the correct position and the corresponding reaction at the end of the unloaded segment determined, the stress in the vertical may be calculated by one of the methods explained for determining the dead-load stress in a vertical. (345 : 4). The first method involves the V component of that particular live-load stress in an inclined chord member that occurs *when the bridge is partially loaded* (to cause the maximum stress in the vertical). Note that this stress in the inclined chord member is not the same as the maximum live-load stress in that chord member. The second method gives the live-load stress in the vertical without reference to the stress in any other member. The possible reversal of stress in a vertical will be considered later.

1. *Illustrative example of the calculation of live-load stresses in verticals.* The following example will illustrate the method just explained. The truss in Fig. 348 is the same as that used in 345 : 4. The live load per panel is 2000 lbs. The lever arm z of Cc with respect to m, the intersection

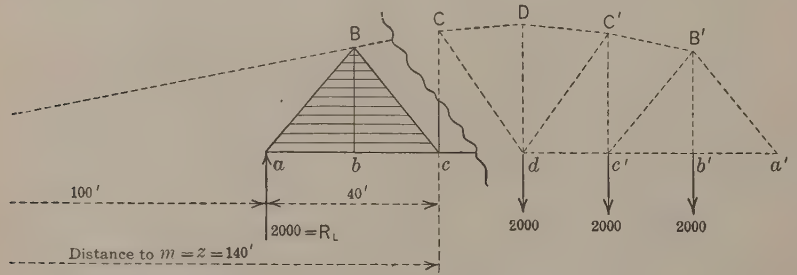


Fig. 348.

of BC and cd was calculated in 346 : 2 and found to be 140 ft. It is assumed that the truss is without counters, hence both the maximum live-load compression and the maximum live-load tension are to be determined. For the latter, loads will be brought on from the left.

2000 lbs. $= (1 + 2 + 3)\frac{1}{6} \times 2000 = R_L$ when loads are at d, c', and b'.

Determine the live-load stress in Cc from the equation:

$$0 = -2000 \times 100 + Cc \times 140. \quad \text{(Second method, 346 : 2.)}$$

1430 lbs. $= Cc$. (Live-load compression.)

3000 lbs. $= (5 + 4)\frac{1}{6} \times 2000 = R_L$ when loads are at b and c.

Determine the live-load stress in Cc from the equation:

$$0 = -3000 \times 100 + 2000\,(120 + 140) + Cc \times 140$$

-1570 lbs. $= Cc$. (Live-load tension.)

2. *Live-load stresses in diagonals.* The criteria for placing the live load are, in general, the same as for the diagonals of a parallel-chord truss, and the same in principle as that just given for verticals. Exceptions will be explained later. For a truss without counters, the maximum live-load tension in a diagonal will occur when all panel points of the longer segment are loaded and there are no loads at the panel points of the other segment; the maximum compression will occur when these conditions are reversed. This is the same criterion as for a truss with parallel chords. (311 : 6.) The live load having been placed in the correct position and the corresponding reaction at the end of the unloaded segment determined, the stress in the diagonal may be calculated by one of the methods explained for determining the dead-load stress in a diagonal. (346 : 5.) As in the case of verticals, one of these methods involves a live-load stress in an inclined chord member which is not a maximum stress; each of the other two methods gives the stress in the diagonal without reference to the stress in any other member. The prevention of the reversal of stress in a diagonal by the introduction of a counter will be considered later.

3. *Illustrative example of the calculation of live-load stresses in diagonals.* The following example will illustrate two methods of determining stresses in diagonals. The truss in Fig. 349 is the same as that in the preceding example. The lever arm z' was calculated in 347 : 2. It is assumed that the truss is without counters, hence both the maximum live-load tension and the maximum live-load compression are to be determined. For the latter, loads will be brought on from the *left.*

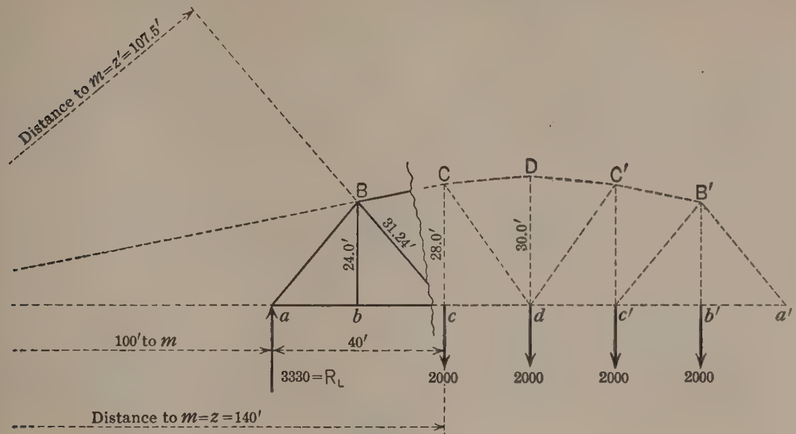


Fig. 349.

3330 lbs. $= (1 + 2 + 3 + 4)\,\frac{1}{6} \times 2000 = R_L$ when loads are at c, d, c', and b'.

Determine the live-load tension by two methods as indicated by the following equations:

$$0 = -3330 \times 100 + Bc_V \text{ (at } c) \times 140. \quad \text{(Second method, 347 : 1.)}$$

$$2380 = Bc_V$$

$$3100 \text{ lbs.} = 2380 \times \frac{31.24}{24.0} = Bc. \quad \text{(Live-load tension.)}$$

$$0 = -3330 \times 100 + Bc \times 107.5. \quad \text{(Third method, 347 : 2.)}$$

$$3100 \text{ lbs.} = Bc. \quad \text{(Live-load tension.)}$$

$$1670 \text{ lbs.} = \tfrac{5}{6} \times 2000 = R_L \text{ when load is at } b.$$

$$0 = -1670 \times 100 + 2000 \times 120 + Bc \times 107.5. \quad \text{(Third method, 347 : 2.)}$$

$$680 \text{ lbs.} = Bc. \quad \text{(Live-load compression.)}$$

1. *Note:* In determining live-load stresses when the load is in a position to cause maximum *negative* shear, it is somewhat less work to calculate the stresses in the web members of the *right-hand* half of the truss, bringing the load on from the right, but still considering the *left-hand* segment as the body in equilibrium. This was the method followed for parallel-chord stresses in CHAPTER XX and it is the one that will be used in the next illustrative problem.

2. *Note:* Since the lever arm z = panel length $\times \frac{h_1}{r}$, and since the lever arm z' is expressed in terms of z, namely, $z \times \frac{h}{k}$, each of these lever arms may be expressed in terms of panel lengths, and it is often advantageous to do so, particularly when $\frac{h_1}{r}$ reduces to a whole number. For example, in the panel of the truss just considered, $\frac{h_1}{r} = \frac{28}{4} = 7$, hence $z = 7p$, and $z' = 7p \times \frac{24}{31.24} = 5.38p$. In deriving formulas for the determination of stresses in web members of trusses with inclined chords, it is usually best to keep the lever arms z and z' in terms of panel lengths.

3. *Question:* Can the V component of the live-load stress in a diagonal be determined readily from the application of $\Sigma V = 0$ to a lower-chord joint as in the case of dead-load stresses? (346 : 10.)

4. **ILLUSTRATIVE PROBLEM IN THE CALCULATION OF DEAD-LOAD AND LIVE-LOAD STRESSES IN A PARKER TRUSS WITHOUT COUNTERS.** The dead-load and live-load stresses in each truss member of a typical panel of a Parker truss will now be calculated, i.e., in each of the members CD, cd, Dd, and Cd of the truss in Fig. 350. Each stress will be determined by at least two methods in order that different methods may be compared. The length of truss is 180 ft., and there are nine panels. The dead load is 1000 lbs. and the live load 1800 lbs. per linear foot of bridge. It is assumed at first that all the dead load is applied to panel points of the lower chord. The calculations are for a truss without counters; later the calculations for a counter in this panel will be explained.

5. *Note:* In this illustrative problem, it is important to study not merely methods, but the relationships which exist between the stresses in different members, particularly members in the same panel or in adjacent panels, and members that meet at the same joint. It is also important constantly to keep in mind the relationships between the stress in each member and the H and V components of that stress. In this connection it will be particularly helpful to use the geometric relationship of *force is to length as force is to length.* (65 : 9.) Since the angles of inclination of the web diagonal and the inclined chords vary for different panels, trigonometric functions such as the secant and the tangent vary, and they cannot be expressed once for all as in a parallel-chord truss. Hence instead of using such functions, it is probably better, at least for the beginner, to express these relations in simple ratios of lengths of members, heights of truss and lengths of panel, in accordance with the geometric principle just quoted. One thus gets a clearer conception of the relationships between the stress in a member and its components. It will be noted that no trigonometric functions, expressed as such, appear in the illustrative problem.

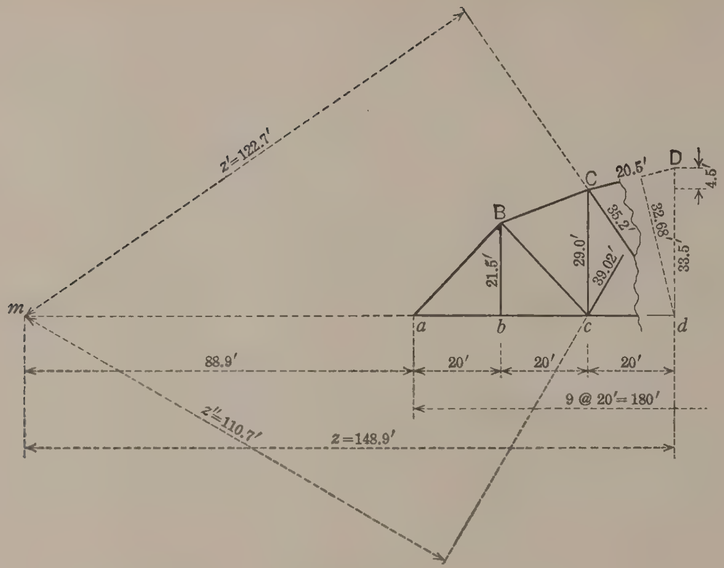


Fig. 350.

1. *Preliminary calculations*

10,000 lbs. $= 1000 \div 2 \times 20 =$ dead load per panel per truss.

18,000 lbs. $= 1800 \div 2 \times 20 =$ live load per panel per truss.

$1.8 =$ ratio of live load to dead load.

$h = 29.0$ ft.; $h_1 = 33.5$ ft.; $r = 4.5$ ft.

$20.5 = \sqrt{20^2 + 4.5^2} =$ length of $CD = i$.

$35.22 = \sqrt{20^2 + 29^2} =$ length of $Cd = k$.

$39.02 = \sqrt{20^2 + 33.5^2} =$ length of $cD = k'$.

$$148.89 = 20 \times \frac{33.5}{4.5} = p\frac{h_1}{r} = \text{lever arm } z.$$

$$122.67 = 148.9\frac{29}{35.2} = z\frac{h}{k} = \text{lever arm } z'.$$

$$32.68 = 20\frac{33.5}{20.5} = p\frac{h_1}{i} = \text{lever arm } n \text{ of } CD.$$

$$110.72 = 128.9\frac{33.5}{39} = (z - 20)\frac{h_1}{k'} = \text{lever arm } z''.$$

40,000 lbs. $= 8 \times 10{,}000 \div 2 = R_L$ due to dead load.

2. *Stresses in chord members CD and cd*

(*CD* by the third method, 345 : 1.)

10,000 lbs. = Dead load per panel.

40,000 lbs. = Dead-load reaction.

55,100 lbs.(C) $= -[40{,}000 \times 60 - 10{,}000\,(20 + 40)] \div 32.68 = CD.$ (Dead-load stress.)

(*CD* by the second method, 344 : 6.)

$-53{,}730 = -[40{,}000 \times 60 - 10{,}000\,(20 + 40)] \div 33.5 = CD_H.$ (From $\Sigma M_d = 0$.)

55,100 lbs.(C) $= 53{,}730 \times 20.5 \div 20 = CD.$ (Dead-load stress.)

$1.8 =$ Ratio of live load to dead load.

99,200 lbs.(C) $= 55{,}100 \times 1.8 = CD.$ (Live-load stress.)

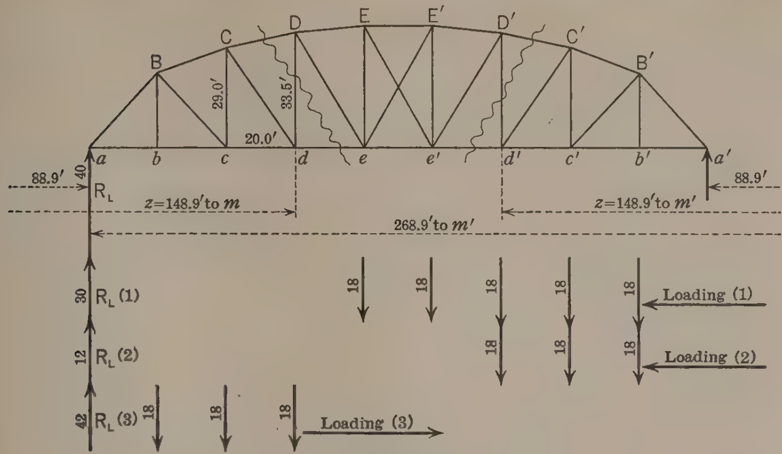
3. *Note:* CD_H is equal to the stress in *de*, and if *de* is known, there is no necessity for calculating CD_H. (First method, 344 : 5.) Note that the equation for *de* would be the same as that given for CD_H.

4. (*cd* by the usual method.)

48,300 lbs.(T) $= -(40{,}000 \times 40 - 10{,}000 \times 20) \div 29 = cd.$ (Dead-load stress.)

86,900 lbs.(T) $= 48{,}300 \times 1.8 = cd.$. (Live-load stress.)

5. *Note:* If the H component of *BC* is known there is no necessity for calculating the stress in *cd*. (Why?)



Loading for live-load stresses in D*d*
Fig. 351.

1. *Stresses in the vertical Dd.* (Fig. 351.)

(First method, dead-load stresses only.)

$$-12{,}100 = -53{,}730 \times 4.5 \div 20 = CD_H \times r \div p = CD_V.$$
$$2100 \text{ lbs.(T)} = -(40{,}000 - 3 \times 10{,}000 - 12{,}100) = Dd \text{ (dead)}.$$

(Second method, dead-load and live-load stresses.)

$$2100 \text{ lbs.(T)} = -[-40{,}000 \times 88.9 + 10{,}000\,(108.9 + 128.9 + 148.9)] \div 148.9 = Dd \text{ (dead)}.$$
$$30{,}000 = \tfrac{15}{9} \times 18{,}000 = R_L \text{ for loading (1)}.$$
$$17{,}900 \text{ lbs.(C)} = -(-30{,}000 \times 88.9) \div 148.9 = Dd \text{ compression (live)}.$$
$$12{,}000 = \tfrac{6}{9} \times 18{,}000 = R_L \text{ for loading (2)}.$$
$$21{,}700 \text{ lbs.(T)} = -[12{,}000 \times (180 + 88.9)] \div 148.9 = D'd' = Dd \text{ tension (live)}.$$

2. *Note:* In the last equation, loading (2) was used, the left-hand segment was the body in equilibrium, and the center of moments was taken to the right of the truss.

For comparison, the following equation is for loading (3), center of moments to the left of the truss.

$$42{,}000 = \tfrac{21}{9} \times 18{,}000 = R_L \text{ for loading (3)}.$$
$$21{,}700 \text{ lbs. (T)} = -[-42{,}000 \times 88.9 \div 148.9 + 18{,}000\,(108.9 + 128.9 + 148.9)] \div 148.9 = Dd.$$

It is seen that bringing loads on from the left involves considerably more work if the left-hand segment is taken as the body in equilibrium. If loading (3) is used, it is better to take the right-hand segment as the body in equilibrium.

3. *Note:* It has been assumed in this illustrative problem that the dead load is considered as applied wholly at lower-chord joints. If a portion of it is considered as applied at upper-chord joints, the only stresses affected by the change will be those in the verticals, and the stresses found from the assumption made in this problem may easily be corrected for the new assumption as explained in connection with dead-load stresses in parallel-chord trusses. (199 : 14.)

4. *Question:* Is dead-load tension less likely to occur in an intermediate vertical if a portion of the dead load is assumed to take effect at joints of the upper chord?

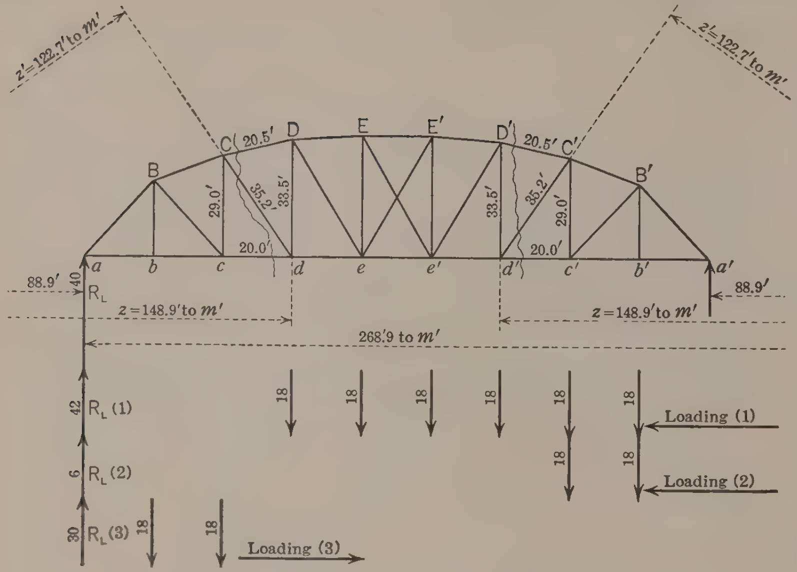
5. *Stresses in diagonal Cd.* (Fig. 352.)

(First method, dead-load stresses only.)

$$-12{,}100 = 53{,}730 \times 4.5 \div 20 = CD_H \times r \div p = CD_V.$$
$$7900 = 40{,}000 - 2 \times 10{,}000 - 12{,}100 = \text{shear} - CD_V.$$
$$0 = 7900 + Cd_V.$$
$$9600 \text{ lbs.(T)} = -7900 \times (35.2 \div 29) = Cd \text{ (dead)}.$$

(Second method, dead-load and live-load stresses.)

$$7900 = -[-40{,}000 \times 88.9 + 10{,}000 \times (108.9 + 128.9)] \div 148.9 = Cd_V.$$
$$9600 \text{ lbs.(T)} = 7900 \times (35.2 \div 29) = Cd \text{ (dead)}.$$
$$42{,}000 = \tfrac{21}{9} \times 18{,}000 = R_L \text{ for loading (1)}.$$
$$25{,}100 = -(-42{,}000 \times 88.9) \div 148.9 = Cd_V.$$
$$30{,}400 \text{ lbs.(T)} = 25{,}100 \times (35.2 \div 29) = Cd \text{ tension (live)}.$$
$$6000 = \tfrac{3}{9} \times 18{,}000 = R_L \text{ for loading (2)}.$$
$$-10{,}800 = -(6000 \times 268.9) \div 148.9 = C'd'_V.$$
$$13{,}200 \text{ lbs.(C)} = -10{,}800 \times (35.2 \div 29) = C'd' = Cd \text{ compression (live)}.$$



Loading for live-load stresses in Cd
Fig. 352.

(Third method, dead-load and live-load stresses.)

$$9600 \text{ lbs.(T)} = -[-40{,}000 \times 88.9 + 10{,}000\ (108.9 + 128.9)] \div 122.7 = Cd \text{ (dead).}$$

$$42{,}000 = \tfrac{21}{9} \times 18{,}000 = R_L \text{ for loading (1).}$$

$$30{,}400 \text{ lbs.(T)} = -(-42{,}000 \times 88.9) \div 122.7 = Cd \text{ tension (live).}$$

$$6000 = \tfrac{3}{9} \times 18{,}000 = R_L \text{ for loading (2).}$$

$$13{,}200 \text{ lbs.(C)} = -(6000 \times 268.9) \div 122.7 = C'd' = Cd \text{ compression (live).}$$

1. *Note:* For comparison with the last equation, the following equation is given for loading (3) and the center of moments to the left of the truss:

$$30{,}000 = \tfrac{15}{9} \times 18{,}000 = R_L \text{ for loading (3)}$$

$$13{,}200 \text{ lbs.(C)} = -[-30{,}000 \times 88.9 + 18{,}000\ (108.9 + 128.9)] \div 122.7 = Cd.$$

It is seen that bringing the loads on from the left involves more work unless the right-hand segment is used as the body in equilibrium.

2. *Parker truss with an odd number of panels.* When there is an odd number of panels in a Parker truss, the upper as well as the lower central chord member is horizontal, hence the stresses in all members in the center panel may be determined by exactly the same methods as those used to determine the stresses in the members in the center panel of a Pratt truss with the same number of panels. (329 : 5 and 330 : 1.)

3. **General Method of Procedure for the Calculation of Dead-Load and Live-Load Stresses in a Parker Truss.** The illustrative examples just explained have afforded an opportunity to compare different methods of determining stresses in a truss with inclined chords. In such a comparison, it is important to note that a method which may be used to advantage for a Parker truss may not be suitable for a truss of a different type. For example, in a Parker truss the H component of the dead-load stress in an inclined chord member is equal to the dead-load stress in one of the horizontal chord members (344 : 5), and this relationship makes it easy to determine the dead-load stress in the inclined chord member from its H component, but in a truss with a triangular web system no such relationship exists, consequently some other method of determining the stress in an inclined chord member may be better. It is obviously impracticable to give a general method of procedure which would be the best for all types of trusses with inclined chords. Moreover, there are usually two or three methods equally good for any given type. The following method of procedure for the calculation of dead-load and live-load stresses in a Parker truss will be found efficient; it is the one used in the illustrative problem at the end of the chapter, and the solution of that problem should be followed through in studying the method of procedure as outlined.

4. *First step.* Draw a truss diagram.

5. *Note:* It is well to draw the diagram carefully to scale in order that some of the preliminary calculations may be checked by scaling distances on the truss diagram. If a "stress sheet" is to be prepared, the diagram on that sheet will serve every purpose.

6. *Second step.* Make the necessary preliminary calculations. Check the lengths of members and the lever arms by scaling the truss diagram. Enter on this diagram all dimensions of the truss, lengths of members, and lengths of lever arms.

1. *Note:* Preliminary calculations usually include: Panel loads (live and dead) per truss, and the ratio of live to dead; lengths of all inclined chord members and web members; lengths of lever arms z for all verticals and lengths of lever arms z' for all main diagonals; if there are counters, the lengths of lever arms z'' for counters. The ratios of lengths of members to height of truss and to panel length may be included in the preliminary calculations if desired, but these trigonometric functions of angles differ for different panels, and it is probably just as well to enter them in the equations as simple ratios without previous calculations. (349 : 5.)

2. *Third step.* Indicate in tabular form the equations from which the stresses in all chord members will be determined.

(*a*) For the dead-load stresses in horizontal chord members, moment equations similar to those for corresponding members in a parallel-chord truss.

(*b*) For the dead-load stress in each inclined chord member, the equations for the geometric ratio between the stress and its H component — the H component being equal to the stress in a horizontal chord member.

(*c*) For all live-load stresses in both horizontal and inclined chords, the equations expressing the dead-load stresses multiplied by the ratio of live load to dead load.

3. *Fourth step.* Indicate the equations from which the stresses in all web members will be determined.

(*a*) For dead-load stresses in verticals, moment equations with centers of moments at intersections of chords, unless the chords in a panel are both horizontal. The lever arm z for each vertical has previously been determined in the preliminary calculations.

(*b*) For dead-load stresses in diagonals, the equations $\Sigma V = 0$, applied at the joints in which the diagonals intersect the horizontal chord in order to obtain the V components of the diagonals, and the geometric ratios from which the stresses in the diagonals are determined from their V components. (From $\Sigma V = 0$, the V component in a diagonal (except an end post) is usually equal to the stress in the vertical at the joint plus the dead load applied at the joint.)

(*c*) For live-load stresses in verticals, moment equations similar to those in (*a*), except that with the live load in a position for maximum stress the live-load reaction is usually the only force in the equation in addition to the unknown stress in the vertical. This reaction, as usual, may be indicated as a fraction (in terms of panel lengths) multiplied by the live load per panel.

(*d*) For live-load stresses in diagonals (except an end post), moment equations in which centers of moments are at intersections of chords, unless the chords in a panel are both horizontal. The lever arm z' for each diagonal has previously been determined in the preliminary calculations. With the live load in the position for maximum stress, the live-load reaction is usually the only force in the equation in addition to the unknown stress in the diagonal. This reaction may be indicated, as usual, in terms of a fraction multiplied by the live load per panel.

4. *Note:* For each main diagonal and for each vertical post of a Parker truss without counters, there may be required two equations for live-load stresses, one for tension, the other for compression. In certain web members, such as a hip vertical, the stress is obvious from inspection. Stresses in web members of a Parker truss with counters are determined by methods to be explained later.

5. *Fifth step.* When all equations for stresses have been indicated and checked by inspection, and not before, carry out all calculations required by the equations, and enter the results (stresses) thus determined in the columns left vacant for that purpose. (See notes 331 : 14 and 332 : 2.) If a portion of the dead load is considered as applied at upper-chord joints, corresponding corrections should be made in dead-load stresses in verticals (351 : 3).

6. *Note:* As a final step, a stress sheet may be prepared. (See note 332 : 7.)

7. **Stresses in Counters.** The theory of stresses in counters was explained in 322 : 4 to 325 : 1. It was shown that the V component of the stress in a counter in a parallel-chord truss is equal to the resultant shear for the panel obtained by combining the dead-load positive shear with the maximum negative live-load shear. This is not true, however, when a chord in the panel is inclined.

8. Let the segment $aBCc$ in Fig. 354 be a body in equilibrium. In addition to the dead loads and dead-load reaction, let there be acting on the segment live loads at b and c and the corresponding live-load reaction. Assuming that the loads at b and c are the only live loads on the whole truss, this is the position of loading that will cause maximum

negative live-load shear, and consequently the position that will cause the maximum stress in the counter cD if such a counter is needed.

1. Assume now that neither diagonal, Cd or cD, will take compression. Which diagonal should be inserted? If under the action of all of the other forces the segment would move upward, the diagonal Cd should be inserted, but if it would move downward, the counter cD would act instead.

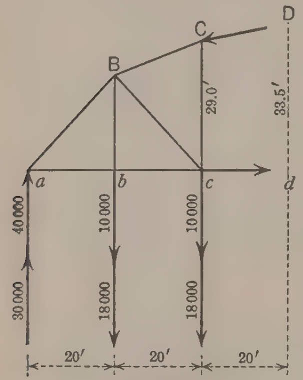


Fig. 354.

2. The dead-load shear is 40,000 − 10,000 − 10,000 = 20,000 lbs.; the live-load shear is 30,000 − 18,000 − 18,000 = −6000; the resultant shear is 20,000 − 6000 = 14,000. Judging from this resultant shear only (as would be done in a parallel-chord truss) the segment would move upward, the diagonal Cd would act, and no counter would be needed. But in this truss the chord member CD has a V component which must be included in $\Sigma V = 0$ applied to the segment. This component is *downward* for both dead and live loads, and acts, like the live load to move the segment downward. It may be great enough to make a counter necessary in spite of the fact that the combined shear is 14,000 lbs. upward. The actual magnitude of CD_V, calculated for the conditions shown in Fig. 354, is 17,400 lbs. This added algebraically to the resultant shear is:

$$14{,}000 - 17{,}400 = -3400 \text{ lbs.}$$

This means that with the eight forces shown in Fig. 354 acting on the segment $aBCc$ (the diagonals being omitted), there would be a resultant force of 3400 lbs. acting to move the segment *downward*, hence the counter cD should be inserted; it would act upward in tension, with a V component of 3400 lbs. If the result above had been positive, no counter would have been required.

3. *Note:* In the explanation just given, no allowance was made for impact or for any increase in the live load that may be specified in place of impact. In practice, the live-load shear would be made to include such an allowance, as was done in the calculation of counter stresses in parallel-chord trusses (322 : 6), and as will be done in the more complete example of a counter in a Parker truss to be given later.

4. For a given loading, the stress in the chord CD will not be the same when Cd is acting as it is when cD is acting. For example, in calculating the dead-load stress in CD, the center of moments is taken at d if Cd is assumed to be in action, but at c if cD is in action. The two dead-load stresses thus obtained for CD will differ. It follows that the V component of the combined dead-load and live-load stress in CD will be one amount when Cd is acting, but quite a different amount when the counter cD is acting. It is the V component of the combined dead-load and live-load stress which is in CD when the counter cD is assumed to be the only diagonal in the panel that must be considered in determining the stress in the counter.

5. The illustrative example has shown that when there is an inclined chord in the panel, the V component of the stress in a counter is not equal to the combined dead-load and live-load shear as it is when both chords are horizontal, but is equal to the algebraic sum of the combined shear and the V component of the combined dead-load and live-load stress that is in the inclined chord *when the counter is in action.* Moreover, this V component may exceed the combined shear when the latter is positive and thus make necessary a counter that would not be required if the combined shear only were considered.

6. Although the V component of the stress in a counter may be obtained by adding algebraically the combined shear and the V component of the combined dead-load and live-load stresses in the inclined chord, as just explained, the calculation of this V component may involve work which can be avoided by some other method. Three such methods will now be explained by means of an illustrative example.

7. *Illustrative example of the methods of determining stresses in counters.* The typical panel used in 349: 4 will be taken as an illustrative example. It will be assumed, however, that the diagonal Cd cannot take compression, and that therefore a counter cD may be needed. Assume also that the specifications require that in determining stresses in counters, the live load shall be increased 50%. Panel loads: Dead = 10,000 lbs.; live = 18,000 + 50% of 18,000 = 27,000 lbs.

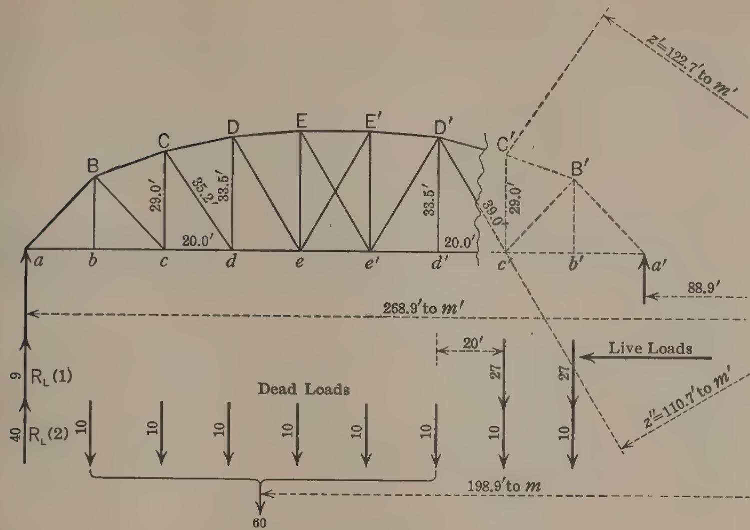


Fig. 355.

1. *First method.*

In Fig. 355 the diagonal $d'C'$ has been replaced by the counter $D'c'$. Live loads are at b' and c', the position to cause maximum stress in the counter $D'c'$. The corresponding reaction $R_L = 9000$ lbs. The dead load is also assumed to be acting with a corresponding reaction $R_L = 40{,}000$ lbs. Consider the segment $aBD'd'$ to be in equilibrium, and determine the stress in $D'c'$ from $\Sigma M = 0$, taking as a center of moments the point m' at the intersection of $D'C'$ and $d'c'$, 89.9 ft. to the right of the joint a'. (This distance and the lever arms z' and z'' were calculated in 350 : 1.)

$$11{,}200 \text{ lbs.(T)} = -[(40{,}000 \times 268.9 - 60{,}000 \times 198.9) + 9000 \times 268.9] \div 110.7 = D'c'.$$

The quantity within the parentheses is equal in magnitude to the dead-load stress in $d'C'$ multiplied by z'. (Why?) Hence if the dead-load stress previously calculated as 9600 lbs.(T) multiplied by 122.7 is substituted in the equation it becomes:

$$11{,}200 \text{ lbs.(T)} = -[(9600 \times 122.7) + 9000 \times 268.9] \div 110.7 = D'c'.$$

The substitution just made is equivalent to replacing all of the dead loads in Fig. 355 by a member $d'C'$ with its dead-load stress. Hence the method may be expressed as follows: Let S' = the stress in a counter; S = the dead-load stress in the main diagonal in the same panel, and M the moment of the live-load reaction with respect to m', then:

$$S' = (S \times z' - M) \div z''$$

This is a direct and simple method that does not involve the stress in the inclined chord.

1. *First method modified.* It was pointed out in 347 : 3 that the lever arms of two diagonals in the same panel are in the inverse ratio of the lengths of those diagonals, i.e., $\frac{z'}{z''} = \frac{k'}{k}$ in which k' = length of the counter $D'c'$, and k the length of the diagonal $d'C'$. The method may then be expressed as follows: (1) Divide the moment of the live-load reaction about m' as a center by the lever arm z'' of the counter. (2) Multiply the dead-load stress in the main diagonal by the ratio of the length k' of the counter to the length k of the diagonal. If the second result is less than the first, a counter is needed and the stress in it is equal to the second result subtracted from the first. If the second result is the larger, no counter is needed. This modified method involves the calculation of the ratio of lengths of diagonals, and saves little if any work, particularly if the lengths of both z' and z'' have previously been used. Applied to the panel in Fig. 355, the work may be indicated as follows:

$$21{,}900\text{(T)} = 9000 \times 268.9 \div 110.7 = R_L \times 268.9 \div z'$$

$$10{,}600\text{(C)} = 9600 \times 39.0 \div 35.2 = \text{Dead-load stress in } C'd' \times \frac{k'}{k}$$

$$11{,}300 \text{ lbs.(T)} = 21{,}900 - 10{,}600 = D'c'$$

Note that 10,600 lbs. is the dead-load compression that would be in the counter if no live load were on the truss and the counters were acting in place of the main diagonal; 21,900 lbs. is the tension in the counter due to live load only. If the dead-load compression had been greater than the live-load tension, no counter would have been needed.

2. *Second method.*

First step: Determine the V component of the dead-load stress in the inclined top chord member that is cut by the section through the counter. This V component should be the V component of the dead-load stress that is in the inclined chord member *when the counter is acting* and not of the dead-load stress as it is usually understood, i.e., the dead-load stress

in the chord member when the main diagonal is acting. (354 : 4.) The required V component may be found from the corresponding H component of the stress in the inclined chord member, and this H component is equal to the dead-load stress in the bottom chord member of the same panel when the main diagonal in that panel is acting. (Why?)

Second step: Add algebraically the dead-load shear for the panel and the V component determined in the first step. The result is that portion of the dead-load shear that would be carried by the counter if it were acting in place of the main diagonal.

Third step: From $\Sigma M = 0$, determine the V component of the live-load stress in the counter. The result is equal in magnitude but opposite in sense to that portion of the live-load shear that is carried by the counter if one is needed.

Fourth step: If the result obtained in the second step is greater than that obtained in the third, no counter is needed; if less, subtract the former result from the latter in order to obtain the V component of the actual stress in the counter due to the combined effect of dead and live loads. (This is equivalent to determining the magnitude of that portion of the combined dead-load and live-load shear that is carried by the counter.) From the V component of the stress in the counter determine the stress.

Illustrative example. Required: The stress in the counter $D'c'$ in Fig. 355.

First step: The dead-load stress in $d'c'$ when the main diagonal $d'C'$ is acting is 48,300 lbs. (calculated in 350 : 4), and this is equal to the H component of the dead-load stress in $D'C'$ when the counter $D'c'$ is acting. The V component is: $D'C'_V = 48{,}300 \times 4.5 \div 20 = 10{,}900$ lbs.

Second step: The dead-load shear for panel $d'c' = 40{,}000 - 6 \times 10{,}000 = -20{,}000$ lbs. The portion of this dead-load shear that would be carried by the counter $D'c'$ if the diagonal $d'C'$ were not acting is: $-20{,}000 + 10{,}900 = -9100$ lbs.

Third step: To calculate the V component of the live-load stress in the counter $D'c'$, let this unknown stress be replaced by its two components $D'c'_H$ and $D'c'_V$ applied at c'. Assume the center of moments at m', the intersection of $D'C'$ and $d'c'$. Then $D'c'_V = -(9000 \times 268.9) \div 128.9 = -18{,}800$ lbs. (away from D', or tension). The portion of the live-load shear carried by the counter is equal to $-D'c'_V = 18{,}800$ lbs.

Fourth step: The combined shear carried by the counter $D'c'$ is: $18{,}800 - 9100 = 9700$ lbs. The V component of the actual stress in the counter is: $D'c'_V = -9700$ lbs., and the stress is $-9700 \times 39 \div 33.5 = 11{,}300$ lbs. tension.

1. **Effect of Counters on Vertical Members.** The effect of counters on the vertical members of a parallel-chord truss was explained in 325 : 3 to 327 : 10. It was shown that for certain positions of the live load, the action of counters might leave the post as the only web member acting at a joint of the truss, and that this change in the form of the truss, for that is what it really is, might reduce the stress in the post to less than the dead-load stress or even to zero. The action of counters in a Parker truss may do still more than this, for they may change the stress in a post from compression to tension. This is because, unlike a parallel-chord truss, the top chord members, if inclined, exert forces upward or downward at the joint at the top of the post.

2. In Fig. 356, Dd represents the center vertical of a truss with an even number of panels when the central top chord members are inclined. Under the action of dead load only, the counters are not in action, and Dd is the only web member acting at D. Applying $\Sigma V = 0$ at D, the V components of both CD and $C'D$ are upward, consequently Dd must act downward in tension. The same conditions exist when the live load covers the whole span, and the live-load tension in Dd for that loading is the maximum live-load tension, since the stresses in CD and $C'D$ are then maximum.

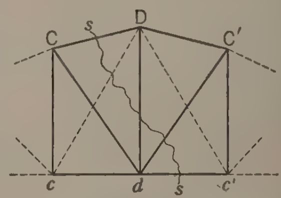


Fig. 356.

The dead-load tension in Dd is equal to the sum of the V components of the dead-load stresses in CD and $C'D$, and the live-load tension in Dd since it occurs for a fully-loaded truss, may be calculated by multiplying the dead-load tension by the ratio of live to dead load. In a truss with an odd number of panels, one of the top chord members at D is horizontal, and therefore only the V component of the other chord member acts to cause tension in Dd. Note that in any case, the maximum tension in a center vertical is the same as if the truss were without counters. In a truss with an even number of panels the upper chord members of the two central panels are often horizontal; in such a truss there can be no tension in the center vertical.

3. *Note:* If a portion of the dead load is considered as applied at D, the dead-load tension will be decreased by the amount of this load from what it otherwise would be. (How would this affect the calculation of the live-load tension from the dead-load tension?)

1. When either of the counters is in action, the stress in *Dd* may change from tension to compression; the maximum compression will occur when the stress in either counter is a maximum. Assume the stress in *Dc′* to be a maximum; the diagonal *dC′* is not in action, hence, from the section *s–s* the stress in *Dd* may be calculated by the method already explained for live-load stresses in verticals. (347 : 7.) Had *Dd* been one of the center verticals in a truss with an odd number of panels, the counter extending from *d* into the center panel would not be in action, and the method of calculating the maximum compression from the section *s–s* would be the same.

2. It remains to consider the effect of a counter on a vertical other than a center vertical. The explanation given for the effect of counters on posts of a Pratt truss (326 : 1) applies, with certain modifications, to similar effects on posts of a Parker truss. The difference is, that as usual, the V components of inclined chords must be taken into account. Assume that it is possible to place the live load on the truss in such a position that the counter *cD* in Fig. 357 is in action, but the counter *bC* is not. The counter *cD* replaces the main diagonal *Cd* (not shown in the figure), and this leaves the vertical *Cc* as the only web member at the joint *C*. Under this condition, the H components of *BC* and *CD* must be equal (from ΣH = 0 at *C*). It follows that the V component of *BC* must be greater than that of *CD*. (Why?) *BC* acts upward at joint *C* and *CD* downward, consequently without the action of *Cc* the joint *C* would move upward. *Cc* must, therefore, act downward in tension equal in amount to BC_V–CD_V. The greater the stresses in *BC* and *CD* the greater the tension. The maximum tension in *Cc* will not occur, for a fully-loaded truss, however, as in the case of a center vertical, for then the counter *cD* would not be in action. Neither does it necessarily occur when the stress in the counter *cD* is a maximum. The maximum tension in *Cc* will occur when as much load as possible is on the truss and yet preserve the condition that the counter *cD* shall be in action but the counter *bC* shall not be. This may mean that instead of loading the truss from the left up to and including joint *c*, the position for maximum stress in the counter *cD*, the loading may extend farther — for example, to include joint *d*. The correct position of the live load may be determined easily by trial. The live-load stresses and the dead-load stresses in *BC* and *CD* *when the live load is in this position* are the stresses that cause (by their V components) the maximum tension in *Cc*. (The dead-load stress in *CD* is different when the counter *cD* is acting than it is when the regular diagonal *Cd* is in action.) (Why?) (354 : 4.)

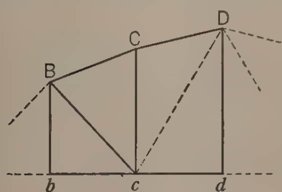


Fig. 357.

3. *Note:* If a portion of the dead load acts at *C*, the tension in *Cc* will be decreased by the amount of this load from what it otherwise would be.

4. *Minimum stresses.* If minimum stresses are desired, they may be determined in much the same manner as in a Pratt truss. In certain members, the minimum stresses are obviously the dead-load stresses. In web diagonals in panels with counters, the minimum stresses are zero, as they also are in verticals that are subject to reversal of stress due to the action of counters. In a panel in which a counter is not needed, the stress in the main diagonal may be reduced to less than the dead-load stress when the live load is in certain positions, particularly if the panel is near the center of the truss.

5. *Note:* Throughout the discussion of the effect of the action of counters on the stresses in other members, it has been assumed that allowance is made for impact or for any increase in live load that may be required by the specifications when stresses in counters are to be determined.

6. **Method of Coefficients Applied to a Parker Truss.** The method of coefficients, used so generally for calculating both dead-load and live-load stresses in parallel-chord trusses, may, with certain modifications, be used also for the calculation of stresses in a truss with inclined chords. In order to use this method, however, with ease and certainty, one must have a thorough knowledge of the method of sections as applied to trusses with inclined chords. It was shown in 221 : 1 that the method of coefficients applied to parallel-chord trusses is practically identical with the method of sections when the equilibrium equations are written in terms of panel loads, panel lengths, and height of truss. In the same way it could be shown that the method of coefficients applied to a truss with inclined chords is merely a short-cut application of the method of sections used thus far in this chapter. The method of procedure will

therefore need comparatively little explanation. All forces, including stresses, are given in kips instead of in pounds. The illustrative problem should be compared throughout with the same problem solved by the method of sections on pages 366 and 367.

1. Given: The same Parker truss as that shown on page 366 with the same dead loads and live loads. Required: To calculate the dead-load and live-load stresses by the method of coefficients.

2. *First step:* Draw a truss diagram, and enter on this diagram the length of each member and the rise of each inclined chord, first making such preliminary calculations as may be necessary in order to do this. (See preliminary calculations on page 366.)

the method of sections used. (Why?) The live-load stress in any chord member is found by multiplying the corresponding dead-load stress by 1.8, the ratio of live load to dead load.

Lower Chord Members

Member	$c \times W_D \times (p \div h) =$	*Dead-load Stress*	*Live-load Stress*
$ab = bc$	$4 \times 10 \times (20 \div 21.5) =$	37.2(T)	67.0(T)
cd	$7 \times 10 \times (20 \div 29.0) =$	48.3(T)	86.9(T)
de	$9 \times 10 \times (20 \div 33.5) =$	53.7(T)	96.7(T)
ee'	$10 \times 10 \times (20 \div 35.0) =$	57.1(T)	102.8(T)

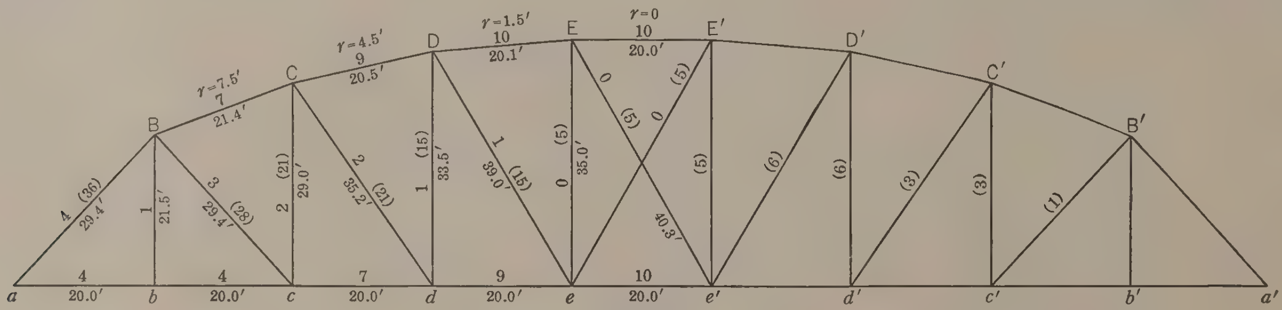


Fig. 358.

3. *Second step:* Place dead-load and live-load coefficients on the truss just as if it were a parallel-chord truss. (Fig. 358.)

4. *Note:* Only the numerators of live-load coefficients are shown in Fig. 358, and these are in parentheses. Where only one coefficient is shown, it is the same for live load as it is for dead load. The live-load coefficients for the diagonals and verticals of the center panel are in accord with the assumption that the shear for the center panel for all positions of the load is equally divided between the two diagonals. (329 : 5.)

5. *Third step:* Calculate the stresses in the lower chord members. The dead-load stress in each chord member is equal to the coefficient c for that member multiplied by the panel load W_D multiplied by the panel length p divided by the height of truss h. The height h is measured at the point that would be the *center of moments for the chord member* were

6. *Note:* Throughout this problem the relation between a stress and its H or V component will be expressed in terms of lengths instead of in terms of the trigonometric functions. Force is to force as length is to length, or a required force is equal to the given force multiplied by the length corresponding to the required force divided by the length corresponding to the given force. (8 : 1.)

7. *Fourth step:* Calculate the stresses in the upper chord members from their H components. The H component of the dead-load stress in an upper chord member in any panel (except the center panel) is equal to the dead-load stress in the lower chord member in the adjacent panel toward the center of the truss. (Why?) The stress in an inclined chord is equal to its H component C_H multiplied by the length L_C of the chord divided by p the panel length. The live-load stresses are calculated directly from the dead-load stresses.

Upper Chord Members

Member	$C_H \times (L_C \div p) =$	*Dead-load Stress*	*Live-load Stress*
BC	$48.3 \times (21.4 \div 20) =$	51.7(C)	93.0(C)
CD	$53.7 \times (20.5 \div 20) =$	55.1(C)	99.1(C)
DE	$57.1 \times (20.1 \div 20) =$	57.4(C)	103.4(C)
$EE' = ee'$	$=$	57.1(C)	102.8(C)

1. *Fifth step:* Calculate the dead-load stresses in the verticals. The dead-load stress in any vertical (except the hip vertical) is equal to the coefficient c multiplied by the panel load W_D minus the V component of the stress in the inclined chord member cut by the section through the vertical. The V component of the stress in the chord member may be found by multiplying the H component C_H (used in the fourth step) by the rise r divided by the panel length p.

Verticals

Member	$c \times W_D - C_H \times (r \div p) =$	*Dead-load Stress*
Bb	$=$	10.0(T)
Cc	$2 \times 10 - 48.3 \times (7.5 \div 20) =$	1.9(C)
Dd	$1 \times 10 - 53.7 \times (4.5 \div 20) =$	−2.1(T)
Ee	$0 \times 10 - 57.1 \times (1.5 \div 20) =$	−4.3(T)

2. *Note:* In the calculation of dead-load stresses in verticals, the dead load was assumed as applied wholly at joints of the lower chord; if a portion of it is assumed as applied at upper joints, corresponding corrections are made in the dead-load stresses in the verticals. (199 : 14.)

3. *Sixth step:* Calculate the dead-load stresses in the diagonals. The V component of the dead-load stress in any diagonal (except the end post) is equal to one panel load W_D plus (or minus) the dead-load stress S_V in the vertical that meets that diagonal at a lower-chord joint. This V component multiplied by the length of the diagonal L_D divided by the height of truss h will equal the dead-load stress in the diagonal. The height h is the vertical side of the right-angled triangle of which the diagonal is the hypotenuse.

Diagonals

Member	$(W_D \pm S_V) \times (L_D \div h) =$	*Dead-load Stress*
Bc	$(10 + 1.9) \times (29.4 \div 21.5) =$	16.3
Cd	$(10 - 2.1) \times (35.2 \div 29.0) =$	9.6
De	$(10 - 4.3) \times (39.0 \div 33.5) =$	6.6
Ee'	$=$	0
aB	$4 \times 10 \times 29.4 \div 21.5 =$	54.7

For each of the diagonals *Cd* and *De* the dead-load stress S_V in the corresponding vertical is subtracted from W_D because the vertical is in tension and therefore acts upward on the segment in the opposite direction to that of W_D.

4. *Live-load stresses in web members.* Before proceeding to the calculation of the live-load stresses in the verticals and diagonals (seventh and eighth steps), the methods of determining such stresses will be considered. When the live load is in a position to cause maximum stress in a web member of a Parker truss, the shear for the *unloaded* segment is equal to the reaction R_L, and this reaction is equal to the coefficient c for the web member multiplied by the live load per panel W_L. If the top-chord member which acts on the segment is inclined, the V component of the live-load stress in the web member is equal to the live-load shear R_L *minus* the V component of the stress in the inclined chord member if that member is acting down on the segment, or *plus* the V component of the stress in the chord member if it is acting up. The stress in the chord member just referred to is the live-load stress *when the load is in a position to cause the maximum stress in the web member*, and is a stress which is not ordinarily computed. Its H component is equal to $(R_L \times Np) \div h_1$, in which N is the number of panels from R_L to the center of moments for the H component, p the panel length, and h_1 the lever arm of the H component. The center of moments is at the joint in which the web member intersects the lower chord, and the lever arm h_1 is the height of the truss (the length of the vertical) at this joint. The V component of the stress in the inclined chord is equal to the H component multiplied by the rise r of the chord divided by p the panel length. Hence C_V, the V component of the stress in the inclined chord, is equal to $R_L \times Nr \div h_1$, or:

$$C_V = c \times W_L \times N \times r \div h_1$$

1. The stress in a *vertical* is equal to the shear for a section through the vertical minus (or plus) C_V.

2. *Stress in a vertical* $= R_L - C_V = c \times W_L + c \times W_L \times (Nr \div h_1)$

$$= c \times W_L \times (h_1 - Nr) \div h_1$$

when the inclined chord member acts downward on the unloaded segment;

$$= c \times W_L \times (h_1 + Nr) \div h_1$$

when the inclined chord member acts upward on the unloaded segment.

3. The expressions just given hold true also for the *V component* of the stress in a *diagonal* except that all terms are for a section through the diagonal and for the corresponding position of the live load, and c is the coefficient for the diagonal instead of for the vertical. The stress in the diagonal is its V component multiplied by $L_D \div h$ in which L_D is the length of the diagonal, and h the vertical side of the right-angled triangle of which the diagonal is the hypotenuse. (Note that $L_D \div h$ is the secant of the angle which the diagonal makes with the vertical.)

4. *Stress in a diagonal* $= [c \times W_L \times (h_1 - Nr) \div h_1] \times (L_D \div h)$

when the inclined chord member acts downward on the unloaded segment;

$$= [c \times W_L \times (h_1 + Nr) \div h_1] \times (L_D \div h)$$

when the inclined chord member acts upward on the unloaded segment.

5. *Note:* These formulas should not be used blindly. They do not apply for example to such web members as a hip vertical or a hanger. Neither do they hold good when the section through the web member in which the stress is required cuts another web member, such, for example, as a section through a vertical of the center panel. In using the formulas, the significance of each term should be kept in mind; for example, h_1 is the lever arm of the H component of the inclined chord cut by the section, whereas h is the vertical side of a triangle used in obtaining the stress in a diagonal from its V component.

6. In the two remaining steps of the calculation of stresses in the Parker truss, namely, in the calculation of the maximum live-load tension and compression in the verticals and in the calculation of the maximum tension and compression in the diagonals, the use of the formulas just derived will be illustrated.

7. *Seventh step:* Calculate the live-load stresses in the verticals.

Verticals

Member	$c \times W_L \times (h_1 \pm N \times r) \div h_1 =$	*Live-load Stress*
Cc	$\frac{21}{9} \times 18 \times (29.0 - 2 \times 7.5) \div 29.0 =$	20.3(C)
C′c′	$\frac{3}{9} \times 18 \times (29.0 + 7 \times 7.5) \div 29.0 =$	16.9(T)
Dd	$\frac{15}{9} \times 18 \times (33.5 - 3 \times 4.5) \div 33.5 =$	17.9(C)
D′d	$\frac{6}{9} \times 18 \times (33.5 + 6 \times 4.5) \div 33.5 =$	21.7(T)
Ee	$\frac{5}{9} \times 18 - \frac{10}{9} \times 18 \times (4 \times 1.5 \div 35.0) =$	6.6(C)
E′e′	$\frac{5}{9} \times 18 + \frac{10}{9} \times 18 \times (4 \times 1.5 \div 35.0) =$	14.3(T)
Bb	One panel load	18.0(T)

For $C'c'$ and $D'd'$ the sign within the parentheses is plus because each of these inclined chords is acting upward on the left-hand or unloaded segment, i.e., it acts with the shear instead of against it. The expressions for Ee and $E'e'$ are slightly modified since a section through each of these verticals cuts another web member which is in action, i.e., "$\frac{5}{9} \times 18$" is the shear, and the rest of the expression is the V component of the stress in the inclined chord, derived from $R_L \times (N \times r \div h_1)$.

8. *Eighth step:* Calculate the live-load stresses in the diagonals.

Diagonals

Member	$[c \times W_L \times (h_1 \pm N \times r) \div h_1] \times (L_D \div h) =$	*Live-load Stress*
Bc	$[\frac{28}{9} \times 18 \times (29.0 - 2 \times 7.5) \div 29.0] \times (29.4 \div 21.5) =$	36.9(T)
B′c′	$[\frac{1}{9} \times 18 \times (29.0 + 7 \times 7.5) \div 29.0] \times (29.4 \div 21.5) =$	7.7(C)
Cd	$[\frac{21}{9} \times 18 \times (33.5 - 3 \times 4.5) \div 33.5] \times (35.2 \div 29.0) =$	30.4(T)
C′d′	$[\frac{3}{9} \times 18 \times (33.5 + 6 \times 4.5) \div 33.5] \times (35.2 \div 29.0) =$	13.1(C)
De	$[\frac{15}{9} \times 18 \times (35.0 - 4 \times 1.5) \div 35.0] \times (39.0 \div 33.5) =$	28.9(T)
D′e′	$[\frac{6}{9} \times 18 \times (35.0 + 5 \times 1.5) \div 35.0] \times (39.0 \div 33.5) =$	17.0(C)
Ee′	$\frac{5}{9} \times 18 \times 40.3 \div 35 =$	11.5(T)
E′e	$=$	11.5(C)
aB	$= 1.8 \times$ dead-load stress in $aB = 1.8 \times 54.7 =$	98.5(C)

The maximum stresses in each of the members Ee' and $E'e$ are 11,500 lbs. tension for one position of the live load and 11,500 lbs. compression for

another position. The sum of the V components of Ee' and $E'e$ is equal to the shear, since the top-chord member cut by a section through these diagonals is horizontal. The sign within the parentheses is plus for members $B'c'$, $C'd'$, and $D'e'$, since these members act upward on the unloaded (left-hand) segment, i.e., the diagonal has to resist both the positive shear and the V component of the inclined chord.

1. *Note:* From inspection of the algebraic headings for the various tabulations it will be seen that the following quantities have been used in determining the stresses in the Parker truss by the method of coefficients:

$(p \div h)$, or the panel length divided by the height of truss at each panel point.
$(L_C \div p)$, or the length of each inclined chord member divided by the panel length.
$(r \div p)$, or the rise of each inclined chord member divided by the panel length.
$(L_D \div h)$, or the length of each diagonal divided by its vertical projection.

Both speed and correctness of results are likely to be gained if the calculations of these quantities are made one after the other as part of the preliminary calculations, and the results are entered on the truss diagram in some convenient and unmistakable form.

2. An idea of the advantages and disadvantages of the method of coefficients as compared with the method of sections may be gained by comparing the work as just outlined in the tabulated forms with the work as outlined on pages 366 and 367. One who is experienced in the calculation of stresses can probably use the method of coefficients to better advantage than the method of sections, but the difference in favor of the former method is not nearly so great when a truss has inclined chords as it is when top and bottom chords are parallel.

3. *Special stress coefficients for trusses with inclined chords.* If for a Parker truss of a given number of panels, fixed ratios of height to panel length are adopted, one ratio at one vertical, another ratio at another vertical, coefficients may be determined which will hold good for any Parker truss which has the same number of panels and the same ratios of height to panel length. The coefficient for each member, for example, may be the actual stress in that member due to a uniform load of one pound per linear foot per truss for a panel length of one foot. For any other load of w lbs. per lin. ft. and a panel length p, the stresses in a Parker truss of the same number of panels and having the same ratios of height to panel length may be calculated by simply multiplying the coefficient for each member by wp. Coefficients may be given in tabular form or on truss diagrams, not only for Parker trusses with different numbers of panels and different ratios of height to panel length, but also for other types of trusses with inclined chords. It is to be noted, however, that coefficients of this character have a different meaning from that attached to the coefficients used in determining stresses by the method just explained in the illustrative example.

4. **Graphic Method of Successive Joints.** For trusses with parallel chords and equal panel lengths, there is no method of determining the stresses as simple and rapid as the algebraic method of coefficients, but when the chords are not parallel or when the panel lengths are unequal, the graphic method of successive joints is considered by many engineers to be superior to the algebraic method, particularly when the conventional system of uniform live load is used. It is a simple method, and, because of its mechanical nature, it is to a certain extent self checking. It is sufficiently accurate for most purposes. For example, the stresses in the Parker truss on page 358, determined by the method of coefficients, were rapidly checked by the graphic method, and, without special effort for accuracy, the results obtained did not vary by more than one hundred pounds for any member.

5. Two force diagrams are necessary. From the dead-load diagram, the live-load stresses in the members in the perimeter of the truss may be obtained, as well as the dead-load stresses in all members. (337 : 3.) From the live-load diagram, live-load stresses for web members may be determined. In the latter diagram, it is well to use a fictitious reaction equal to one panel of live load. (337 : 5.) For example, a fictitious reaction of 18,000 lbs. was used for the live-load diagram in checking the stresses in the Parker truss.

6. Graphic methods may often be combined with algebraic methods in determining stresses in trusses with inclined chords. If the truss diagram is drawn carefully to scale, lengths of members and lever arms may often be scaled with sufficient accuracy, or, when computed, they may be checked by such measurements. A simple force diagram may be used to determine or to check a particular system of forces in equilibrium, as, for example, the relatively small number of forces that act on a joint or on a segment to hold it in equilibrium.

7. The graphic method, when applied to a single joint is merely the graphic solution of Case B, concurrent forces, (page 29); when applied

to several joints in succession, the method of procedure is the same as that explained in CHAPTER XVI and used for roof trusses throughout CHAPTER XVII.

1. **THE DETERMINATION OF STRESSES IN VARIOUS TYPES OF TRUSSES WITH POLYGONAL CHORDS.** Throughout this chapter thus far, the Parker truss has been used to illustrate methods of determining stresses in a truss with inclined chords. This type of truss was selected, not only because it is by far the most common type of truss with polygonal chord, but also because nearly every question that can arise in the determination of stresses due to uniform load is involved in the determination of stresses in a Parker truss. The general methods already explained may be used for any ordinary type of truss, though in certain types, some slight modifications are necessary.

2. The criterion for placing the live load in a position to cause maximum tension or compression in a web member holds good for any type of truss, provided the point in which the chord members cut by the section intersect does not lie anywhere between the supports. When this point of intersection is between supports, the maximum stress in the web member cut by the section will occur when the truss is fully loaded. This latter condition occurs in only a few types of trusses, such, for example, as in certain forms of the three-hinged arch.

3. Any modification of the general method of determining stresses that may be necessary is usually very simple. When the web system is similar to that of a Warren truss, with or without verticals, no modification of the general method is required, and the work of determining the stresses is simpler, if anything, than that required for a Parker truss. The Petit truss (pages 103 and 104) is a Baltimore truss with a polygonal top chord. Where a section cuts only three members, the stresses in these members may be determined by exactly the same methods as those used for a Parker truss. When a section cuts four members, one of these members will be a half-diagonal, a sub-strut, or a sub-tie, and to one who understands the methods used for the Baltimore truss under similar conditions, the variations in methods due to inclined chords present no special difficulties. The K-truss, with one chord polygonal, requires much the same analysis as the K-truss with parallel chords (332 : 8) except that the portion of the shear in any panel carried by the inclined chord in that panel must be taken into account, just as in any truss with polygonal chord. Trusses with both chords, top and bottom, polygonal are now rarely used for bridges, except for very long spans, when a more complex type of truss may be desirable, such, for example, as some form of cantilever truss. Should it be required, however, to determine the stresses in a simple truss with both chords polygonal, the modifications of the general method that may be necessary will be simple and obvious.

4. The intent in the preceding paragraph is to emphasize the fact that a general method (algebraic or graphic) holds good for all types of trusses with polygonal chords. There is no necessity, therefore, for explaining the application of these general methods to various forms of trusses; to do so might give an impression, already too prevalent, that each type of truss presents special difficulties which render the determination of stresses in that type a distinct and different problem from the determination of stresses in another type.

5. **THE EXACT METHOD OF DETERMINING STRESSES.** When the exact method of loading is used in place of the conventional method, the only stresses affected are those in web members. In order to determine the stress in any web member, it is necessary to determine first how far the uniform load should extend into the panel of the loaded chord that is cut by the usual section through that member. The point in the panel to which the load should extend when one of the chord members in the panel is inclined is not the same as the neutral point when both chord members are horizontal. The position of the uniform load that will result in the maximum stress in a typical diagonal will be considered first.

6. If the ordinate dd' in Fig. 363 is laid off to any convenient scale to represent the stress in the diagonal Cd due to a load of unity placed at joint d of the truss, and cc' is laid off to the same scale to represent the stress in the same diagonal when a load of unity is placed at c, the line $ac'd'a'$ is the influence line for the stress in the diagonal. (Why?) To obtain the maximum live-load tension in Cd, the uniform live load should extend from a' to n, and to obtain the maximum live-load compression in Cd, the uniform live load should extend from a to n. A single concentrated load placed at n could cause neither tension nor compression, hence the stress in Cd due to a concentrated load at n is zero. The point n may be considered, therefore, as the neutral point for stress in Cd. This point

is *not* the same as the neutral point for shear for the panel cd as obtained in 298 : 4. In the case of a parallel-chord truss, the neutral point for stress in a diagonal and the neutral point for shear for the panel that

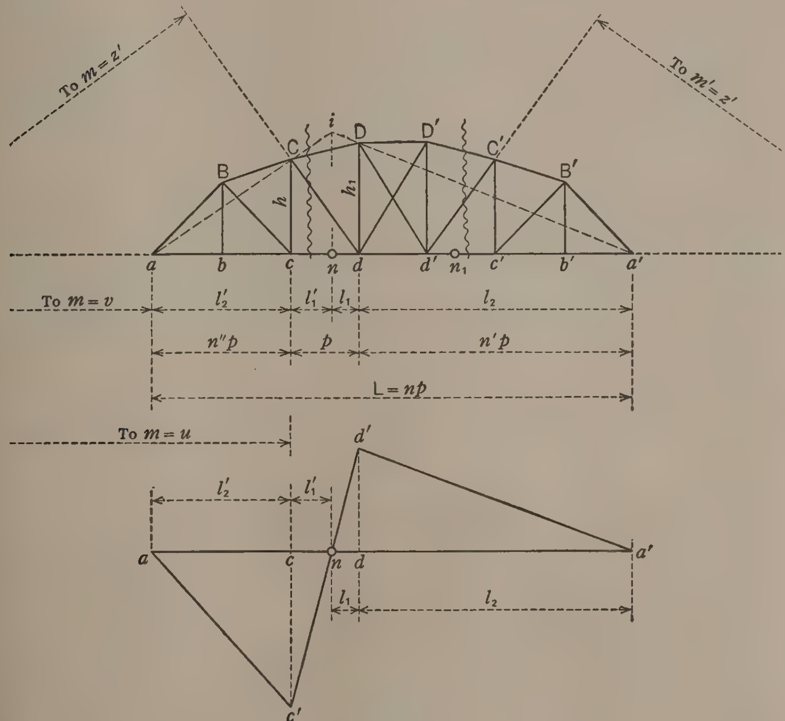


Fig. 363.

contains the diagonal are the same, but in the case of a truss with inclined chords, it is necessary to keep in mind the distinction between the neutral point for stress and the neutral point for shear, except in panels in which both chords, upper and lower, are horizontal. (Compare the influence line in Fig. 363 with that in Fig. 298.)

1. *To determine the neutral point for stress in a diagonal.* In Fig. 363 let v represent the distance from a to the point m in which the lines CD and cd produced intersect. (Point m not shown.) Let u represent the distance from c to the same point m; let z' represent the lever arm of Cd with respect to m as a center of moments; let n be the neutral point for stress in the diagonal Cd. The distance l_1 from d to n may be determined as follows: For a single concentrated load W at n the stress in Cd is zero, hence $Cd \times z' = -R_L \times v + W_c \times u = 0$ in which R_L is the reaction at a due to the concentrated load at n and W_c is the floor-beam load at c due to the same concentrated load.

$$R_L = W \times (l_2 + l_1) \div L, \quad \text{and} \quad W_c = W \times l_1 \div p$$

$$\Sigma M_m = 0 = -\frac{W\,(l_2 + l_1)}{L} \times v + \frac{W l_1}{p} \times u$$

$$l_1 = \left(\frac{pv}{Lu - pv}\right) l_2, \text{ and for equal panels } l_1 = \left(\frac{v}{nu - v}\right) l_2$$

(n in the last expression represents the number of panels in the truss.)

2. Another expression for l_1 more convenient for use in the usual case of equal panels, may be obtained by using the ratio $\frac{h_1}{h}$ of the heights of the verticals at d and c. Let n' and n'' represent respectively the number of panels in the right-hand and left-hand segments.

$$\frac{h_1}{h} = \frac{v + n''p + p}{v + n''p}, \quad \text{or} \quad v = \frac{n''p\left(1 - \frac{h_1}{h}\right) + p}{\frac{h_1}{h} - 1}$$

$$u = v + n''p$$

Substituting in the final expression for l_1 obtained above the values for v and u and reducing, the expression for l_1 becomes:

$$l_1 = \left(\frac{n}{n''\frac{h_1}{h} + n'} - 1\right) l_2$$

3. The position of n may be determined graphically as follows: Draw a line through joints a and C and a line through joints a' and D. Through

the point of intersection i of these two lines draw a vertical line; it will intersect aa' at n. This graphic method is similar to that for parallel-chord trusses (298 : 5) but the result is different, since the lengths of the verticals at c and d are not equal as they are in a parallel-chord truss.

1. *To determine the stress in Cd.* To determine the maximum live-load tension in Cd, assume the uniform live load to extend from a' to n. Full panel loads will be at joints a', b', c', and d', but only partial panel loads, W_c and W_d, at joints c and d. The reaction R_L may be determined as if the uniform load were on a beam without panels, and the load W_c may be determined from the load in the panel cd as if cd were a simple beam. These two forces R_L and W_c are the only external forces on the left-hand segment, and the stress in Cd may be determined from the moment equation (center of moments at m the intersection of CD and cd):

$$Cd \times z' - R_L \times v + W_c \times u = 0$$

To determine the maximum live-load compression in Cd, it will be convenient to work with $C'd'$, using as a center of moments the intersection of $C'D'$ and $c'd'$ produced. (349 : 1.) The load will extend from a' to the neutral point n_1 for the stress in $C'd'$. This neutral point will be as far from c' as the neutral point n is from c. The body in equilibrium will be the left-hand segment, and the only external forces on that segment are the reaction at a and the partial panel load at d'.

2. *Note:* The exact method of determining the stress in Cd differs from the conventional method as follows: The amount of uniform live load on the bridge is slightly different hence the magnitude of the reaction is slightly different. The external forces on the left-hand segment are the reaction and a partial panel load, whereas in the conventional method there is only one external force — a reaction. These differences between the two methods are typical — they hold true for practically all web members except a hanger or some other vertical which carries only the load at a joint.

3. *Exercise:* In 339 : 2 an expression was given for the greatest shear for any panel of a parallel-chord truss, determined by the exact method. Similar expressions will now be given for the maximum stresses in each of the two diagonals of a panel in which the top chord is inclined. Let these two diagonals be Cd and cD (the counter) in Fig. 363. Let w represent the uniform live load per linear foot.

$$\text{Maximum tension in } Cd = (\tfrac{1}{2}w \times l_1 \times l_2) \times \frac{h \text{ sec. } cCd}{h_1 p}$$

$$\text{Maximum compression in } Cd = (\tfrac{1}{2}w \times l_1' \times l_2') \times \frac{\text{sec. } cCd}{p}$$

$$\text{Maximum tension in } cD = (\tfrac{1}{2}w \times l_1' \times l_2') \times \frac{h_1 \text{ sec. } CcD}{hp}$$

$$\text{Maximum compression in } cD = (\tfrac{1}{2}w \times l_1 \times l_2) \times \frac{\text{sec. } CcD}{p}$$

When the uniform load is an equivalent uniform load obtained by means of an empirical formula based on the lengths of the segments of the base of the influence triangle (339 : 3), the value of w in the second and third expressions is that derived from l_1' and l_2', whereas the value of w in the first and fourth expressions is that derived from l_1 and l_2.

The derivation of the expressions for stresses just given is left as an exercise for the student.

4. *Stress in a vertical web member.* Let it be required to determine the maximum live-load stress in the vertical Cc. It is necessary to determine first the neutral point between c and d for the stress in Cc. This will not be the same point as the neutral point for the stress in Cd. It may be determined, however, from one of the expressions for l_1, provided that the distance v' from a to the intersection m' of BC and cd produced be substituted for v, and that the distance u' from c to the same intersection be substituted for u thus:

$$l_1 = \left(\frac{pv'}{Lu' - pv'}\right) l_2 = \left(\frac{v'}{nu - v'}\right) l_2 = \left(\frac{n}{n''\dfrac{h_1'}{h'} + n'}\right) l_2$$

In the last expression for l_1, the ratio $\dfrac{h_1'}{h'}$ is the ratio of the length of the verticals at c and b instead of the ratio of the verticals at d and c used in determining l_1 for the diagonal Cd. This is because the section through Cc cuts the inclined chord BC instead of the inclined chord CD.

5. The neutral point of stress for Cc may be determined graphically as follows: Produce BC until it intersects Dd produced. Through this point of intersection and a' draw a line until it intersects a line drawn through a and C. This second point of intersection will be directly above the required neutral point for the stress in Cc. It will be slightly to the right of the neutral point for the stress in Cd.

6. The neutral point having been determined, the load is assumed to cover the span from this neutral point to the right-hand support; the reaction and the partial panel load on the left-hand segment are then determined, and the stress is calculated from a moment equation with the

center of moments at the intersection of the lines of action of the top and bottom chord members cut by the section.

1. **Impact.** The statements regarding impact made in connection with parallel-chord bridges in 339 : 4 hold true for bridges with inclined chords. When the loaded length is $l_1 + l_2$ (or $l_1' + l_2'$), the distance l_1 (or l_1') is determined by the method explained in 363 : 1 or 364 : 4. Some engineers, when using the conventional method of uniform loading, regard it as sufficiently accurate to use l_2 (or l_2') as the loaded length. (339 : 8.)

2. **Use of Formulas.** The calculation of stresses in parallel-chord trusses by the method of coefficients is so simple and direct that nothing is gained by the use of formulas. Algebraic methods of determining stresses in a truss with inclined chords may be shortened, however, by the use of formulas which express the various steps in the method of procedure reduced to simplest terms. But until the methods themselves are thoroughly understood, formulas should not be used; once methods are understood it is a simple matter to devise formulas which will facilitate their application. For this reason very few formulas have been given in this chapter.

3. Formulas for determining stresses in trusses with inclined chords, may, in general, be classified as follows:

(*a*) Formulas for determining lengths and distances, such, for example, as that for determining the lever arm of a diagonal. (347 : 2.)

(*b*) Formulas for determining the portion of the shear for a panel carried by the inclined chord in that panel, such, for example, as that derived in 345 : 5.

(*c*) Formulas for obtaining the stress in a chord member, a vertical member, or a diagonal web member, such as those in 364 : 3.

4. Formulas for lengths or distances usually involve two or more of the following terms: Panel length; height of truss; difference in heights of truss at two successive joints or the rise of an inclined chord; length of a truss member.

5. Formulas for shear and for the stress in a truss member usually involve two or more of the following terms: Reaction; bending moment; shear; panel length; height of truss; rise of an inclined chord; number of panels between the support and the center of moments; lever arm of a truss member expressed in general terms; the known stress or the component of the stress in a member other than that for which the stress is required.

6. *Note:* Many formulas of the character just described may be found in standard books of reference and in various engineering periodicals. It is an excellent exercise for the student to derive formulas for himself, and afterwards to compare them with those derived by others.

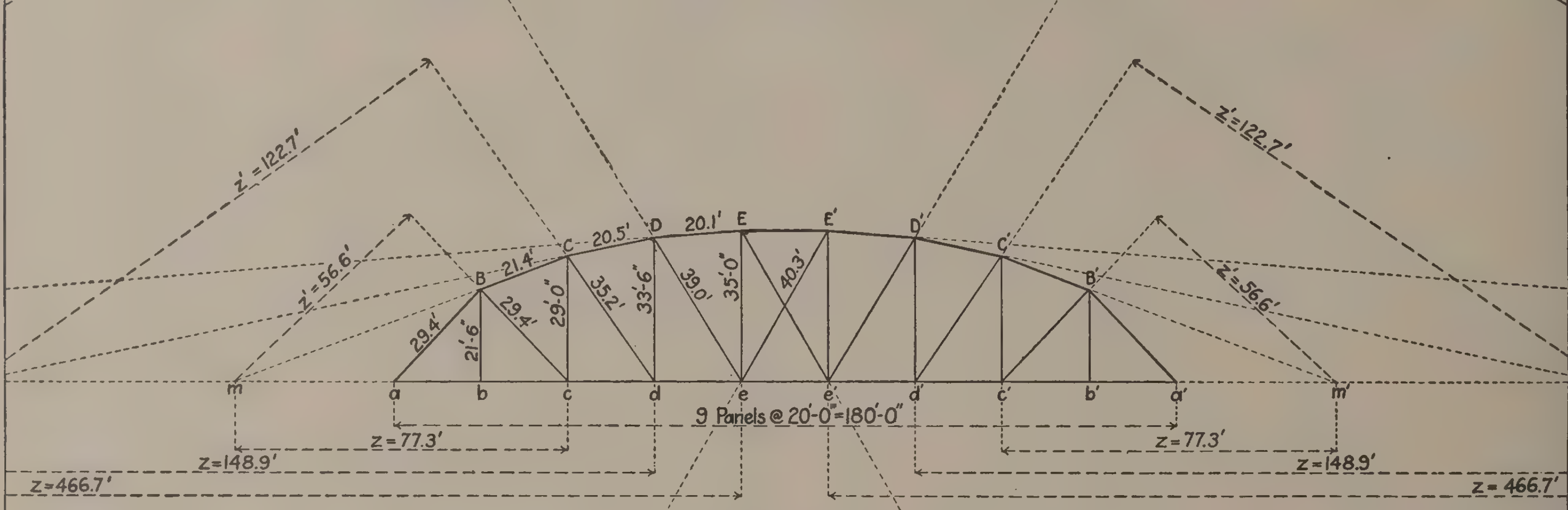
7. **Systematic Method of Procedure and Arrangement of Work — Illustrative Example.** It is even more important to follow some good method of procedure and arrangement of work in the calculation of stresses in a truss with a polygonal chord than it is in calculating stresses in a parallel-chord truss. One such method is illustrated on the two pages that follow. In order that the method may be easily followed throughout by the student, the arrangement is somewhat different from that used by an experienced engineer. It should serve, however, to illustrate the essential features of any good method of procedure and corresponding method of arrangement. It is to be noted that lengths of members and various lever arms are entered on the truss diagram; even more preliminary calculations may be made, and the results entered on this diagram, as suggested in 353 : 1.

8. Any method of procedure and arrangement of work should be such as not only to facilitate computations, but also to disclose, as far as possible, obvious mistakes. For example, by scanning the algebraic expressions from which stresses in members of the same general character are obtained, certain terms will be seen to vary by a fixed amount, or in accordance with some obvious relation, and tabulated forms, such as those shown on page 367, enable one to detect any obviously incorrect variation,

9. The method of procedure shown in the illustrative example is that outlined for the method of sections in 352 : 3. It should be studied until thoroughly understood, and then it should be compared with the method of coefficients outlined in 357 : 6. The truss is like that shown in the photograph on page 102, except that it has two more panels.

10. *Question:* If a portion of the dead load is assumed to take effect at upper joints of the truss, what changes should be made in the stresses calculated in the illustrative example under the assumption that all of the dead load takes effect at joints of the lower chord?

ILLUSTRATIVE PROBLEM IN THE CALCULATION OF DEAD- AND LIVE-LOAD STRESSES IN A THROUGH PARKER TRUSS WITHOUT COUNTERS



Preliminary Calculations

Dead Load per lin. ft. of bridge = 1000 lbs. applied at lower chord; live load per lin. ft. = 1800 lbs.

$10 = 1.0 \times 20 \div 2 = W_D$ = dead load per panel per truss

$18 = 1.8 \times 20 \div 2 = W_L$ = live load per panel per truss

1.8 = ratio of live to dead load

Lengths of Chords

$BC = 21.4 = \sqrt{(20)^2 + (7\frac{1}{2})^2}$

$CD = 20.5 = \sqrt{(20)^2 + (4\frac{1}{2})^2}$

$DE = 20.1 = \sqrt{(20)^2 + (1\frac{1}{2})^2}$

Lengths of Diagonals

$Bc = 29.4 = \sqrt{(20)^2 + (21\frac{1}{2})^2}$

$Cd = 35.2 = \sqrt{(20)^2 + (29)^2}$

$De = 39.0 = \sqrt{(20)^2 + (33\frac{1}{2})^2}$

$Ee' = 40.3 = \sqrt{(20)^2 + (35)^2}$

Lever arms z for verticals.

$Cc = 77.3 = 20 \times 29 \div 7\frac{1}{2}$

$Dd = 148.9 = 20 \times 33\frac{1}{2} \div 4\frac{1}{2}$

$Ee = 466.7 = 20 \times 35 \div 1\frac{1}{2}$

Lever arms z' for diagonals

$Bc = 56.6 = 21.5 \times 77.3 \div 29.4$

$Cd = 122.7 = 29.0 \times 148.9 \div 35.2$

$De = 400.9 = 33.5 \times 466.7 \div 39.0$

$Ee' = 405.3 = 35.0 \times 466.7 \div 40.3$

Stresses in Lower Chord Members

Member		Equation	Dead-Load Stress	Live-Load Stress D.L. Stress × 1.8
ab = bc	$\Sigma M_B = 0$	$40 \times 20 \div 21\frac{1}{2}$	37.2(T)	67.0(T)
cd = BC_H	$\Sigma M_C = 0$	$[40 \times 40 - 10 \times 20] \div 29$	48.3(T)	86.9(T)
de = CD_H	$\Sigma M_D = 0$	$[40 \times 60 - 10(40 + 20)] \div 33\frac{1}{2}$	53.7(T)	96.7(T)
ee′ = DE_H	$\Sigma M_E = 0$	$[40 \times 80 - 10(60 + 40 + 20)] \div 35$	57.1(T)	102.8(T)

Stresses in Upper Chord Members

Member	From H-component	Dead-Load Stress	Live-Load Stress D.L. Stress × 1.8
BC	$48.3 \times 21.4 \div 20$	51.7(C)	93.0(C)
CD	$53.7 \times 20.5 \div 20$	55.1(C)	99.1(C)
DE	$57.1 \times 20.1 \div 20$	57.4(C)	103.4(C)
EE′	= ee′	57.1(C)	102.8(C)

Dead-Load Stresses in Vertical Members

Member	Stress		Equation
Bb	10.0(T)		
Cc	1.9(C)	$\Sigma M_m = 0$	$[40 \times 37.3 - 10(57.3 + 77.3)] \div 77.3$
Dd	2.1(T)	"	$[40 \times 88.9 - 10(108.9 + 128.9 + 148.9)] \div 148.9$
Ee	4.3(T)	"	$[40 \times 386.7 - 10(406.7 + 426.7 + 446.7 + 466.7)] \div 466.7$

Dead-Load Stresses in Diagonal Members

Member	Stress	Equation
aB	54.7(C)	$-40 \times 29.4 \div 21.5$
Bc	16.3(T)	$(1.9 + 10) \times 29.4 \div 21.5$
Cd	9.6(T)	$(-2.1 + 10) \times 35.2 \div 29$
De	6.6(T)	$(-4.3 + 0 + 10) \times 39.0 \div 33.5$
Ee′	0	$(-40 + 10 \times 4) \times 40.3 \div 35$

Live-Load Stresses in Vertical Members

Member	Stress		Equation
Bb	18.0(T)		
Cc	20.3(C)	$\Sigma M_m = 0$	$[\frac{21}{9} \times 18 \times 37.3] \div 77.3$
Dd	17.9(C)	"	$[\frac{15}{9} \times 18 \times 88.9] \div 148.9$
Ee	6.6(C)	"	$[\frac{10}{9} \times 18 \times 386.7 - 11.5 \times 405.3] \div 466.7$
E′e′	14.3(T)	$\Sigma M_{m'} = 0$	$[-\frac{10}{9} \times 18 \times 566.7 + 11.5 \times 405.3] \div 466.7$
D′d′	21.7(T)	"	$[-\frac{6}{9} \times 18 \times 268.9] \div 148.9$
C′c′	16.9(T)	"	$[-\frac{3}{9} \times 18 \times 217.3] \div 77.3$

Live-Load Stresses in Diagonal Members

Member	Stress		Equation
aB	98.5(C)		54.7×1.8
Bc	36.9(T)	$\Sigma M_m = 0$	$(\frac{28}{9} \times 18 \times 37.3) \div 56.6$
Cd	30.4(T)	"	$(\frac{21}{9} \times 18 \times 88.9) \div 122.7$
De	28.9(T)	"	$(\frac{15}{9} \times 18 \times 386.7) \div 400.9$
Ee′	11.5(T)	$\Sigma V = 0$	$\frac{1}{2}(-\frac{10}{9} \times 18 \times 40.3) \div 35$
E′e	11.5(C)	"	$\frac{1}{2}(-\frac{10}{9} \times 18 \times 40.3) \div 35$
D′e′	17.0(C)	$\Sigma M_{m'} = 0$	$(-\frac{6}{9} \times 18 \times 566.7) \div 400.9$
C′d′	13.1(C)	"	$(-\frac{3}{9} \times 18 \times 268.9) \div 122.7$
B′c′	7.7(C)	"	$(-\frac{1}{9} \times 18 \times 217.3) \div 56.6$

ASSIGNMENTS

(1) Derive formulas to be used in calculating stresses in trusses with inclined chords, and report on similar formulas in general use. (365 : 2.)

(2) Report on special formulas for determining the proportion of the shear taken by a diagonal. (See *Engineering News-Record, Vol. 97*, p. 465.)

(3) Report on special stress coefficients for bridge trusses with inclined chords. (361 : 3.)

(4) Determine the dead-load and live-load stresses in a Parker truss with at least eight panels, first by the algebraic method of coefficients, and then by the graphic method of successive joints. Do the same for some other type of truss with polygonal top chord. Report on the relative advantages and disadvantages of the algebraic and graphic method.

(5) Report on the essential differences between the calculation of the stresses in a Parker truss and the calculation of stresses in a Petit truss. Discuss particularly stresses in members of a Petit truss that require special analyses — corresponding to similar analyses for a Baltimore truss.

(6) Report on the application of the general method of sections to a truss in which the bottom chord as well as the top chord is polygonal.

(7) Prove that the criteria used throughout the chapter for the critical positions of live load hold true so long as the chords cut by a section intersect beyond a support, but that when they intersect between supports, the maximum stress in the web member cut by the section will occur when the truss is fully loaded.

(8) Report on the question raised in 346 : 8.

CHAPTER XXII

CONCENTRATED LIVE LOADS

BEAMS AND GIRDERS

In this chapter are explained the methods of determining reactions, shears, and bending moments when the specified live load is a conventional system of wheel loads.

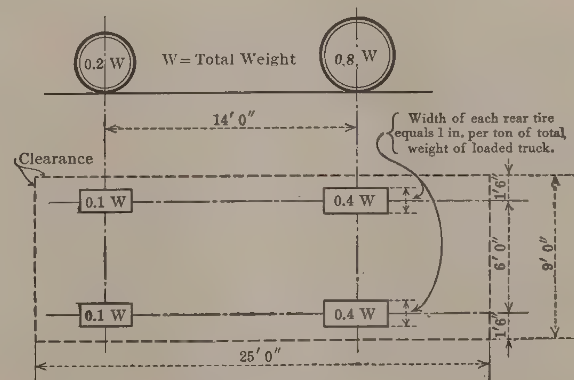
1. **Conventional Wheel-Load Systems.** A conventional wheel-load system is a series of concentrated loads that correspond approximately in weights and spacing to the actual loads on the wheels of a moving load such as an automobile truck, an electric car, or a locomotive. An axle load in such a system, is a concentrated load on one axle, assumed to be divided equally between two wheels, one at either end of the axle; a wheel load, therefore, is equal to one-half of the corresponding axle load.

2. **Wheel-Load Systems for Highway Bridges.** The wheel loads usually specified for highway bridges are those that correspond to the loads on the four wheels of a typical automobile truck. The typical truck shown in Fig. 369 is taken from the "Specifications for Steel Highway Bridges," published in the *Transactions of the American Society of Civil Engineers, Volume LXXXVII*, page 1277. To provide for different classes of bridges, three values for the total weight W are given in the specifications, namely:

Class A Bridges	H–20	20-ton trucks.
Class B Bridges	H–15	15-ton trucks.
Class C Bridges	H–13	13-ton trucks.

The spacing and dimensions shown in the figure hold good for all three loadings. Two-tenths of the total load is distributed to the front axle and eight-tenths to the rear. For example, in the H–20 loading there are two tons on each front wheel and eight tons on each rear wheel. There may be as many trucks side by side as the width of roadway will accommodate, but since the panel length of a highway bridge rarely exceeds 25 feet, not more than one truck can be in the same traffic lane in the same panel. (See assignment at the end of the chapter for distribution of loads.)

3. *Note:* The stresses in the main girders or trusses of a highway bridge are frequently determined for a combination of concentrated loads and uniform load. Various combinations of this character will be given in the next chapter.



Typical Truck
Fig. 369.

4. In the same specifications typical electric cars are specified as follows: "A train on each track shall be composed of two double-truck cars coupled together, with wheel concentrations and spacing of axles as shown in Fig.

370 (*a*) or Fig. 370 (*b*)." The loads indicated are axle loads or loads per track. The loads per rail are half as great. The axle loads are given in thousands of pounds (kips).

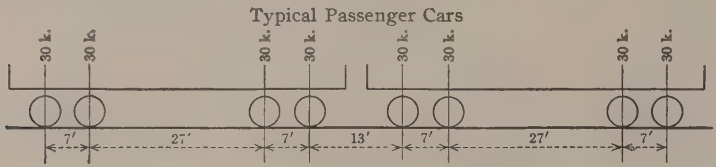


Fig. 370 (*a*).

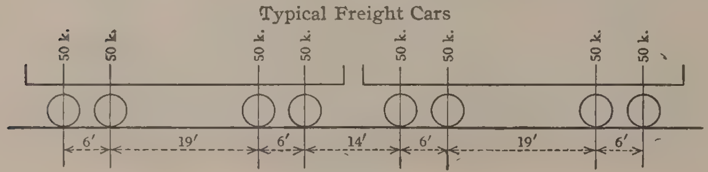


Fig. 370 (*b*).

1. **Wheel-Load Systems for Railway Bridges.** The wheel loads usually specified for railway bridges are those that correspond to the loads on the wheels of a typical locomotive. Two systems of locomotive loadings are in general use, namely, that first proposed by Theodore Cooper in 1894,* and that proposed by D. B. Steinman in 1922.† Cooper's system (Fig. 370 (*c*)) consists of eighteen concentrated loads (axle concentrations) of such weight and spacing that they represent the loads on the axles of two locomotives coupled together, four driving axles in each locomotive. Steinman's system (Fig. 370 (*d*)) consists of eleven concentrated loads representing a locomotive of the Mallet type with two groups of drivers. In each system the locomotive loading is followed by a train load which is assumed to be uniformly distributed. The axle loads are given in thousands of pounds (kips), and the distances between axles in feet. Neither locomotive diagram is intended to represent an actual locomotive, but rather a composite of locomotives of approximately the same type and weight as the locomotive shown in the diagram. The stresses determined from the locomotive loading will then be average stresses that correspond to those caused by existing locomotives of the same class. Each system provides for several classes, i.e., for heavy, light, and medium weight locomotives. Class E-10 (Fig. 370 (*c*)) and Class M-10 (Fig. 370 (*d*)) are not loadings that would be used in practice, but are merely basic units. The various classes of loadings actually used, such as E-40, E-50, and E-60, are multiples of these units. For example, in Class E-40 each axle load is four times as great as the corresponding axle load in Class E-10, and hence the axle load for each pair of drivers is 40,000 lbs. or 40 kips; in Class E-50 each axle load is five times as great, and the axle load for drivers is 50 kips; in Class E-60 each axle load is six times as great, and the axle load for drivers is 60 kips. The spacing of axles is the same for all classes, hence reactions, shears, bending moments, or stresses obtained for one class of loading may easily be changed by simple proportion to those for another class. For example, stresses for E-40 are $\frac{4}{5}$ of those for E-50, and $\frac{4}{6}$ of those for E-60. The number used to designate any class in Cooper's system is the number of kips that represents the axle load for each pair of drivers.

* *Transactions of the American Society of Civil Engineers, Vol. XXXI* (1894) p. 174.

† *Transactions of the American Society of Civil Engineers, Vol. LXXXVI* (1923) p. 606.

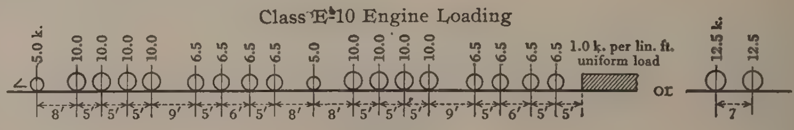


Fig. 370 (*c*).

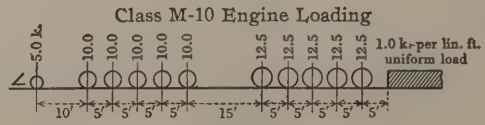


Fig. 370 (*d*).

2. *Note:* When Cooper's system was first proposed, Class E-40 corresponded to the heaviest locomotives in common use. To provide for modern heavy locomotives, it has become necessary to extend the number of classes to include such loadings as E-60, E-65, E-70, and E-75. Class E-60 has been used extensively as a basic unit, and many tables and diagrams have been prepared based on this unit. It is somewhat more logical to adopt E-10 as a basic unit, and in some of the more recent specifications this has been done.

1. The various classes of engine loading in Steinman's system are obtained from the unit loading M-10 (Fig. 370 (*d*)). By multiplying this unit loading by 4, 5, or 6, the loadings M-40, M-50, or M-60 are obtained. As in the case of Cooper's loading, the spacing of axles is the same for all classes, and reactions, shears, bending moments, or stresses determined for one class may be changed by simple proportion to those for another class. The number used to designate any class in Steinman's system is the number of kips that is the axle load for each pair of drivers *in the first group* of driving wheels.

2. *Note:* Loading M-60 will give stresses for all spans very close to the maximum stresses producible by the heaviest existing locomotive loadings. It is approximately equivalent to Cooper's E-75 for short spans and to Cooper's E-60 for long spans.

3. *Note:* Since the loads indicated in the unit systems E-10 (Fig. 370 (*c*)) and M-10 (Fig. 370 (*d*)) are *axle* loads, the loading in each system is the total loading *per track*. The loading *per rail* is one half as great.

4. *Note:* In the "Specifications for Steel Railway Bridge Superstructure," published in the *Transactions of the American Society of Civil Engineers, Volume LXXXVI*, page 477, it is stated that "the live load for each track shall consist of typical engines followed by uniform train load, according to Class E series or Class M series as may be specified by the Engineer. It shall be a multiple of one or the other of the loads with wheel spacing as shown in Figs. 370 (*c*) and 370 (*d*). Loading E-60 or Loading M-50 is recommended for main-line bridges of American railways." The two loadings thus recommended will be used in this book. It should be noted, however, that it is not uncommon for a railroad company to have its own standard system of locomotive loading that is specified in place of the E-system or the M-system because it represents more nearly the locomotives used on its lines.

5. *Alternative loading.* In addition to a regular locomotive loading, an alternative or special loading of *two* axle loads at a fixed distance apart is sometimes specified, this loading to be used in place of the locomotive loading for relatively short beams, such, for example, as stringers. In the E-10 system this alternative loading consists of two concentrated loads of 12,500 lbs. each spaced 7 ft. apart. For other Cooper loadings, these loads are to be increased in the same proportion as that for the regular locomotive loadings. For example, for E-60 loading, the two axle loads are each six times as great, i.e., 75,000 lbs. each per track or 37,500 lbs. each per rail. Note that each of these axle loads is one-fourth greater than the axle load for drivers.

6. **Maximum Reactions, Shears, and Bending Moments.** When a maximum live-load reaction is required, the concentrated live loads are placed in position to cause that reaction, and the reaction is then calculated as if the loads were static in that position.

7. The live-load shear generally required for a given segment of a beam is the *greatest* shear for that segment that the concentrated loads can cause as they move across the beam. There is at least one position in which these loads will produce this greatest shear. The loads are placed in this position, and the shear is then calculated as if the loads were static in that position. Frequently the *maximum shear* required is the maximum of all shears. In this case the section for which this shear occurs must be determined first (it is usually a section at or near the end of the beam), and the greatest shear for a corresponding segment may then be calculated just as for any other segment.

8. The live-load bending moment generally required for a given segment is the *greatest* bending moment for that segment that the concentrated live loads can cause. The loads are placed in position to produce this bending moment, and the bending moment is then calculated as if the loads were static in that position. Frequently the *maximum bending moment* required is the maximum of all bending moments. In this case the section for which this maximum bending moment occurs must be determined first (it is usually a section at or near the center of the beam), and the greatest bending moment for a corresponding segment may then be calculated just as for any other segment.

9. A maximum shear determined when the moving concentrated loads are treated as if they were static loads is not so great as the actual shear caused by the loads when they are in motion. In order to obtain a maximum shear which is as nearly as possible equal to the actual shear, an allowance for impact is added. This allowance is usually determined by merely multiplying the calculated shear by an empirical percentage. (267 : 5.) In like manner, the impact allowance to be added to a maximum bending moment or to a maximum reaction is determined by an empirical percentage. The method of determining these percentages will be explained at the end of this chapter. Since calculations of impact allowances are very simple and are entirely independent of the methods of determining reactions, shears, and bending moments, such calculations

will be omitted from the illustrative problems in this chapter. It should be understood, however, that specifications for bridges, both railway and highway, require that impact allowances be included in determining the shears and bending moments due to live load.

1. **General Methods.** The general methods of determining reactions, shears, and bending moments due to two moving concentrated loads were explained in Chapter XIX. When there is a series of such concentrated loads, these methods are inadequate. To determine a maximum reaction, shear, or bending moment due to the wheel loads of a line of trucks, an electric car, or a locomotive, one must know, first of all, in what position to place the loads in order to obtain the maximum result required.

2. *Position of the loads.* It was shown in Chapter XIX that the maximum reaction at a support due to two moving concentrated loads occurs when one of the loads is at the support, and that the maximum shear, or maximum bending moment, for a given section occurs when one of the loads is at the section. The same statements hold true for a series of wheel loads, but it is not always apparent *which* wheel should be at the support or at the section. When in doubt, it frequently is possible to determine this **critical wheel** by some general criterion, but in some cases it is necessary to determine the critical wheel by trial.

3. *Calculation of reactions, shears, and bending moments.* Once the critical wheel has been determined and this wheel has been placed at the support or at the section, as the case may be, the methods of calculation explained in Chapters XIII and XIX could be used, but for the relatively large number of wheel loads that are likely to be on a beam, these methods are cumbersome and tedious. The methods explained in this chapter are based on those of Chapter XIX, but are more efficient for determining reactions, shears, and bending moments when the live load is a series of wheel loads.

4. *Note:* Locomotive loadings will be used exclusively in this chapter. When one has learned the methods of determining reactions, shears, and bending moments due to systems of locomotive loading, he can easily adapt these methods to corresponding calculations when the live load is a line of trucks or an electric car.

5. *Illustrative example.* The following example will serve to illustrate the difference between the methods of Chapter XIX and those to be used in this chapter. Let *ab* in Fig. 372 represent a beam on which there are ten wheel loads numbered as shown. Let it be required to determine the bending moment for a section taken at *s* assuming that wheel 5 is at the section.

First step. Determine the reaction R_L. By the method explained in Chapter XII and used in Chapter XIX, it is necessary in determining R_L to calculate the total moment of all ten wheel loads about *b*, and then to divide the result by the length of the beam. This involves considerable numerical work. In this chapter a table will be used which enables one to obtain the total moment about *b* by adding to a tabular value the result of a single multiplication.

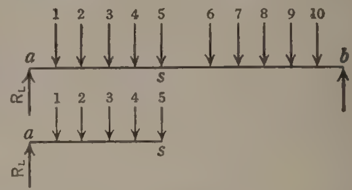


Fig. 372.

Second step. Find the resultant moment about *s* of R_L and the four wheel loads 1, 2, 3, and 4. The table just referred to gives the moment of the four wheel loads about *s*, and thus reduces the amount of calculation required.

6. The use of the table will be explained later. It will be sufficient for the present to state that from this table the total weight on any group of wheels may be read, as, for example, the total weight on wheels 2 to 6, or 1 to 5, or 2 to 9; also the total moment of any group of wheels about a center of moments at the extreme right-hand wheel of the group may be obtained from the table, as, for example, the moment of wheels 2 to 6 about wheel 6, or the moment of wheels 1 to 10 about wheel 10.

7. **Notation.** Locomotives will be considered to be headed toward the left, and the axle loads or wheels will be numbered 1, 2, 3, and so on in order, beginning with the axle of the pilot wheels at the extreme left. The last wheel in the E-system is wheel 18, the last wheel of the second engine; the last wheel in the M-system is wheel 11, the last wheel of the second group of drivers.

8. *Wheel.* The tables used in this chapter are for loads on wheels on one rail, not for axle loads. The term **wheel** will be used to designate the concentrated load on one wheel.

9. *Wheel* O: The critical wheel, i.e., the wheel which is placed at a support in order to determine a maximum reaction at that support, or at a section in order to determine the maximum shear or bending moment for that section.

10. *Wheel* N: The wheel next to wheel O on the left.

11. *Wheel* A: The wheel on the beam nearest to the left-hand support when the locomotive loading has been placed in any desired position. This first wheel on the beam is not necessarily wheel 1, but it may be

any wheel, depending upon the length of the beam or the length of the segment.

1. *Wheel* Z: The wheel on the beam nearest the right-hand support when the locomotive has been placed in any desired position. This last wheel on the beam is not necessarily wheel 18 of the E-system, or wheel 11 of the M-system, but it may be any wheel.

2. *Point* U: The beginning of the uniform train load when such load is on the beam.

3. *Length L:* The length of the beam.

4. *Length l:* The length of that portion of the beam that is covered by uniform train load when such load is on the beam.

5. *Length S:* The length of a segment, usually the left-hand segment.

6. *Point s:* The point through which the section is taken.

7. *Weight W:* The total weight on the beam including uniform train load if any is on the beam.

8. *Weight w:* The weight per linear foot of uniform train load.

9. *Weight of one wheel:* W(1), W(2), W(10), etc. represents the weight of wheel 1, wheel 2, wheel 10, etc.

10. *Weight of a group of wheel loads:* W(2...8) represents the total weight on wheels 2 to 8 inclusive; W(1...9) the weight on wheels 1 to 9 inclusive; W(A...Z) the weight on all wheels on the beam including the first wheel A and the last wheel Z. The weight of any group of wheels may be represented in a similar manner.

11. *Shear:* V represents the shear for a given segment or section.

12. *Moment:* M represents total moment (not bending moment) about some center. The center is designated by a subscript as, for example, M_a or M_b. The symbol M_B, however, is reserved for bending moment.

13. *Bending moment:* Represented by M_B. When it is desired to indicate the point where the section is taken a subscript is added. For example M_{B_s} represents the bending moment for a section at s.

14. *Moment of a group of wheels:* $M(1...4)_4$ represents the total moment of wheels 1 to 4 inclusive about a center at wheel 4; $M(4...9)_9$, represents the moment of wheels 4 to 9 inclusive about wheel 9. (Note that $M(1...3)_4$ and $M(1...4)_4$ are equal, likewise $M(4...8)_9$ and $M(4...9)_9$ are equal.) $M(4...18)_U$ represents the moment of wheels 4 to 18 inclusive about the beginning of the uniform train load; $M(A...Z)_Z$ represents the moment of all the wheels on the beam, including the first wheel A and the last wheel Z, about the last wheel Z; $M(A...Z)_U$ represents the moment of all wheels on the beam about the beginning of the uniform load. The moment of any group of wheels may be represented in a similar manner.

15. **To Change the Moment of a Group of Wheels from One Center of Moments to Another.** Let M be the moment of a group of wheels about any center of moments m, W the total weight of the wheels in the group, and d the horizontal distance from the center m to a vertical line through another center of moments m'. Let M' represent the moment of the group of wheels about m', and assume that m' is farther from the center of gravity of the forces than is m. In accordance with the principle explained in 32 : 7, the moment M' may be calculated from the moment M as follows:

$$M' = M + W \times d.$$

This principle is used constantly in calculating the moment of all of the wheels on a beam about a center at either support, particularly about a center at the right-hand support. For example, it is desired to determine the moment of all of the wheels on the beam ab (Fig. 373) about b as a center. The moment of wheels 7 to 11 about wheel 11 can be read from a moment table as will be explained later. If to this moment is added the weight of wheels 7 to 11 multiplied by the lever arm difference d, the result will be the moment of wheels 7 to 11 about b.

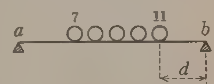


Fig. 373.

$$M(7...11)_b = M(7...11)_{11} + W(7...11) \times d.$$

16. In a moment table the moments of any group of wheels are only given for centers at other wheels and at the beginning of the uniform load. It is necessary therefore in calculating the moment of wheels on a beam about a center at a support to use the method just described, *unless a wheel or the beginning of the uniform load happens to be exactly at that support.*

17. **Practice in Indicating Reactions, Shears, and Bending Moments.** Before the use of tables of shears and bending moments is explained, it will be good practice merely to *indicate* the methods of calculating reactions, shears, and bending moments as illustrated in the

accompanying problems. In checking the work as indicated, it should be borne in mind that the weight of any group of wheels and the moment of any group of wheels about an extreme wheel of the group are quantities that may be read from the tables. Such quantities are printed in boldface type in the equations in order that they may be distinguished at a glance from other quantities. In each problem it is required to determine (1) the left-hand reaction, (2) the shear, and (3) the bending moment. The section for shear and bending moment will be taken through a wheel. Except in the figures for general cases, only three wheels will be shown in each figure, namely, the first and last wheels on the beam, and the wheel at the section. It is to be understood, however, that between any two of these wheels there are as many other wheels as the numbering of the two wheels implies. For example, if the number of one wheel is 6 and of the other is 12, it is implied that between these two wheels are wheels 7, 8, 9, 10, and 11. If there is uniform load on the beam, the length of that load will be indicated. Expressions for shears and bending moments are for *left-hand* segments. In determining shear for a left-hand segment, the wheel at the section is considered to be on the other segment, i.e., to be at an indefinitely small distance to the right of the section *s*. Neither the shear or the bending moment in any example is necessarily a maximum, but merely the shear or bending moment for the loads in the position shown. The notation of 372 : 7 is used throughout. Equations should be regarded not merely as equations but as expressions of methods.

1. *Beam in Fig. 374* (*a*)

$$R_L = \mathbf{M(A\ldots Z)_Z} \div L$$

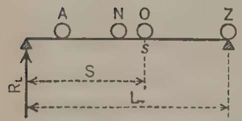


Fig. 374 (*a*).

Reaction = Moment of wheels A to Z inclusive about wheel Z divided by the length of the span.

$$V = R_L - \mathbf{W(A\ldots N)}$$

Shear = Reaction − (weight of the wheels on segment).

$$M_B = R_L \times S - \mathbf{M(A\ldots O)_O}$$

Bending moment = (Reaction × length of segment) − (moment of wheels A to O about wheel O).

2. *Beam in Fig. 374* (*b*)

$$R_L = \mathbf{M(7\ldots 11)_{11}} \div 24$$

Reaction = Moment of wheels 7 to 11 about wheel 11 divided by length of span.

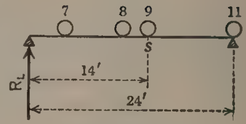


Fig. 374 (*b*).

$$V = R_L - \mathbf{W(7\ldots 8)}$$

Shear = Reaction minus weight of wheels 7 and 8.

$$M_B = R_L \times 14 - \mathbf{M(7\ldots 9)_9}$$

Bending moment = (Reaction × length of segment) − (moment of wheels 7 to 9 about wheel 9).

3. *Beam in Fig. 374* (*c*)

$$R_L = [\mathbf{M(A\ldots Z)_Z} + \mathbf{W(A\ldots Z)} \times d] \div L$$

Reaction = [Moment of wheels A to Z about wheel Z plus the weight of wheels A to Z multiplied by the distance from Z to the right-hand support] ÷ [Length of span].

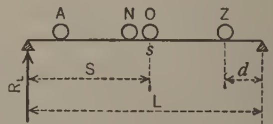


Fig. 374 (*c*).

$$V = R_L - \mathbf{W(A\ldots N)}$$

$$M_B = R_L \times S - \mathbf{M(A\ldots O)_O}$$

4. *Beam in Fig. 374* (*d*)

$$R_L = [\mathbf{M(10\ldots 16)_{16}} + \mathbf{W(10\ldots 16)} \times 3] \div 42$$

Reaction = [Moment of wheels 10 to 16 about wheel 16 plus the weight of wheels 10 to 16 multiplied by the distance from wheel 16 to the right-hand support] ÷ [Length of span].

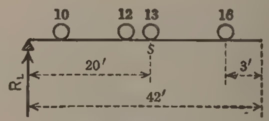


Fig. 374 (*d*).

$$V = R_L - \mathbf{W(10\ldots 12)}$$

$$M_B = R_L \times 20 - \mathbf{M(10\ldots 13)_{13}}$$

1. *Beam in Fig. 375 (a)*

$$R_L = \left[\mathbf{M(A\ldots Z)_U} + \mathbf{W(A\ldots Z)} \times l + w \times l \times \frac{l}{2}\right] \div L$$

Reaction = [Moment of wheels A to Z about the beginning of the uniform load plus the weight of wheels A to Z multiplied by the length of the uniform load plus the weight of the uniform load per lin. ft. multiplied by the length of the uniform load multiplied by half its length] ÷ [Length of span].

$$V = R_L - \mathbf{W(A\ldots N)}$$

$$M_B = R_L \times S - \mathbf{M(A\ldots O)_O}$$

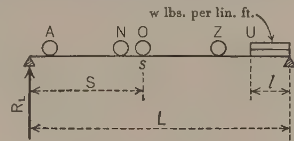


Fig. 375 (*a*).

2. *Beam in Fig. 375 (b)*

$$R_L = [\mathbf{M(6\ldots18)_U} + \mathbf{W(6\ldots18)} \times 6 + 3000 \times 6 \times 3] \div 91$$

R_L = [Moment of wheels 6 to 18 about the beginning of the uniform load plus the weight of wheels 6 to 18 multiplied by the length of the uniform load plus the weight per lin. ft. of uniform load multiplied by its length multiplied by half its length] ÷ [Length of span].

$$V = R_L - \mathbf{W(6\ldots11)}$$

$$M_B = R_L \times 45 - \mathbf{M(6\ldots12)_{12}}$$

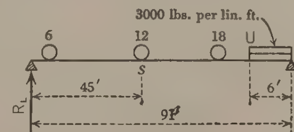


Fig. 375 (*b*).

3. *Beam in Fig. 375 (c)*

$$R_L = [\mathbf{M(3\ldots11)_U} + \mathbf{W(3\ldots11)} \times 8 + 2500 \times 8 \times 4] \div 64$$

$$V = R_L - \mathbf{W(3\ldots6)}$$

$$M_B = R_L \times 31 - \mathbf{M(3\ldots7)_7}$$

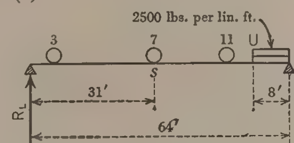


Fig. 375 (*c*).

4. *Beam in Fig. 375 (d)*

$$R_L = [\mathbf{M(3\ldots18)_U} + \mathbf{W(3\ldots18)} \times 4 + 3000 \times 4 \times 2] \div 100$$

$$V = R_L - \mathbf{W(3\ldots10)}$$

$$M_B = R_L \times 51 - \mathbf{M(3\ldots11)_{11}}$$

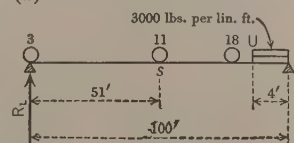


Fig. 375 (*d*).

5. *Practice problems.* Following the general method used in the illustrative problems just given, indicate the reaction at *a*, the shear for the segment *as*, and the bending moment for segment *as*, for each of the beams shown in Fig. 375 (*e*). Underline in each equation any quantity which is either the weight of a group of wheels or the moment of a group of wheels about a center at a wheel, i.e., quantities that can be read from tables.

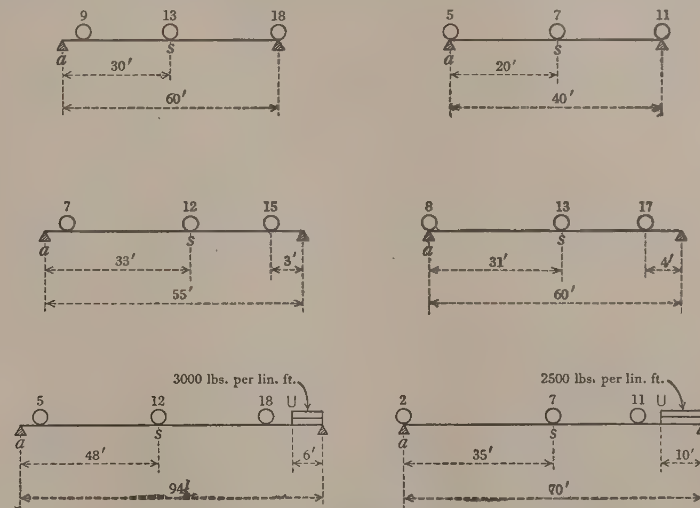


Fig. 375 (*e*).

6. **General Methods of Procedure.** Throughout this chapter the method of procedure in determining reactions, shears, and bending moments will be indicated by means of algebraic expressions similar to those just used in the illustrative problems. These expressions have the appearance of formulas, but they should not be regarded as such, and they certainly should not be used as such. They are all based on a few simple general methods, and in every case the algebraic expression of a method of procedure should be studied for the sole purpose of understanding the general method that it is intended to illustrate.

The methods of procedure in determining reactions, shears, and bending moments are based on the principles explained in Chapters XIII and XIX. These general methods may be stated as follows:

1. *To determine the left-hand reaction when there is no uniform train load on the beam:* (1) *From a moment table read the moment of all the wheels on the beam about the last wheel, i.e.,* $M(A \dots Z)_Z$. (2) *Add to this moment the product of the weight of all wheels on the beam by the distance d from the last wheel to the support, and divide the result by the length of span, i.e.,* $[M(A \dots Z)_Z + W(A \dots Z) \times d] \div L = R_L$.

2. *Note:* If the last wheel Z happens to be at the right-hand support, the reaction is equal to the moment of all the wheels on the beam about the last wheel divided by the length of span, i.e., $M(A \dots Z)_Z \div L = R_L$.

3. *To determine the left-hand reaction when there is a uniform train load on the beam:* (1) *From a moment table read the moment of all the wheels on the beam about the beginning of the uniform load, i.e.,* $M(A \dots Z)_U$. (2) *Multiply the weight of all wheels on the beam by the length of the uniform load, i.e.,* $W(A \dots Z) \times l$. (3) *Multiply the weight of the uniform load by its lever arm* $\frac{l}{2}$, *i.e.,* $w \times l \times \frac{l}{2}$. (4) *Add the results obtained in* (1), (2), *and* (3), *and divide this sum by the length of span, i.e.,*

$$\left[M(A \dots Z)_U + W(A \dots Z) \times l + w \times l \times \frac{l}{2}\right] \div L = R_L.$$

4. *Note:* If the beginning of the uniform load happens to be at the right-hand support the reaction $R_L = M(A \dots Z)_U \div L$.

5. *To determine a right-hand reaction.* The reaction that is required in the majority of problems is the left-hand reaction, calculated, as just explained, from $\Sigma M = 0$, with the center of moments at the right-hand support. A right-hand reaction may be calculated in a similar manner from $\Sigma M = 0$, with the center of moments at the left-hand support, provided it is possible to read from the moment table the moment of all wheels on the beam about the extreme left-hand wheel on the beam; when this cannot be done, it is usually best to calculate the left-hand reaction and subtract the result from the total load on the beam.

6. *To determine the shear for a given segment: From a table of shears read the total weight of all wheels on the segment, and subtract this total weight from the reaction on the segment.*

7. *Note:* The critical wheel at the section is not considered as on the segment and its weight is therefore not included.

8. *Note:* If the shear is end shear, i.e., if the section is indefinitely close to the support, the shear is equal to the reaction.

9. *To determine the bending moment for a given segment:* (1) *Multiply the reaction on the segment by the length S of the segment, i.e.,* $R_L \times S$. (2) *From a moment table read the moment of all wheels on the segment about the wheel O at the section and subtract this quantity from the moment found in* (1), *i.e.,* $R_L \times S - M(A \dots O)_O = M_B$.

10. None of the general methods just outlined can be used until the concentrated live loads have been placed in the correct position, whatever that may be. The remainder of this chapter will be devoted (1) to an explanation of the use of moment and shear tables, (2) to the methods of determining the correct position of the live load, and (3) to the application of the general methods of calculating reactions, shears, and bending moments, once the live load has been placed in the correct position.

11. *Note:* Before studying the explanation of moment and shear tables, it is well to become thoroughly accustomed to the methods of indicating reactions, shears, and bending moments used in 373 : 17. These methods are best acquired by means of practice problems similar to those of 375 : 5.

12. SHEAR AND MOMENT TABLES FOR LOCOMOTIVE LOADINGS. There are various forms of shear and moment tables for locomotive loadings, but in all of the more useful forms, the following quantities are given: (1) The *distance* between any two consecutive wheels; also the total distance between any two wheels, and between any wheel and the beginning of the uniform load. (2) The *weight* on each wheel and the total weight on any group or combination of consecutive wheels. (3) The *moment* of any group of consecutive wheels about a center at the extreme right-hand wheel of the group; also the moment of any group of consecutive wheels about a center at the beginning of the uniform load, provided the right-hand extreme wheel of the group is adjacent to the uniform load. (4) The *moment* of any group of consecutive wheels about a center at the extreme left-hand wheel of the group.

13. *Tables for Cooper's E-60 loading and Steinman's E-50 loading compared.* Two shear and moment tables will be used in this book, namely, that for Cooper's E-60 loading on page 377* and that for Steinman's

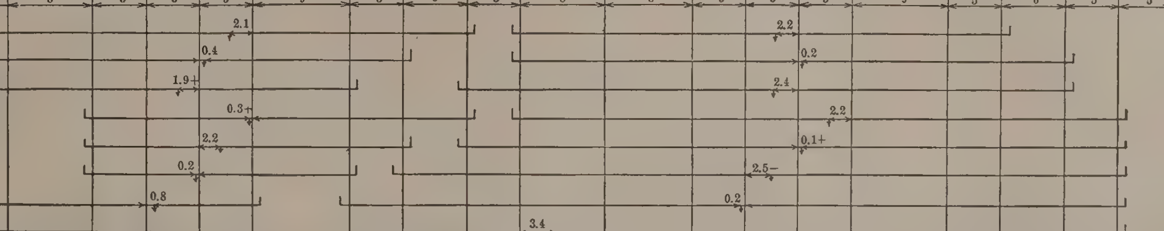
* The table for E-60 on page 377 is the same as that on page 318 of Professor C. T. Bishop's book on "Structural Drafting and Design."

SHEAR AND MOMENT TABLE FOR COOPER'S E60 LOADING.

ALL LOADS, SHEARS, AND MOMENTS ARE FOR EACH RAIL. HORIZONTAL DISTANCES ARE PLOTTED TO THE SCALE 1 IN. = 16 FT. THE VALUES GIVEN IN THE TABLE ARE FOR COOPER'S CONVENTIONAL E60 LOADING. VALUES FOR COOPER'S OTHER LOADINGS MAY BE OBTAINED FROM THOSE IN THE TABLE BY PROPORTION, AS FIVE-SIXTHS FOR E50, OR TWO-THIRDS FOR E40.

	FIRST ENGINE: PILOT	DRIVERS				TENDER				PILOT	SECOND ENGINE: DRIVERS				TENDER				TRAIN UNIFORM LOAD
WHEEL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
WHEEL LOADS ON EACH RAIL IN THOUSANDS OF POUNDS	15.0	30.0	30.0	30.0	30.0	19.5	19.5	19.5	19.5	15.0	30.0	30.0	30.0	30.0	19.5	19.5	19.5	19.5	3.0 PER LIN. FT.
DIST. C. TO C. WHEELS IN FEET	8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5	

DISTANCES TO THE CENTERS OF GRAVITY OF DIFFERENT COMBINATIONS OF LOADS, MEASURED FROM THE NEAREST LOADS.



SHEARS

EACH VALUE ON THE LEFT OF ANY VERTICAL OF THE ZIG-ZAG LINE IS THE SUM, IN THOUSANDS OF POUNDS, OF THE LOAD DIRECTLY OVER THIS VERTICAL, OF THE LOAD OVER THE FIRST LINE AT THE LEFT OF THE VALUE, AND OF ALL LOADS BETWEEN THESE TWO.

EACH VALUE PRINTED VERTICALLY IS THE DISTANCE FROM THE ZIG-ZAG LINE TO THE LINE NEAREST THE VALUE.

EACH VALUE ON THE RIGHT OF ANY VERTICAL OF THE ZIG-ZAG LINE IS THE SUM, IN THOUSANDS OF POUNDS, OF THE LOAD DIRECTLY OVER THIS VERTICAL, OF THE LOAD OVER THE FIRST LINE AT THE RIGHT OF THE VALUE, AND OF ALL LOADS BETWEEN THESE TWO.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
109	101	96	91	86	77	72	66	61	53	45	40	35	30	21	16	10	5
104 426	96 411	91 381	86 351	81 321	72 291	67 271.5	61 252	56 232.5	48 213	40 198	35 168	30 138	25 108	16 78	11 58.5	5 39	
99 406.5	91 391.5	86 361.5	81 331.5	76 301.5	67 271.5	62 252	56 232.5	51 213	43 193.5	35 178.5	30 148.5	25 118.5	20 88.5	11 58.5	6 39	39 5	
93 387	85 372	80 342	75 312	70 282	61 252	56 232.5	50 213	45 193.5	37 174	29 159	24 129	19 99	14 69	5 39	39 6	58.5	
88 367.5	80 352.5	75 322.5	70 292.5	65 262.5	56 232.5	51 213	45 193.5	40 174	32 154.5	24 139.5	19 109.5	14 79.5	9 49.5	39 5	58.5 11	78 16	
79 348	71 333	66 303	61 273	56 243	47 213	42 193.5	36 174	31 154.5	23 135	15 120	10 90	5 60	49.5 9	69 14	88.5 20	108 25	
74 318	66 303	61 273	56 243	51 213	42 183	37 163.5	31 144	26 124.5	18 105	10 90	5 60	60 5	79.5 14	99 19	118.5 25	138 30	
69 288	61 273	56 243	51 213	46 183	37 153	32 133.5	26 114	21 94.5	13 75	5 60	60 5	90 10	109.5 19	129 24	148.5 30	168 35	
64 258	56 243	51 213	46 183	41 153	32 123	27 103.5	21 84	16 64.5	8 45	60 5	90 10	120 15	139.5 24	159 29	178.5 35	198 40	
56 228	48 213	43 183	38 153	33 123	24 93	19 73.5	13 54	8 34.5	45 8	75 13	105 18	135 23	154.5 32	174 37	193.5 43	213 48	

MOMENTS

EACH VERTICAL OF THE ZIG-ZAG LINE CONTAINS THE POINT OF MOMENTS FOR ALL VALUES ON THE LEFT OF THIS LINE. EACH VALUE IS THE MOMENT, IN THOUSANDS OF POUND-FEET, OF ALL LOADS FROM THE POINT OF MOMENTS TO AND INCLUDING THE LOAD OVER THE FIRST LINE AT THE LEFT OF THE VALUE.

EACH VERTICAL OF THE ZIG-ZAG LINE CONTAINS THE POINT OF MOMENTS FOR ALL VALUES ON THE RIGHT OF THIS LINE. EACH VALUE IS THE MOMENT, IN THOUSANDS OF POUND-FEET, OF ALL LOADS FROM THE POINT OF MOMENTS TO AND INCLUDING THE LOAD OVER THE FIRST LINE AT THE RIGHT OF THE VALUE.

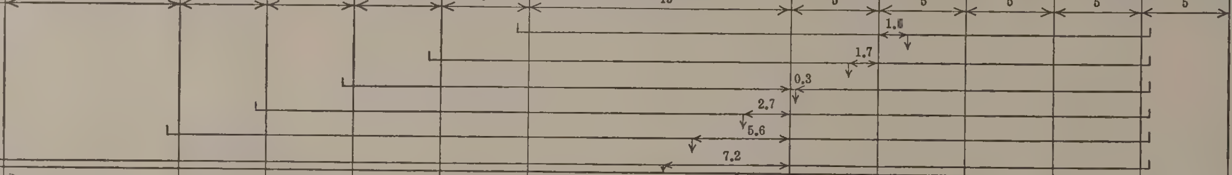
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
24,500	22,900	19,900	17,000	14,300	11,700	10,200	8,790	7,500	6,310	5,510	4,160	2,960	1,910	1,010	605	293	97.5
22,400	20,900	18,000	15,200	12,700	10,200	8,830	7,530	6,340	5,240	4,520	3,320	2,270	1,370	624	312	97.5	
20,400	18,900	16,200	13,600	11,200	8,880	7,570	6,360	5,270	4,280	3,630	2,580	1,680	932	332	117	97.5	
18,100	16,700	14,100	11,700	9,470	7,370	6,180	5,090	4,110	3,230	2,680	1,810	1,090	518	97.5	117	332	
16,200	14,900	12,500	10,300	8,150	6,200	5,110	4,120	3,240	2,460	1,980	1,260	690	270	97.5	312	624	
13,100	11,900	9,780	7,800	5,970	4,290	3,370	2,550	1,850	1,250	900	450	150	176	449	839	1,330	
11,500	10,400	8,410	6,580	4,900	3,370	2,550	1,830	1,230	720	450	150	150	423	794	1,280	1,870	
10,100	9,030	7,200	5,520	3,990	2,610	1,890	1,260	755	345	150	150	450	821	1,290	1,870	2,560	
8,770	7,810	6,130	4,600	3,220	1,990	1,370	842	432	120	150	450	900	1,370	1,930	2,620	3,400	
6,950	6,110	4,670	3,380	2,240	1,250	780	410	156	240	630	1,170	1,860	2,480	3,210	4,040	4,980	

SHEAR AND MOMENT TABLE FOR STEINMAN'S M50 LOADING.

ALL LOADS, SHEARS, AND MOMENTS ARE FOR EACH RAIL. HORIZONTAL DISTANCES ARE PLOTTED TO THE SCALE 1 IN.=10 FT. THE VALUES GIVEN IN THE TABLE ARE FOR STEINMAN'S CONVENTIONAL M50 LOADING. VALUES FOR STEINMAN'S OTHER LOADINGS MAY BE OBTAINED FROM THOSE IN THE TABLE BY PROPORTION, AS SIX-FIFTHS FOR M60 OR FOUR-FIFTHS FOR M40.

	ENGINE											TRAIN
	PILOT	DRIVERS						DRIVERS				UNIFORM LOAD
WHEEL LOADS ON EACH RAIL IN THOUSANDS OF POUNDS	12.5	25	25	25	25	25	31.25	31.25	31.25	31.25	31.25	2.5 PER LIN. FT.
	1	2	3	4	5	6	7	8	9	10	11	
DIST. C. TO C. WHEELS IN FEET	10	5	5	5	5	15	5	5	5	5	5	

DISTANCES TO THE CENTERS OF GRAVITY OF DIFFERENT COMBINATIONS OF LOADS, MEASURED FROM THE NEAREST LOADS.



SHEARS

EACH VALUE ON THE LEFT OF ANY VERTICAL OF THE ZIG-ZAG LINE IS THE SUM, IN THOUSANDS OF POUNDS, OF THE LOAD DIRECTLY OVER THIS VERTICAL, OF THE LOAD OVER THE FIRST LINE AT THE LEFT OF THE VALUE, AND OF ALL LOADS BETWEEN THESE TWO.

EACH VALUE PRINTED VERTICALLY IS THE DISTANCE FROM THE ZIG-ZAG LINE TO THE LINE NEAREST THE VALUE.

EACH VALUE ON THE RIGHT OF ANY VERTICAL OF THE ZIG-ZAG LINE IS THE SUM, IN THOUSANDS OF POUNDS, OF THE LOAD DIRECTLY OVER THIS VERTICAL, OF THE LOAD OVER THE FIRST LINE AT THE RIGHT OF THE VALUE, AND OF ALL LOADS BETWEEN THESE TWO.

70	60	55	50	45	40	25	20	15	10	5
65 293.8	55 231.3	50 256.3	45 231.3	40 206.3	35 181.3	20 156.3	15 125	10 93.8	5 62.5	
60 262.5	50 250	45 225	40 200	35 175	30 150	15 125	10 93.8	5 62.5	62.5 5	
55 231.3	45 218.8	40 193.8	35 168.8	30 143.8	25 118.8	10 93.8	5 62.5	62.5 5	93.8 10	
50 200	40 187.5	35 162.5	30 137.5	25 112.5	20 87.5	5 62.5	62.5 5	93.8 10	125 15	
45 168.3	35 156.3	30 131.3	25 106.3	20 81.3	15 56.3	62.5 5	93.8 10	125 15	156.3 20	
30 137.5	20 125	15 100	10 75	5 50	56.3 15	87.5 20	118.8 25	150 30	181.3 35	
25 112.5	15 100	10 75	5 50	50 5	81.3 20	112.5 25	143.8 30	175 35	206.3 40	
20 87.5	10 75	5 50	50 5	75 10	106.3 25	137.5 30	168.8 35	200 40	231.3 45	
15 62.5	5 50	50 5	75 10	100 15	131.3 30	162.5 35	193.8 40	225 45	256.3 50	
10 37.5	50 5	75 10	100 15	125. 20	156.3 35	187.5 40	218.8 45	250 50	281.3 55	
37.5 10	62.5 15	87.5 20	112.5 25	137.5 30	168.8 45	200 50	231.3 55	262.5 60	293.8 65	

MOMENTS

EACH VERTICAL OF THE ZIG-ZAG LINE CONTAINS THE POINT OF MOMENTS FOR ALL VALUES ON THE LEFT OF THIS LINE. EACH VALUE IS THE MOMENT, IN THOUSANDS OF POUND-FEET, OF ALL LOADS FROM THE POINT OF MOMENTS TO AND INCLUDING THE LOAD OVER THE FIRST LINE AT THE LEFT OF THE VALUE.

EACH VERTICAL OF THE ZIG-ZAG LINE CONTAINS THE POINT OF MOMENTS FOR ALL VALUES ON THE RIGHT OF THIS LINE. EACH VALUE IS THE MOMENT, IN THOUSANDS OF POUND-FEET, OF ALL LOADS FROM THE POINT OF MOMENTS TO AND INCLUDING THE LOAD OVER THE FIRST LINE AT THE RIGHT OF THE VALUE.

9470	8590	7090	5720	4470	3340	2340	1560	938	469	156
8000	7190	5810	4560	3440	2440	1560	938	469	156	
6690	5940	4690	3560	2560	1690	938	469	156	156	
5530	4840	3720	2720	1840	1090	469	156	156	469	
4530	3910	2910	2030	1280	656	156	156	469	938	
3690	3130	2250	1500	875	375	156	469	938	1560	
1630	1250	750	375	125	469	1090	1880	2810	3910	
1060	750	375	125	125	750	1530	2470	3560	4810	
625	375	125	125	375	1160	2090	3190	4440	5840	
313	125	125	375	750	1690	2780	4030	5440	7000	
125	125	375	750	1250	2340	3590	5000	6560	8230	
250	625	1130	1750	2500	3910	5470	7190	9060	11100	

E-50 loading on page 378. The tables are similar in form but they differ in details as follows: In E-60 there are two engines — in M-50 there is only one. In E-60 there are eighteen wheels — in M-50 there are eleven. In E-60 there are four driving wheels in each group — in M-50 there are five. In E-60 the two groups of drivers belong to two engines and the loads on the two groups are the same — in M-50 the two groups of drivers belong to one engine and the loads on the second group are greater than those on the first. In E-60 the distance between the first wheel of the first engine and the last wheel of the second engine is 104 ft. — in M-50 the distance between the first wheel and the last is 65 ft. In E-60 the total weight of one engine is 106.5 tons, or 213 tons for the two engines — in M-50 the weight of the engine is 146.9 tons. In E-60 the uniform train load is 3000 lbs. per linear foot — in M-50 it is 2500 lbs. per linear foot.

In spite of the differences in details in the two tables, there is no essential difference in the use of these tables for the purpose of determining shears and moments.

1. **Explanation of the Use of Shear and Moment Tables.** In the tables on pages 377 and 378, distances are given in feet, loads in thousands of pounds, and bending moments in thousands of pound-feet.

2. *To find the weight on any wheel.* Look directly above the wheel. The weight given is not an axle load but the load on one rail. For example, W(8) is 19,500 lbs. for E-60, and 31,250 lbs. for M-50.

3. *To find the distance between two adjacent wheels.* Look directly below the space between the two wheels. For example, the distance between wheels 5 and 6 is 9 ft. for E-60, and 5 ft. for M-50.

4. *To find the center of gravity of any group of wheels.* Look for the bracket which embraces the first and last wheels of the group. The distance is given in feet from the center of gravity of the group to the wheel nearest to that center of gravity. For example, the center of gravity of the group from wheels 8 to 17 of E-60 is 2.4 ft. to the left of wheel 13; the center of gravity of wheels 4 to 11 of M-50 is 0.3 ft. to the right of wheel 7.

5. *Note:* There are two heavy zigzag lines, upper and lower, shown in the table. The short verticals in these zigzag lines are constantly used as starting points in looking up quantities in the table. For the sake of brevity, these short verticals will be referred to as **risers** — an **upper riser** is a vertical line in the upper zigzag line and a **lower riser** is a vertical line in the lower zigzag line.

6. *To find the distance of one wheel from any other wheel.* Find the upper riser that is directly under one of the two wheels. Move the eye along the corresponding horizontal space until the vertical line through the other wheel is found. The required distance is printed vertically along this vertical line.

7. *Illustration.* What is the distance in the E-system from wheel 5 to wheel 14? There is no upper riser under wheel 5, but one is directly under wheel 14. Follow the corresponding horizontal space to the left to the vertical line through wheel 5. Answer: 56 ft.

8. *Illustration.* What is the distance in the M-system from wheel 4 to wheel 10? There is an upper riser under each wheel, hence the distance may be found in two ways:

(1) Find the upper riser under wheel 10 and follow horizontally to the left to the vertical through wheel 4. Answer: 40 ft.

(2) Find the upper riser under wheel 4, and follow horizontally to the right to the vertical through wheel 10. Answer: 40 ft.

9. *Illustration.* What is the distance in each system from wheel 4 to the beginning of the uniform load? Find the upper riser under the beginning of the uniform load, and follow horizontally to the vertical through wheel 4. Answer: 91 ft. for the E-system; 50 ft. for the M-system.

10. *To find which wheel loads are on a given span when the position of some wheel on the span is fixed.* The method will be explained by means of the following illustrative examples:

11. *Illustration.* Wheel 2 of the M-system is at the left-hand end of a 48-ft. beam. What is the last wheel on the beam? Follow down the vertical line through wheel 2 until the quantity 45 (printed vertically) is reached. This is the largest quantity along the vertical line which is *less* than 48 ft., the length of the beam. Follow horizontally from this quantity to the corresponding riser, and note that this riser is directly under wheel 9. Wheel 9, therefore, is the last wheel on the beam; it is 45 ft. from the left-hand end (or wheel 2), and 3 ft. from the right-hand end.

12. *Illustration.* Wheel 2 of the E-system is at the left-hand end of a 36-ft. beam. What is the last wheel on the beam? Follow down the vertical line through wheel 2. The last distance given is 48 ft. which is considerably longer than the beam. Shift to wheel 11, the wheel of the second locomotive which corresponds to wheel 2. Follow down the vertical line through wheel 11 to 35, the largest quantity less than 36 ft., the length of the beam, then follow horizontally to the right to the corresponding riser, and note that this riser is under wheel 17. Wheel 17 is, therefore, the last wheel on the beam, and it is 1 ft. from the right-hand end. The same result may also be obtained by starting from the upper riser under wheel 11, and following horizontally to the right to 35, the largest number less than 36. In certain cases, the second engine is used instead of the first for reasons to be explained later.

1. *Illustration.* Wheel 12 of the E-system is at the center of a 62-ft. beam. What is the first and what is the last wheel on the beam? Any wheel 31 ft. or less to the left or to the right of wheel 12 is on the beam. Starting from the upper riser under wheel 12, follow horizontally to the left to 26 along the vertical line through wheel 8. The next number to the left is 32 under wheel 7, but as this is greater than 31, wheel 7 is off the beam. Wheel 8 is, therefore, the first wheel on the beam, and it is 5 ft. from the left-hand end. Starting again from the upper riser under wheel 12, follow horizontally to the right to 30 along the vertical line through wheel 17. Wheel 17 is, therefore, the last wheel on the beam, and is 1 ft. from the right-hand end. (This is also shown by the "30" along the vertical line through wheel 12.)

2. *Illustration.* Wheel 9 of the M-system is at the center of a 62-ft. girder. What loading is on the girder? Starting from the upper riser under wheel 9 and following horizontally to the left to 30, the largest number less than 31, wheel 5 is found to be the first wheel on, 1 ft. from the left-hand end. Starting again from the upper riser under wheel 9 and following horizontally to the right, wheel 11 is found to be 10 ft. from wheel 9; the end of the uniform load is, therefore, 5 ft. further, or 15 ft. from wheel 9. (This is also shown by the "15" along the vertical line through wheel 9.) The length of that portion of the girder covered with uniform load is 30 − 15 = 15 ft.

3. *Note:* A scale of $\frac{1}{16}$ inch = 1 ft. may be constructed on the edge of a piece of cardboard, and by sliding this scale along the diagram on the shear and moment table, the number and position of wheel loads on any given span may be determined from inspection. It is doubtful, however, if the use of such a scale is of any decided advantage, once one has become accustomed to reading distances from the table by the method just explained.

4. *To find the sum of the weights on any group of consecutive wheels.* Find the upper riser under one of the extreme wheels of the group, and follow the corresponding horizontal space toward the other extreme wheel; just before reaching the vertical line through this latter wheel, the required sum or total weight will be found printed in the horizontal space. This method of procedure is similar to that just explained for reading distances, but the required quantity will be found printed in a horizontal instead of in a vertical direction.

5. *Illustration.* What is the sum of the weights for both E-60 and M-50 of all wheels from 3 to 10 inclusive, i.e., W(3...10)? Find the upper riser directly below wheel 10. (There is no riser in the table for E-60 below wheel 7.) Follow horizontally to the left toward wheel 3; the result, 183,000 lbs. for E-60 or 225,000 lbs. for M-50, is found in the horizontal space just before the vertical through wheel 3 is reached. For M-50, the result may also be found by starting at the upper riser below wheel 3 and following horizontally to the right until the result (225,000 lbs.) is read just before reaching the vertical line through wheel 10.

6. *Illustration.* What is the sum of the weights on all wheels of both engines of E-60, i.e., W(1...18)? Find the upper riser under the last wheel, i.e., wheel 18. Follow horizontally to the left toward wheel 1; the result, 426,000 lbs., is found in the horizontal space just before the vertical through wheel 1 is reached. The weight of all wheels of M-50 is found, in a similar manner, to be 293,800 lbs.

7. *Illustration.* What is the sum of the weights of all wheels from 11 to 17 inclusive of E-60, i.e., W(11...17)? (Two methods.)

(1) Find the upper riser under wheel 17. Follow horizontally to the left toward wheel 11; the result, 178,500 lbs., is found just before the vertical through wheel 11 is reached.

(2) Find the upper riser under wheel 11. Follow horizontally to the right toward wheel 17; the result, 178,500 lbs., is found just before the vertical through wheel 17 is reached.

8. *Illustration.* The first wheel on a girder is wheel 4 of E-60 and the uniform load extends on the girder a distance of 16 feet. What is the total load? Find the riser under wheel 18. Follow horizontally to the left toward wheel 4. The weight on wheels 4 to 18 is found just before the vertical through wheel 4 is reached, and is 351,000 lbs. The weight of the uniform load is $16 \times 3000 = 48{,}000$ lbs. The total weight = 351,000 + 48,000 = 399,000 lbs. For a similar position of M-50 loading: $W(4\ldots11) + 2.5 \times 16 = 231{,}300 + 40{,}000 = 271{,}300$ lbs.

9. *To find the sum of the moments of any group of consecutive wheels about any center of moments that is in a vertical line through one of the extreme wheels of the group.* (The moment is usually referred to as the moment of the given wheels about one of the extreme wheels of the group, as for example, moment of wheels 11 to 17 about wheel 17, or $M(11\ldots17)_{17}$.) *The center of moments may always be considered as in a riser of the lower zigzag line.* Find the *lower* riser that lies in the vertical line through the center of moments, i.e., the riser under one of the extreme wheels. Follow the corresponding horizontal space toward the other extreme wheel; just before reaching the vertical line through this latter wheel, the required moment will be found printed in the horizontal space. This method of procedure is exactly like that for finding the total weight of a group of consecutive wheels, except that the lower part of the table is used instead of the upper.

10. *Illustration.* What is the moment of wheels 9 to 16 inclusive of E-60 about a center in a vertical through wheel 16, i.e., $M(9\ldots16)_{16}$? Find the center of moments in the *lower* riser under wheel 16. Follow horizontally to the left toward wheel 9; the required moment, 4,110,000 lb.-ft., is found in the horizontal space just before the vertical through wheel 9 is reached.

1. *Note:* Note that the moment of wheels 9 to 16 about wheel 16 is the same as the moment of wheels 9 to 15 about wheel 16. This is because the lever arm of wheel 16 is zero for any center of moments in a vertical line through wheel 16.

2. *Illustration.* What is the moment of wheels 3 to 9 inclusive of M-50 about wheel 9, i.e., $M(3\ldots9)_9$? Find the center of moments in the *lower* riser under wheel 9 and follow horizontally to the left toward wheel 3; the required moment, 3,720,000 lb.-ft., is found just before the vertical through wheel 3 is reached.

3. What is the moment of the same group of wheels about a center at the other extreme wheel, i.e., $M(9\ldots3)_3$? Find the center of moments in the *lower* riser under wheel 3 and follow horizontally to the right toward wheel 9; the required moment, 4,030,000 lb.-ft., is found just before the vertical through wheel 9 is reached. Notice that the result is not the same as that obtained when the point of moments was taken at wheel 9. (Why?)

4. *Illustration.* What is the moment of all of the wheels of both engines of E-60 about the last wheel, i.e., $M(1\ldots18)_{18}$, and about the beginning of the uniform load, i.e., $M(1\ldots18)_U$? Find the *lower* riser under wheel 18 and follow horizontally to the left toward wheel 1; the result, 22,400,000 lb.-ft., is found in the horizontal space just before the vertical through wheel 1 is reached. Similarly, to find $M(1\ldots18)_U$, start from the *lower* riser under the beginning of the uniform load; the moment is found to be 24,500,000 lb.-ft. For M-50: $M(1\ldots11)_{11}$ = 8,000,000 lb.-ft., and $M(1\ldots11)_U$ = 9,470,000 lb.-ft.

5. *Illustration.* What is the moment of wheels 6 to 18 inclusive of E-60 about a center at the beginning of the uniform load, i.e., $M(6\ldots18)_U$? Find the center of moments in the *lower* riser under the beginning of the uniform load and follow horizontally to the left toward wheel 6; the result, 11,700,000 lb.-ft., is found just before reaching the vertical through wheel 6. Similarly for M-50, $M(6\ldots11)_U$ = 3,340,000 lb.-ft.

6. *Use of the second engine of E-60 in place of the first.* Weights and moments for the first engine, which if given in the table would merely duplicate corresponding values for the second engine, have been omitted. Hence, it is necessary to use the second engine in place of the first when the span is too short to hold more than one engine.

7. **Maximum Reaction.** The maximum reaction for a simple beam supported at each end is the greatest reaction that can occur as the live load moves across the beam. This reaction may occur at the left-hand support or at the right-hand support, depending upon the length of the beam. When the locomotive is in the position to cause the maximum reaction, one of its wheels, called the **critical wheel,** will be at the support at which the reaction occurs. The critical wheel for a beam of a given length may be different from the critical wheel for a beam of another length. The first step, therefore, in any given case, is to determine the critical wheel, and the next step is to determine the reaction. The critical wheel for the maximum reaction for a beam is the same as the critical wheel for the maximum end shear for that beam, and the method of procedure used in determining the maximum reaction is identical with that used in determining the maximum end shear. This method will be explained in the next article.

8. **Maximum Shear (End Shear).** It was shown in Chapter XIX that, for simple beams, the maximum live-load shear will occur for a section so close to one of the supports that the only force acting on the indefinitely short segment at the support is the reaction, i.e., the maximum shear is the *end shear.* (286 : 3.) The maximum shear due to any system of locomotive loading is the end shear when the loading is placed in such a position that it will cause the greatest possible reaction. For any given length of beam, this maximum reaction will occur when some wheel of the engine is indefinitely close to the support (practically *at* the support), and this wheel is the **critical wheel** for that length of beam. The critical wheel must be such that when placed at the support it will bring as much load as possible near that support. This means that one of the groups of driving wheels should be placed at one end of the beam, and to bring the locomotive into this position, the critical wheel must be the first or the last wheel of the group.

9. To correspond to the shear and moment tables, the locomotive loading on a beam will always be placed with the engines headed toward the *left.* When the critical wheel is the *first* wheel of a group of drivers, it is placed at the *left-hand* support and the end shear is the left-hand reaction; when it is the *last* wheel of a group, it is placed at the *right-hand* support, and the end shear is the right-hand reaction.

10. In determining the maximum end shear for a given beam, two steps are necessary, namely, (1) to determine the critical wheel for the beam, and (2) to calculate the reaction caused by the locomotive loading when the critical wheel is at the corresponding support.

11. *To determine the critical wheel for maximum end shear.* Critical wheels for different lengths of span will be given first for the E-system of loading, and then for the M-system. It is assumed that locomotives are headed toward the *left.* In each system there is only one range of spans in which the critical wheel is placed at the right-hand support and the end

shear is the right-hand reaction. Even in such cases, the locomotives are still headed toward the left. For spans under 12.5 feet, the critical wheel for the E-system is one of the two wheels of the alternative loading. (371 : 5.)

SPAN IN FEET	CRITICAL WHEEL	END AT WHICH WHEEL IS PLACED
	E-System of Loading	
0 to 12.5	Alternative	Left
12.5 to 23.0	11	Left
23.0 to 27.3	14	Right
27.3 to 62.6	11	Left
62.6 and longer	2	Left
	M-System of Loading	
0 to 50.9	7	Left
50.9 to 94.2	11	Right
94.2 and longer	2	Left

1. *Note:* The critical wheel for the M-system may also be ascertained from the table on page 389. In using this table to determine the critical wheel for a maximum reaction, the shorter segment l_1 is zero and the longer segment l_2 is equal to the length of the beam. For example, the critical wheel for a beam 40 ft. long, as ascertained from the table on page 389, is wheel 7, and for a beam 80 ft. long, it is wheel 11. In the latter case the number " 11 " is overlined in the table and wheel 11 should therefore be placed at the right-hand end of the beam, with the locomotive still headed toward the left. Both of these results are in accord with those which would have been obtained by the use of the limits in the table given above.

2. *General method of calculating the end shear.* Once the critical wheel O has been determined and placed at the support, the general method of calculating the corresponding reaction (equal to the end shear) is that already given in 376 : 1 to 5. It may be outlined as follows:

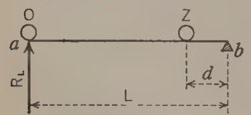


Fig. 382 (*a*).

3. When the end shear is at the *left-hand* support.

First step: Place the critical wheel O at the support *a*. (Fig. 382 (*a*).) From the table ascertain which wheel is the last wheel on (wheel Z), and calculate the distance *d* of this wheel from the support *b*.

Second step: Determine the end shear (equal to R_L). Using the notation given in 372 : 7:

$$V = R_L = [M(O\ldots Z)_Z + W(O\ldots Z) \times d] \div L. \quad (376 : 1.)$$

4. When there is uniform load on the beam as indicated in Fig. 382 (*b*), the first step is the same as above, except that it is necessary to determine l the distance covered by the uniform load instead of the distance d. The second step is then modified as follows:

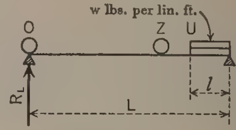


Fig. 382 (*b*).

Second step:

$$V = R_L = \left[M(O\ldots Z)_U + W(O\ldots Z) \times l + w \times l \times \frac{l}{2}\right] \div L. \quad (376 : 3.)$$

5. When the end shear is at the *right-hand* support and the critical wheel has been placed at that support, there are two methods of procedure, namely:

(*a*) First method: Calculate the left-hand reaction R_L by the usual method, and subtract it from the total weight W on the beam in order to obtain the required *right-hand* reaction R_R, i.e., $R_R = W - R_L$.

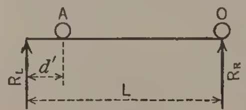


Fig. 382 (*c*).

(*b*) Second method: Calculate the right-hand reaction R_R directly from a moment equation with the center of moments at the left-hand support. (Fig. 382 (*c*).) In this case the moments are read from the moment table *below* the *lower* zigzag line.

The two methods may be indicated as follows:

First method: $V = R_R = W - R_L = W(A\ldots O) - [M(A\ldots O)_O] \div L$

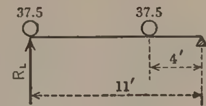
Second method: $V = R_R = [M(O\ldots A)_A + W(A\ldots O) \times d'] \div L.$

6. *Illustrative problems in maximum end shears.* Methods of determining maximum end shears for spans of several different lengths are indicated on the next page. The shear in each case is determined first for E-60 and then for M-50. The notation used is that given in 372 : 7. Results are given in kips. Tables on pages 377 and 378 were used.

Span of 11 ft.

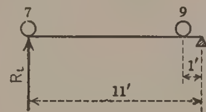
For the E-system, the critical wheel is the first of the alternative system of loading. (Page 382.)

$$R_L = V = 37.5 + (37.5 \times 4) \div 11$$
$$51.1 = V$$



For the M-system the critical wheel is wheel 7 at the left-hand support. (Page 382.)

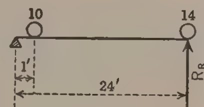
$$R_L = V = [M(7\ldots9)_9 + W(7\ldots9) \times 1] \div 11$$
$$51.2 = V = [\quad 469 \quad + \quad 93.8 \quad] \div 11$$



Span of 24 ft.

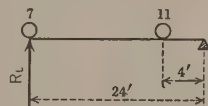
For the E-system the critical wheel is wheel 14 at the right-hand support. (Page 382.)

$$R_R = V = [M(14\ldots10)_{10} + W(10\ldots14) \times 1] \div 24$$
$$83.1 = V = [\quad 1860 \quad + \quad 135 \quad] \div 24$$



For the M-system the critical wheel is wheel 7 at the left-hand support. (Page 382.)

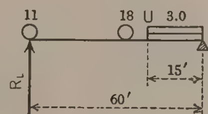
$$R_L = V = [M(7\ldots11)_{11} + W(7\ldots11) \times 4] \div 24$$
$$91.1 = V = [\quad 1560 \quad + \quad 156.3 \times 4] \div 24$$



Span of 60 ft.

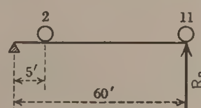
For the E-system the critical wheel is wheel 11 at the left-hand support.

$$R_L = V = [M(11\ldots18)_U + W(11\ldots18) \times 15 + 3.0 \times 15 \times 7.5] \div 60$$
$$147.0 = V = [\quad 5510 \quad + \quad 198 \times 15 + \quad 338 \quad] \div 60$$



For the M-system the critical wheel is wheel 11 at the right-hand support.

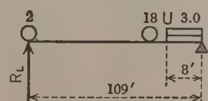
$$R_R = V = [M(11\ldots2)_2 + W(2\ldots11) \times 5] \div 60$$
$$161.4 = V = [\quad 8280 \quad + \quad 281.3 \times 5] \div 60$$



Span of 109 ft.

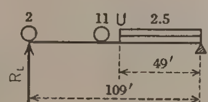
For the E-system the critical wheel is wheel 2 at the left-hand support.

$$R_L = V = [M(2\ldots18)_U + W(2\ldots18) \times 8 + 3.0 \times 8 \times 4] \div 109$$
$$241.1 = V = [\quad 22{,}900 \quad + \quad 411 \times 8 \quad + \quad 96 \quad] \div 109$$



For the M-system the critical wheel is wheel 2 at the left-hand support.

$$R_L = V = [M(2\ldots11)_U + W(2\ldots11) \times 49 + 2.5 \times 49 \times 24.5] \div 109$$
$$232.5 = V = [\quad 8590 \quad + \quad 281.3 \times 49 + \quad 3000 \quad] \div 109$$



1. **Greatest Shear for a Given Section.** In Chapter XIX it was shown that the greatest shear for a given section will occur when there is little or no load on the shorter segment and as much load as possible on the longer segment, with the center of gravity of the load on the longer segment as near as possible to the section. (286 : 2.) This principle holds true for locomotive loadings. The greatest shear for any section will occur, in general, when there is little or no load on the shorter segment and as many of the largest wheel loads (drivers) as possible are on the longer segment, placed as near to the section as possible. To fulfill this condition, the locomotive loading is placed with one of the extreme wheels of a group of drivers on the longer segment indefinitely close to the section. This frequently brings a whole group of driving wheels on the longer segment near the section, and, at the same time, brings no wheel, or at the most one wheel, on the shorter segment. The driving wheel indefinitely close to the section is called the **critical wheel,** and it is usually referred to as *at* the section. It should not be included among the external forces that act on the shorter segment, but should be considered as acting on the longer segment. The locomotive loading in this position causes not only the greatest positive shear for the shorter segment, but also the greatest negative shear for the longer segment, and these two shears are equal.

2. The locomotive loading on a beam is placed with the engine headed toward the *left.* When the critical wheel is the *first* wheel of a group of drivers, the shorter segment of the beam is assumed to be a *left-hand* segment; when it is the *last* wheel of a group, the shorter segment is assumed to be a *right-hand* segment.

3. *Note:* It frequently happens that when a locomotive loading is placed in a position to cause the greatest shear for a given section, the engine will be headed toward the *right.* Instead of using the given section in such a case, it is convenient to use the corresponding section at the other end of the beam in order that the engine may be headed toward the left, as it is in the shear and moment tables. The magnitude of the shear thus obtained will be the same as if the given section had been used. This may be illustrated as follows:

Fig. 383 (*a*).

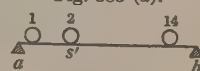


Fig. 383 (*b*).

4. Let it be required to determine the greatest shear for a section at *s* in Fig. 383 (*a*), and assume that this shear would occur when wheel 2, the first of a group of drivers, is the critical wheel. To bring the other drivers in the group on the *longer* segment, the engines must be headed toward the *right* as shown in Fig. 383 (*a*). It is convenient to assume that

the section is at s' instead of at s (Fig. 383 (b)), making as' equal to sb. Wheel 2 may now be placed at s' (Fig. 383 (b)) with the engines headed toward the *left* as shown. The shear for a section at s' for the position of the load shown in Fig. 383 (b) will be equal to the shear for a section at s for the position of the load shown in Fig. 383 (a).

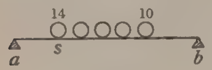


Fig. 384 (a).

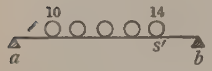


Fig. 384 (b).

1. Let it be required to determine the greatest shear for a section at s in Fig. 384 (a), and assume that wheel 14, the last of a group of drivers, is the critical wheel. To bring the other drivers in the group on the longer segment, the engine must be headed toward the *right* as shown in Fig. 384 (a). It is more convenient to assume that the section is at s' (Fig. 384 (b)), making $s'b$ equal to as. The engine may then be placed as shown in Fig. 384 (b), headed toward the left, with the drivers on the left-hand (longer) segment.

2. *To determine the critical wheel for the greatest shear for a given section.* Two conditions determine which driving wheel shall be placed at a given section in order to bring the locomotive loading into position to cause the greatest shear for that section. These two conditions are (1) the length of span and (2) the lengths of the longer and shorter segments. Critical wheels for different lengths of span and varying lengths of the two segments will be given, first for the E-system of loading and then for the M-system. It is assumed that engines are headed toward the left.

Critical wheels, E-system of loading

3. In the E-system of loading, the critical wheel for the greatest shear for a given section will be one of three wheels, namely, (1) one of the wheels of the alternative system of loading, (2) wheel 2 of the first locomotive (or wheel 11, the corresponding wheel of the second locomotive), and (3) wheel 5 (or wheel 14). When wheel 5 (or 14) is the critical wheel, the shorter segment is assumed to be the right-hand segment; for other wheels, the shorter segment is the left-hand segment. (383 : 2.)

4. *Note:* When the critical wheel may be either a wheel of the first locomotive or the corresponding wheel of the second locomotive, as often happens in short spans, the wheel of the second locomotive should be chosen for convenience in the use of the shear and moment table. For this reason, only one wheel will be named in any of the following rules for the selection of the critical wheel.

5. When the longer segment is 12.5 ft. or less, one of the two wheels of the alternative system of loading. (371 : 5.) (The loading per rail is 37.5 kips on each of two wheels spaced 7 ft. apart.) Shorter segment — left-hand.

6. When the longer segment is between 23.0 and 27.3 ft. and the shorter segment is 9 ft. or less, wheel 14. Shorter segment — right-hand.

7. When the length of a beam is less than 34.5 ft. and the lengths of the two segments are such that neither the conditions in (384 : 5) or in (384 : 6) are fulfilled, it may be necessary to try first one then the other of *two* critical wheels, namely, (1) one of the two concentrated loads in the alternative system of loading (371 : 5), or (2) wheel 2 (or wheel 11). Shorter segment — left-hand.

8. *Note:* In order to use the tables it is convenient to place wheel 11 at the section as the critical wheel and then to ignore wheel 9 of the first engine should that wheel be on the shorter segment.

9. When the length of a beam is greater than 34.5 ft. there are two cases, namely:

(a) When the shorter segment is less than 16 ft. and the longer segment less than 62.6 ft., wheel 11. Shorter segment — left-hand.

(b) When the shorter segment exceeds 16 ft. or the longer segment exceeds 62.6 ft., wheel 2. Shorter segment — left-hand.

10. *Note:* When the shorter segment exceeds 16 ft. and the longer segment is less than 48 ft., wheel 2 is the critical wheel, but in order to use the tables, it is convenient to place wheel 11 at the section and then to ignore any load in front of wheel 11 or behind wheel 18.

Critical wheels, M-system of loading

11. In the M-system of loading, the critical wheel for the greatest shear for a given section will be one of three wheels, namely, wheel 2, wheel 7, or wheel 11. When wheel 11 (the last of a group of driving wheels) is the critical wheel, the shorter segment is assumed to be the right-hand segment; for either wheel 2 or wheel 7, the shorter segment is assumed to be the left-hand segment.

12. When the longer segment does not exceed 50.9 ft., wheel 7, with the shorter segment to the left, unless the shorter segment exceeds 15 ft., when wheel 11 may be the critical wheel — shorter segment to the right.

13. When the longer segment is between 50.9 and 94.2 ft., wheel 11; shorter segment — right-hand.

1. When the longer segment exceeds 94.2 ft., wheel 2; shorter segment — left-hand.

2. *Note:* These limits are practically the same as those given on page 382 for the critical wheels for maximum end shear, but for end shear the longer segment is equal to the length of the beam.

3. *Note:* Critical wheels for the M-system may also be determined from the table on page 389. In using this table to determine the critical wheel for the greatest positive shear for a given section, the shorter segment l_1 of the base of the influence triangle $ss'b$ (Fig. 385(a)) is zero and the longer segment l_2 is sb, i.e., the length of the longer segment of the beam. For example, let sections be taken 25 ft. and 15 ft. from the left-hand end of a beam 70 ft. long. For the first section the longer segment is 45 ft. and the critical wheel, as ascertained from the table on page 389, is wheel 7; for the second section the longer segment is 55 ft. and the critical wheel is wheel 11, but since this number is overlined in the table, the section should be taken 15 ft. from the right-hand end, with the locomotive still headed toward the left. Both of these results are in accord with those which would have been obtained by the use of the limits given above.

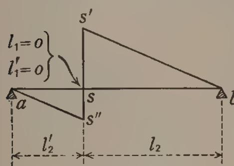


Fig. 385 (a).

4. *Criterion for the position of the loading for the greatest shear for a given section.* The critical wheel for any system of locomotive loading for a given section of a given beam may be determined as follows: Let W_1, W_2, W_3, and W_4 represent the weights respectively of wheels 1, 2, 3, and 4, and let W represent the total load on the beam for any given position of the loading. Let a, b, and c represent, respectively, the distances between successive wheels as shown in Fig. 385 (b). The critical wheel for the maximum positive shear for a section at any point s of the left-hand half of the beam may be determined by means of the following criteria:

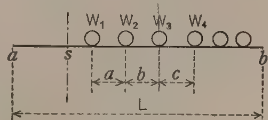


Fig. 385 (b).

W_1 is the critical wheel if when it is placed at s

$$W > 0 \text{ and } < \frac{L}{a} W_1$$

W_2 is the critical wheel if when it is placed at s

$$W > \frac{L}{a} W_1 \text{ and } < \frac{L}{b} W_2$$

W_3 is the critical wheel if when it is placed at s

$$W > \frac{L}{b} W_2 \text{ and } < \frac{L}{c} W_3$$

Successive wheels may be tested in this manner until one is found which, when placed at s, brings the loading into a position such that the criterion is satisfied. In shifting the loading to bring the different wheels to s in succession, the total load W on the span will usually vary, and hence it may be possible that the criterion will be fulfilled for two successive wheels. In such a case, the shear must be calculated for each of the two corresponding positions of the loading in order to determine the greatest shear. In most locomotive loadings the critical wheel is usually the first or the last wheel of a group of driving wheels, as is the case in the E- and M-systems of loading.

5. *Note:* In Fig. 385 (b) the left-hand segment as is the shorter segment. In some cases, the greatest shear is determined when the shorter segment is to the right of the section. (383 : 2 and 384 : 1.) A criterion, exactly the same in principle will hold good in such cases.

6. *Exercise:* Prove that the general criterion indicated above is correct.

7. *General method of determining the greatest shear for a given section.* In determining the greatest shear for a given section of a beam three steps may be necessary, namely, (1) to determine the critical wheel for the given length of beam and given lengths of segments; (2) to calculate the reaction that acts on the shorter segment; (3) to subtract from the reaction any load that may be on the shorter segment. (In many cases there is no load on shorter segment; the reaction is then the shear.) The general method of procedure may be indicated as follows:

8. First step: From the rules given in 384 : 3 to 385 : 1 determine the critical wheel. Assume the shorter segment to be a left-hand or a right-hand segment according to whether the critical wheel is the first or the last wheel of a group of drivers. (383 : 2.)

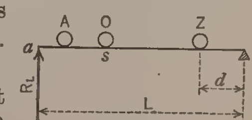


Fig. 385 (c).

9. Second step: (When the shorter segment is a left-hand segment.) Place the critical wheel O at the section s. (Fig. 385 (c).) Ascertain from the table which is the first wheel A and which is the last wheel Z on the beam (379 : 10); determine also the distance d of the last wheel from the

right-hand support. Calculate the reaction R_L on the shorter segment by the method given in 376 : 1.

$$R_L = [\mathrm{M(A\ldots Z)_Z} + \mathrm{W(A\ldots Z)} \times d] \div L$$

1. Third step: Subtract from the reaction R_L any load that may be on the shorter segment. The result is the required shear.

$$\mathrm{V} = R_L - \text{Load on wheel A.}$$

If, in addition to wheel A, there are other wheels on segment *as*, these loads should also be subtracted from R_L.

2. When the shorter segment is a right-hand segment, the right-hand reaction is determined in the second step instead of the left-hand, and this right-hand reaction is used in the third step. In calculating the right-hand reaction, either of the methods outlined in 376 : 5 may be used.

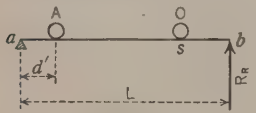


Fig. 386 (*a*).

Shear for the shorter segment *sb* in Fig. 386 (*a*) may be indicated as follows:

$$\mathrm{V} = R_R = [\mathrm{M(O\ldots A)_A} + \mathrm{W(A\ldots O)} \times d'] \div L$$

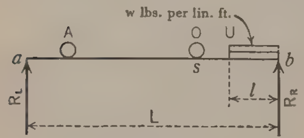


Fig. 386 (*b*).

Shear for the shorter segment *sb* in Fig. 386 (*b*) may be indicated as follows:

$$\mathrm{V} = R_R - w \times l = W - R_L - w \times l$$

$$\mathrm{V} = [\mathrm{W(A\ldots O)} + w \times l] - \left[\mathrm{M(A\ldots O)_U} + \mathrm{W(A\ldots O)} \times l + w \times l \times \frac{l}{2}\right] \div L - w \times l$$

3. *Notes:* When the E-system of loading is used, there will be no wheel on the shorter segment *as* in Fig. 385 (*c*) unless the length of that segment exceeds 8 ft., and in any case, there will be not more than one wheel on the shorter segment, namely wheel 1 (or 10). When only one locomotive can be on the beam, the second should be that one.

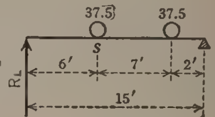
4. When the M-system is used, there will be no wheel on the shorter segment *as* in Fig. 385 (*c*) unless the length of that segment exceeds 10 ft. if wheel 2 is at the section, or 15 ft. if wheel 7 is at the section. When wheel 11 is at the section and the shorter segment exceeds 5 ft., there will be uniform load on that segment as indicated in Fig. 386 (*b*).

5. When uniform load is on either segment, the general method of calculating the left-hand reaction is that given in 376 : 3.

6. When a right-hand reaction is determined by taking moments about a center in the line of action of the left-hand reaction, the table of moments *below* the *lower* zigzag line is used. (376 : 5.)

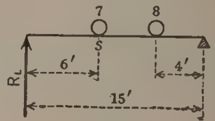
7. *Illustrative problems in determining the greatest shear for a given segment.* Methods of determining the greatest shears for different lengths of span and different lengths of segment are indicated below. In each problem the given segment is the shorter. The shear in each case is determined first for E-60 and then for M-50. The notation used is that given in 372 : 7. Results are given in kips. Use tables on pages 377 and 378.

Span of 15 ft.; segment 6 ft.



For the E-system the critical wheel is the first of the alternative system of loading. (384 : 5.)

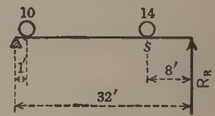
$R_L = \mathrm{V} = [37.5 \times (9+2)] \div 15$
$27.5 = \mathrm{V} = [\ \ 337.5 + 75\ \ ] \div 15$



For the M-system the critical wheel is wheel 7 with the shorter segment to the left. (384 : 12.)

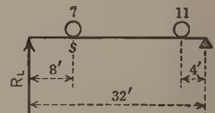
$R_L = \mathrm{V} = [\mathrm{M}(7\ldots8)_8 + \mathrm{W}(7\ldots8) \times 4] \div 15$
$27.1 = \mathrm{V} = [\quad 156 \quad + \quad 62.5 \times 4] \div 15$

Span of 32 ft.; segment 8 ft.



For the E-system the critical wheel is wheel 14 with the shorter segment to the right. (384 : 6.)

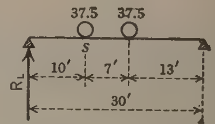
$R_R = \mathrm{V} = [\mathrm{M}(14\ldots10)_{10} + \mathrm{W}(10\ldots14) \times 1] \div 32$
$62.3 = \mathrm{V} = [\quad 1860 \quad + \quad 135 \quad] \div 32$



For the M-system the critical wheel is wheel 7 with the shorter segment to the left. (384 : 12.)

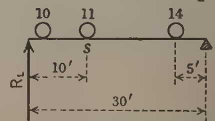
$R_L = \mathrm{V} = [\mathrm{M}(7\ldots11)_{11} + \mathrm{W}(7\ldots11) \times 4] \div 32$
$68.3 = \mathrm{V} = [\quad 1560 \quad + \quad 156.3 \times 4] \div 32$

Span of 30 ft.; segment 10 ft.



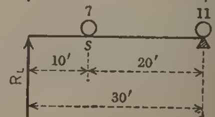
For the E-system two trials are necessary, one for the alternative system and one for wheel 11 as the critical wheel with the shorter segment to the left. (384 : 7.)

$R_L = \mathrm{V} = [37.5 \times (20+13)] \div 30$
$41.3 = \mathrm{V}$



Second trial:

$R_L = [\mathrm{M}(10\ldots14)_{14} + \mathrm{W}(10\ldots14) \times 5] \div 30$
$64.2 = R_L = [\ \ 1250 \quad + \quad 135 \times 5] \div 30$
$49.2 = \mathrm{V} \ \ = R_L - \mathrm{W}(10) = 64.2 - 15$

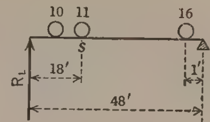


For the M-system the critical wheel is wheel 7 with the shorter segment to the left. (384 : 12.)

$R_L = \mathrm{V} = \mathrm{M}(7\ldots11)_{11} \div 30$
$52.0 = \mathrm{V} = \quad 1560 \div 30$

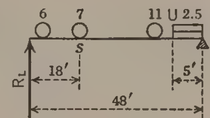
Span of 48 ft.; segment 18 ft.

For the E-system the critical wheel is wheel 2 (384 : 9 (*b*)), but use wheel 11 and ignore wheel 9. (384 : 10.) Shorter segment is to the left.



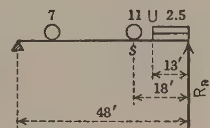
$$R_L = [M(10\ldots16)_{16} + W(10\ldots16)\times 1] \div 48$$
$$70.9 = R_L = [\quad 3230 \quad + \quad 174 \quad] \div 48$$
$$55.9 = V \ = R_L - W(10) = 70.9 - 15$$

For the M-system the critical wheel may be wheel 7 with the shorter segment to the left or it may be wheel 11 with the shorter segment to the right. (384 : 12.)



$$R_L = [M(6\ldots11)_U + W(6\ldots11)\times 5 + 2.5\times 5\times 2.5] \div 48$$
$$89.1 = R_L = [\quad 3340 \quad + \quad 181.3\times 5 + \quad 31.25 \quad] \div 48$$
$$64.1 = V \ = R_L - W(6) = 89.1 - 25$$

Second trial:

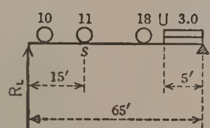


$$R_L = [M(7\ldots11)_U + W(7\ldots11)\times 13 + 2.5\times 13\times 6.5] \div 48$$
$$95.5 = R_L = [\quad 2340 \quad + \quad 156.3\times 13 + \quad 211.25 \quad] \div 48$$
$$60.8 = V \ = -[R_L - W(7\ldots11)] = -95.5 + 156.3$$

(Why is wheel 11 considered to be on the longer segment?)

Span of 65 ft.; segment 15 ft.

For the E-system the critical wheel is wheel 11 with the shorter segment to the left. (384 : 9 (*a*).)

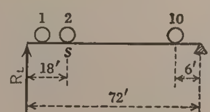


$$R_L = [M(10\ldots18)_U + W(10\ldots18)\times 5 + 3.0\times 5\times 2.5] \div 65$$
$$114.0 = R_L = [\quad 6310 \quad + \quad 213 \quad \times 5 + \quad 37.5 \quad] \div 65$$
$$99.0 = V \ = R_L - W(10) = 114 - 15$$

The determination of the shear for the same span and segment for the M-system is left as an exercise for the student.

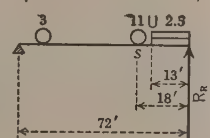
Span of 72 ft.; segment 18 ft.

For the E-system the critical wheel is wheel 2 with the shorter segment to the left. (384 : 9 (*b*).)



$$R_L = [M(1\ldots10)_{10} + W(1\ldots10)\times 6] \div 72$$
$$115.5 = R_L = [\quad 6950 \quad + \quad 228\times 6] \div 72$$
$$100.5 = V \ = R_L - W(1) = 115.5 - 15$$

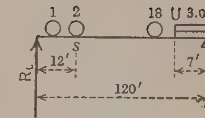
For the M-system the critical wheel is wheel 11 with the shorter segment to the right. (384 : 13.)



$$R_L = [M(3\ldots11)_U + W(3\ldots11)\times 13 + 2.5\times 13\times 6.5] \div 72$$
$$147.6 = R_L = [\quad 7090 \quad + 256.3 \quad \times 13 + \quad 211.25 \quad] \div 72$$
$$108.7 = V \ = -[R_L - W(3\ldots11)] = -147.6 + 256.3$$

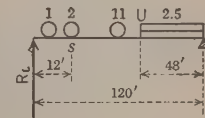
Span of 120 ft.; segment 12 ft.

For the E-system the critical wheel is wheel 2 with the shorter segment to the left. (384 : 9 (*b*).)



$$R_L = [M(1\ldots18)_U + W(1\ldots18)\times 7 + 3.0\times 7\times 3.5] \div 120$$
$$229.6 = R_L = [\quad 24{,}500 \quad + \quad 426\times 7 + \quad 73.5 \quad] \div 120$$
$$214.6 = V \ = R_L - W(1) = 229.6 - 15$$

For the M-system the critical wheel is wheel 2 with the shorter segment to the left. (385 : 1.)



$$R_L = [M(1\ldots11)_U + W(1\ldots11)\times 48 + 2.5\times 48\times 24] \div 120$$
$$220.4 = R_L = [\quad 9470 \quad + \quad 293.8\times 48 + \quad 2881 \quad] \div 120$$
$$207.9 = V \ = R_L - W(1) = 220.4 - 12.5$$

1. **Greatest Bending Moment for a Given Section.** When a simple beam is divided into two segments by any section, the greatest moment for either segment will occur when some wheel (critical wheel) is at the section. (286 : 5.) It will be assumed that in all cases locomotives will be headed toward the *left*. Under this condition, the greatest moment for a section will usually occur when the shorter segment is a left-hand segment, but in some cases it will occur when the shorter segment is a right-hand segment.

2. *Criterion for position of the loading for the greatest bending moment for any section: The average load on one segment should be greater than the average load on the whole beam when the critical wheel is considered to be on that segment, but less when the critical wheel is considered to be on the other segment.* The average load on a segment means the total load on that segment divided by the length of the segment, and, similarly, the average load on the whole beam means the total load on the beam divided by the length of the beam. It is immaterial to which of the two segments the criterion is applied. Using the notation of 372 : 7 and letting W_S represent the weight on a segment of length S, the criterion may be indicated as follows:

$$W_S \div S = W \div L$$

For purposes of calculation a more convenient form is:

$$W_S \gtrless W \times \frac{S}{L}$$

This criterion applied to Fig. 388 (a) becomes:

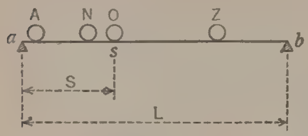


Fig. 388 (a).

$$\begin{matrix} W(A\ldots O) > \\ W(A\ldots N) < \end{matrix} W(A\ldots Z) \times \frac{S}{L}$$

If any uniform load is on the right-hand segment its weight should be added to the weight of the wheels from A to Z. Frequently the criterion is satisfied by more than one wheel. In such a case the bending moment must be calculated for each of the corresponding positions of the loading in order to determine the greatest bending moment. It is sometimes possible to determine from inspection without the use of the criterion which is the critical wheel for a given section.

1. *Ambiguity concerning the inclusion in a criterion of a wheel exactly at a support or at a panel point.* It frequently happens that when the critical wheel O is at a section, the first wheel A is exactly at the left-hand support, or the last wheel Z is exactly at the right-hand support, or that both of these conditions exist. When shall a wheel that is directly over a support be considered on the beam and when off in applying a criterion? Assume that a section is taken at c in Fig. 388 (b), and that, when the critical wheel O is at the section, wheels A and Z are each directly over the corresponding support. The wheels O and A cannot both be on segment ac at the same time. When wheel O is moved ever so little to the left so that it may be considered on that segment, wheel A moves off the beam, consequently when wheel O is considered a part of W_S, the load on the segment ac, wheel A should not be considered a part of W, the total load on the beam. Wheel Z, however, by the movement of wheel O to the left has been brought on the beam and is a part of W. Similarly, when wheel O is not considered a part of W_S, i.e., when it has been moved ever so little to the right to segment cg, wheel Z moves off the beam and is no longer a part of W, but wheel A is brought on the beam and is, therefore, included in W.

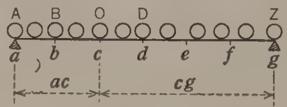


Fig. 388 (b).

2. Later on, a similar ambiguity will arise in applying criteria to girders and trusses with panels. Let the critical wheel O be at the panel point c in Fig. 388 (b). It may happen that wheel B is exactly at panel point b, or wheel D is exactly at panel point d. When wheel O is considered as a part of the load W_p in panel bc, wheel A is not on the bridge, wheel B is not in the panel bc, wheel D is in panel cd, and wheel Z is on the bridge. When wheel O is not considered a part of the load in panel bc, wheel A is on the bridge, wheel B is in panel bc, wheel D is in panel de, and wheel Z is off the bridge. Numerous cases in which one or more wheels are at a panel point or a support occur in practice, though seldom, if ever, is there a case with so many as are shown in the figure.

3. When in doubt whether to count a wheel at a support as on or off a beam, or a wheel at a panel point as on or off a panel, it is well to consider the effect on that wheel of an indefinitely short movement of the critical wheel to the left or right.

4. **Tables of Critical Wheels.** Tables of critical wheels, one for the M-system and one for the E-system, are given on the next page. These tables enable one to select the critical wheel that should be placed at any given section of any given beam in order to bring the locomotive loading into the position for the greatest bending moment for that segment. In Fig. 388 (c) is shown a typical influence diagram for bending moment for a given section at s. This diagram is a triangle; a perpendicular to the base of the triangle divides the base into two segments l_1 and l_2. Each of these two segments of the base is equal in length to the corresponding segment of the beam, i.e., $l_1 = as$ and $l_2 = sb$. From the values of l_1 and l_2 in any given case, the critical wheel may be read from the table. In using either table it is to be assumed that the engine is always headed toward the left. Except when a wheel is overlined in the table, it is understood that the shorter segment is the left-hand segment.

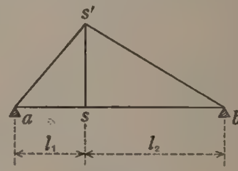


Fig. 388 (c).

5. *Note:* It will be explained later that the tables may be used in selecting the critical wheel for the greatest shear for a section anywhere between two consecutive panel points of a girder or truss with floor beams. In such a case, the segments of the base of the influence triangle are not the same in length as the segments of the girder or truss. To avoid ambiguity that might easily lead to mistakes when using the tables, l_1 and l_2 will always be considered as representing the shorter and longer segments, re-

Critical Wheels that Determine the Positions of Locomotive Loadings for Maximum Shears, Bending Moments, and Stresses

Steinman's M-System

Shorter Segment l_1 of the Base of the Influence Triangle

Longer Segment l_2 of the Base of the Influence Triangle

Segments	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	100	110
1000 to 300	2	2	3	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	10	11
290 to 250	2	2	3	3	3	4	4	5	5	6	7	7	7	8	8	9	9	9	10	10	11
240	2	2	3	3	3	4	4	5	6	6	7	7	7	8	8	9	9	9	10	10	11
230	2	2	3	3	3	4	4	5	6	6	7	7	7	8	8	9	9	9	10	10	11
220	2	2	3	3	3	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	11
210	2	2	3	3	3	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	11
200 to 160	2	2	3	3	3	4	5	5	6	6	7	7	8	8	8	9	9	9	10	10	11
150	2	2	3	3	3	4	5	5	6	6	7	7	8	8	8	9	9	9	10	10	11
140	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	11
130	2	2	3	3	4	4	5	5	6	7	7	7	8	8	9	9	9	10	10	10	11
120	2	2	3	3	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
110	2	2	3	4	4	4	5	5	6	7	7	8	8	8	9	9	9	10	10	10	11
100	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	11
95	2	2	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10	10	
90	$\overline{11}$	$\overline{11}$	3	4	4	4	5	6	6	7	7	8	8	8	9	9	9	10	10		
85	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	7	7	7	8	8	9	9	9	9	10			
80	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	7	8	8	9	9	9	10				
75	$\overline{11}$	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	8	8	9	9	9					
70	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	8	8	9	9						
65	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	9							
60	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	8								
55	$\overline{11}$	$\overline{11}$	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$									
50	7	7	$\overline{10}$	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$	$\overline{8}$										
45	7	7	8	$\overline{10}$	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$	$\overline{8}$											
40	7	7	8	8	$\overline{9}$	$\overline{9}$	$\overline{9}$	$\overline{8}$	$\overline{8}$												
35	7	7	8	8	9	$\overline{9}$	$\overline{9}$	$\overline{8}$													
30	7	7	8	9	9	9	$\overline{9}$														
25	7	7	8	9	9	9															
20	7	7	8	9	9																
15	7	7	8	9																	
10	7	7	8																		
5	7	7																			

The shorter segment is ahead followed by the longer one except where wheel is overlined. For $l_1 > 117.5$ place head of uniform load 117.5 from head of span.

Cooper's E-System

Shorter Segment l_1 of the Base of the Influence Triangle

Longer Segment l_2 of the Base of the Influence Triangle

Segments	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	130	140
300 to 260	2	2	3	3	4	4	5	5	6	7	7	8	9	10	10	11	11	12	12	13	14	15	17	18
250 to 200	2	2	3	3	4	4	5	5	6	7	8	8	9	10	11	11	12	12	12	13	14	15	17	18
190 to 150	2	2	3	3	4	4	5	5	6	7	8	9	9	11	11	12	12	12	12	13	14	15	17	18
140	2	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	12	12	12	13	14	15	17	18
130	2	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	12	12	12	13	14	15	17	
120	2	3	3	3	4	4	5	5	6	7	8	9	10	11	12	12	12	13	13	13	14	15		
110	2	3	3	3	4	4	5	6	7	7	8	9	10	11	12	12	12	13	13	13	14			
100	2	3	3	3	4	5	5	6	$\overline{14}$	$\overline{14}$	$\overline{14}$	$\overline{13}$	$\overline{13}$	11	12	12	12	13	13	13				
95	2	3	3	4	4	5	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	12	12	12	13	13					
90	2	3	3	4	4	5	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	12	12	13						
85	2	3	3	4	4	5	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	12	12							
80	2	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	12								
75	2	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	12									
70	2	3	3	4	4	$\overline{13}$	$\overline{13}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	11										
65	2	3	3	4	4	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{12}$	$\overline{11}$	11	11											
60	11	3	3	4	4	5	$\overline{13}$	$\overline{12}$	$\overline{11}$	$\overline{11}$	$\overline{11}$	11												
55	11	12	12	12	4	12	$\overline{13}$	$\overline{12}$	$\overline{12}$	13	11													
50	11	12	12	12	12	12	$\overline{13}$	$\overline{13}$	$\overline{13}$	12														
45	2	3	12	12	12	12	$\overline{13}$	$\overline{13}$	13															
40	2	3	3	3	12	12	$\overline{13}$	13																
35	2	3	3	4	4	13	13																	
30	2	3	3	4	4	13																		
25	2	3	3	4	4																			
20	2	$\overline{4}$	3	4																				
15	2	3	3																					
10	2	3																						
5	2																							

The shorter segment is ahead followed by the longer one except where wheel is overlined. When each of two segments is greater than 142 ft., advance the load on the longer segment first, and upon the shorter segment until wheel 1 is within 33 ft. of the far end of the shorter segment.

Reactions and Shears

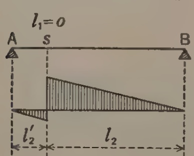


Fig. (a)

Chord Stresses, Bending Moments and Floor-beam Reactions

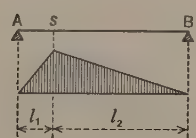


Fig. (b)

Stresses in Diagonals Shears for Panels

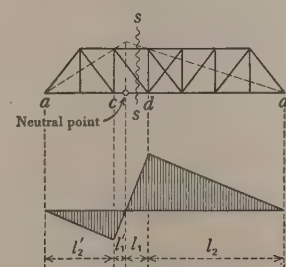


Fig. (c)

Stresses in Diagonals

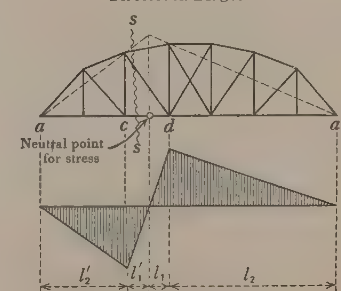


Fig. (d)

spectively, of the *base of the influence triangle,* and not, necessarily, the two segments into which the beam is divided by the given section.

1. *Illustrations of the use of the tables.* (*a*) Span 50 ft.; segments 20 and 30 ft. What is the critical wheel for greatest bending moment? Answers: For the M-system, wheel 9; for the E-system, wheel 4. The shorter segment is the left-hand segment for both wheels.

(*b*) Span 120 ft.; segments 40 and 80 ft. What is the critical wheel for the greatest bending moment? Answers: For the M-system, wheel 10 with the shorter segment the right-hand segment, since wheel 10 is overlined; for the E-system, wheel 13 with the shorter segment a right-hand segment, since wheel 13 is overlined.

2. *General method of determining the greatest bending moment for a given section.* The general method of procedure may be outlined as follows:

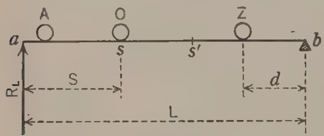


Fig. 390.

3. First step: Select the critical wheel by means of the criterion in 387 : 2, or from the table on page 389, and place this wheel O at the section. (Fig. 390.) Determine which wheel is wheel A, the first wheel on the beam and which is wheel Z, the last on; also the distance d from Z to the right-hand end.

4. Second step: Calculate the reaction R_L.

$$R_L = [M(A\ldots Z)_Z + W(A\ldots Z) \times d] \div L. \quad (376:1.)$$

5. Third step: Calculate the bending moment for the segment *as*

$$M_B = R_L \times S - M(A\ldots O)_O$$

6. When the length of span is such that uniform load is on the beam the method of procedure is the same except that the uniform load must be taken into account in calculating R_L as explained in 376 : 3.

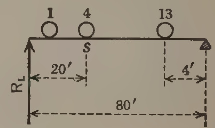
7. When the shorter segment is the right-hand segment, it is still convenient to calculate the bending moment for the left-hand segment, even though it is the longer. The method of procedure, will, therefore, be exactly the same as that indicated above.

8. Let s' be as far from the center of the span as is s. It frequently happens that a greater bending moment may be obtained for a section at s' than for one at s. (Engine headed toward the left in both cases.)

9. *Illustrative problem in the greatest bending moment for a given segment.* Given: A girder 80 ft. long. Required: The greatest bending moment for a segment 20 ft. long, first for E-60 and then for M-50. The notation used is that given in 372 : 7. Reactions are given in kips, and bending moments in thousands of pound-feet.

The critical wheel for the E-system may be ascertained from the table on page 389. Entering the table with $l_1 = 20$ and $l_2 = 60$, wheel 4 is found to be the critical wheel. It may be well, however, to illustrate how the criterion in 387 : 2 would be applied to wheel 4.

$$W(1\ldots 4) > W(1\ldots 13)\tfrac{20}{80}$$
$$105 > 318\times\tfrac{1}{4}$$
$$W(1\ldots 3) < W(1\ldots 13)\tfrac{20}{80}$$
$$75 < 79.5$$

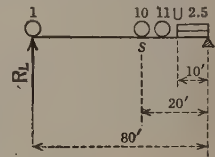


The criterion is satisfied by wheel 4.

$$R_L = [M(1\ldots 13)_{13} + W(1\ldots 13)\times 4]\div 80$$
$$159.7 = R_L = [\quad 11{,}500 \quad + \quad 318\times 4]\div 80$$
$$2474 = M_B = R_L\times 20 - M(1\ldots 4)_4 = 159.7\times 20 - 720$$

The critical wheel for the M-system is found by entering the table on page 389 (with $l_1 = 20$ and $l_2 = 60$) to be wheel 10 with the shorter segment to the right. (Overlined.)

$$R_L = [M(1\ldots 11)_U + W(1\ldots 11)\times 10 + 2.5\times 10\times 5]\div 80$$
$$156.7 = R_L = [\quad 9470 \quad + \quad 293.8\times 10 + \quad 125 \quad]\div 80$$
$$2712 = M_B = R_L\times 60 - M(1\ldots 10)_{10} = 156.7\times 60 - 6690$$



10. *Note:* When there are no tables at hand from which to select the critical wheel and it becomes necessary to apply the criterion, it should be remembered that the criterion may be satisfied by more than one critical wheel. For example, in the problem just solved, the criterion is satisfied by four different wheels of the M-system, namely, wheels 4 and 5 with the shorter segment to the left, and wheels 9 and 10 with the shorter segment to the right. Had the greatest bending moment been calculated for each of the four wheels as the critical wheel, the four results would have been: Wheel 4 = 2,507,000 lb.-ft.; wheel 5 = 2,522,000 lb.-ft.; wheel 9 = 2,696,000 lb.-ft.; wheel 10 = 2,712,000 lb.-ft. Wheel 10 therefore gives the greatest possible bending moment for the segment.

It is interesting to note that for the E-system the greatest bending moment for the 20-ft. segment occurs when that segment is to the left, whereas for the M-system the greatest bending moment for a 20-ft. segment occurs when that segment is to the right.

11. **Maximum Bending Moment.** The maximum bending moment for a simple beam will occur for a section near but not necessarily at the center of the beam. One of the wheels, called the critical wheel, will be at the section wherever it may be. *The section should be taken at such a point that when the critical wheel is at that point it is far on one side of the center of the beam as the center of gravity of all the loads on the beam is on the*

other side of the center. This is the same principle as that used in determining the maximum bending moment due to two moving concentrated loads (287 : 2 (*b*)).

1. *Note:* When a span is short, the critical wheel may be in a position to satisfy the criterion and yet the corresponding bending moment will not be the absolute maximum. This was shown in 286 : 10 (*c*) and 287 : 2 (*c*).

2. *Critical wheels for maximum bending moment.* For beams of ordinary length, the critical wheel will be one of the heaviest drivers, because, when placed at the section near the center, it will bring the heaviest part of the engine near the center. Critical wheels for different lengths of span will be given, first for the E-system and then for the M-system. It is assumed that locomotives are headed toward the left.

SPAN IN FEET	CRITICAL WHEEL
E-System of Loading	
0 to 11.4	Alternative loading
11.4 to 18.7	12 or 13, either one
18.7 to 35.0	12
35.0 to 70.0	13
M-System of Loading	
0 to 61.5	9
61.5 to 118.0	8
118 or over	9

3. Girders seldom exceed 120 feet in length. For spans between 70 and 120 ft. the greatest bending moment for a section *at the center* of the beam determined by the method explained in 387 : 1 to 390 : 10 will not vary greatly from the absolute maximum bending moment. For the E-system of loading this variation is less than one per cent, hence for most practical purposes it is sufficiently accurate when a span exceeds 70 ft. to determine the greatest bending moment for a section at the center. In such a case the critical wheel is given in the table on page 389. (l_1 will equal l_2.)

4. The critical wheel for any length of span and for any system of loading may be determined by trial as follows: Select tentatively some wheel which is likely to be the critical wheel. Place this wheel at the section which satisfies the criterion of 390 : 11, and calculate the bending moment. In like manner, determine the bending moment when first one and then the other of the wheels on either side of the wheel first selected is treated as the critical wheel. If each of the two bending moments thus obtained is less than that determined for the wheel first selected, this wheel is the critical wheel; if either is greater, then the process must be repeated, until some wheel is found such that, when treated as a critical wheel, the corresponding bending moment is greater than that obtained by considering the wheel on either side as a critical wheel.

5. *General method of determining the maximum bending moment.* The general method of procedure in determining the maximum bending moment for a beam of given length may be outlined as follows:

6. First step: Select the critical wheel from one of the tables given on this page, and place this wheel O temporarily at the center c of the span in order to ascertain which wheel is wheel A, and which is wheel Z, the first and last wheels, respectively, on the beam. (Fig. 391 (*a*).)

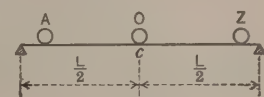


Fig. 391 (*a*).

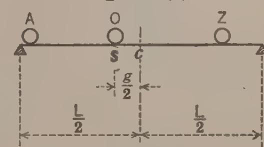


Fig. 391 (*b*).

7. Second step: From the table on page 377 (or 378) determine the distance g from the critical wheel O to the center of gravity of all of the loads on the beam, and shift the loading until this distance is bisected by the center of the span, thus bringing the critical wheel and the section s as far on one side of the center as the center of gravity is on the other side. (Fig. 391 (*b*).) It is assumed, for the present, that this shifting causes no load to move onto or off of the beam.

8. *Note:* The critical load is shown in Fig. 391 (*b*) to have been moved to the left of the center. This indicates that the center of gravity is to the right of the critical wheel; when it is to the left of the critical wheel, that wheel is moved to the right of the center. The method of procedure is the same whichever side of the center the critical wheel may be.

9. *Note:* In many forms of tables, distances to centers of gravity are not given. It may then be necessary to determine the center of gravity of all loads on the beam by the method given on page 42.

1. Third step: Determine the distance d of the last wheel Z from the right-hand end of the beam, and the distance S from the left-hand end to the section s under the critical wheel O (Fig. 392 (a)). Show these distances on a sketch.

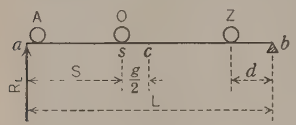


Fig. 392 (a).

2. Fourth step: Calculate the left-hand reaction (376 : 1).

$$R_L = [\mathrm{M(A\ldots Z)_Z} + \mathrm{W(A\ldots Z)} \times d] \div L.$$

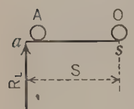


Fig. 392 (b).

3. Fifth step: Calculate the bending moment for the segment as. (Fig. 392 (b).)

$$\mathrm{M_B} = R_L \times S - \mathrm{M(A\ldots O)_O}$$

4. *Note:* If in shifting the loading in the second step a wheel moves onto or off of the beam, the distance g will be changed. The new distance g will be that between the critical wheel and the center of gravity of the wheels that are on the beam *after* the shift. The loading must be shifted until in its final position the center of gravity of the wheels that are on the beam when the loading is in that position is as far on one side of the center as the critical wheel is on the other.

5. When the length of span is such that uniform train load is on the beam the general method of procedure is modified as follows:

6. First step: Select the critical wheel. If uncertain as to the correct wheel, select one that seems likely to be the critical wheel. Place this wheel at the center of the beam, and determine which wheel is wheel A and how far the uniform load extends onto the beam (distance l). (Fig. 392 (c).)

7. Second step: Determine the distance x from the center of the uniform load to the center of gravity of *all* of the load on the beam including the uniform load.

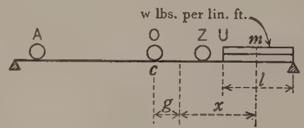


Fig. 392 (c).

$$x = \left[\left(\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times \frac{l}{2}\right) \div (\mathrm{W(A\ldots Z)} + w \times l)\right]. \quad (376:3.)$$

From x determine g. Assuming that the center of gravity is to the right of the critical wheel O, the entire loading must be shifted toward the left. As it is shifted, more uniform load moves onto the beam, and if no wheel moves off, the distance g will be slightly increased by this additional uniform load. The loading should be shifted to the left until the distance from the critical wheel O to the center is slightly greater than the distance $\frac{g}{2}$ as first determined, so that when the loading is in its final position the criterion will be satisfied. Similarly, when the center of gravity is to the left of the critical wheel, the loading is shifted to the right until the distance from the critical wheel O to the center is slightly less than the distance $\frac{g}{2}$ as first determined, and until the criterion is satisfied.

8. The remaining steps are similar to those already outlined. Assuming that the loading has been placed in its final position in Fig. 392 (d), the reaction and bending moment are determined by the usual methods.

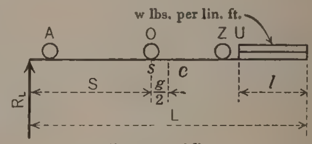


Fig. 392 (d).

$$R_L = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + w \times l \times \frac{l}{2}\right] \div L. \quad (376:3.)$$

$$\mathrm{M_B} = R_L \times S - \mathrm{M(A\ldots O)_O}$$

9. *Notes:* If it is not certain that the wheel used as a critical wheel is the correct wheel, other wheels may be tried, until one is found which, when placed to satisfy the criterion, will bring the loading into a position such that the resulting bending moment is greater than that obtained when any other wheel is used as the critical wheel. (391 : 4.)

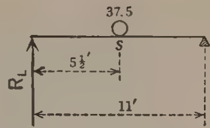
10. If in shifting the loading any wheel moves onto or off of the left-hand end of the beam, an entirely new distance g must be used as explained in 392 : 4.

11. The method of procedure just outlined is theoretically correct, but often unnecessary. As stated in 391 : 3, when the span is over 70 ft., it is frequently sufficiently accurate for all practical purposes to place the critical wheel at the center of the beam, and to calculate the bending moment for a section at the center by the method explained in 387 : 1 to 390 : 10.

12. *Illustrative problems in maximum bending moments.* Methods of determining maximum bending moments for simple beams of different lengths will now be indicated. The maximum moment in each case is determined first for the E-system and then for the M-system. The notation used is that given in 372 : 7. Reactions are given in kips and bending moments in thousands of pound-feet. (Use tables on pages 377 and 378.)

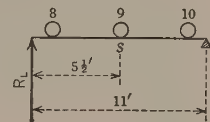
Span of 11 ft.

For the E-system the alternative loading is used with one wheel at the center. (391 : 2.)



$$103 = M_B = (37.5 \div 2) \times 5.5$$

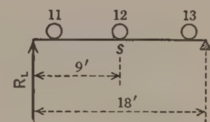
For the M-system the critical wheel is wheel 9 at the center of the beam. (Why?)



$$46.9 = R_L = W(8\ldots10) \div 2 = 93.8 \div 2$$
$$102 = M_B = R_L \times 5.5 - M(8\ldots9)_9 = 46.9 \times 5.5 - 156$$

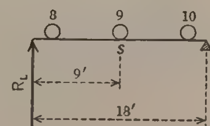
Span of 18 ft.

For the E-system the critical wheel is wheel 12 at the center of the beam. (391 : 2.)



$$45.0 = R_L = W(11\ldots13) \div 2 = 90 \div 2$$
$$255 = M_B = R_L \times 9 - M(11\ldots12)_{12} = 45 \times 9 - 150$$

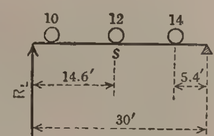
For the M-system the critical wheel is wheel 9 at the center of the beam. (391 : 2.)



$$46.9 = R_L = W(8\ldots10) \div 2 = 93.8 \div 2$$
$$266 = M_B = R_L \times 9 - M(8\ldots9)_9 = 46.9 \times 9 - 156$$

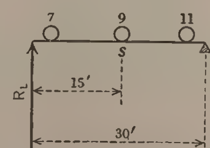
Span of 30 ft.

For the E-system the critical wheel is wheel 12. (391 : 2.) Wheels on the beam 10 to 14. Center of gravity of 10 to 14 is 0.8 ft. to the right of wheel 12. Section and wheel 12 is 0.4 ft. to the left of the center. (391 : 7.)



$$R_L = [M(10\ldots14)_{14} + W(10\ldots14) \times 5.4] \div 30$$
$$66.0 = R_L = [\quad 1250 \quad + \quad 135 \times 5.4] \div 30$$
$$619 = M_B = R_L \times 14.6 - M(10\ldots12)_{12} = 66.0 \times 14.6 - 345$$

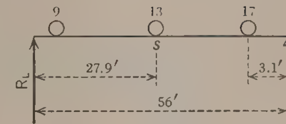
For the M-system the critical wheel is wheel 9 at the center (391 : 2).



$$78.2 = R_L = W(7\ldots11) \div 2 = 156.3 \div 2$$
$$704 = M_B = R_L \times 15 - M(7\ldots9)_9 = 78.2 \times 15 - 469$$

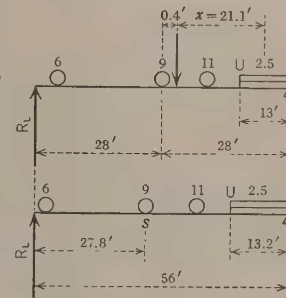
Span of 56 ft.

For the E-system the critical wheel is wheel 13 (391 : 2); wheels on are 9 to 17; center of gravity of 9 to 17 is 0.2 to the right of wheel 13. Place wheel 13 0.1 ft. to the left of the center of the beam.



$$R_L = [M(9\ldots17)_{17} + W(9\ldots17) \times 3.1] \div 56$$
$$106.0 = R_L = [\quad 5270 \quad + \quad 213 \times 3.1] \div 56$$
$$1727 = M_B = R_L \times 27.9 - M(9\ldots13)_{13} = 106 \times 27.9 - 1230$$

For the M-system the critical wheel is wheel 9. To find x, the distance of the center of gravity from the center of the uniform load when wheel 9 is at the center of the beam.



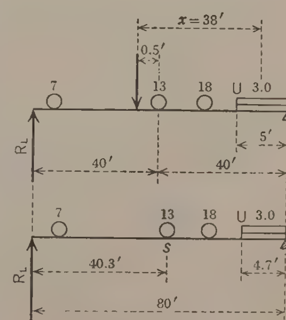
$$x = [M(6\ldots11)_U + W(6\ldots11) \times 6.5] \div [W(6\ldots11) + 2.5 \times 13]$$
$$21.1 = x = [\quad 3340 \quad + \quad 181.3 \times 6.5] \div [181.3 + 32.5]$$

Center of gravity is $21.5 - 21.1 = 0.4$ ft. to the right of wheel 9. Place wheel 9 a distance 0.2 ft. to the left of the center.

$$R_L = [M(6\ldots11)_U + W(6\ldots11) \times 13.2 + 2.5 \times 13.2 \times 6.6] \div 56$$
$$106.3 = R_L = [\quad 3340 \quad + \quad 181.3 \times 13.2 + \ 214.5] \div 56$$
$$1865 = M_B = R_L \times 27.8 - M(6\ldots9)_9 = 106.3 \times 27.8 - 1090$$

Span of 80 ft.

Assume that for the E-system the critical wheel is unknown. It will be one of the drivers, and there should be as much load as possible on the beam, hence for a span of 80 ft. the critical wheel is likely to be one of the drivers of the second locomotive. It will be determined by trial as explained in 391 : 4. Try wheel 13:



$$x = [M(7\ldots18)_U + W(7\ldots18) \times 2.5] \div [W(7\ldots18) + 3.0 \times 5]$$
$$38.0 = x = [\quad 10{,}200 + \quad 271.5 \times 2.5] \div [\quad 271.5 + 15 \quad]$$

Center of gravity is 0.5 ft. to the left of wheel 13. In shifting the loading to the right, some of the

uniform load will move off and the center of gravity will be slightly farther from wheel 13, say 0.6 ft. Place wheel 13 a distance 0.3 ft. to the right of the center.

$$R_L = [M(7\ldots18)_U + W(7\ldots18)\times4.7 + 3.0\times4.7\times2.35]\div80$$
$$143.9 = R_L = [\ 10{,}200 \quad + \quad 271.5\times4.7 \quad + \quad 33 \quad]\div80$$
$$3250 = M_B = R_L\times40.3 - M(7\ldots13)_{13} = 143.9\times40.3 - 2550$$

Try wheel 12:

$$R_L = [M(6\ldots18)_{18} + W(6\ldots18)\times4.9]\div80$$
$$145.3 = R_L = [\ 10{,}200 \quad + \quad 291\times4.9]\div80$$
$$3217 = M_B = R_L\times40.1 - M(6..\ 12)_{12} = 145.3\times40.1 - 2610$$

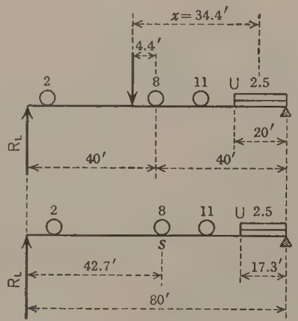
Try wheel 14:

$$x = [M(8\ldots18)_U + W(8\ldots18)\times5]\div[W(8\ldots18)+3.0\times10]$$
$$35.6 = x = [\quad 8790 \quad + \quad 252\times5]\div[\quad 252+30]$$
$$R_L = [M(8\ldots18)_U + W(8\ldots18)\times9.8 + 3.0\times9.8\times4.9]\div80$$
$$142.5 = R_L = [\quad 8790 + \quad 252 \quad \times9.8 + \quad 144 \quad]\div80$$
$$3178 = M_B = R_L\times40.2 - M(8\ldots14)_{14} = 142.5\times40.2 - 2550$$

The bending moment for wheel 13 is greater than that for the wheel on either side, therefore it is the maximum of maximums.

For the M-system the critical wheel is wheel 8.

$$x = [M(2\ldots11)_U + W(2\ldots11)\times10]\div[W(2\ldots11)+2.5\times20]$$
$$34.4 = x = [\quad 8590 \quad + \quad 281.3 \quad \times10]\div[\ 281.3 \quad + \quad 50\ ]$$



Center of gravity is 4.4 ft. to the left of wheel 8. It will be found that in shifting the load to the right this distance is increased to about 5.4 ft., hence wheel 8 should be about 2.7 to the right of the center.

$$R_L = [M(2\ldots11)_U + W(2\ldots11)\times17.3 + 2.5\times17.3\times8.7]\div80$$
$$172.9 = R_L = [\quad 8590 \quad + \quad 281.3\times17.3 \quad + \quad 376 \quad]\div80$$
$$3473 = M_B = R_L\times42.7 - M(2\ldots8)_8 = 172.9\times42.7 - 3910$$

1. **Girders with Floor Beams.** The wheel loads have been treated thus far as if they rolled along the top of a beam and hence were applied directly to the beam. This is an approximately correct assumption when the track ties rest directly on the beam as, for example, on the top of a stringer or on top of a deck plate girder, though even in such a case the weight of a wheel is distributed by the rail to several ties. In through bridges the loads are carried by the stringers to the floor beams and by the floor beams to the girders or trusses (97 : 1), hence live loads take effect only at panel points. It is necessary therefore to modify slightly some of the methods of finding shears and bending moments.

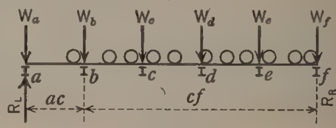


Fig. 394.

2. Let Fig. 394 represent a bridge of five panels. Let a, b, c, d, e, and f, represent panel points at which floor beams are connected to a girder, those at a and f being end floor beams. Each wheel load is assumed to act on a stringer at the point where the wheel is shown, but the loads as a whole cause **floor-beam loads** or **panel loads** W_a, W_b, W_c, W_d, W_e, and W_f. These panel loads are usually unequal, but any panel load can be found, if necessary, for any given position of the locomotive by methods to be explained later.

3. *When floor beams may be ignored.* In certain calculations the floor beams may be ignored and the wheel loads treated just as if they were rolling along the top of a simple beam of length af. (307 : 2.) The most important of these calculations are:

(*a*) Calculation of the reaction at either end of the girder. (R_L or R_R.)

(*b*) Calculation of the greatest shear for a section exactly at a panel point.

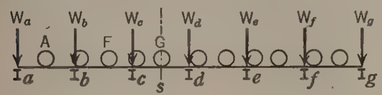
(*c*) Calculation of the greatest bending moment for a section exactly at a panel point.

When a section is exactly at a panel point the two segments meet at that point. For example, if the section is at c, the segments are ac and cf, just as for a girder without floor beams.

4. *Cases in which floor beams must be taken into account.* When a section is taken *between* floor beams, the methods of determining shear and bending moment are not the same as the methods of determining shear and bending moment for any segment of a beam without floor beams.

5. Let W_a, W_b, W_c, W_d, W_e, W_f, and W_g in Fig. 395 represent the floor-beam loads or panel loads due to a locomotive loading in a given position. Let it be required to determine the shear and bending moment

for a section at s *between* panel points c and d. In the first step, that of calculating the reaction R_L, the floor beams may be ignored, and the result, obtained by the method of 376 : 1, will be the same as that obtained by calculating the reaction directly from the panel loads.



1. The shear and bending moment for the segment *as* may be obtained from the reaction and panel loads (Fig. 395) as follows:

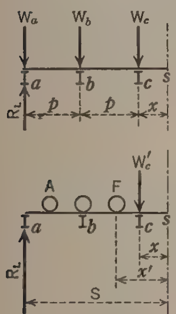


Fig. 395.

$$V = R_L - W_a - W_b - W_c$$

$$M_B = R_L(2p + x) - W_a(2p + x) - W_b(p + x) - W_c(x)$$

These methods, however, involve the calculation of W_a, W_b, and W_c; they may be modified to permit the use of a shear and moment table and thus to reduce the work of calculation, as follows:

2. Let W'_c be the floor-beam load at c due to the loads in the panel cd. W'_c differs from W_c since the latter is the floor-beam load due to the loads in both panels bc and cd. The shear and bending moment may be obtained as follows (Fig. 395):

$$V = R_L - \mathbf{W}(\mathbf{A}\ldots\mathbf{F}) - W'_c$$

$$\begin{aligned} M_B &= R_L \times S - \mathrm{M}(\mathrm{A}\ldots\mathrm{F})_s - W'_c \times x \\ &= R_L \times S - \mathbf{M}(\mathbf{A}\ldots\mathbf{F})_\mathbf{F} + \mathbf{W}(\mathbf{A}\ldots\mathbf{F}) \times x' - W'_c \times x \end{aligned}$$

The quantities in bold-face type may be read from a shear and moment table. It is to be noted that in the expressions for both shear and bending moment, the loads in panels ab and bc are treated just as if they rested on the top of the girder, but that the loads in panel cd are replaced by the floor-beam loads which they cause at the points c and d. Only one of these floor-beam loads, namely W'_c acts on the segment, hence only this load enters into the expressions for bending moment and shear. This reaction W'_c may be calculated from *all* of the wheel loads in the panel cd just as if cd were a simple beam. (The load W'_c is brought to the floor beam at c by the stringers between c and d.)

3. The methods just explained may be given in a more general form for use in calculating the shear and bending moment for any section *between* two successive panel points:

4. *The reaction at either end of a girder with floor beams may be calculated from the loading on the entire girder just as if that loading were applied directly to the girder, i.e., floor beams may be ignored.* (394 : 3 (*a*).)

5. *In calculating the shear for any section between two successive panel points, the downward shearing forces on either segment due to the loads on that segment that are* **not** *in the panel cut by the section may be taken as the loads themselves, just as if they were applied directly to the segment — it is not necessary to determine the corresponding floor-beam loads at panel points, but the downward shearing force due to loads* **in** *the panel cut by the section is the partial panel load on the segment at the end of that panel due to all of the loads in the panel.*

6. *In calculating the bending moment for a center between two successive panel points, the downward forces on either segment are the downward shearing forces described in the preceding paragraph, namely, all loads on the segment that are not in the panel cut by the section and the partial panel load on the segment due to all loads in the panel cut by the section.*

7. *Bending moment for a section at the center of a panel.* When the section is exactly at the center of a panel, the bending moment for a given position of the loading may be determined without regard to floor-beam loads or panel loads as follows: Calculate the bending moment for a section at one end of the panel just as if there were no floor beams; similarly, calculate the bending moment for a section at the other end of the panel; the average of these two bending moments will be equal to the bending moment for a section at the center of the panel.

8. *Note:* The effect of floor beams on a girder may be summed up in the following general statement: End reactions on girders are calculated exactly as for girders without floor beams. The shear and bending moment for a given section are calculated exactly as for a girder without floor beams, except when the section is between panel points; even then, the only floor-beam load that is used in the calculations is a partial floor-beam load at one end of the panel in which the section is taken, namely, the single force on the segment due to all of the loads in the panel. When the section is at the center of a panel, the bending moment may be determined without using any part of a floor-beam load by calculating the bending moments for sections at the ends of the panel and finding the average of these two bending moments. The proof of this whole general statement is left to the student.

1. The methods of calculating shears and bending moments for girders with floor beams are also used for calculating shears and bending moments for trusses with floor beams. The calculations of stresses in trusses due to locomotive loading are based very largely on these methods, as explained in the next chapter. Because these methods are so important, they will now be given in considerable detail.

2. **Girders with Floor Beams — Greatest Shear for a Given Section.** The greatest shear for a section exactly at a panel point is determined as if there were no floor beams by the method explained in 383 : 1 to 386 : 2.

3. Let it be required to determine the greatest shear for a section *between* two adjacent panel points as, for example, between b and c in Fig. 396. This greatest shear will occur when some wheel (critical wheel) is at c. Ordinarily there will be wheels in the panel bc in which the section is taken, but none to the left of that panel, although if panels are exceptionally short, the loading may extend into the next panel ab.

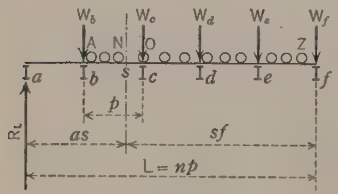


Fig. 396.

4. Assume that the loading in Fig. 396 is in a position to cause the greatest shear for a section at s, i.e., the greatest positive shear for the segment as, or the greatest negative shear for the segment sf. Since the loading can take effect at panel points only, let it be replaced by the floor-beam loads W_b, W_c, W_d, W_e, and W_f. The shear for all sections between two successive panel loads is constant (172 : 3), hence the shear for *any* section between b and c is the same as the shear for the section at s, and it is equal to $R_L - W_b$. Treating bc as a beam, W_b is equal and opposite to the reaction at b due to the loads in the panel bc, or, using the notation of 372 : 7, $W_b = \mathrm{M(A\ldots O)_O} \div p$.

Shear for segment as from floor-beam loads $= R_L - \mathrm{M(A\ldots O)_O} \div p$

Shear for segment as from loads on a beam af without floor beams $= R_L - \mathrm{W(A\ldots N)}$.

Not only do these two shears differ, but the former is the shear for *any* section between b and c, whereas the latter is the shear only for sections taken between wheels N and O. For any section between panels, therefore, the floor beams cannot be ignored either in determining the criterion for the position of the loading, or in calculating the shear, once the loading is in the correct position.

5. *Criterion for greatest shear.* The criterion for the position of the loading which will cause the greatest shear for a given section *between* two successive panel points may be stated as follows:

When the critical wheel is at one end of the panel, the average load on the panel will be greater than the average load on the entire girder when that critical wheel is included in the load on the panel, but less when the critical wheel is not included in the load on the panel. For example, if in Fig. 396 wheel O is the correct (critical) wheel to be at c, then:

$$\mathrm{W(A\ldots O)} \div p > \mathrm{W(A\ldots Z)} \div L$$
$$\mathrm{W(A\ldots N)} \div p < \mathrm{W(A\ldots Z)} \div L$$

When panels are equal, as is usually the case, a more useful form of the criterion is

$$\begin{matrix}\mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} <\end{matrix} \mathrm{W(A\ldots Z)} \div n \qquad (n = \text{number of panels})$$

The criterion is frequently given in the following form:

$$W_p = W\frac{p}{L} \quad \text{or} \quad W_p = \frac{W}{n} \quad \text{or} \quad W_p \times n = W$$

W_p = load on the panel in which the section is taken; W = total load on the girder; p = panel length; L = length of girder; n = number of panels. Although the criterion is expressed as an equation, it is used as if it were expressed in the form $W_p \gtrless W\frac{L}{p}$, according to whether or not the critical wheel is included in W_p.

6. When uniform train load is on the bridge, its total weight should be included in the total load. If l is the length of this uniform load and w its weight per linear foot the criterion in 396 : 3 becomes:

$$\begin{matrix}\mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} <\end{matrix} [\mathrm{W(A\ldots Z)} + wl] \div n$$

7. When the greatest positive shear is desired for a left-hand segment, the critical wheel should be placed at the right-hand end of the panel

through which the section is taken, regardless of whether that panel is to the left or to the right of the center of the girder. When, on the other hand, the greatest positive shear is desired for a right-hand segment *and the panel through which the section is taken is to the right of the center*, the critical wheel should be placed at the left-hand end of that panel. These statements hold true for the usual assumption that the locomotive loading moves onto the beam from the right and is headed toward the left.

1. *Note:* When the panel through which the section is taken is to the left of the center, the greatest positive shear for a right-hand segment cannot be obtained with the locomotive loading headed toward the left — it must be headed toward the right. The necessity for determining such a shear does not arise.

2. When a locomotive loading is in the position to cause the greatest positive shear for a right-hand segment and the panel through which the section is taken is to the right of the center of the girder, there will be live load on that segment, since the load moves on from right to left. The criterion still holds true, but any such load on the segment is not in the panel cut by the section, and should not, therefore, be included in W_p, the load in that panel; it should, of course, be included in W, the total load on the girder. An example of this will be found in the illustrative problem in 399 : 1, in the calculations for the M-system of loading. In applying the criterion in this problem, the uniform live load in panel *ef* is included in the total load but not in the load in panel *de*.

3. It occasionally happens that when a section is taken through a panel to the left of the center, one or more loads will be on the segment to the left of the panel when the loading is in a position to satisfy the criterion. Such loads are included in W but not in W_p.

4. *Critical wheels.* Assume that engines are headed toward the left and that the greatest positive shear for a left-hand segment is required. For ordinary spans and panel lengths the critical wheel will be one of the first four or five, depending on the length of span, the panel length and the panel in which the section is taken. This critical wheel will be placed at the right-hand end of the panel in which the section is taken. Let the given section be in panel *fg* in Fig. 397 (*a*). The critical wheel O will be at *g*. The total load on the girder will be relatively small, hence, in order to satisfy the criterion, the load in panel *fg* must be relatively small, probably not more than one wheel. Let the given section be in panel *bc*. The critical wheel O will be at *c*. The total load on the girder will be relatively large, hence the load in panel *bc* must be relatively large, probably two or three wheels, possibly more. For a given girder with floor beams, the nearer the section is to the right-hand end, the nearer to the front of the engine will be the critical wheel.

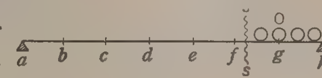


Fig. 397 (*a*).

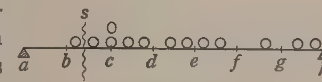


Fig. 397 (*b*).

As an illustration of an extreme range for the E-system, a truss of 10 panels of 25 ft. each may be taken. For a section in the first panel from the left-hand end the critical wheel is wheel 4, whereas for a section in the ninth panel, the critical wheel is wheel 1. Under certain conditions the criterion is satisfied by more than one wheel. In such a case it may be necessary to determine the shear first when one wheel and then the other is regarded as the critical wheel.

5. Assume that the engines are headed toward the left but that the greatest positive shear is required for a right-hand instead of a left-hand segment. Assume also that the panel through which the section is taken is to the right of the center of the girder. The critical wheel is placed at the left-hand end of that panel. This critical wheel must be one that will bring as much load as possible on the left-hand segment (because it is the longer segment), and, at the same time, as much weight as possible near the right-hand end of that segment. (Why?) Only a driver of the second group will satisfy these requirements, hence one of these drivers will be the critical wheel. An example of this is shown in Fig. 399 (*b*) of the illustrative problem. The section is through the panel *de*, the greatest positive shear is required for the right-hand segment, and the critical wheel (placed at *d*, the left-hand end of the panel *de*) is wheel 10, which is next to the last driving wheel of the M-system.

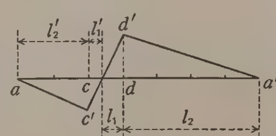


Fig. 397 (c).

6. *To select the critical wheel for greatest shear for a panel from the tables on page 389.* In Fig. 397 (*c*) is shown the influence diagram for shear for any section in the third panel *cd* of a girder with seven panels. That part of the influence diagram that corresponds to positive shear is the triangle *above* the base line. The seg-

ments of the base of this triangle are l_1 and l_2. The length of the segment l_1 is equal to the distance from d to the neutral point, and this distance l_1 may be calculated from $l_1 = \frac{n'}{n-1} p$. (298 : 4.) For this girder, l_1 is $\frac{4}{6} \times p$. The length l_2 is equal to the length of the right-hand segment da', but note that the length l_1 is *not* equal to the length of the left-hand segment ac. Having determined the values for l_1 and l_2, enter the table on page 389 with these values and read the number of the critical wheel in the usual way. The same general method may be used for any section in any panel.

1. When a number in the table is overlined, the panel through which the section is taken should be to the right of the center (unless it is a center panel). If the assumed section is through a panel to the left of the center, it should be changed to pass through the corresponding panel to the right of the center. (For reasons explained in 384 : 1.) The wheel that corresponds to the overlined number should always be placed at the left-hand end of the panel; the greatest positive shear will then be for the right-hand segment. (Why?) Note that an overlined wheel is always one of the second group of drivers as already explained.

2. *Illustration of the use of the tables on page 389.* Let the panel length in Fig. 397 (c) be 24 ft. (a) What wheel placed at d will be the critical wheel for positive shear for the panel cd? $l_1 = \frac{4}{6} \times 24 = 16$ ft.; $l_2 = 4 \times 24 = 96$ ft. Answers: Entering the tables with $l_1 = 16$ and $l_2 = 96$, the critical wheels are found to be wheel 4 for the M-system and wheel 3 for the E-system.

(b) What is the critical wheel for negative shear? $l_2' = 2 \times 24 = 48$ ft.; $l_1' = 8$ ft. Answers: For the M-system, wheel 8; for the E-system, wheel 3. Note that, with the locomotive headed toward the left, the section should be taken through the third panel from the right-hand end, and the critical wheel should be at the right-hand end of that panel.

(c) What is the critical wheel for shear for the center panel? $l_2 = 3 \times 24 = 72$; $l_1 = \frac{3}{6} \times 24 = 12$. Answers: For the M-system, wheel 10; for the E-system, wheel 3. (Wheel 10 should be placed at the left-hand end of the panel (398 : 1).)

(d) A girder 120 ft. long is divided into five equal panels. It is required to determine the greatest shear for a section through the second panel. The loading is M-50. What is the critical wheel? $l_1 = \frac{3}{4} \times 24$ ft. $= 18$ ft.; $l_2 = 3 \times 24$ ft. $= 72$ ft. For these values, the table gives wheel 10 which is overlined. The section should be taken, therefore, through the second panel from the right-hand end, and wheel 10 should be placed at the left-hand end of that panel. (398 : 1.)

3. *General method of determining the greatest shear for any section in a given panel.* The general method of procedure will be outlined first for the case in which the shear required is the greatest positive shear for a left-hand segment. It is assumed that the locomotive loading is headed toward the left.

First step: Determine the critical wheel either by means of the criterion (396 : 5) or from a table such as that on page 389. This wheel O will be at the right-hand end of the panel in which the section is taken. Determine the distance d of the last wheel Z on the beam from the right-hand end of the beam.

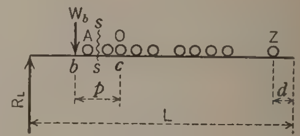


Fig. 398.

Second step: Calculate the reaction R_L:

$$R_L = [\mathrm{M(A\ldots Z)_Z} + \mathrm{W(A\ldots Z)} \times d] \div L. \quad (376:1.)$$

Third step: Calculate the floor-beam load W_b:

$W_b = \mathrm{M(A\ldots O)_O} \div p$. (The panel bc is treated as a beam supported at b and at c; this is equivalent to determining the reaction at the floor beam due to the loads on the stringer bc.)

Fourth step: Determine the shear for the section s–s:

$$V = R_L - W_b$$

Note: When the panels are unusually short, there may be a wheel load in the panel to the left of the panel in which the section is taken. If so, this wheel load as well as W_b should be subtracted from R_L in calculating the shear.

When the length of span is such that uniform load is on the beam, the method of procedure is the same except that the uniform load must be taken into account in calculating R_L as explained in 376 : 3.

4. It frequently happens that the greatest positive shear for the left-hand segment when the section is through a given panel to the left of the center of the girder is less than the greatest positive shear for the right-hand segment when the section is through the corresponding panel to the right of the center of the girder. (Locomotive loading headed toward the left in both cases.) The shear should then be calculated for the section through the panel to the right of the center, with the critical wheel placed at the *left-hand* end of that panel. (397 : 5.) The method of procedure is essentially the same as that outlined above, except that it is more convenient

to determine the shear for the right-hand segment. The forces acting on that segment are the right-hand reaction, the floor-beam load at the right-hand end of the panel cut by the section, and usually floor-beam loads at other panel points of the right-hand segment, due to live load on that segment. If a partial floor-beam load at the right-hand end of the panel cut by the section is determined wholly from the live load in that panel as if there were no other live load on the right-hand segment, all other live load on that segment may be considered as one force which, together with the partial panel load, is to be subtracted from the right-hand reaction in order to determine the shear for the right-hand segment. An example of this is the uniform load in Fig. 399 (*b*), treated as one force. (See note 399 : 2.)

1. *Illustrative problem in the greatest shear for a section anywhere through a given panel.* Given: A girder 100 ft. long with panels each 20 ft. long. Required: The greatest shear for a section through the second panel, first for E-60 and then for M-50. The notation used is that given in 372 : 7. Results are given in kips.

To ascertain the critical wheel from the table on page 389, it is necessary first to determine the lengths l_1 and l_2 (397 : 6). $l_1 = \frac{3}{4} \times 20 = 15$ ft., and $l_2 = 3 \times 20 = 60$ ft. The critical wheel for the E-system read from the table for $l_1 = 15$ and $l_2 = 60$ is wheel 3, placed at c. If no table was at hand, the criterion (396 : 5) may be applied as follows:

$$\begin{aligned} W(1\ldots3) &> W(1\ldots12) \div 5 \\ 75 &> 288 \div 5 \\ W(1\ldots2) &< W(1\ldots12) \div 5 \\ 45 &< 288 \div 5 \end{aligned}$$

The criterion is satisfied by wheel 3. (Show that it is not satisfied by either wheel 2 or wheel 4.)

$$\begin{aligned} R_L &= [M(1\ldots12)_{12} + W(1\ldots12) \times 4] \div 100 \\ 112.52 = R_L &= [\ \ 10{,}100 \ \ + \ \ 288 \times 4] \div 100 \\ 17.25 = W_b &= M(1\ldots3)_3 \div 20 = 345 \div 20 \\ 95.3 = V &= R_L - W_b = 112.52 - 17.25 \end{aligned}$$

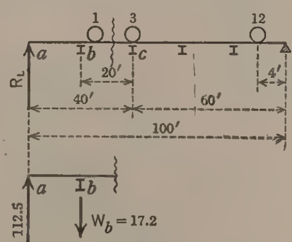


Fig. 399 (*a*).

For the M-system the critical wheel from the table on page 389 is wheel 10 with the shorter segment to the right. (Overlined.) The section is taken through the fourth panel, and wheel 10 is at d. (Fig. 399 (*b*).) If no table is at hand the criterion may be applied as follows:

$$\begin{aligned} [W(10\ldots11) + 2.5 \times 10] &> [W(1\ldots11) + 2.5 \times 30] \div 5 \\ [\ \ 62.5 + 25\ \ ] &> [\ \ 293.8 + 75\ \ ] \div 5 \\ [W(11) \ \ + 2.5 \times 10] &< [W(2\ldots11) + 75\ \ ] \div 5 \\ [\ \ 31.2 + 25\ \ ] &< [\ \ 281.3 + 75\ \ ] \div 5 \end{aligned}$$

The criterion is satisfied by wheel 10. Note that in applying the criterion, the uniform load in panel *ef* is included in the total load on the girder but not in the load in the panel *de*. (397 : 2.) Note also that when wheel 10 is exactly at d, wheel 1 is exactly at a, therefore when wheel 10 is considered in panel *de*, wheel 1 is on the girder, but when wheel 10 is considered ever so little to the left of d, wheel 1 moves off the girder. This is why in one case the total weight is $W(1\ldots11) + 75$ whereas in the other case it is $W(2\ldots11) + 75$. (388 : 1.)

$$\begin{aligned} R_R &= [M(11\ldots1)_1 + 2.5 \ \ \times 30 \times 85] \div 100 \\ 174.8 = R_R &= [\ \ 11{,}100 \ \ + \ \ 6375 \ \ ] \div 100 \\ W_e &= [M(11\ldots10)_{10} + 2.5 \times 10 \times 15] \div 20 + 2.5 \\ &\qquad \times 20 \div 2 \\ 51.5 = W_e &= [\ \ 156 \ \ + \ \ 375 \ \ ] \div 20 + 25 \\ 25.0 = W_f &= 2.5 \times 20 \div 2 \\ 98.3 = V \ &= R_R - W_e - W_f = 174.8 - 51.5 - 25 \end{aligned}$$

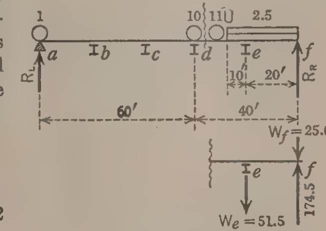


Fig. 399 (*b*).

Note that in obtaining the shear for the shorter segment, the floor-beam load at f was included in the forces that were subtracted from R_R. (Why?)

The criterion is satisfied also by wheel 3 and by wheel 4 of the M-system, with the shorter segment to the left (section between b and c), but with either of these two wheels at c, the shear is 94,000 lbs. or less than that obtained from wheel 10 at d.

Exercise: Check the shear 98,300 lbs. for wheel 10 by calculating the shear for the longer (left-hand) segment.

2. *Note.* For the M-system, the shear was determined by subtracting from R_R the actual floor-beam concentrations at e and f. It might have been determined by the method suggested in 398 : 4 as follows: Calculate the portion of the load at e due to the live load in the panel $de = W_e' = [M(1\ldots10)_{10} + 2.5 \times 10 \times 15] \div 20 = 26.5$. Calculate the load in panel $ef = 2.5 \times 20 = 50$. The shear $= R_R - W_e' - 50 = 174.8 - 26.5 - 50 = 98.3$.

3. **Girders with Floor Beams — Greatest Bending Moment for a Given Section.** *The greatest bending moment for a section exactly at a panel point* is determined as if there were no floor beams by the method explained in 387 : 1 to 390 : 2. When the panels are of equal length, as they usually are, the criterion $W_S = W\dfrac{S}{L}$ may be used in the following form:

$$W_S = W\frac{n'}{n}$$

W_S = total load on either segment; S = length of that segment; W = total load on the girder; n' = number of panels in the segment, and n = number of panels in the length L of the girder.

When either the M-system or the E-system of loading is used, the critical wheel may be determined by means of one of the tables on page 389. The segments l_1 and l_2 of the base of the influence triangle will be equal to the corresponding lengths of the segments of the girder or truss, just as in the case of a simple beam without floor beams. (388 : 4.)

1. *When the section lies between two adjacent panel points*, as for example the section at s in Fig. 400 (a), the bending moment is the resultant moment of the reaction and all floor-beam loads on either segment. For example, for a given position of the loading on the girder ag the bending moment for the segment as is $M_B = R_L \times S - W_b \times bs - W_c \times cs$, in which W_b and W_c are floor-beam loads. It can be shown by a method similar to that used for shear in 396 : 4, that the bending moment thus obtained is not the same as that which would be obtained by assuming the wheels to act directly on the girder as if it were a beam without floor beams.

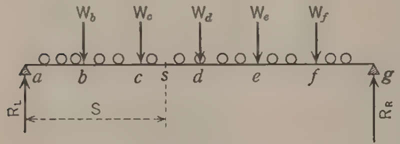


Fig. 400 (a).

2. *Criterion for greatest bending moment.* When the loading is in the position to cause the greatest bending moment for a section between two adjacent panel points, the critical wheel will be at one of these panel points, but it is not always evident which should be selected. It may be necessary, therefore, to try a wheel at each of the panel points. The criterion will be given first on the assumption that the critical wheel is at the right-hand end of the panel and then on the assumption that it is at the left-hand end.

3. Let it be required to determine the position of the loading which will cause the greatest bending moment for a section at s between the panel points c and d in Fig. 400 (b). Assume first that the critical wheel O is at d. Let F be the wheel in panel bc nearest to c, and G the wheel in panel cd nearest to c. If O is the critical wheel, the following criterion will be fulfilled (notation that of 372 : 7):

$$\begin{array}{l} \mathrm{W(A\ldots F)} + \mathrm{W(G\ldots O)}\dfrac{k}{p} > \\ \mathrm{W(A\ldots F)} + \mathrm{W(G\ldots N)}\dfrac{k}{p} < \end{array} \mathrm{W(A\ldots Z)}\frac{S}{L}$$

4. When the section s is at the center of the panel, $k = \frac{1}{2}p$. Letting n' and n represent the number of panels in S and L respectively, the criterion becomes:

$$\begin{array}{l} \mathrm{W(A\ldots F)} + \frac{1}{2}\mathrm{W(G\ldots O)} > \\ \mathrm{W(A\ldots F)} + \frac{1}{2}\mathrm{W(G\ldots N)} < \end{array} \mathrm{W(A\ldots Z)}\frac{n'}{n}$$

In Fig. 400 (b), $\dfrac{n'}{n} = 2\frac{1}{2} \div 6 = \frac{5}{12}$.

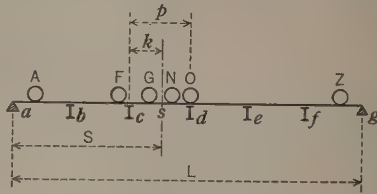


Fig. 400 (b).

5. Assume now that the critical wheel is at c as shown in Fig. 400 (c). Let P and T be the wheels in panel cd nearest to c and d respectively. If O is the critical wheel, the following criterion will be fulfilled.

$$\begin{array}{l} \mathrm{W(A\ldots O)} + \mathrm{W(P\ldots T)}\dfrac{k}{p} > \\ \mathrm{W(A\ldots N)} + \mathrm{W(O\ldots T)}\dfrac{k}{p} < \end{array} \mathrm{W(A\ldots Z)}\frac{S}{L}$$

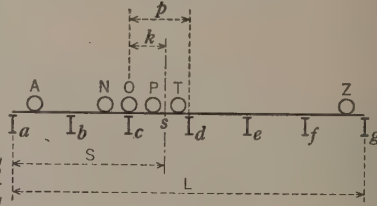


Fig. 400 (c).

When the section s is at the center of the panel the criterion becomes:

$$\begin{array}{l} \mathrm{W(A\ldots O)} + \frac{1}{2}\mathrm{W(P\ldots T)} > \\ \mathrm{W(A\ldots N)} + \frac{1}{2}\mathrm{W(O\ldots T)} < \end{array} \mathrm{W(A\ldots Z)}\frac{n'}{n}$$

6. When uniform train load is on the girder, its total weight should be added to the weight of all of the wheels on the beam before multiplying by $\dfrac{S}{L}$ or $\dfrac{n'}{n}$, so that the right-hand side of each inequality becomes $(\mathrm{W(A\ldots Z)} + wl)$ multiplied by $\dfrac{n'}{n}$ or $\dfrac{S}{L}$ according to whether the center of moments is or is not at the center of the panel. (w is the weight per linear foot and l the length of the uniform load.)

7. The criterion for the greatest bending moment for a section between panel points is frequently given in the following form:

$$W' + W_p\frac{k}{p} = W\frac{S}{L}$$

W' = total load to the left of the panel in which the section is taken; W_p the load on the panel in which the section is taken; W = total load on the girder including uniform load if there is any. Although the criterion is expressed as an equation, it is used as if it were expressed in the form $W' + W_p\frac{k}{p} \gtrless W\frac{S}{L}$ according to whether the critical wheel is included in W_p or in W'. (Show that the criterion, when expressed as an equation is equivalent to stating that the average load to the left of the section is equal to the average load on the whole beam.) This criterion will be derived later. Note that when the section is at the center of a panel the criterion becomes:

$$W\frac{S}{L} = W' + \tfrac{1}{2}W_p.$$

The criterion may be satisfied when more than one wheel is regarded as the critical wheel. In such a case it may be necessary to determine the bending moment when first one and then the other of the critical wheels is placed in position.

1. *General method of determining the greatest bending moment for any section between two successive panel points.* The general method of procedure may be outlined as follows:

First step: From the criterion (400 : 7) determine the critical wheel. This wheel will be at one end or the other of the panel in which the section is taken, and its position will determine which are the first and the last wheels on the girder, i.e., which are wheels A and Z.

Second step: With the loading in the position as determined by the critical wheel, calculate the reaction R_L at the end of the girder. (376 : 1.)

Third step: Calculate the bending moment. The calculation will be indicated here for the condition that the critical wheel is at the right-hand end of the panel.

Let the section be anywhere in the panel cd in Fig. 401 a distance k from c. When the critical wheel is at d the load W'_c at c due to all of the loads in the panel cd is: $W'_c = \mathrm{M(G\ldots O)_O} \div p$.

The bending moment for the segment as is:

$$\mathrm{M_B} = R_L \times S - \mathrm{M(A\ldots F)_s} - W'_c \times k$$
$$= R_L \times S - [\mathrm{M(A\ldots F)_F} + \mathrm{W(A\ldots F)} \times h] - [\mathrm{M(G\ldots O)_O} \div p] \times k$$

When the critical wheel is at c instead of at d, the method of procedure is practically the same, except that in determining the load W'_c it may be more convenient to assume the center of moments at c.

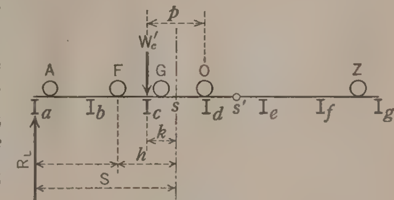


Fig. 401.

2. When uniform load is on the beam the reaction is calculated by the method explained in 376 : 3, but with this exception, the method of calculating the bending moment is the same as that just given.

3. Let s' be as far from the center of the span as is s. It frequently happens that a greater bending moment may be obtained for a section at s' than for one at s. (Engine headed toward the left in both cases.)

4. When the panel is a center panel and there is an odd number of panels, it may be necessary to determine the bending moment, first when a critical wheel is at one end of the center panel, and then when some other critical wheel is at the other end; there may also be more than one critical wheel for either end of the center panel.

5. *Bending moment for a center of moments anywhere in a vertical line through the center of a panel.* The criterion for the critical wheel is the last one given in 400 : 7. When the load has been placed in position, the bending moment is determined as follows: (1) The bending moment is calculated for a center of moments at one end of the panel; (2) the bending moment is calculated (with the load in the same position) for a center of moments at the other end of the panel; (3) the required bending moment is the average of the two bending moments obtained in (1) and (2). (395 : 7.)

The method just given will be explained more in detail in the next chapter where it is used in determining the stress in a lower chord member of a through Warren truss. (419 : 1.)

6. **Maximum Reaction at a Floor Beam.** Two stringers in the same longitudinal line, one in one panel the other in an adjacent panel, are connected to a floor beam at the same point. It is required to determine the maximum concentrated load that the two stringers can bring to that point due to a locomotive loading extending over the two panels. This live-load concentration is equal and opposite to the sum of two

reactions, namely, the reaction R_R (Fig. 402 (*a*)) at the right-hand end of the left-hand stringer *ab* and the reaction R'_L at the left-hand end of the right-hand stringer *bc*, or if R_F represents the total reaction at the floor beam: $R_F = R_R + R'_L$.

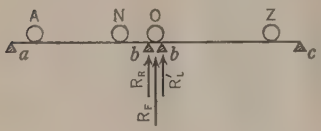


Fig. 402 (*a*).

1. *Critical wheel.* It can be demonstrated that the total reaction R_F will be a maximum when the locomotive loading is in a position to cause the greatest bending moment for a section at the point *b*, provided the double panel length *ac* is treated as if it were a simple beam supported only at *a* and at *c*. This means that it will occur when some wheel (critical wheel) is at *b*, and that this critical wheel may be selected by means of the criterion in 387 : 2. When the E-system or the M-system is used, the critical wheel may be read from one of the tables on page 389. (l_1 will equal the length of one panel and l_2 the length of the other panel; in the usual case of equal panels $l_1 = l_2$.)

2. *General methods of determining the maximum reaction at a floor beam.* The first step is to determine the critical wheel O, to place this wheel at the floor-beam point *b*, and then to determine which wheel (A) is on the stringer *ab* nearest to *a*, and which wheel (Z) is on the stringer *bc* nearest to *c*; also the distance *d* from the wheel A to *a* and the distance *d′* from the wheel Z to *c*. (Fig. 402 (*b*).) This having been done, there are three methods of procedure any one of which may be followed: Using the notation of 372 : 7 these methods may be outlined as follows:

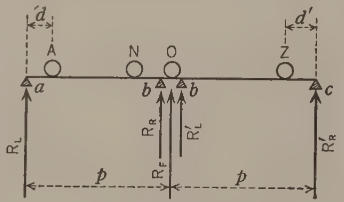


Fig. 402 (*b*).

First method

3. Determine the reactions R_R and R'_L and add them together. These two reactions may be determined by the methods of 376 : 5 and 376 : 1.

$$
\begin{aligned}
R_R &= \mathrm{W(A\ldots N)} - R_L = \mathrm{W(A\ldots N)} - \mathrm{M(A\ldots O)_O} \div p \\
R'_L &= \mathrm{W(O\ldots Z)} - R'_R = \mathrm{W(O\ldots Z)} - \mathrm{M(Z\ldots O)_O} \div p \\
R_F &= R_R + R'_L
\end{aligned}
$$

4. *Notes:* (*a*) The wheel load O may be considered as on either stringer but not on both. In the equations just given it was considered as on stringer *bc*. (*b*) The reaction R_R may also be calculated directly from a moment equation with *a* as a center, and the reaction R'_L from a moment equation with *c* as a center.

Second method

5. It can be demonstrated that the reaction R_F in Fig. 402 (*b*) is equal to twice the bending moment for a section at *b* divided by the panel length *p* or:

$$R_F = 2\,\mathrm{M_{B_b}} \div p$$

when the double panel length *ac* is treated as a simple beam supported only at *a* and at *c*.

$$R_F = \left[2 \times \left(\frac{\mathrm{M(A\ldots Z)_Z} + \mathrm{W(A\ldots Z)} \times d'}{2\,p} \times p\right) - 2 \times \mathrm{M(A\ldots O)_O}\right] \div p$$

Third method

6. From the equation just given, it is evident that R_F is equal to the moment of all of the wheels on both panels about *c* as a center minus twice the moment of the wheels on panel *ab* about *b* as a center, the whole divided by the panel length *p* or:

$$R_F = [\mathrm{M_c} - 2\,\mathrm{M_b}] \div p$$

in which $\mathrm{M_c}$ is the moment of all of the wheels in both panels from *a* to *c* about *c* as a center, and $\mathrm{M_b}$ is the moment of all the wheels in panel *ab* about *b* as a center.

$$
\begin{aligned}
R_F &= [\mathrm{M(A\ldots Z)_c} - 2 \times \mathrm{M(A\ldots O)_O}] \div p \\
&= [\mathrm{M(A\ldots Z)_Z} + \mathrm{W(A\ldots Z)} \times d' - 2 \times \mathrm{M(A\ldots O)_O}] \div p
\end{aligned}
$$

This last expression is obviously equivalent to the expression for the second method but it is simpler.

7. *Maximum reaction at a floor beam when the panels are unequal.* Assume that the two panels in Fig. 402 (*b*) are unequal. Let l_1 and l_2 represent respectively the lengths *ab* and *bc*. Let $\mathrm{M_{B_b}}$ represent the greatest bending moment for a section at *b* when the two panels are treated as a simple beam equal in length to $l_1 + l_2$, and supported only at *a* and at *c*. Then

$$R_F = \mathrm{M_{B_b}}\left(\frac{l_1 + l_2}{l_1 l_2}\right). \quad \text{(Prove.)}$$

1. *Maximum live load transmitted by a floor beam.* The maximum live-load concentration, equal and opposite to the total reaction R_F at the floor beam, is also the *maximum* live load which the floor beam can transmit to a girder or a truss of a single track bridge, i.e., it is the maximum **floor-beam load** at a panel point or the maximum **panel concentration.**

2. *Note:* When a pier supports the ends of trusses in two adjacent spans, the **pier reaction** is equal to the reaction at the right-hand end of the left-hand truss plus the reaction at the left-hand end of the right-hand truss. This pier reaction can be determined by methods similar to those just given for floor-beam reactions.

3. *Illustrative problem in maximum reaction at a floor beam.* Given: A railroad bridge in which the panel length is 25 ft. Required: The maximum reaction at a floor beam, first for E-60 and then for M-50. The notation is that given in 372 : 7. Bending moments are given in thousands of pound-feet and reactions in kips.

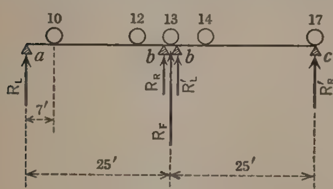


Fig. 403 (*a*).

For the E-system the critical wheel for $l_1 = 25$ and $l_2 = 25$, from the table on page 389, is wheel 4 — use wheel 13. If no table were at hand, the criterion would be applied as follows:

$$W(10\ldots13) > W(10\ldots17)\tfrac{25}{50}$$
$$105 > 193.5\times\tfrac{1}{2}$$
$$W(10\ldots12) < W(10\ldots16)\tfrac{1}{2}$$
$$75 < 174\times\tfrac{1}{2}$$

The criterion is satisfied for 13. If applied to wheels 12 and 14 it would be found that it is not satisfied by either of these wheels. Note that when wheel 13 is at b, wheel 17 is exactly at c, therefore when wheel 13 is considered in panel ab, wheel 17 is considered in panel bc, but when wheel 13 is considered to have moved ever so little into panel bc, wheel 17 moves off the beam. This is why in one case the total weight is taken as W(10...17), whereas in the other case it is W(10...16). (388 : 1.)

First method. (402 : 3.)

$$R_R = [M(12\ldots10)_{10} + W(12\ldots10)\times7]\div25$$
$$46.2 = R_R = [\quad 630 + \quad 75\times7]\div25$$
$$67.2 = R'_L = M(13\ldots17)_{17}\div25 = 1680\div25$$
$$113.4 = R_F = R_R + R'_L = 46.2 + 67.2$$

Second method. (402 : 5.)

Consider ac as a simple beam supported only at a and at c, and R'_L the reaction at a due to all loads between a and c. Let M_B represent the bending moment for a section at b.

$$85.6 = R'_L = M(10\ldots17)_{17}\div50 = 4280\div50$$
$$1420 = M_B = R_L\times25 - M(10\ldots13)_{13} = 85.6\times25 - 720$$
$$113.6 = R_F = 2\times M_B\div p = 2\times1420\div25$$

Third method. (402 : 6.)

Let M_c represent the sum of the moments of all loads about c and M_b the sum of the moments about b of all loads to the left of b.

$$113.6 = R_F = (M_c - 2M_b)\div p = [M(10\ldots17)_{17} - 2M(10\ldots13)_{13}]\div25 = (4280 - 2\times720)\div25$$

For the M-system the critical wheel for $l_1 = 25$ and $l_2 = 25$, from the table on page 389, is wheel 9. If no table were at hand, the criterion would be applied as follows:

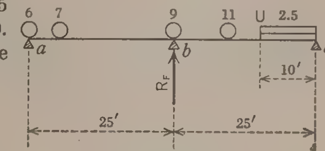


Fig. 403 (*b*).

$$W(7\ldots9) > [W(7\ldots11) + 2.5\times10]\times\tfrac{25}{50}$$
$$93.8 > [156.3 \quad + \quad 50 \quad]\times\tfrac{1}{2}$$
$$W(6\ldots8) < [W(6\ldots11) + 2.5\times10]\times\tfrac{25}{50}$$
$$87.5 < [\quad 181.3 \quad + \quad 50 \quad]\times\tfrac{1}{2}$$

The criterion is satisfied by wheel 9. Note how both sides of the inequality are affected by the fact that when wheel 9 is at b, wheel 6 is exactly at a. Using the third method:

$$M_c = M(6\ldots11)_U + W(6\ldots11)\times10 + 2.5\times10\times5$$
$$5278 = M_c = \quad 3340 \quad + \quad 181.3\times10 + 125$$
$$1090 = M_b = M(6\ldots9)_9$$
$$123.9 = R_F = (M_c - 2M_B)\div p = (5278 - 2\times1090)\div25$$

4. *Exercise:* Show that the same result will be obtained for R_F if, in calculating M_c and M_b, wheel 6 is considered as not on the panel ab and therefore not to be included among the forces.

5. **The Derivation of Various Criteria.** Thus far in this chapter, none of the various criteria has been derived. A general method of derivation will now be given:

First step: Determine the algebraic expression for shear or for bending moment, as the case may be, in terms of groups of loads as, for example, in terms of W the total load on the girder, W' the total load on the girder to the left of some critical point such as the center of moments or the end of a panel in which the section is taken, and W_p, the load in the panel in which the section is taken.

Second step: Consider that the load groups are moved an indefinitely small distance d. Determine the algebraic expression for the shear or for the bending moment for the new position. This expression will differ from the original expression principally in the lengths of lever arms, each of which will be changed by an amount equal to d.

Third step: Determine a third expression by subtracting the original expression in the first step from the expression determined in the second

step. The result will represent the change in the shear or in the bending moment due to the small movement of the loading.

Fourth step: Divide the third expression by d. The result will be the *rate of change per unit of length.* If this rate of change is equated to zero, the equation will be the criterion for the maximum shear or bending moment, as the case may be. (Why?)

1. *Criteria determined by means of calculus.* The method just given may be carried out more simply by means of the calculus. Determine the algebraic expression for shear or for bending moment, as the case may be, in terms of groups of loads, representing by x, x', x'' and so on every lever arm that will be changed by a slight movement of the loading. Make the first differential of the expression equal to zero. (If there is more than one variable, dx', dx'', and so on will equal dx because all loads are moved the same distance.) The result will be the criterion.

2. *Illustrative example. Criterion for greatest bending moment for a given section of a simple beam.* Let W represent the total load on the beam in Fig. 404 (a) and W_S the total load to the left of the section s.

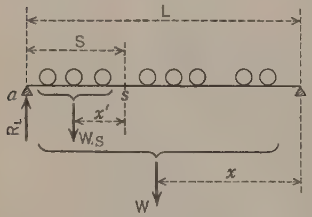


Fig. 404 (a).

$$M_B = R_L \times S - W_S \times x' = W\frac{x}{L} \times S - W_S \times x'$$

Move the loading an indefinitely small distance d to the left.

$$M_B = W\left(\frac{x+d}{L}\right) \times S - W_S \times (x'+d)$$

Subtract the first expression for bending moment from the second, divide the result by d and equate to zero:

$$W\frac{S}{L} - W_S = 0 \quad \text{or} \quad \frac{W}{L} = \frac{W_S}{S} \quad \text{(The criterion given in 387: 2.)}$$

By calculus:

$$M_B = W\frac{x}{L} \times S - W_S \times x'$$

$$\frac{dM_B}{dx} = W\frac{S}{L} - W_S = 0.$$

3. *Illustrative example. Criterion for greatest bending moment for any section between two successive panel points.* Let W represent the total load on the girder in Fig. 404 (b), W_p the total load in the panel cd in which the section is taken, and W' the total load to the left of that panel. Let W_c be the partial floor-beam load at c due to all loads in the panel cd. Let p represent the length of the panel cd. The bending moment for a section at s is:

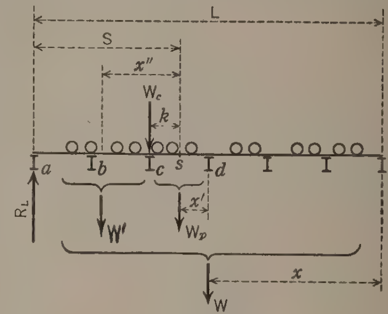


Fig. 404 (b).

$$M_B = W\frac{x}{L} \times S - W' \times x'' - W_c \times k$$

or

$$M_B = W\frac{x}{L} \times S - W' \times x'' - W_p\frac{x'}{p} \times k$$

Move the loading an indefinitely small distance d to the left.

$$M_B = W\frac{x+d}{L} \times S - W' \times (x''+d) - W_p\left(\frac{x'+d}{p}\right) \times k$$

Subtract the first expression for bending moment from the second, divide the result by d and equate to zero:

$$\frac{W}{L} \times S - W' - W_p\frac{k}{p} = 0, \quad \text{or} \quad W' + W_p\frac{k}{p} = W\frac{S}{L}$$

(The criterion given in (400 : 7).)

By calculus:

$$M_B = W\frac{x}{L} \times S - W' \times x'' - W_p\frac{x'}{p} \times k$$

$$\frac{dM_B}{dx} = W\frac{S}{L} - W' - W_p\frac{k}{p} = 0, \quad \text{or} \quad W' + W_p\frac{k}{p} = W\frac{S}{L}$$

4. *Note:* When the student has become familiar with the use of the various criteria given in this chapter he should derive or verify them by methods similar to those just explained. (See assignment at the end of the chapter.)

1. **Equivalent Uniform Live Load.** Shears, bending moments, and stresses are determined more easily, as a general rule, when the live load is uniformly distributed than when it is a series of concentrated loads. If a uniform load can be determined which, under given conditions, will give results approximately the same as those obtained from a given system of concentrated live loads, the work of calculating shears, bending moments and stresses may be simplified by using this **equivalent uniform live load** in place of the system of concentrated loads.

2. For any given system of concentrated live loads, an equivalent uniform live load may be determined for any given span that will cause the same maximum shear or bending moment for a given section or the same stress in a given member of the structure as that caused by the given system of concentrated loads. There would be no advantage, however, in such a uniform load, unless it could be used in place of the concentrated loads in determining shears and bending moments for other sections, or stresses in other parts of the structure. There can be no single uniform live load which will give exactly the same results for all sections or members as the system of concentrated live loads, but the errors involved in the use of a single equivalent uniform load in determining all shears, bending moments, or stresses in a given beam, girder, or truss are often negligible. Several methods of determining equivalent uniform loads will now be given.

3. *Equivalent uniform live load determined from bending moments.* For a girder or truss with floor beams, the centers of moments for bending moments are taken, with few exceptions, at panel points, and the bending moments are determined exactly as for a simple beam without panels. Let l_1 and l_2 represent the lengths of the two segments of a simple beam formed by any section, and w the uniform live load per linear foot. The greatest bending moment due to the uniform live load occurs when it covers the whole beam and is equal to:

$$\frac{w}{2} \times l_1 \times l_2. \quad (167:1.)$$

4. The greatest bending moment for any section of any simple beam may be determined for any given system of concentrated live loads by the methods of 390 : 2 and its value substitute for M_B in the equation just given. The section determines the lengths l_1 and l_2, hence the only unknown in the equation is w; the value of w found by solving the equation is the uniform load per linear foot that will cause the same greatest bending moment for a given section of a given beam as that caused by the given system of concentrated loads.

5. Let it be desired to determine the equivalent uniform live load for E-60 from the bending moment for a section at the quarter point of a beam 80 ft. long. In the illustrative problem in 390 : 9, the greatest bending moment for a section 20 ft. from the end of a beam 80 ft. long was found to be 2,474,000 lb.-ft.

$$2{,}474{,}000 = \frac{w}{2} \times 20 \times 60$$

Solving for w, the equivalent uniform load that will cause the same bending moment for the section at the quarter point is found to be 4120 lbs. per linear foot.

6. *Exercise:* Show that for M-50 the corresponding equivalent uniform live load is 4520 lbs. per linear foot.

7. The bending moment at the center of the same beam due to the E-60 loading is 3,246,000 lb.-ft.

$$3{,}246{,}000 = \frac{w}{2} \times 40 \times 40$$

Solving for w, the equivalent uniform load that will cause the same bending moment for the section at the center is found to be 4060 lbs.

8. *Exercise:* Show that for M-50 the corresponding equivalent uniform live load is 4310 lbs. per linear foot.

9. *Equivalent uniform live load determined from shear.* Let x represent the distance from the left-hand end of a simple beam to a given section, V the greatest shear for that section due to a given system of concentrated live loads, w the equivalent uniform live load per linear foot, and L the length of the beam. The following equation may be easily verified:

$$w = \frac{2\,VL}{(L - x)^2}$$

The greatest shear for the given section may be determined for any given system of concentrated loads by the method of 385 : 7, and its value substituted for V in the equation. The value of w, found by solving the equation is the uniform load per linear foot that will cause the same shear for the given section as that caused by the given system of concentrated loads.

1. Let n represent the number of panels in a truss, p the panel length, V the greatest shear in the *first* panel due to a given system of concentrated loads, and w the equivalent uniform load per linear foot. The greatest shear for any section in the first panel due to uniform load will occur when the truss is fully loaded, a condition that exists when maximum stresses occur in chord members; this shear is equal to the reaction or $\frac{1}{2}\, w(n - 1) \times p$. Equating this to V and solving for w:

$$w = \frac{2\,\mathrm{V}}{(n - 1)\,p}$$

The greatest shear for the end panel may be determined for any given system of concentrated loads by the method of 398 : 3 and its value substituted for V in the equation. The value of w found by solving the equation is the uniform load per linear foot that will cause the same shear for a section in the first panel as that caused by the given system of concentrated loads.

2. *Three different methods of determining equivalent uniform live load compared.* Any equivalent uniform live load as determined by any one of the methods just explained is not exactly equivalent to the corresponding system of concentrated loads except in producing the greatest bending moment or shear, as the case may be, for the section used in determining that equivalent load. It is important to know whether the results obtained from the equivalent load for other sections are greater or smaller than those obtained for the same sections from the concentrated loads. If greater, they are on the side of safety, otherwise they are not. The three most common methods of determining equivalent uniform live loads are (1) from the bending moment for a section at the center (405 : 5), (2) from the bending moment for a section at a quarter point (405 : 7), and (3) from the shear for a section anywhere in the first panel. (406 : 1.) The results obtained by each of these three methods will now be compared with the results obtained from the use of locomotive loading.

3. The equivalent uniform live load determined from the bending moment for a section at the center gives somewhat smaller shears and bending moments except for sections near the center of the span.

4. The equivalent uniform live load determined from the bending moment for a section at the quarter point gives approximately the same bending moments for sections near the center of the span but smaller bending moments for sections near the ends of the span; also negative shears that are nearly the same but positive shears that are smaller, particularly for sections near the end of the span.

5. The equivalent uniform live load determined from the shear in the end panel gives approximately the same bending moments for sections near the end of the span, but greater bending moments for sections near the center; also approximately the same positive shears for sections near the end of the span, but greater negative shears. It is to be noted that of the three methods, this is the only one in which all results are either approximately the same or greater than those obtained by the concentrated load, and hence it is the method in which all results are on the side of safety.

6. *Empirical formulas for equivalent uniform load.* In order to obtain a greater agreement between results from equivalent uniform load and those from the corresponding system of concentrated load, an empirical formula may be used for determining the equivalent load. The best of such formulas are based on two variables, namely, l_1 and l_2, the lengths of the segments of the base of the influence triangle for shear or bending moment as the case may be. (388 : 1 and 2; also page 389.)

7. An example of such an empirical formula is that proposed by D. B. Steinman in the *Transactions of the American Society of Civil Engineers, Vol. LXXXVI*, page 635. The proposed loading formula is $q = 833 + 10{,}000\,\dfrac{\sqrt{l_2}}{l_1 + l_2}$ for M-10 loading. (l_1 = shorter segment of the base of the influence triangle; l_2 = longer segment of the base of the influence triangle; q = equivalent uniform load for the values of l_1 and l_2.) For any other M-loading the values of the constants are increased in proportion to the increased axle loads. For example, for M-50 loading:

$$q = 4166 + 50{,}000\,\frac{\sqrt{l_2}}{l_1 + l_2}.$$

1. *Use of equivalent uniform live load.* Tables or diagrams may be prepared for any loading that is frequently used, such as the E-system or the M-system, from which may be read the equivalent uniform loads for different lengths of spans and for different values of l_1 and l_2. By the use of such tables the work of calculating shears, bending moments, or stresses is reduced to a minimum. Equivalent uniform load is used more for long girders and for trusses than for short beams. There is a difference of opinion among engineers concerning the extent to which equivalent load should be used. This will be discussed in the next chapter.

2. **TABLES OF REACTIONS, SHEARS, AND BENDING MOMENTS.** For systems of concentrated loads in common use, various tables may be found, in structural engineers handbooks and elsewhere, that give for various lengths of span and for trusses with varying lengths and numbers of panels quantities that have been calculated by methods explained in this chapter. These quantities include maximum reactions at the supports, maximum floor-beam reactions and pier reactions, greatest shear for sections at the center and quarter points of simple beams, maximum end shear, maximum bending moments in beams and girders without floor beams, greatest shears for different panels of trusses, and greatest bending moments for centers of moment at different panel points.

IMPACT. The impact allowance for stringers and other short beams is sometimes specified as a definite percentage, but usually the impact percentage is determined by means of an empirical formula. (267 : 8.) For locomotive loadings headed toward the left, the loaded length L in the impact formula is the distance from the right-hand support to the first wheel at the head of the loading, unless this wheel has passed the left-hand support, in which case the length L is the length of span. In either case, L is taken to the nearest foot.

Approximate loaded length. Useless refinements in determining the loaded length should be avoided. For example, in determining the impact allowance for bending moment for a short beam such as a stringer, the loaded length is usually taken as the length of beam, even though the first wheel has not reached the left-hand support. Similarly, in determining the impact allowance for a maximum reaction at a floor beam, (401 : 6), the loaded length of stringers may be taken as the sum of the lengths of the two panels, or, in the case of equal panels, twice the panel length. In calculating the impact allowance for bending moment for any segment of a girder with floor beams, it is usually sufficiently accurate to use the length of span as the loaded length.

The actual loaded length from the right-hand support to the first wheel (if that wheel has not passed the left-hand support) is generally used in the following cases: (*a*) Impact for shear for a given segment of a beam with or without floor beams. (*b*) Impact for bending moment for a segment of a deck plate girder.

ASSIGNMENTS

(1) Prove that the bending moment for a section exactly at any panel point of a girder with floor beams may be obtained exactly as for a beam without floor beams, i.e., as if the wheel loads were applied directly to the top of the girder.

(2) Prove that the second and third methods of determining the maximum reaction at a floor beam (402 : 5 and 6) are each equivalent to the first method.

(3) Using the general method of 403 : 5, derive each of the criteria used in this chapter for determining critical wheels.

(4) Report on the method of determining criteria by means of influence lines.

(5) Report on methods of determining pier reactions for several spans.

(6) Report on the different methods of determining equivalent uniform live load for beams without panels, and on tables and diagrams for such equivalent load.

(7) Report on the distribution of typical truck loads to the floor system of a highway bridge.

(8) Report on the paper on "Locomotive Loadings for Railway Bridges" by D. B. Steinman and on the discussion on this paper in the *Transactions of the American Society of Civil Engineers, Vol. LXXXVI.*

CHAPTER XXIII

CONCENTRATED LIVE LOADS

STRESSES IN BRIDGE TRUSSES

In this chapter are explained methods of determining stresses in standard types of bridge trusses due to conventional systems of moving concentrated loads, particularly stresses in railway-bridge trusses due to systems of locomotive loadings.

1. **Concentrated Live Loads for Trusses.** *Highway bridges.* The simplest form of concentrated loading for trusses of a highway bridge is a single concentrated load which may be placed anywhere on the truss. A common form of specification for loading is a total load per lane of traffic composed of a uniform load per linear foot and a single concentrated load, as, for example, a total load on each traffic lane composed of a uniform load of 600 pounds per linear foot and a single concentrated load of 28,000 pounds. The stresses due to uniform load are determined by the methods explained in Chapters XX and XXI. In order to determine the stress in any member due to the concentrated load, the load is first placed at the point in the truss where it will cause the maximum stress in that member. For example, if the stress is that in a chord member, the concentrated load is placed where it will cause the greatest *bending moment* for the corresponding segment (method of sections), i.e., at the center of moments for the chord member; if the stress is that in a web member, the concentrated load is placed where it will cause the greatest *shear* for the segment, usually at the joint of the loaded chord immediately to the right or to the left of the section. The concentrated load may be conceived as an excess load which may be moved along the top of the uniform load to the critical position for the maximum stress in a member. Once it is placed in this position the stress which it causes in the member may be calculated most readily by the method of sections.

2. Another form of loading for highway bridges is composed of two concentrated loads a fixed distance apart, followed by, or preceded by, or both followed and preceded by a uniform live load.

3. Whether the concentrated live load is a single load or two loads, the general methods of determining maximum shears and bending moments are those explained in Chapter XIX.

4. To provide for stresses caused by electric cars crossing a bridge, a system of eight concentrated loads is usually specified. (369 : 4.) The methods of determining stresses due to locomotive loads, explained in this chapter, are readily adapted to the calculation of stresses due to electric-railway wheel loads. For this reason only locomotive loadings will be considered in the remainder of the chapter.

5. *Railway bridges.* The concentrated live loads for trusses of railway bridges are those which correspond to axle loads of locomotives. The E-system and the M-system used for illustrative examples in the preceding chapter will also be used in this chapter.

6. **Impact.** In combining dead- and live-load shears or stresses in order to determine the actual stress in a counter, or in studying the effect of counters on other members, the impact allowance or the specified increase in the live load should be added to or included in the live-load stress. (265 : 6.)

In determining the impact stress in any member due to the movement of engines and cars, the loaded length L in the impact formula (267 : 7 to 268 : 7) may be taken as follows: For any chord member, L is equal to

the length of span; for any web member, L is equal to the distance from the first left-hand wheel to the right-hand end of the span when the load is in the position to cause the live-load stress in that member. (268 : 2.)

1. **Uniform Live Load and Locomotive Loading Compared.** In calculating stresses due to the conventional system of uniform live load, it is assumed that the loads brought by floor beams to joints of the loaded chord of a truss are equal. For locomotive loadings these panel loads are unequal. This unequal loading makes it more difficult to determine the position of the loading for the maximum stress in a given member. Moreover, in the use of uniform live load, the panel load is determined once for all, whereas for locomotive loading, the panel loads vary as the position of the loading changes, and it is frequently necessary to determine one or more panel loads that are caused by the loading in different positions. The three steps in the calculation of stresses due to locomotive loading are: (1) Determination of the position of the loading. (2) Determination of external forces on the truss due to the loading in the position determined in (1). (3) Determination of the stresses due to the external forces determined in (2). It is the first of these steps that will require the most consideration. The second and third steps, though more complex than the corresponding steps in the calculation of stresses due to uniform live load, are essentially the same in principle.

2. **Notation.** The methods of indicating shear and bending moments used in the preceding chapter will be followed in this chapter, and the same symbols will be used. (372 : 7.)

3. *Notation for criteria.* In the general forms of criteria, the following notation will be used: L = length of span; p = panel length; l = length of uniform load on the truss; n = the number of panels in the truss; n' = the number of panels in the distance from the left-hand support to a vertical through the joint of the truss that is used as a center of moments in determining the stress in a chord member; W = total load on the truss, including uniform load if there is any; W_p = total load on the panel through which the section is taken, usually but not always immediately to the left of the panel point at which the critical wheel is placed.

4. In the special forms of criteria, the wheels on the truss are indicated as follows: O represents the critical wheel. A represents the wheel nearest the left-hand end of the truss, and Z the wheel nearest the right-hand end. Wheel A may or may not be the first wheel of the locomotive and wheel Z may or may not be the last, depending on the length of truss, and the position of the load. N represents the wheel immediately to the left of the critical wheel O.

5. The symbols just given, both for the general and the special forms of criteria, are those already used to some extent in the preceding chapter; they are used so frequently in this chapter that they are given together here for ready reference.

6. **Position of Concentrated Loads.** A truss is a beam with panels and should be so considered when determining the shears and bending moments from which the stresses are calculated. The criteria for positions of locomotive loadings or train loads are, therefore, either those already given in the preceding chapter for beams with panels, or are simple modifications of those criteria. The two criteria most used are (1) for maximum shear for a section between floor beams, namely,

$$W_p = W\frac{p}{L} \quad \text{or} \quad W_p = \frac{W}{n}, \ (396 : 5),$$

and (2) for maximum bending moment for a section at a floor beam, namely,

$$W_S = W\frac{S}{L} \quad \text{or} \quad W_S = W\frac{n'}{n}, \ (399 : 3).$$

These are the basic criteria; other criteria are merely modified forms to meet special conditions.

7. **Critical Wheels.** The critical wheel will always be at some joint of the loaded chord. This will be true regardless of whether the stress is determined from shear or from bending moment, and regardless of whether the section is at a panel point or between panel points.

8. The critical wheel may be determined by means of a criterion. In some cases more than one wheel will satisfy the criterion; in order to obtain the maximum stress, it may be necessary to determine the stress, first when one wheel, and then the other, is considered as the critical wheel. It will frequently be found, however, that when the criterion for the maximum stress in a member is satisfied by more than one critical wheel, the stresses in the member determined for the different critical wheels will be so nearly equal that it is immaterial which of the stresses is used in de-

signing the member. In some cases the critical wheel may be ascertained by means of a previously prepared table of critical wheels such as the tables on page 389. Frequently it is unnecessary to use a criterion or to consult a table because one can judge from inspection which is the critical wheel. As the length of span increases, the use of a criterion or table becomes more necessary.

1. In each of the tables of critical wheels on page 389, the critical wheel is determined by the lengths l_1 and l_2 of the shorter and longer segments of the base of an influence triangle. If the critical wheel for the greatest bending moment for a center at a panel point is required, the lengths l_1 and l_2 are those of the segments of the base of the influence triangle for bending moment, as explained in 388 : 4. If the critical wheel for the greatest shear for a section through a panel is required, the lengths l_1 and l_2 are those of the segments of the base of the influence triangle for shear, as explained in 397 : 6.

2. When the center of moments or the section for shear is in the right-hand half of the truss, the tables on page 389 do not necessarily apply, and the critical wheel is best determined by means of a criterion.

3. **Methods of Determining External Forces.** The concentrated live loads having been placed in the position to cause the maximum stress in a given member, the next step is to determine the external forces that are acting on the *left-hand* segment (method of sections) when the loading is in that position. One of these forces is the reaction R_L, and this reaction may be calculated by the same method as that used for a beam *without* panels. (376 : 1 to 376 : 3.)

4. In the calculation of the stress in any member by the method of sections, the two segments of the truss are separated by the panel cut by the section. All loads, concentrated or uniform, that are on either segment, may be treated as if there were no panels — it is not necessary to determine the floor-beam concentrations due to these loads. This statement holds true regardless of whether the stress to be determined is that in a chord member or that in a web member, i.e., regardless of whether the calculation is for bending moment or for shear.

5. It remains to consider the loads *in* the panel cut by the section. In the calculation of the stress in a chord member from bending moment, the center of moments is usually in a vertical through one end or the other of the panel cut by the section. If this center is at the right-hand end of the panel and the left-hand segment is the body in equilibrium, the loads in the panel as well as those on the segment may be treated as if they were on a girder without floor beams. (Why?) If the center of moments is at the left-hand end of the panel, the loads in the panel do not enter into the equation for bending moment. (Why?) It occasionally happens that the center of moments is *between* the two ends of the panel cut by the section. In such a case, it is necessary to determine the floor-beam concentration on the left-hand segment due to the loads in the panel.

6. In the calculation of the stress in a web member from shear, it is necessary to determine the shearing force on the left-hand segment due to the loads in the panel cut by the section, and this shearing force is the floor-beam concentration on the segment due to the loads in the panel, as explained in 398 : 3. The other downward shearing forces are the actual live loads on the segment, if there are any, and these live loads as previously stated, may be treated as if there were no panels.

7. From the statements just made, it is evident that it is not necessary to determine the floor-beam concentration at any joint of a truss except at an end of the panel cut by the section, and then only, as a general rule, when the shear is to be calculated. Two exceptions may be noted, (1) when, as occasionally happens, the center of moments used in calculating the stress in a chord member lies between panel points, and (2) when the stress in a web member in a truss with inclined top chord is determined from a moment equation. These exceptions will be treated later.

8. *Note:* The statements have been made to apply to a left-hand segment because that is the one generally used in determining stresses due to locomotive loadings.

9. When the floor-beam concentration at one end of a panel due to loads in that panel is required, it may be determined by the method used in calculating the reaction at the end of a simple beam equal in length to the panel length, as explained in the third step of 398 : 3.

10. **Methods of Determining Stresses.** The method of sections is generally used for the determination of stresses due to concentrated

loads. Stresses in chord members are calculated from bending moments, as in the case of uniform live load. Stresses in web members are usually calculated from shears, although in some types of trusses, as, for example, in the Parker truss, they may be determined more easily by the moment equation of equilibrium.

1. In most trusses the center of moments for a chord member is either at a floor-beam joint of the lower chord, or at a joint of the top chord directly above a floor-beam joint. In either case, the bending moment is determined exactly as for a beam *without* panels. (390 : 2.) Shear, on the other hand, must generally be determined as for a beam with panels. (398 : 3.) Once the bending moment or the shear has been calculated, the stress in the member is determined by exactly the same method as that used for uniform live load.

2. In a truss without counters, two stresses, maximum tension and maximum compression, must be determined for each web member in which reversal of stress is possible. Methods given in CHAPTERS XX and XXI for determining stresses in counters, minimum stresses, stresses in members of a center panel, and other stresses due to special conditions also hold good for concentrated live loads.

3. Throughout this chapter the method of procedure in determining stresses is indicated by means of algebraic expressions as was done in the preceding chapter. The resulting equations have the appearance of formulas, but they should not be regarded as such, and they certainly should not be used as such. Although these expressions may differ slightly for different trusses and for different members in the same truss, they are all based on a few simple general methods, and in every case the algebraic expression of a method of procedure should be studied for the sole purpose of understanding the general method that it is intended to outline. The notation used in these algebraic expressions is that used for a similar purpose in the preceding chapter; it is explained in 372 : 7 to 373 : 14. In studying the general methods for the simpler types of trusses, such as the Pratt truss and the Warren truss, it should be kept in mind that the methods of determining bending moments and shears, including criteria, are exactly the same as the corresponding methods explained in the preceding chapter for girders with floor beams.

THROUGH PRATT TRUSS WITHOUT COUNTERS

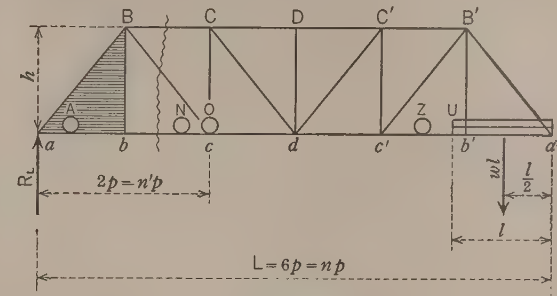


Fig. 411.

4. *To determine the stress in an upper chord member of a through Pratt truss*, as, for example, the stress in BC in Fig. 411. The bending moment required is that for a center of moments at c. The general method of procedure is exactly the same as that given in 399 : 3 and 390 : 2. The critical wheel O will be at c. Let W_S represent the total load to the left of c. The general form of the criterion for the position of the loading is:

$$W_S = W\frac{n'}{n}. \quad (399:3.)$$

The special form is:

$$\begin{matrix} \mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} < \end{matrix} [\mathrm{W(A\ldots Z)} + wl] \times \tfrac{2}{6}.$$

First step: From the criterion determine which wheel is the critical wheel O. (Or determine the critical wheel from a table of critical wheels.) Place the loading in position.

Second step: Calculate the total moment $\mathrm{M}_{a'}$ of all loads on the truss about a':

$$\mathrm{M}_{a'} = \mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}$$

Third step: Calculate the bending moment with c as a center:

$$\mathrm{M_B} = R_L \times 2p - \mathrm{M(A\ldots O)_O}$$
$$= \frac{\mathrm{M}_{a'}}{6\cancel{p}} \times 2\cancel{p} - \mathrm{M(A\ldots O)_O}$$

1. *Note:* The reaction R_L is equal to $M_{a'}$ divided by the length of the span, but it is usually better not to calculate the reaction because both the length of span and the lever arm of R_L in the equation for bending moment can be expressed in terms of panel lengths. If $M_{a'}$, is then multiplied by the ratio $\frac{n'}{n}$ (which, in this case, is $\frac{2}{6}$), the result will be that portion of the bending moment due to R_L.

Fourth step: Determine the stress in $BC = M_B \div h$.

2. *Note:* Compare the first three steps with the three steps in the general method of procedure used in determining the greatest bending moment for a given section of a beam without panels. (390 : 2.)

3. *To determine the stress in a lower chord member.* The method of procedure is exactly the same. For example, to determine the stress in *cd*, the center of moments is *C*. Since this is in a vertical line through *c*, the center of moments for *BC*, the calculations are identical with those just given, i.e., the maximum stress in *cd* is equal in magnitude to that in *BC*.

Comments

4. The criterion may also be satisfied by another position of the loading, i.e., for some other wheel at *c*. (390 : 10.) For a standard wheel-load system, the critical wheel for a given section may be determined from a table or diagram prepared for that purpose. (388 : 4.)

5. Since the span is short, a critical wheel at c' may give a greater stress in $B'C'$ than that found for *BC*. (390 : 8.) If so, this latter stress is the same as the maximum stress in *BC* when the train comes on from the left headed toward the right, and is the value used for maximum stresses in four members, namely, *BC*, *cd*, $B'C'$, and $c'd$. For example, if the span shown in Fig. 411 is 150 ft., wheels 7 and 8 of Cooper's loading both satisfy the criterion when *c* is the center of moments, and the corresponding stresses in *BC* are nearly equal, but neither is as great as the stress in $B'C''$ determined when wheel 14 is at c', hence the last mentioned stress is the required stress for the four members.

6. The maximum stress in the lower chord member *ab* in the end panel is equal to that for *bc* when *B* is the center of moments, and the position of the train load that causes this stress is the same as that which causes the maximum stress in the end post *aB* as will be shown later. This is as it should be, since the maximum stress in *ab* should equal the H component of the maximum stress in *aB*.

7. *Illustrative problem in determining the stress in a chord member of a Pratt truss.* Given: A through Pratt-truss bridge. Span = 150 ft.; panel length = 25 ft.; height of truss = 28 ft. Required: The stress in the top chord member *BC* due to E-60 loading.

8. The stress in *BC* will be determined from the greatest bending moment for *c* as a center of moments; $l_1 = 50$ ft. and $l_2 = 100$ ft. (388 : 4.) From the table on page 389, the critical wheel for these values is wheel 14. Since this wheel number is overlined in the table, the center of moments should

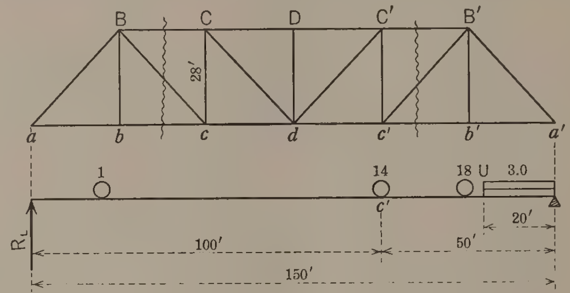


Fig. 412.

be taken at c' instead of at *c*, and the stress calculated for $B'C'$. If no table were at hand the criterion would be applied to wheel 14 as follows:

$$W(1\ldots14) > [W(1\ldots18) + 3.0 \times 20] \times \tfrac{100}{150} \qquad W(1\ldots13) < 324$$
$$348 > [\quad 426 \quad + \quad 60 \quad] \times \tfrac{1}{3} = 324 \qquad 318 \quad < 324$$

Wheel 14 satisfies the criterion. The bending moment for c' as a center is determined as follows:

$$R_L = [M(1\ldots18)_U + W(1\ldots18) \times 20 + 3.0 \times 20 \times 10] \div 150$$
$$224.1 = R_L = [\quad 24{,}500 \quad + \quad 426 \times 20 + \quad 600 \quad] \div 150$$
$$9310 = M_B = R_L \times 100 - M(1\ldots14)_{14} = 224.1 \times 100 - 13{,}100.$$

The stress in $B'C' = BC = 9{,}310{,}000 \div 28 = 332{,}500$ lbs. compression.

9. Three other wheels satisfy the criterion, namely, wheel 13 for the center of moments at c', and wheels 7 and 8 for the center of moments at *c*. Were it not for the knowledge gained from the table that wheel 14 gives a maximum, it would be necessary, theoretically, to determine the bending moment for these three wheels also, in order to ascertain which wheel gives the greatest. It would be found that the bending moment for wheel 13 is the same as for wheel 14; for wheel 7 it is 9,305,000 lb.-ft., and for wheel 8 it is 9,295,000 lb.-ft. No wheel, therefore, gives a larger bend-

ing moment than wheel 14 — as was learned from the table in the beginning. It should be noted that the discrepancy between 9,310,000 lb.-ft., the largest result, and 9,295,000 lb.-ft., the smallest, is only 15,000 lb.-ft. This corresponds to a difference of about 500 lbs. in the stress in BC, a difference that is negligible.

1. In calculating the percentage for impact allowance, the loaded length to be substituted for "L" in the impact formula is the length of the span = 150 ft. (408 : 6.)

2. *To determine the stress in a diagonal of a through Pratt truss*, as, for example, the stress in Bc in Fig. 413. The shear required is that for a section in the panel bc, i.e., the shear for the segment aBb. The general

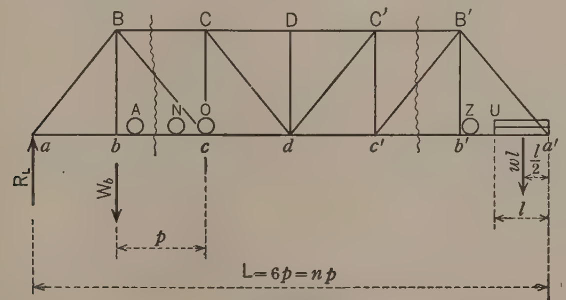


Fig. 413.

method of procedure is exactly the same as that given in 398 : 3. The critical wheel O will be at c. The general form of the criterion for the position of the loading is:

$$W_p = \frac{W}{n}. \quad (396 : 5.)$$

The special form is:

$$\begin{matrix} \mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} < \end{matrix} [\mathrm{W(A\ldots Z)} + wl] \div 6$$

First step: From the criterion determine which wheel is the critical wheel O. (Or determine the critical wheel from a table of critical wheels.) Place the loading in position.

Second step: Calculate the reaction R_L:

$$R_L = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right] \div L$$

Third step: Calculate the floor-beam load at b due to the loads in the panel bc:

$$W_b = \mathrm{M(A\ldots O)_O} \div p$$

Fourth step: Calculate the shear V for the segment aBb:

$$\mathrm{V} = R_L - W_b$$

Fifth step: Calculate the stress in Bc:

$$Bc = \mathrm{V} \times \text{secant of the truss angle.}$$

3. *Note:* Compare the first four steps with the four steps in the general method of determining the greatest shear for any section in a given panel. (398 : 3.)

4. In determining the maximum live-load compression in Bc, it is convenient to work with the corresponding diagonal $B'c'$. The load is brought on from the right as before, and the critical wheel is placed at b'. The method of procedure is then essentially the same. Having determined the critical wheel, the reaction R_L at a and the floor-beam load $W_{c'}$ at c' (due to loads in panel $c'b'$) are determined. Then the shear V for the segment $aBC'c'$ will be $\mathrm{V} = R_L - W_{c'}$; this will be a positive shear, hence $B'c'$ will be in compression, and the stress will equal $V \times$ *secant of truss angle.*

Comments

5. The critical wheel O at c for the maximum stress in Bc will usually be different from the critical wheel O at c for the maximum stress in BC. (Why?)

6. More than one wheel may satisfy the criterion for shear. (397 : 4.) For a standard wheel-load system, the critical wheel for a given section may be determined from a diagram or table prepared for that purpose, unless the section is through a panel in the right-hand half of the truss. (388 : 4.) In using the table on page 389, the length of l_1 is determined by the position of the neutral point in the panel. (397 : 6.)

7. The maximum stress in the end post aB is determined, not from the maximum reaction at a, but from the reaction which is at a when the shear for the panel ab is a maximum. The position of the live load which will cause this shear, determined by the criterion for shear, will be the same as the position which will cause the maximum stress in bc, determined by the criterion for bending moment. (Prove this statement.) The critical wheel for the shear for a section through aB will be at b.

8. If the lever arms of R_L about a' and of W_b about c are expressed in panel lengths, note that the shear is: $\mathrm{V} = \frac{\mathrm{M_{a'}}}{6p} - \frac{\mathrm{M_c}}{p} = \frac{1}{p}\left(\frac{\mathrm{M_{a'}}}{6} - \mathrm{M_c}\right)$.

1. *To determine the stress in a vertical of a through Pratt truss.* The maximum stress in either hip vertical Bb or $B'b'$ is equal to the maximum load which the floor beam brings to the joint b or b'. The method of determining this load is explained in 403 : 1 and 401 : 6 to 402 : 7.

2. The live-load stress in Dd is zero.

3. The maximum stress in any other vertical is equal to the V component of the maximum stress in that diagonal that meets it at a joint of the unloaded chord, just as in the case of uniform live load. For example, if the maximum compression in Cc were to be calculated by the method of sections, the section through Cc would make d the joint at which the critical wheel would be placed. But this is also the joint at which the critical wheel would be placed in calculating the maximum tension for Cd. The criterion for shear is the same for both cases, hence the critical wheel is the same, and the resulting shear is the same, i.e., the maximum compression in Cc = V component of the maximum tension in Cd. Similarly, the maximum tension in Cc = V component of the maximum compression in Cd.

Illustrative problem. What is the maximum stress in the hip vertical Bb due to the E-system of loading. Answer: 113,500 lbs. (See 403 : 3 for the solution.)

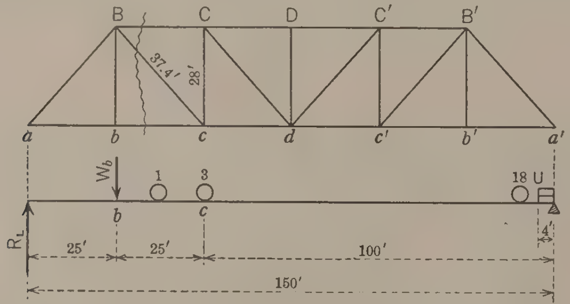


Fig. 414 (*a*)

4. *Illustrative problem in determining the stress in a diagonal of a Pratt truss without counters.* Given: A through Pratt-truss bridge without counters. Span = 150 ft.; panel length = 25 ft.; height of truss = 28 ft. Required: The maximum live-load tension and compression in the diagonal Bc due to E-60 loading.

5. The maximum tension in Bc will be determined from the greatest positive shear for a section through panel bc; $l_1 = \frac{4}{5} \times 25 = 20$ ft. and $l_2 = 100$ ft. (397 : 6.) From the table on page 389, the critical wheel is wheel 3. If no table were at hand, the criterion would be applied as follows:

$$W(1\ldots3) > [W(1\ldots18) + 3.0 \times 4] \times \tfrac{25}{150} \qquad W(1\ldots2) < 73$$
$$75 > [\;426 \;+\; 12\;] \times \tfrac{1}{6} = 73 \qquad 45 < 73$$

Wheel 3 satisfies the criterion. The shear for panel bc with wheel 3 at c is determined as follows:

$$R_L = [M(1\ldots18)_U + W(1\ldots18) \times 4 + 3.0 \times 4 \times 2] \div 150$$
$$174.9 = R_L = [\;2450 \;+\; 426 \times 4 + \;24\;] \div 150$$
$$13.8 = W_b = M(1\ldots3)_3 \div 25 = 345 \div 25$$
$$161.1 = V \;= R_L - W_b = 174.9 - 13.8$$

The stress in Bc = 161,100 × (37.4 ÷ 28) = 215,000 lbs. tension. Wheel 4 also satisfies the criterion and gives practically the same shear as that just found for wheel 3.

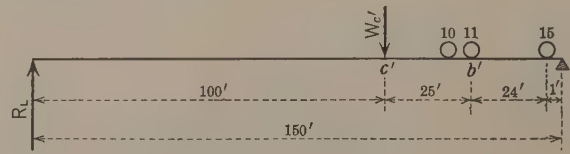


Fig. 414 (*b*).

To determine the greatest live-load compression in Bc, it will be convenient to work with the corresponding member $B'c'$, with the critical wheel at b'. (Fig. 414 (*b*).) Since the section is in the right-hand half of the truss, the table on page 389 does not necessarily apply. The critical wheel will be near the front of the locomotive. (397 : 4.) Try the criterion for wheel 2. (Use wheel 11 and disregard all wheels in front of wheel 10.)

$$W(10\ldots11) > W(10\ldots15) \times \tfrac{25}{150} \qquad W(10) < 29$$
$$45 > 154.5 \times \tfrac{1}{6} = 29 \qquad 15 < 29$$

Wheel 2 satisfies the criterion; it would be found that no other wheel would satisfy it. The shear for panel $b'c'$ with wheel 2 at b' is determined as follows:

$$
\begin{aligned}
R_L &= [M(10\ldots15)_{15} + W(10\ldots15) \times 1] \div 150 \\
17.4 = R_L &= [\quad 2460 \quad + \quad 154.5] \div 150 \\
4.8 = W_{c'} &= M(10\ldots11)_{11} \div 25 = 120 \div 25 \\
12.6 = V \; &= R_L - W_{c'} = 17.4 - 4.8
\end{aligned}
$$

The stress in $B'c' = Bc = 12{,}600 \times (37.4 \div 28) = 16{,}800$ lbs. compression.

1. In calculating the percentage for impact allowance, the loaded length to be substituted for " L " in the impact formula is from the end of the span to wheel 1. (408 : 6.) For the tension in Bc this length is 113 ft.; for compression in Bc it is 33 ft.

2. *Pratt truss with an odd number of panels and without counters.* The stresses in a Pratt truss with an odd number of panels are determined exactly as for a Pratt truss with an even number, with the exception of the stresses in the two diagonals, the two verticals and the two chords in the center panel. The methods of determining the stresses in these six members are the same in principle as those explained for determining stresses in the same members when the live load is uniformly distributed. (329 : 5.)

3. To determine the stresses in the two diagonals, calculate the maximum live-load shear for a section through the panel just as for any section. The V component of the stress in each diagonal will then be one-half of this shear; this V component is also the maximum live-load stress in each of the two verticals. The assumption upon which this division of the shear is based, namely, that since each diagonal is capable of taking either tension or compression, each may be assumed to resist one-half of the shear, is sufficiently correct for practical purposes.

4. The stresses in the top and bottom chord members of the *center* panel are equal. To determine the stress in either chord member, take the center of moments at the center of the other chord member. The moments of the two diagonals about this center will cancel each other. (200 : 6.) Since the center of moments is midway between floor-beam joints, the position of the live load must be determined by the criterion in 400 : 2. The criterion may be satisfied by one or more critical wheels at one end of the center panel and also by one or more critical wheels at the other end. Once the loading has been placed in position, the maximum bending moment for the load in that position and for a section at the center of the center panel is equal to the average of the moments obtained by taking the center of moments first at one end of the center panel and then at the other end. (401 : 5.) This average of the two bending moments divided by the height of truss will equal the stress in each of the two chord members in the center panel. It is to be noted that the bending moments for the two centers of moments, one at either end of the center panel, are calculated just as for any floor-beam joint. This method will be explained more fully later in connection with the determination of stresses in the members of the *loaded* chord of a Warren truss without verticals.

5. **Through Pratt Truss with Counters.** Since the calculations of the live-load stresses in Pratt trusses with counters are for most members identical with the calculations of stresses in Pratt trusses without counters, it will be necessary to consider only those members in which the stresses are affected by the action of counters.

6. *Pratt truss with an even number of panels.* The stresses in the chord members and the end posts of a Pratt truss with an even number of panels are not affected by the action of counters; neither are the stresses in web members except in panels in which there are counters. It will be necessary, therefore, to consider only the main diagonal, the vertical, and the counter in a typical panel containing a counter.

7. To obtain the maximum live-load stress in a counter, conceive the main diagonal to be removed, and then determine the stress in the counter just as for any other diagonal. It will be convenient to work with the counters in the right-hand half of the truss in order that the live load may be brought on from the right, headed toward the left.

8. Once the live-load stress in a counter has been determined, all questions pertaining to the action of counters are similar to those already discussed in connection with uniform live load. For example, the actual stress in a counter is the live-load stress minus the dead-load tension in the main diagonal which must be reduced to zero before the counter can act. (323 : 5.) If minimum stresses are desired, the effect of counters on other members must be considered. The action of counters may, for

example, not only reduce the stress in a diagonal to zero, but may also reduce the stress in a vertical to less than the dead-load stress and often to zero, as was explained in 325 : 3. In determining the actual stress in a counter and its effect on other members, any impact or increase in the live load that may be required by the specifications should be taken into account. (267 : 3 and 7.)

1. *Pratt truss with an odd number of panels.* In a Pratt truss with an odd number of panels, the live-load shear for the center panel is not divided between the two diagonals in that panel, as was the case in a truss without counters, but, on the contrary, it is assumed that when one counter is in action the other is not, and that, therefore, the entire live-load shear for the panel is carried by whichever counter, *acting in tension*, can resist that shear. The stress in the counter may then be determined just as if the other counter did not exist, and the method is the same as for any other diagonal. The entire method of procedure is analogous to that for the determination of stresses in the counters of a center panel when the live load is uniformly distributed. (325 : 1.)

2. The only other difference in the determination of stresses in members in the center panel arises from the effect of the action of counters in that panel on the stresses in the chord members at the center of the truss. A section through the center panel cuts two counters as well as two chord members, and since the center of moments used in determining the stress in a chord member should lie in the line of action of whichever counter is in stress, and since there may be some question as to which counter is in action when the live load is in a position to cause maximum stress in the chord member, there may be some uncertainty as to which joint of the truss should be taken as a center of moments. Let the three accompanying figures represent the three panels in the center of any Pratt truss with an odd number of panels. In Fig. 416 (a) the counters are represented by broken lines, and when there is no live load on the bridge, none of these counters are in action. In Fig. 416 (b) only those members are shown which are in action when the live-load shear for the left-hand segment (section s–s) is positive, and in Fig. 416 (c) only those members are shown which are in action when the live-load shear for the same segment is negative. In the last two figures, the counters bC and $C'b'$ in the two panels adjacent to the center are omitted because when the live load is in a position to cause the maximum stress in any chord member neither of these counters is in action.

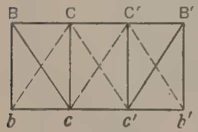


Fig. 416 (a).

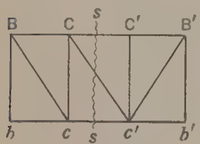


Fig. 416 (b).

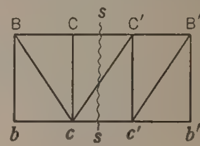


Fig. 416 (c).

3. The stress in the upper chord member CC' will be considered first. If the members shown in Fig. 416 (b) are in action, the center of moments for determining the stress in CC' will be c', but if the members shown in Fig. 416 (c) are in action the center of moments will be c. If the live load were uniformly distributed, it would be immaterial which of these centers were taken, but for a system of concentrated wheel loads, the moment may be greater for one center than for the other, hence in order to determine the maximum stress in CC' it is necessary to calculate the maximum moment for each of these centers. It is to be noted that if the greater of these two maximum moments occurs when c' is assumed as the center of moments (Fig. 416 (b)), the resulting stress in CC' is also the stress in $C'B$ for the corresponding position of the live load. Similarly, if the greater moment is for c as a center (Fig. 416 (c)), the resulting stress in CC' is also the stress in BC for the corresponding position of the live load.

4. The question may be asked: "Is it certain that if the greater of the two moments occurs when c' is the center, the counter Cc' is in action, or if it occurs when c is the center, the counter cC' is in action?" If the bending moment for c' is greater than that for c, the shear for any section between c and c' is positive (Why?), and hence Cc' must be the counter in action, as it was assumed to be when c' was taken as the center of moments (Fig. 416 (b)), but if the bending moment for c is greater than for c', the shear for a section in the center panel is negative, and hence cC' must be the counter in action, as it was assumed to be when c was taken as a center of moments (Fig. 416 (c)).

5. It remains to consider the stress in the lower chord member cc'. If the members shown in Fig. 416 (b) are in action, the maximum bending moment will occur for a center of moments at c' or for any point as C' in a vertical line through c', as explained in the preceding paragraph.

But neither c' nor C' is the center of moments for determining the stress in cc' when the counter Cc' is in action (Fig. 416 (*b*)) since neither point is at the intersection of Cc' and CC'. Similarly, if the members shown in Fig. 416 (*c*) are in action, the center of moments must be in a vertical line through c, but when the counter cC' is in action (Fig. 416 (*c*), neither c nor C is the center of moments for determining the stress in cc'. Because of these conditions, the maximum stress in cc' is determined from one of the following moments: (1) Moment with C as a center but with the live load placed as if it were desired to obtain the maximum bending moment for a section at c' (members in action shown in Fig. 416 (*b*)); (2) moment with C' as a center but with the live load placed as if it were desired to obtain the maximum bending moment for a section through c (members in action shown in Fig. 416 (*c*)); (3) moment with either C or C' as a center, and with the live load placed in such a position that the shear in the center panel is zero. (This last position is found by trial; when the live load is thus placed, neither counter is in action and the moments at C and C' are equal.) In order to determine the maximum stress in cc', it is necessary, theoretically, to determine all three moments, and then to divide the greatest of the three by the height of truss. For practical purposes, however, the maximum stress in cc' can be considered equal to the maximum stress in CC'; it is always less, but the error, which is on the side of safety, is slight — usually not more than from one to two per cent.

1. **Deck Pratt Trusses.** The methods of determining stresses in deck Pratt trusses do not differ from those already explained for through Pratt trusses. If a deck truss and a through truss have the same general dimensions, the stresses in the chord members and diagonals due to any system of wheel loads will be the same except that if the end diagonal of the deck truss slopes in the opposite direction from the end post of a through truss, the tension in the former will be equal to the compression in the latter, and the compression in the end panel of the upper chord of the deck truss will be equal to the tension in the end panel of the lower chord of the through truss. The live-load stresses in all verticals except the end verticals of the deck truss will be greater than the stresses in the corresponding verticals of the through truss, because for a section through a given vertical, there will be one more loaded joint on the right-hand segment of the deck truss than on the right-hand segment of the through truss, just as in the case of uniform live load. (327 : 10.)

2. **Warren Trusses.** The stresses in a Warren truss with verticals are determined by the same methods as those already explained for a Pratt truss without counters. The problem is simpler if anything, since the live-load stress in a vertical of a Warren truss is equal to the maximum floor-beam concentration if the vertical is the only web member at a joint of the loaded chord, or to zero if the vertical meets two diagonals at a joint of the loaded chord. Methods of determining stresses in typical members are illustrated in the following problem.

3. *Illustrative problem in determining the stresses in typical members of a Warren truss with verticals.* Given: a through Warren truss with verticals. Span = 200 ft.; panel length = 25 ft.; height of truss = 32 ft. Required: The stresses in the lower chord member de and the diagonal cD due to M-50 loading.

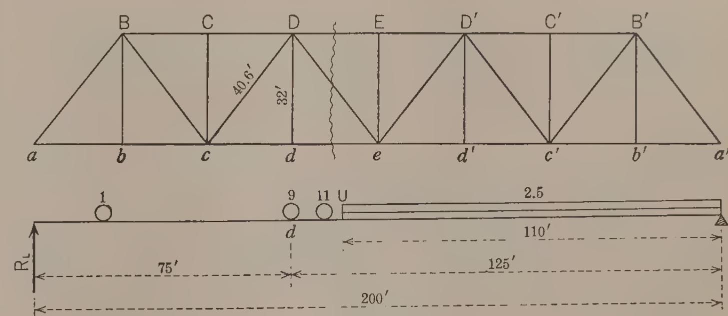


Fig. 417.

4. The stress in de will be determined from the greatest bending moment for D as a center of moments (same as for a center at d); $l_1 = 75$ ft. and $l_2 = 125$ ft. From the table on page 389, the critical wheel for these two values is wheel 9. If no table were at hand the criterion (409 : 6) would be applied as follows:

$$W(1\ldots9) > [W(1\ldots11) + 2.5 \times 110] \times \tfrac{75}{200} \qquad W(1\ldots8) < 213$$

$$231.3 > [\ 293.8 \quad + \quad 275\ ] \times \tfrac{3}{8} = 213 \qquad 200 < 213$$

Wheel 9 satisfies the criterion; it would be found that no other wheel does. The bending moment for D (or d) as a center is determined as follows:

$$R_L = [\mathrm{M}(1\ldots11)_{\mathrm{U}} + \mathrm{W}(1\ldots11) \times 110 + 2.5 \times 110 \times 55] \div 200$$

$$284.6 = R_L = [\quad 9470 \quad + \quad 293.8 \times 110 + \quad 15{,}125 \quad] \div 200$$

$$15{,}815 = \mathrm{M_B} = R_L \times 75 - \mathrm{M}(1\ldots9)_9 = 284.6 \times 75 - 5530$$

The stress in de is 15,815,000 ÷ 32 = 494,000 lbs. tension.

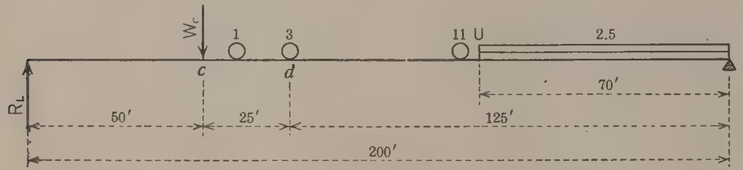


Fig. 418 (*a*).

1. The maximum compression in cD (Fig. 417) will be determined from the greatest shear for a section through panel cd in Fig. 418 (a); $l_1 = \frac{5}{7} \times 25 = 18$ ft. and $l_2 = 125$ ft. (397 : 6.) From the table on page 389, the critical wheel is wheel 3 or 4. To make sure, apply the criterion (409 : 6):

$$\mathrm{W}(1\ldots3) > [\mathrm{W}(1\ldots11) + 2.5 \times 70] \times \tfrac{1}{8} \qquad \mathrm{W}(1\ldots2) < 58$$

$$62.5 > [\quad 293.8 + 175 \quad] \times \tfrac{1}{8} = 58 \qquad 37.5 < 58$$

Wheel 3 satisfies the criterion; it would be found that no other wheel does. The shear for the panel cd with wheel 3 at d is determined as follows:

$$R_L = [\mathrm{M}(1\ldots11)_{\mathrm{U}} + \mathrm{W}(1\ldots11) \times 70 + 2.5 \times 70 \times 35] \div 200$$

$$180.8 = R_L = [\quad 9470 \quad + \quad 293.8 \times 70 + \quad 6125 \quad] \div 200$$

$$12.5 = W_c = \mathrm{M}(1\ldots3)_3 \div 25 = 313 \div 25$$

$$168.3 = \mathrm{V} = R_L - W_c = 180.8 - 12.5.$$

The stress in cD = 168,300 × (40.6 ÷ 32) = 213,600 lbs. compression.

2. To determine the maximum tension in cD, it will be convenient to work with the corresponding member $c'D'$, with the critical wheel at c'. Since the section is in the right-hand half of the truss, the table on page 389 does not necessarily apply. The critical wheel will be one near the front of the locomotive. (Why?) (397 : 4.) Try the criterion for wheel 2 in Fig. 418 (b):

$$\mathrm{W}(1\ldots2) > \mathrm{W}(1\ldots10) \times \tfrac{1}{8} \qquad \mathrm{W}(1) < \mathrm{W}(1\ldots9) \times \tfrac{1}{8}$$

$$37.5 > \quad 262.5 \times \tfrac{1}{8} = 33 \qquad 12.5 < \quad 218.8 \times \tfrac{1}{8} = 27$$

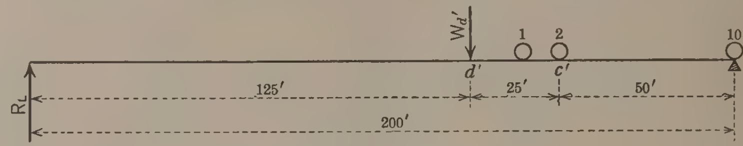


Fig. 418 (*b*).

Wheel 2 satisfies the criterion; it would be found that no other wheel does. The shear for panel $c'd'$ with wheel 2 at c' is determined as follows:

$$33.5 = R_L = \mathrm{M}(1\ldots10)_{10} \div 200 = 6690 \div 200$$

$$5.0 = W_{d'} = \mathrm{M}(1\ldots2)_2 \div 25 = 125 \div 25$$

$$28.5 = \mathrm{V} = R_L - W_{d'} = 33.5 - 5$$

The stress in $c'D'$ = cD = 28,500 × (40.7 ÷ 32) = 36,200 lbs. tension.

In calculating the percentage for impact allowance the loaded lengths are: 200 ft. for the stress in de; for compression in cD, 140 ft.; for tension in cD, 60 ft. (408 : 6.)

The stress in an upper chord member may be determined by the same general method as that used for a lower chord member. The stress in Bb, Dd, $D'd'$, or $B'b'$ is equal to the greatest floor-beam concentration, and this was determined in 403 : 3 to be 123,900 lbs. for 25-ft. panels. The live-load stress in Cc, Ee, or $C'c'$ is obviously zero.

3. *The stresses in a Warren truss without verticals* are also determined by the same methods as those used for calculating stresses in the chords and diagonals of a Pratt truss without counters, with the exception of the stresses in the members of the *loaded* chord. For each of these members, the center of moments lies in a vertical line midway *between* two successive floor-beam joints, and consequently the general method of determining bending moments is that explained in 401 : 5. It has already been explained in connection with the stresses in the two chord members in the center panel of a Pratt truss without counters. (415 : 4.)

1. *To determine the stress in a member of the loaded chord of a through Warren truss,* as, for example, the stress in cd in Fig. 419. The bending moment required is that for a center at D. The general method of procedure is the same as that given in 401 : 1, except that since the center

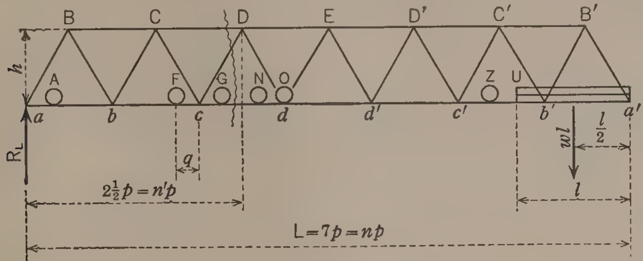


Fig. 419.

is in a vertical half-way between panel points c and d, the bending moment is obtained by taking the average of the two bending moments for centers at c and d. (401 : 5.) The critical wheel O will be either at c or at d. Let the first trial be at d. Let W' represent the total load to the left of the panel cd, and S the distance from a to a vertical through $D = 2\frac{1}{2}\,p$. The general form of the criterion for the position of the loading is that given on page 401, namely:

$$W' + \tfrac{1}{2}\,W_p = W\frac{S}{L} = W\frac{n'}{n}$$

The special form is:

$$\begin{matrix} W(A\ldots F) + \frac{1}{2}\,W(G\ldots O) > \\ W(A\ldots F) + \frac{1}{2}\,W(G\ldots N) < \end{matrix} [W(A\ldots Z) + wl]\frac{2\frac{1}{2}}{6}$$

(Wheels F and G are the wheels on either side of c nearest to c.)

First step: From the criterion determine which wheel is the critical wheel O.

Second step: Calculate the total moment of all loads on the truss about a'.

$$M_{a'} = M(A\ldots Z)_U + W(A\ldots Z) \times l + wl \times \frac{l}{2}$$

The reaction R_L is equal to $\dfrac{M_{a'}}{7p}$. (Do not solve.)

Third step: Calculate the bending moment with d as a center:

$$M_B \text{ for } d = \frac{M_{a'}}{7\cancel{p}} \times 3\cancel{p} - M(A\ldots O)_O$$

Fourth step: Calculate the bending moment with c as a center:

$$M_B \text{ for } c = \frac{M_{a'}}{7\cancel{p}} \times 2\cancel{p} - [M(A\ldots F)_F + W(A\ldots F) \times q]$$

Fifth step: Calculate the bending moment with D as a center:

M_B for $D = \frac{1}{2}(M_B \text{ for } c + M_B \text{ for } d)$. (401 : 5.)

Sixth step: Determine the stress in $cd = (M_B \text{ for } D) \div h$.

Comments

2. More than one wheel at d may satisfy the criterion.

3. A greater bending moment may be obtained with a critical wheel at c. Note that when a wheel tried at c is counted as in the panel to the left of c it becomes part of the entire load W' to the left of the panel cd, and though the general criterion remains the same the expression for the special form is slightly different. (400 : 3 to 400 : 6.)

4. The stress in $d'c'$, the corresponding member on the other side of the center of the truss may be determined in a similar manner, first from the average of the moments for d' and c' when a critical wheel is at d', and from the average of the moments for the same two centers when another critical wheel is at c'. The greater of the two stresses in $c'd'$ thus found may be greater than any stress that occurs in cd when a critical wheel is at d or another critical wheel is at c.

5. From the three comments preceding it would seem that in order to obtain the maximum stress in the chord member cd, it may be necessary to calculate sixteen bending moments, namely, for c and d as centers for each of two critical wheels at d and for each of two critical wheels at c, and for c' and d' as centers for each of two critical wheels at d' and for each of two critical wheels at c'. This would mean, however, that the criterion would be satisfied eight times, i.e., when one or another of eight critical wheels is placed at one of the four joints c, d, d', or c'. It will be found that in general such is not the case. Not only is the number of critical wheels which satisfy the criterion usually much less, but in some cases the moment desired for a given joint as a center is identical with that for the same joint required in the calculation of the stress in a member of the opposite (unloaded) chord. For example the moment for b as a center used in calculating the stress in ab is identical with the moment for the same joint b used in calculating the stress in BC. (See next comment.)

6. The stress in the end panel ab of the loaded chord may be found from the moment for B obtained by averaging the moments for a and b when the critical wheel is at b. But the moment at a is zero, hence the moment for B is one half that for b. (Show that the criterion for maximum moment for b as a center is the same when the stress in ab is desired as it is when the stress in BC is desired, and that, therefore, the maximum stress in ab is one-half that in BC.)

1. *Deck Warren trusses.* For each of the members of the *upper* chord of a deck Warren truss without verticals the center of moments lies in a vertical line midway *between* two successive floor-beam joints, and hence it is for determining stresses in those members that the method just explained for calculating stresses in the lower-chord members of a through Warren truss is used. The stresses in all other chord members and in the diagonals are determined by the methods used for the calculation of stresses in the chords and diagonals of a Pratt truss without counters. (Pages 411 to 413.)

2. The stresses in the members of a deck Warren truss with verticals are determined by exactly the same methods as those used for a through Warren truss with verticals. (417 : 3.)

3. **Baltimore Truss.** Three types of the Baltimore truss will be considered, namely, the through truss with sub-struts, the through truss with sub-ties, and the deck truss with sub-ties. The following general statements hold true for all three types.

4. The maximum live-load stress in a sub-vertical is equal to the maximum floor-beam concentration from loads on two adjacent panels as determined by one of the methods of 402 : 2 to 402 : 6.

5. The maximum live-load stress in a sub-diagonal is equal to one-half of the maximum floor-beam concentration multiplied by the secant of the truss angle, except in the case of a sub-diagonal that, for certain positions of the live load, acts as a counter. In the latter case, the counter stress in the sub-diagonal may exceed the regular stress in it due to a single floor-beam concentration.

6. When a section through any web member can be taken without cutting any other web member, the criterion for the position of the loads and the method of determining the stress for that web member are the same as those used for the corresponding web member in a Pratt truss of the same number of panels.

7. When there is no sub-diagonal at either end of a chord member, the criterion for the position of the loads and the method of determining the stress in that chord member are the same as for the corresponding chord member in a Pratt truss of the same number of panels. In applying this principle, two chord members in a double panel, one on either side of a sub-vertical, should be considered as one chord member (the stress in one is always equal to the stress in the other), but in counting the number of panels in a Baltimore truss each space between floor beams is considered as one panel.

8. From the statements already made, it is evident that stresses in the following members of a Baltimore truss may be determined by methods used for the corresponding members in a Pratt truss.

(a) In a truss with sub-struts: all members of the *upper* chord; the lower portion of each main diagonal including the lower portion of the end post.

(b) In a truss with sub-ties: all members of the *lower* chord; the upper portion of each main diagonal except the upper portion of the end post; the lower portion of the end post except in deck bridges.

9. *Maximum and minimum stresses.* Only methods of determining maximum stresses will be explained. Minimum stresses in different types of the Baltimore truss due to uniform live load, particularly those caused by the action of counters, were considered in Chapter XX. Methods of determining minimum stresses due to the effect of moving concentrated loads are so nearly analogous, that they will be obvious to one who understands the methods of determining minimum stresses due to uniform live load.

Through Baltimore Truss with Sub-Struts

10. *To determine the stress in an upper chord member*, as, for example, the stress in EG in Fig. 421 (a). No sub-diagonal is at either end of EG, therefore the stress in EG is determined exactly as for a top chord member of a Pratt truss. (411 : 4.) The center of moments is at g. Determine the maximum bending moment for g (399 : 3 and 390 : 2); divide this moment by the height of truss.

11. *To determine the stress in a lower chord member*, as, for example, the stress in fg in Fig. 421 (a). There is a sub-diagonal at one end of efg. The bending moment required is that for a center at E. The critical wheel O will be at f. The general form of the criterion for the position of the loading is:

$$W_S - W_p = W\frac{n'}{n}$$

W_S represents the load to the left of f, and W_p the load in the panel fg to the right of O; n' = the number of panels from a to the center of moments E. The special form of the criterion is:

$$\left.\begin{array}{l}\mathrm{W(A\ldots O)} - \mathrm{W(Q\ldots T)} > \\ \mathrm{W(A\ldots N)} - \mathrm{W(O\ldots T)} < \end{array}\right[\mathrm{W(A\ldots Z)} + wl]\tfrac{4}{12}$$

(Wheels Q and T are the wheels in panel fg nearest respectively to f and g.)

First step: From the criterion, determine which is the critical wheel O.

Second step: Determine the bending moment for the segment $aCEf'f$ with E as a center of moments. The external forces acting on this segment

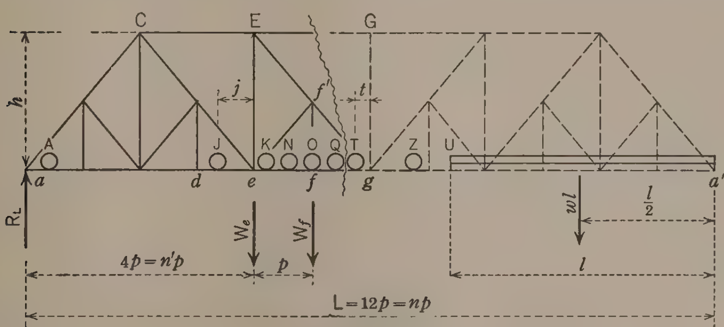


Fig. 421 (*a*).

are the reaction R_L, the wheel loads A to J, the *partial* floor-beam load W_e due to the wheel loads in the panel ef, and the total floor-beam load W_f due to the wheel loads in the two panels ef and fg.

$$\mathrm{M}_{\mathrm{B_E}} = R_L \times 4\,p - \mathrm{M(A\ldots J)_E} + W_e \times 0 + W_f \times p$$

$$R_L \times 4\,p = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right]\frac{4}{12}$$

$$\mathrm{M(A\ldots J)_E} = [\mathrm{M(A\ldots J)_J} + \mathrm{W(A\ldots J)} \times j]$$

$$W_f = [\mathrm{M(K\ldots T)_g} - 2 \times \mathrm{M(K\ldots O)_O}] \div p \quad (402 : 6.)$$

$$W_f \times p = [\mathrm{M(K\ldots T)_T} + \mathrm{W(K\ldots T)} \times t - 2\,\mathrm{M(K\ldots O)_O}]$$

Third step: Determine the stress in $fg = ef = \mathrm{M}_{\mathrm{B_E}} \div h$.

Comments

1. The criterion may also be satisfied by another position of the loading — some other critical wheel at f.

2. Should wheel T happen to be exactly at g, it should be included in the weight W(Q...T) in the criterion but not in the weight W(O...T). This is because when wheel O is moved slightly to the left of f, wheel T moves onto the panel fg, but when O is moved to the right of f, wheel T moves to the right of g or off the panel fg. (388 : 1.)

3. *To determine the stress in the lower half of a main diagonal*, as, for example, the stress in $f'g$ in Fig. 421 (*b*). Since a section through $f'g$ cuts no other inclined member in the panel fg, the maximum stress in $f'g$ will

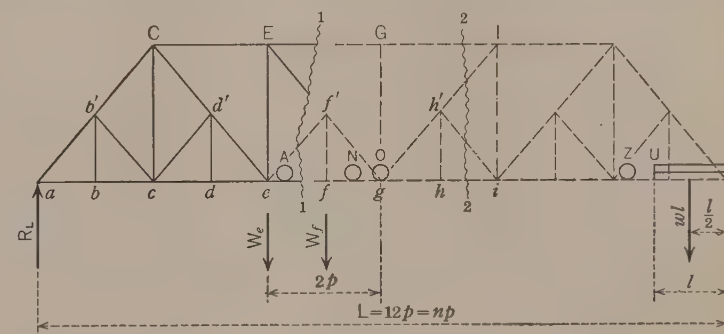


Fig. 421 (*b*).

occur when the live-load shear for the panel fg is greatest. This greatest shear will occur when the critical wheel O is at g. The criterion for the greatest shear in a panel is $W_p = W \div n$. (396 : 5.) Determine this shear by the usual method and multiply it by the secant of the truss angle. The result is the maximum live-load stress in $f'g$. Note that the method of procedure is exactly the same as for the diagonal of a Pratt truss — in this case a Pratt truss of 12 panels.

4. *To determine the stress in the upper half of a main diagonal*, as, for example, the stress in Ef' in Fig. 421 (*b*). The shear required is that for a section in the panel ef, i.e., for the segment $aCEe$. The critical wheel O will be at g. The first wheel A may be in the panel ef or it may

be in the panel fg. Let W_S represent the total load to the left of g. The general form of the criterion for the position of the loading is:

$$\frac{W_S}{2p} = \frac{W}{L} \quad \text{or} \quad W_S = W\frac{2}{n}$$

The special form is:

$$\begin{matrix} \mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} < \end{matrix} [\mathrm{W(A\ldots Z)} + wl] \times \tfrac{2}{12}$$

First step: From the criterion, determine which wheel is the critical wheel O. Place the load in position with the critical wheel at g.

Second step: Calculate the reaction R_L:

$$R_L = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right] \div 12\,p$$

Third step: Calculate $Ef'_V = R_L - W_e - ef'_V = R_L - W_e - \frac{1}{2} W_f$:

W_e = M of all wheels in panel ef about f divided by $p = \mathrm{M}_f \div p$

W_f = M of all wheels in the double panel eg about g minus twice the moment of all wheels in panel ef about f, the whole divided by $p = (\mathrm{M_g} - 2\,\mathrm{M_f}) \div p$. (402 : 6.)

$$\text{Hence } -W_e - \tfrac{1}{2} W_f = -\frac{\mathrm{M_f}}{p} - \frac{\mathrm{M_g}}{2p} + \frac{\mathrm{M_f}}{p} = -\frac{\mathrm{M_g}}{2p}$$

$$Ef'_V = R_L - \mathrm{M_g} \div 2\,p = R_L - \mathrm{M(A\ldots O)_O} \div 2\,p.$$

Fourth step: Calculate the stress in $Ef' = Ef'_V \times$ secant of the truss angle. (Tension.)

Comments

1. The criterion may also be satisfied by another position of the load — some other critical wheel at g. Note that the criterion for the position of loading for the maximum stress in Ef' is the same as for the maximum stress in the diagonal Eg of the Pratt truss formed by omitting sub-verticals and sub-struts in Fig. 421 (b), — a Pratt truss with six panels.

2. There may or may not be wheel loads in the panel ef, the left-hand half of the double panel. Whether there are or not, the method of determining the stress in Ef', as well as the criterion for the position of the loads, is the same. (Prove that the expression $Ef'_V = R_L - \frac{\mathrm{M_g}}{2\,p}$ still holds true.)

3. In a truss without counters it will be necessary to obtain the maximum compression in a diagonal as well as the maximum tension. To obtain the maximum compression in Ef', it will be convenient to work with the corresponding diagonal $h'I$. The loading is brought on from the right, the critical wheel is placed at i, the same criterion as was used for Ef' is applied, and, once the criterion is satisfied, the same method of determining the stress is used.

4. *Stresses in the end post.* From $\Sigma\mathrm{H} = 0$ applied at joint a, $-ab'_H = ab$, hence the maximum stress in ab' may be determined from its maximum H component which is equal to the maximum stress in ab. It may also be determined from the maximum live-load shear for the panel ab. In this case the critical load will be at b, and the position of the load will be the same as for the maximum stress in ab. (Why?) The stress in $b'C$ may be determined from the maximum live-load shear for panel bc, but it is simpler to use a moment equation of equilibrium with c as a center. The load will then be placed in a position to cause the maximum bending moment for c. (399 : 3.)

5. *Stresses in verticals.* The stress in a sub-vertical is equal to the maximum floor-beam concentration. (402 : 6.)

The maximum live-load compression in Ee is equal to the V component of the maximum live-load tension in Ef'. In a truss without counters, the maximum live-load tension in Ee is equal to the V component of the maximum live-load compression in Ef', just as in a Pratt truss without counters.

6. The maximum stress in Cc is equal to the panel load at c, plus the V components of cb' and cd'. The V component of cb' is equal to one-half of whatever load is concentrated at b, and the V component of cd' to one-half of whatever load is concentrated at d. (201 : 5.) The stress in Cc is due wholly, therefore, to loads at the three joints b, c, and d, and these loads are due, in turn, to that portion of the entire load in the *four* panels from a to e that is brought to the truss by the three floor-beams at b, c, and d. Let $\mathrm{M_{Bc}}$ represent the bending moment of all loads on ae (considered as a beam supported only at a and at e) with respect to c as a center, and p the panel length, then from 402 : 5 and 402 : 6

$$\text{Stress in } Cc = \mathrm{M_{Bc}} \div p \quad \text{or} \quad \frac{\mathrm{M_e} - 2\,\mathrm{M_c}}{2\,p}$$

$\mathrm{M_e}$ = moment of all wheel loads on ae with respect to e and $\mathrm{M_c}$ = moment of wheel loads on ac with respect to c. The position of the load that will

result in a maximum value for M_{B_c} is determined as for the center of any beam equal in length to four panels, and the bending moment is calculated by the usual method. (399 : 3.) Note the analogy between the method of determining the maximum stress in the hip vertical Cc and the method of determining the stress in a sub-vertical.

1. The live-load stress in Gg is zero in a truss without counters.

2. *Counters.* In a truss with counters, the members $f'G$ and ef' in Fig. 421 (*b*) act together as counters to prevent compression in either Ef' or $f'g$. When $f'G$ is in action there is no live-load stress in $f'g$. (Why?) Hence in determining the live-load stress in $f'G$ the member $f'g$ may be ignored as in the case of uniform load. With $f'g$ omitted, a section through $f'G$ will cut no other web member and the live-load stress in $f'G$ may be determined by a method similar to that used for $f'g$. (421 : 3.) It will be convenient to work with the corresponding counter Gh'. With the load brought on from the right, the critical wheel will be at h, and the V component of the live-load stress in Gh' will be equal to the maximum shear for the panel gh. The general criterion is $W_p = W \div n$. (396 : 5.) The actual stress in Gh' will be the live-load tension in Gh' minus the dead-load tension in gh' which must be reduced to zero before the counter can act. (334 : 5.)

3. To determine the maximum live-load tension in ef' when it is acting with $f'G$ as a counter, it will be convenient to work with the corresponding member $h'i$ when it is acting with Gh' as a counter. With the load brought on from the right, the critical wheel will be at i, and the general criterion is that used for Ef' in 421 : 4, namely, $W_S = W\frac{2}{n}$. The method of determining the live-load stress in $h'i$ is also the same as that used for Ef'. Let V represent the shear for a section through the panel hi; W_h the floor-beam load at h and W_g the floor-beam load at g when the critical wheel is at i. Assuming that the diagonals can take tension only, there will be no stress in gh' when Gh' and $h'i$ are acting together as counters. The V component of the live-load stress in $h'I$ will then equal $\frac{1}{2}W_h$, and $h'I$ will be in tension. The V component of the stress in $h'i$ will equal the shear for the panel plus $h'I_V$ or:

$$h'i_V = V + h'I_V = R_L - W_g - W_h + h'I_V$$

This expression may be reduced to:

$$h'i_V = R_L - \frac{M_i}{2\,p}$$

in which M_i is the moment of all the loads to the left of the critical wheel O about wheel O when that wheel is at i. (Prove.) This expression is similar to that used in determining the stress in Ef'. (421 : 4.) (Prove that if there should be no load on panel gh, the expression would still hold good.)

4. To determine the actual counter stress in $h'i$, the dead-load compression in $h'i$ must be subtracted from the live-load tension in $h'i$. (334 : 7.)

5. *Note:* In combining dead- and live-load stresses, proper allowance should be made for impact or for increase in live load.

THROUGH BALTIMORE TRUSS WITH SUB-TIES

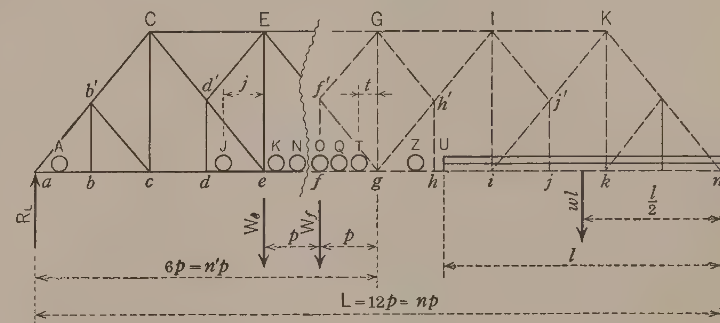


Fig. 423.

6. *To determine the stress in a lower chord member*, as, for example, the stress in efg in Fig. 423. No sub-diagonal is at either end of efg, therefore the stress in eg is determined exactly as for a lower chord member of a Pratt truss. (412 : 3.) Center of moments at E. Determine the greatest bending moment for E (399 : 3), and divide this moment by the height of truss.

7. *To determine the stress in an upper chord member*, as, for example, the stress in EG in Fig. 423. A sub-diagonal is at one end of EG. The bending moment required is that for a center at g. The critical wheel

O will be at f. The general form for the criterion for the position of the loading is:

$$W' + 2\,W_p = W\frac{n'}{n}$$

W' = weight of all loads in the panels from a to e to the left of the double panel eg; W_p = the weight of all loads in the panel ef immediately to the left of the critical wheel O. n' = the number of panels from a to the center of moments g.

Using the notation given in 372 : 7, the special form of the criterion is:

$$\left.\begin{array}{l}\mathrm{W(A\ldots J)} + 2\,\mathrm{W(K\ldots O)} > \\ \mathrm{W(A\ldots J)} + 2\,\mathrm{W(K\ldots N)} < \end{array}\right. [\mathrm{W(A\ldots Z)} + wl] \times \tfrac{6}{12}$$

First step: From the criterion determine which wheel is the critical wheel, and place the loading in position.

Second step: Determine the stress in EG from the equation:

$$\Sigma \mathrm{M}_g = 0 = R_L \times 6\,p - \mathrm{M(A\ldots J)}_g - W_e \times 2\,p + EG \times h.$$

(W_e is the partial floor-beam load at e due to the wheel loads in the panel ef.) The values of the terms in the equation may be determined as follows:

$$R_L \times 6\,p = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right]\frac{6}{12}$$

$$\mathrm{M(A\ldots J)}_g = \mathrm{M(A\ldots J)_J} + \mathrm{W(A\ldots J)} \times (j + 2\,p)$$

$$W_e \times 2\,p = \mathrm{M(K\ldots O)_O} \times \frac{2\cancel{p}}{\cancel{p}}$$

1. Alternative method: Determine the stress in EG from the equation

$$EG = (\mathrm{M_B} + W_f \times p) \div h$$

$\mathrm{M_B}$ = Bending moment of the entire live load on the truss with respect to g as a center. W_f = Floor-beam load at f due to all loads in the double panel efg.

$$\mathrm{M_B} = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right]\frac{6}{12}$$
$$-[\mathrm{M(A\ldots T)_T} + \mathrm{W(A\ldots T)} \times t]$$

$$W_f \times p = \mathrm{M(K\ldots T)_T} + \mathrm{W(K\ldots T)} \times t - 2\,\mathrm{M(K\ldots O)_O}. \quad (402 : 6.)$$

Comments

2. The criterion may be satisfied by more than one position of the loading — by some other wheel at f.

3. The alternative method is similar to that used in determining the stress in the bottom chord of a Baltimore truss with sub-struts. (420 : 11.) When wheel T happens to fall exactly at g this method can be used to advantage.

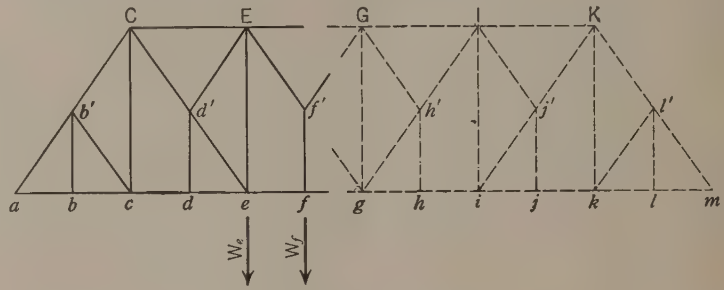


Fig. 424.

4. *Diagonals.* The live-load stress in a *sub-tie* may be obtained from its V component which is equal to one-half the maximum floor-beam concentration. For example, $d'E_V$ = one-half maximum floor-beam concentration at $d = \frac{1}{2}\left(\frac{\mathrm{M}_e - 2\,\mathrm{M}_d}{p}\right)$. (402 : 6.) $d'E = d'E_V \times$ secant truss angle. When a sub-tie acts as a counter the stress in it may be greater than when it acts merely as a sub-tie.

5. The live-load stress in the *upper half of a main diagonal* may be obtained by the method used for the diagonal of a Pratt truss or for the lower half of the main diagonal of a Baltimore truss with sub-struts. For example, since a section through Ef' cuts no other web member, the maximum live-load stress in Ef' may be determined from the maximum live-load shear for the panel ef. The critical wheel at f must be such that the general criterion $W_p = W \div n$ is satisfied. (396 : 5.)

6. The live-load stress in the *lower half of a main diagonal* may be found by the method used for the upper half of a main diagonal of a Baltimore truss with struts. For example, the maximum live-load tension in $f'g$ will occur when the critical wheel is at g and the general criterion

$W_S = W\frac{2}{n}$ is satisfied. (421 : 4.) The stress in $f'g$ is calculated from its V component, $f'g_V = R_L - W_f - W_e + f'G'_V = R_L - \frac{M_g}{2\,p}$. This equation corresponds exactly to the equation used for obtaining the stress in the counter $h'i$. (423 : 3.)

Comments

1. Note that in a Baltimore truss with sub-struts, the maximum tension in either Ef' or $f'g$ occurs when the critical wheel is at g notwithstanding the difference in the criteria for the two stresses, but in a Baltimore truss with sub-ties, the maximum tension in Ef' occurs when the critical wheel is at f and the maximum tension in $f'g$ when the critical wheel is at g.
2. The criterion for either Ef' or $f'g$ may be satisfied for more than one critical wheel.
3. When the loading is in a position to cause maximum stress in $f'g$, there may or may not be loads on the panel ef. Whether there are or not, the method of determining the stress in $f'g$ as well as the criterion for the position of the loads is the same.
4. Since the sub-diagonal in the panel bc is a sub-strut, the method of calculating the stress in $b'C$, the upper portion of the end post, is the same as for a Baltimore truss with sub-struts. (422 : 4.) The stress in ab' is determined as in a Pratt truss, from the maximum shear for panel ab with the critical wheel at b. (422 : 4.)

5. *Counters.* The maximum live-load stress in the counter ef' is equal to that in $h'i$. When $h'i$ is in action, the member Gh' also acts with it as a counter, consequently there is no stress in gh'. Under these conditions the method of determining the live-load stress in $h'i$ is similar to that just explained for $f'g$. (Critical wheel at i; criterion: $W_S = W\frac{2}{n}$; method: $h'i_V = R_L - \frac{M_i}{2\,p}$). The actual stress in $h'i$ is the live-load tension in $h'i$ minus the dead-load tension in $h'I$. (Compare with the counter stress as determined in 423 : 3.)

6. Although the member Gh' acts as a counter when the loads are in the position to cause a maximum stress in $h'i$ as just explained, the live-load stress in Gh' at that time is not a maximum; it will become a maximum when the critical wheel is at h, and the general criterion $W_p = W \div n$ is satisfied. Gh'_V is then equal to the shear in panel gh. The criterion and the method are analogous to those for Ef'. (424 : 5.) The actual stress in Gh' is equal to the live-load stress in Gh' minus the dead-load stress in gh'.

7. *Note:* In combining dead- and live-load stresses, proper allowance should be made for increase in live load (267 : 3) or for impact. (267 : 6.)

8. *Verticals.* The maximum live-load stress in a *sub-vertical* is equal to the maximum floor-beam concentration. (402 : 6.)

9. To determine the maximum stress in Cc, consider the three panels from a to d to act as a beam supported only at a and at d, and determine M_{Bc} the maximum bending moment for a section at c due to all loads on the three panels. The maximum stress in $Cc = \frac{3\,M_{Bc}}{2\,p}$. (The stress in Cc in a Baltimore truss with sub-struts is $\frac{M_{Bc}}{p}$ (422 : 6).)

10. The maximum live-load stress in the vertical Ee is ordinarily equal to the V component of the maximum stress in Ef'. (When the load is in a position to cause this stress, the live-load stress in $d'E$ is zero since there is no load at d.) In a truss of more than twelve panels, however, the load may extend so far to the left that there is a load at joint d. The position of the loads must then be such as to satisfy the general criterion:

$$\frac{W_1}{2} - \frac{W_2}{2} + W_3 = \frac{W}{n}$$

W_1 = load in panel cd; W_2 = load in panel de; W_3 = load in panel ef. The critical wheel will be either at d or at f. When the load has been placed so as to satisfy the criterion, the stress in Ee will be equal to $R_L - (W_1 + W_2) - W_e + d'E_V$ in which W_e = the reaction at e due to W_3, and $d'E_V = \frac{1}{2}$ the load at d due to W_1 and W_2.

11. The maximum live-load stress in Gg is ordinarily equal to the V component of the maximum stress in Gh', but, in a truss with a large number of panels it may be necessary to use the criterion just given for Ee and the corresponding method of calculating the stress.

12. **Deck Baltimore Truss.** The stresses in the members of the deck Baltimore truss with sub-ties are, with few exceptions, determined exactly as for corresponding members of a through Baltimore truss with sub-ties. The exceptions are the stresses in the members ab' and $b'C$, and in the verticals Aa, Cc, Ee, and Gg. Each sub-vertical is in compression instead of in tension, but the magnitude of the stress is the same as for a sub-vertical in the through truss.

1. *Stress in Aa.* The upper chord from A to C may be considered as a beam. The stress in Aa is equal to the maximum reaction at A due to all loads on the beam AC including any load at A. It will be a maximum when the loads on AC are in a position to cause maximum end shear. (One of the heaviest wheels will be at A. (381 : 11).) The stress in Aa may be determined by dividing the moment of all the loads on AC about C by $2p$.

2. *Stress in ab′.* From $\Sigma H = 0$ applied at a, $-ab'_H = ac$, hence the stress in ab' will be a maximum when the stress in $ac = ce$ is a maximum, i.e., when the load, with a critical wheel at C, is in a position to cause the

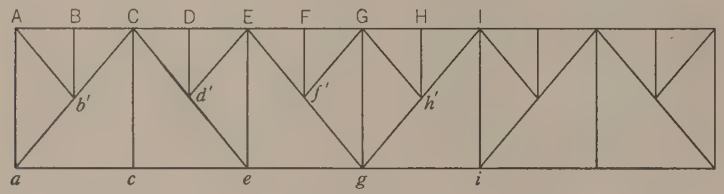


Fig. 426.

greatest bending moment for C. (387 : 2.) The stress in ab' may be calculated from its H component, i.e., from the previously calculated stress in ac.

3. *The stress in b′C* will occur when the critical wheel is at C, and the loads are in a position to cause maximum shear in panel BC. (Criterion: $W_p = W \div n$.) The V component of $b'C$ is equal to this maximum shear.

4. *The stress in Cc* due to live load is obviously zero.

5. *Stress in Ee.* From $\Sigma V = 0$ applied at joint e, the maximum live-load stress in Ee is equal to the V component of the maximum live-load stress in $d'e$. (Compare with the stress in Ee for a through truss. (425 : 10).)

6. *Stress in Gg.* The stress in Gg is equal to the full floor-beam concentration at G, plus one-half the floor-beam concentration at F, plus one-half the floor-beam concentration at H. The method of determining the stress in Gg is therefore that used for determining the stress in the member Cc of a through Baltimore truss with sub-struts. (422 : 6.) The stress in Gg is compression whereas that in Cc is in tension; this is the only difference. The loads in panels E to I are placed to produce a maximum bending moment for a section at G, when the four panels E to I are considered as a beam supported only at E and at I, and the stress is equal to $\frac{M_{B_G}}{p}$ or $\frac{M_I - 2\,M_G}{2\,p}$ in which M_{B_G} = bending moment for section at G due to all loads on the four panels $EFGHI$, M_I = moment of the same loads about I, and M_G = moment of all loads in the double panel EFG about G.

7. In a truss with more than twelve panels, an equal or greater stress may be obtained in the center post by placing the load in such a position that it causes the maximum stress in the left-hand diagonal at the foot of the post. The stress in the center post will then be equal to the V component of the stress in that diagonal. (Why?) (336 : 2.) Note that in this case the method of determining the stress in the center post is similar to that used for the post Ee.

8. **Parker Truss.** The methods of determining the stresses in a Parker truss due to moving concentrated loads differ very little from those used for a Pratt truss of the same number of panels, as may be judged from the following general statements.

(*a*) The stress in any member of the bottom chord is determined exactly as for the corresponding member of a Pratt truss.

(*b*) The stress in any member of the top chord is determined as for the corresponding member of a Pratt truss, except that the lever arm of an inclined chord member of a Parker truss is not the height of truss. (344 : 1.)

(*c*) The criterion for the position of concentrated loads for maximum stresses in web members differs but slightly from that for web members of a Pratt truss. Once the load is placed the stress in a web member is determined by a method that is analogous to that used for determining the stress in a Parker truss when the live load is uniformly distributed. (344 : 4.)

9. The criterion for the position of the concentrated loads for the maximum stress in any web member, except the hip vertical, is: $W_p = \frac{W}{n} \times \frac{v}{u}$ in which W_p is the total load in the panel of the *loaded* chord cut by the section, u is the distance from the left-hand end of that panel to the center of moments m taken at the intersection of the upper and lower

chords cut by the section (see Fig. 427), v the distance from the left-hand support to the same point of moments, and W the total load on the span. The critical wheel is placed at the joint of the lower chord immediately to the right of the section, and this wheel is ordinarily one so near to the first wheel that no load extends beyond the joint of the lower chord immediately to the left of the section.

1. *Note:* For the parallel chords of a Pratt truss, v equals infinity and the term $\frac{v}{u}$ in the criterion just given becomes unity. The criterion then becomes $W_p = \frac{W}{n}$ which is that for the position of the load that will cause the maximum live-load stress in a web member of a Pratt truss.

2. When a web member is in the right-hand half of the truss the center of moments m' is to the right of the truss, and the term $\frac{v}{u}$ in the criterion becomes $\frac{v'}{u'}$. (Fig. 427.) Note that v' like v extends from the left-hand support to the center of moments; this is because each is the lever arm of R_L in a moment equation of equilibrium. Similarly u' like u extends from the left-hand end of the lower-chord panel cut by the section; this is because each is the lever arm of the floor-beam concentration at that end of the panel.

3. When the distance v is equal to an even number of panel lengths, it is advantageous to express v, v', u, and u' in terms of panel lengths.

4. *To determine the stress in the lower chord member cd* in Fig. 427. The center of moments is C, the critical wheel is at c, the criterion is $W_S = W\frac{n'}{n}$ and the stress in $cd = \mathrm{M}_{\mathrm{Bc}} \div h$ (at c), exactly as for the member cd of a Pratt truss. (412 : 3.)

5. *To determine the stress in the upper chord member BC.* The center of moments is c, the critical wheel is at c, the criterion is the same as that just given, and the stress in $cd = \mathrm{M}_{\mathrm{Bc}}$ divided by the lever arm of BC with respect to c. Note that the bending moment M_{Bc} is the same as that for the stress in cd, and is obtained as indicated in 411 : 4. The stress in BC may also be obtained by calculating from the bending moment at c the H component of BC and calculating BC from BC_H. (344 : 6.) If the maximum stress in cd has previously been calculated, this stress is equal to the H component of the maximum stress in BC.

6. *To determine the maximum tension in the diagonal Bc.* (Fig. 427.)

Critical wheel at c. Criterion: $W_p = \frac{W}{n} \times \frac{v}{u}$ (426 : 9.)

$$\begin{array}{l} \mathrm{W(A\ldots O)} > \\ \mathrm{W(A\ldots N)} < \end{array} \frac{\mathrm{W(A\ldots Z)} + wl}{7} \times \frac{v}{u}$$

When the critical wheel O has been determined, calculate $R_L = \left[\mathrm{M(A\ldots Z)_U} + \mathrm{W(A\ldots Z)} \times l + wl \times \frac{l}{2}\right] \div 7p$, and W_b the floor-beam load at b due to loads in panel bc, i.e., $W_b = \mathrm{M(A\ldots O)_O} \div p$. Stress in $Bc = (-R_L \times v + W_b \times u) \div z$. The distance v and the lever arm z of Bc are determined as explained in 346 : 2 and 347 : 2.

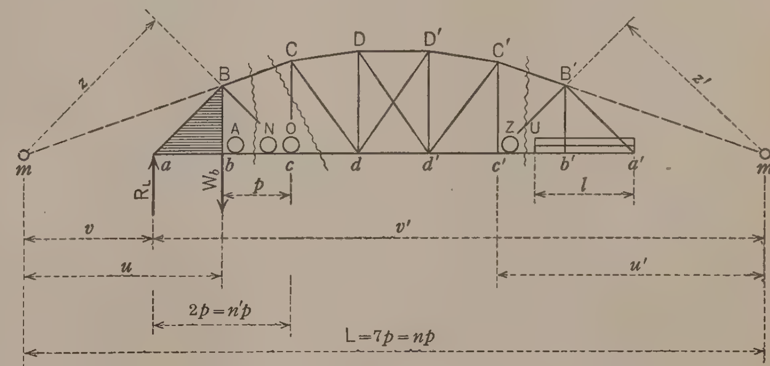


Fig. 427.

7. *To determine the maximum compression in the diagonal Bc for a truss without counters.* It will be convenient to work with the corresponding diagonal $c'B'$. The center of moments m' will be the point of intersection of $C'B'$ and $c'b'$ and therefore to the *right* of the truss. The critical wheel will be at b', and the criterion will be similar to that just used for Bc, namely, $W_p = \frac{W}{n} \times \frac{v'}{u'}$, in which W_p = total load in panel $c'b'$, v' = distance from a to m', and u' the distance from c' to m'. The method of

calculating the stress in $c'B'$, once the loads are placed, is practically the same as that used for Bc.

$$\Sigma M_{m'} = R_L \times v' - W_{c'} \times u' + c'B' \times z' = 0$$

in which $W_{c'}$ is the floor-beam load at c' due to the loads in panel $c'b'$.

1. *Note:* The stress in a diagonal may be determined from the maximum shear as in the case of uniform live load (346 : 6), but since a portion of the shear is resisted by the inclined top chord member, this method involves the V component of the stress that is in that top chord member when the load is in the position to cause maximum stress in the diagonal. Since this stress in the chord member is not a maximum stress, it would be necessary to determine it (or its V component) as a step in the calculation of the stress in the diagonal. For this reason it is simpler to use $\Sigma M = 0$, as illustrated above, than to use $\Sigma V = 0$.

2. *To determine the live-load stress in the counter cD.* Assume that the truss in Fig. 427 is one with counters. To determine the stress in the counter cD it will be convenient to work with the corresponding counter $D'c'$. Conceive the diagonal $d'C'$ omitted, and determine the maximum live-load tension in $D'c'$ by the method explained for Bc. (427 : 6.) The actual stress in the counter, due to combined dead and live load, is obtained by the method explained for uniform live load. (355 : 1.)

3. *To determine the stress in the vertical Cc.* The critical wheel is at d, the center of moments at m and the criterion is the same as for a diagonal: $W_p = \frac{W}{n} \times \frac{v}{u}$ in which W_p is the load in the panel cd, W the total load on the span, n the number of panels in the span, v the distance from R_L to m, and u the distance from W_c, the floor-beam load at c, to m.

$$\Sigma M_m = -R_L \times v + W_c \times u + Cc \times u = 0$$

Note that the distance u is not the same as that that is indicated in Fig. 427. The stress obtained by solving the equation is the maximum live-load compression. The maximum live-load tension in Cc may be obtained from

$$\Sigma M_{m'} = R_L \times v' - W_{d'} \times u' + C'c' \times (u' - p)$$

in which u' is the distance from the floor-beam load $W_{d'}$ at d' to m'. Note that this value of u' differs from that used in determining the compression in the diagonal $B'c'$ and shown in Fig. 427.

4. *The stresses in aB and Bb* are determined exactly as for a Pratt truss. (413 : 7 and 414 : 1.)

5. *The stresses in members of the center panel* are determined exactly as for the center panel of a Pratt truss with the same number of panels. (415 : 2 and 416 : 1.)

6. *Effect of counters, tension in verticals, and minimum stresses* have been treated in CHAPTERS XX and XXI in connection with uniform live load. (325 : 3 to 328 : 2 and 356 : 1 to 357 : 5.) General methods of analysis are the same when the live loads are concentrated.

7. **SKEW BRIDGES.** Conditions at a bridge site are frequently such that a skew bridge is necessary. In Fig. 428 is shown the general plan of a common type of skew bridge.

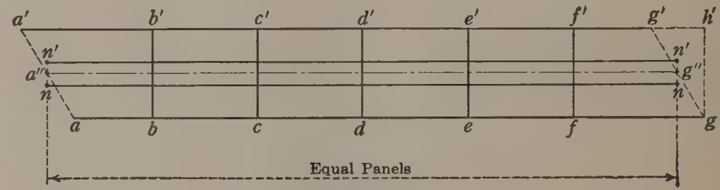


Fig. 428.

The line ag represents the lower chord of one truss and the line $a'g'$ represents the lower chord of the other truss. These two trusses are of the same length. The distance $g'h'$ is the **skew** and the angle $g'g''a''$ is the **skew angle.** The skew is the same at each end of the bridge. The intermediate floor beams are at right angles to the axis $a''g''$ of the bridge. There are no end floor beams; instead, the masonry supports the end stringers at n and n'. The end stringers are equal in length to the stringers in the intermediate panels, hence the six panels measured on the axis $a''g''$ are equal. The panels ab and $f'g'$ in the trusses, however, are shorter and the panels $a'b'$ and fg are longer than the regular panel length. Each truss is therefore unsymmetrical. It is desirable that all four end posts should have equal inclinations. For example, if the type of truss adopted is the Pratt, it may be modified slightly as shown in Fig. 429 (a), in order that the end posts aB and Gg may have equal inclinations. A similar modification is made in the opposite truss except that the long panel is at the left-hand end.

1. *Note:* It may not be practicable in some cases to make the length of the end stringers the same as that of the intermediate stringers, but even in such a case it is desirable that the two stringers at either end of the bridge should be equal in length. Occasionally it is possible to make the skew equal to the regular panel length, in which case the methods of determining stresses are practically the same as for a bridge with

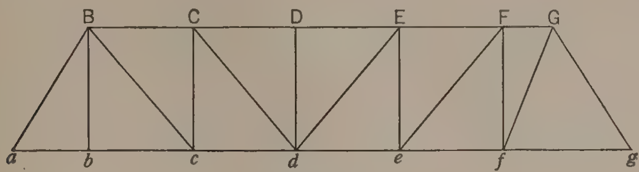


Fig. 429 (a).

square ends. Occasionally it is necessary to make the skew at one end of a bridge different from that at the other. The methods of determining stresses in this more general case do not differ essentially from those to be explained for the common case just described, and hence only the common case will be treated here.

2. *Dead-load stresses.* The dead-load concentrations at lower joints b and f' in Fig. 429 (b) are theoretically less and those at b' and f are theo-

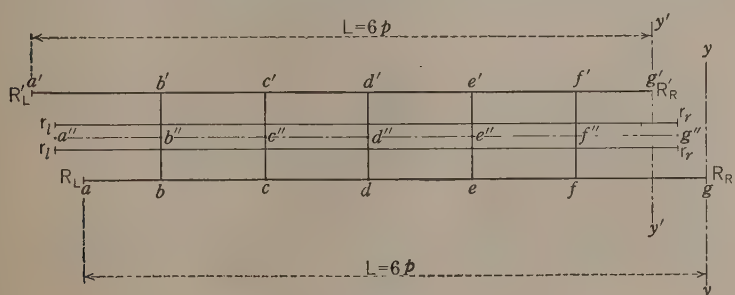


Fig. 429 (b).

retically greater than the regular concentrations at intermediate panel points. The differences, however, may be so slight as to be negligible. The dead loads at the joints of trusses having been determined, the stresses in each truss may be found by one of the methods explained in **PART II.** Since each truss is unsymmetrical, the method of sections is generally used. This method is essentially the same for unsymmetrical as it is for symmetrical trusses.

3. *Live-load stresses.* The general method of determining the maximum stress in a given member due to locomotive loading is as follows:

4. First step: Place the locomotive loading in the position in which it will cause the maximum stress in the given member. For this purpose, the loading may be treated as if it were applied along the axis $a''g''$ in Fig. 429 (b) and as if the ends of the bridge passing through a'' and g'' were square instead of on a skew. The critical wheel for the greatest shear for a given section or for the greatest bending moment for a given center may then be ascertained either from one of the tables on page 389 or by means of a criterion, exactly as for a square bridge. For example, for the greatest shear through panel bc or panel $b'c'$, the critical wheel will be at c'', and it may be determined as explained in 398 : 3; for the greatest bending moment for a center at c or at c', the critical wheel will be at c'', and it may be determined as explained in 399 : 3.

5. Before proceeding to the second step, it should be noted that the external forces on the bridge due to live load are: (a) the locomotive loading in the position determined in the first step; (b) the two equal reactions r_l and r_l at the ends of the stringers at the left-hand support and the two equal reactions r_r and r_r at the ends of the stringers at the right-hand support; (c) the reactions R_L and R_R at the ends of the nearer truss and the reactions R'_L and R'_R at the ends of the farther truss.

6. Second step: Determine the reactions r_l, r_r, R_L, and R_R. Conceive the bridge as divided into two parts by a vertical plane through the axis $a''g''$. Consider the portion in front of the axis $a''g''$ (shown in Fig. 430) as a body in equilibrium. The external forces that act on this body are the locomotive loading on one rail applied on the line of stringers, the reactions r_l, r_r, R_L, and R_R, and the forces in each floor beam where it is cut by the section through $a''g''$.

7. The reaction r_l may be determined from the portion of the locomotive loading that is in panel $a''b''$. For certain positions of the locomotive loading headed toward the left, there will be no load in this panel, and consequently r_l will then be zero. The reaction r_r may be determined from the loading in panel $f''g''$. For engines headed toward the left there will always be load in this panel.

1. The reaction R_L in Fig. 430 may be determined from the sum of the moments of the external forces about the axis yy, i.e., from:

$$\Sigma M = R_L \times L + r_l \times (L + s) + r_r \times s - M_y = 0$$

In this moment equation, M_y represents the moment of the locomotive and train loads (in the position determined in the first step) with respect to the axis yy. Note that any portion of the loading between g'' and the axis yy is not on the bridge and is not, therefore, to be included in the moment.

2. *Note:* Since each floor beam is symmetrically loaded with respect to its center, the live-load shear for any section between the two lines of stringers is zero, therefore there can be no vertical forces where the vertical section through $a''g''$ cuts a floor beam. The horizontal forces at such a section are parallel to the axis yy and consequently have no moments with respect to that axis.

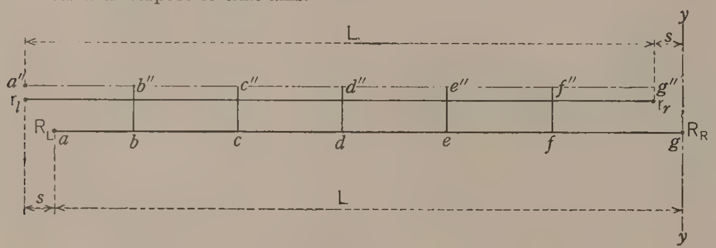


Fig. 430.

3. The reaction R'_L may be determined in a similar manner by considering the portion of the bridge that is behind the axis $a''g''$ as the body in equilibrium. From Fig. 429 (*a*) it is evident that the moment equation with respect to the axis $y'y'$ is:

$$\Sigma M = R'_L \times L + r_l \times (L - s) - r \times s - M_{y'} = 0$$

In this moment equation $M_{y'}$ is the moment of the locomotive and train loads with respect to the axis $y'y'$, but note that any portion of the loading between g'' and $y'y'$ causes a positive instead of a negative moment.

4. Third step: Determine the required stress in the given member. If this member is a *chord member*, the stress may be determined from the bending moment about a horizontal axis perpendicular to the axis of the bridge and passing through the center of moments for that member. The external forces on the segment will be the truss reaction R_L, the stringer reaction r_l and the live load. For example, let the given member be the top-chord member for which the joint c in Fig. 430 is the center of moments. The axis of moments will be $c'c''$. The stress S_c will be equal to the bending moment divided by the height of truss h or:

$$S_c = [R_L \times (2p - s) + r_l \times 2p - M_c] \div h$$

in which M_c is the moment of the live load on one rail from the left-hand end of the bridge to the floor beam at c'' about cc'' as an axis.

5. If the given member is a web member, the stress may be determined, as usual, from the shear. The external forces on the segment will be the truss reaction R_L and the floor-beam concentration at the left-hand end of the panel through which the section is taken, just as for any standard bridge truss. For example, let the given member be the diagonal in the panel bc in Fig. 430. The stress S_d will be equal to the shear multiplied by $\frac{i}{h}$, in which i is the length of the diagonal, or

$$S_d = [\mathrm{R_L} - \mathrm{W_b}] \times \frac{i}{h}$$

in which W_b is the floor-beam concentration at b when the critical wheel is at c.

6. The stresses in members of the farther truss may be determined by similar methods. The stresses in the right-hand half of the nearer truss for the loading headed toward the right will be the same in corresponding members as the stresses in the left-hand half of the farther truss for the loading headed toward the left, and similarly, the stresses in the right-hand half of the farther truss for the loading headed toward the right will be the same in corresponding members as the stresses in the left-hand half of the nearer truss for the loading headed toward the left.

7. It will be observed that the methods of determining the stresses in the trusses of a skew bridge, once the reactions are known, do not differ essentially from those used in determining the stresses in the trusses of a bridge with square ends.

8. *Note:* When the locomotive loading is in the position to cause the maximum stress in a web member, the stringer reaction r_l is usually zero, and the stringer reaction r_r is relatively small. When the locomotive is in the position to cause the maximum stress in a chord member, the stringer reactions r_l and r_r may be so small compared with the total live load, that they do not materially affect the determination of the truss reactions R_L and R'_L. One soon learns from practical experience to what extent these stringer reactions may be disregarded.

1. *Alternative method of determining live-load stresses in the trusses of a skew bridge.* Consider the locomotive loading *per track* as applied along the axis $a''g''$ of the bridge in Fig. 429 (b). When the loading has been placed in the position to cause the maximum stress in the given member, determine the concentration per track at *each* floor beam by the method explained in 402 : 6. Divide the concentration at each floor beam equally between the corresponding truss joints at the ends of the floor beam. With the loads thus determined at the joints of the loaded chord of a truss, the stress in the given member may be calculated by the method of sections, just as if the loads were dead loads. This method usually involves more work than the method previously explained, because, for each position of the loading, concentrations at all floor beams must be determined.

2. *Stresses due to uniform live load* may be found by a method similar to that just explained in the preceding paragraph. When determining the stresses in web members, the conventional method of distributing the uniform live load to joints of the loaded chord of a truss is sufficiently accurate for ordinary bridges.

3. **Bridges on Curves.** When the tracks on a bridge are curved, it may be necessary to make the width center to center of trusses greater than when the tracks are straight, although for slight curvature the standard spacing may suffice.

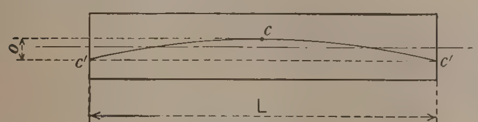


Fig. 431 (a).

4. *Spacing of trusses for a deck bridge.* In Fig. 431 (a) the two full horizontal lines represent the center lines of the girders or trusses of a deck bridge of length L. The curve $c'cc'$ is the center line of the track. The distance o is the middle ordinate of the curve for a chord of length L. If R is the radius of curvature:

$$o = R - \sqrt{R^2 - \frac{L^2}{4}}$$

The following approximate formula is more easily applied, and the results are sufficiently accurate for practical purposes.

$$o = \frac{DL^2}{3820} = \frac{3}{2}\frac{L^2}{R}$$

D = degree of curve; L = chord in feet; R = radius in feet. In order to equalize as nearly as possible the live load on the two main girders or trusses, the track is placed so that the point c is from $\frac{1}{3}\,o$ to $\frac{1}{2}\,o$ from the axis of the bridge. To provide for the middle ordinate o in a deck bridge, it may be necessary to increase the minimum distance between the plate girders given in 91 : 2 or the minimum distance between trusses given in 110 : 3.

5. *Spacing of trusses for a through bridge.* When the track on a single-track bridge is straight, the width in the clear between trusses, as given in recent specifications is not less than 16 ft. (110 : 2.) When the track is curved, it may be necessary to increase this width in order to provide not only for the curvature of the track but also for the tilting and the swinging of the cars.

6. The Pratt truss in Fig. 431 (b) represents a railway bridge with a single curved track. In Fig. a is shown the cross section of a car tilted to

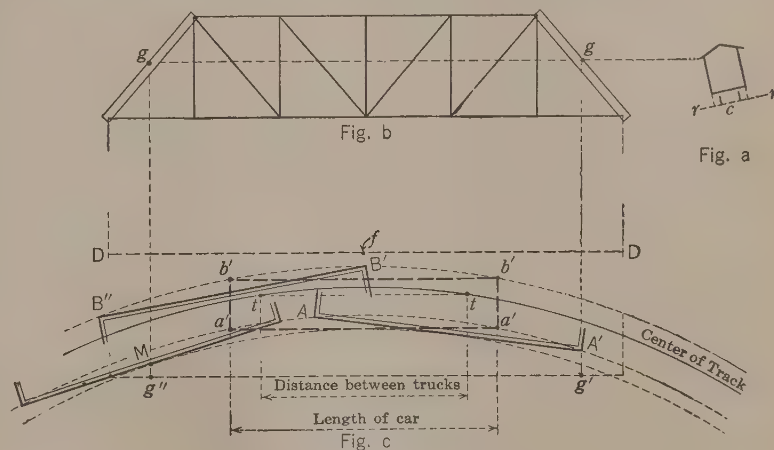


Fig. 431 (b).

correspond to the superelevation of the outer rail on the curve. The line rr represents a line across the tops of the rails. In Fig. c the curve through tt represents the center of the track, and the points t and t represent the centers of the trucks of a car $a'a'b'b'$ when it is at the center of the bridge.

1. The curve through $b'b'$ represents the horizontal projection of the path followed by the outer corners B'' and B' of the car as it swings around the curve. The distance of this curve from the curve representing the center of the track may be determined by the extreme half-width of the car either at the top or at the bottom, depending upon the variable car dimensions and the amount of tilting due to the superelevation of the outer rail. For flat curves, the top width will govern, probably the width to the outside of the rear signal light but possibly to the eaves; for sharper curves, the bottom width may govern, probably the width to the outer edge of the lower step of a Pullman car, but possibly to the lower corner of the car body or to the grab handles. The curve through $b'b'$ is nearest to the outer truss at the midpoint f, and the distance from B' to f, therefore, should not be less than the minimum allowable clearance.

2. The curve through $a'a'$ represents the horizontal projection of the path followed by the inner corners A'' and A' of the car as it swings around the curve, and the curve through M represents the horizontal projection of the path of the midpoint of the inner side of the car. The inner side of the car as it swings around the curve will pass nearest to the inner truss at an end post. Whether the curve through $A''A'$ or that through M is nearer the end posts depends upon the inclination of the end posts, the variable car dimensions, and the amount of superelevation. If the end post is vertical, the signal light (following the curve through AA') or the midpoint of the eaves (following the curve through M) will pass nearest to the end post; if the end post is inclined, the signal light or the lower step at the end (following the curve through AA') or the midpoint of the eaves or of the battery box or of the water tank or of the trussing rods (following the curve through M) may pass nearest to the end post. Whichever curve is nearest, the distance from that curve to the nearest point g' (or g'') of an end post should not be less than the minimum allowable clearance.

3. The width between the clearance line DD of the outer truss and the clearance line $g''g'$ of the inner truss is the minimum allowable width in the clear between trusses.

4. *Note:* The minimum distance in the clear between bridge trusses when the track is curved is usually determined by long Pullman cars rather than by freight cars, although specially wide coal cars or low hopper cars must be considered. For any type of car the critical dimensions vary considerably. In recent specifications it is stated that on curves provision shall be made for a car 80 ft. long, 60 ft. between truck centers, and 14 ft. high above the top of a 6-inch rail; that unless otherwise specified, the superelevation of the outer rail shall be $\frac{3}{4}$ inch for each degree of curvature, with a maximum of 6 inches. Typical limiting widths of cars measured at various elevations above the *tops* of the rails are as follows: Between eaves, 10 ft. 5 in. measured at elevation 11 ft. 4 in.; out to out of signal lights, 10 ft. 10 in. (sometimes 11 ft. 2 in.) measured at elevation 10 ft. 6 in.; out to out of lower steps, 10 ft. 0 in. at elevation 1 ft. 6 in.; bottom of the car body, 9 ft. 10 in. at elevation 3 ft. $7\frac{1}{2}$ in.; out to out of grip handles, 10 ft. 4 in. at elevation 4 ft. 1 in.; width at the trussing rods (or battery boxes) under the car, 9 ft. 7 in. at elevation 1 ft. 2 in.

5. *Note:* The positions of different portions of the car as it swings around the curve depends not only upon the curvature of the track, but also upon the distance between truck centers as is evident from Fig. 431 (b). When this distance is 60 ft., the ordinate as determined from the formula $o = \frac{DL^2}{3820}$ in 431 : 3 is approximately 1 inch for each degree of curvature.

Note: Great refinement in the calculation of clearances for degrees of curvature that commonly occur on main-line bridges is not usually necessary because ample clearances have been standardized, but for bridges on branch lines and for trolley bridges it may be essential to determine clearances accurately. Moreover, railroad companies are called upon constantly to ascertain if special cars of all kinds can be routed over their lines.

6. *Spacing of stringers.* There are two general methods of spacing stringers to provide for the curvature of the track. When the curvature is comparatively small, each of the two lines of stringers is straight from end to end of bridge but the distance between them may be greater than for a straight track. When the curvature is so large that this arrangement would make the distance between the two lines of stringers too great, the pair of stringers in each panel may be offset to conform to the curve as shown in Fig. 432 (a).

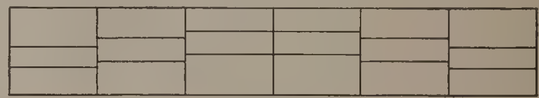


Fig. 432 (a).

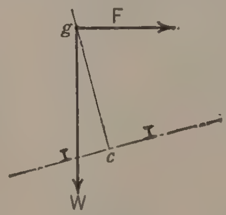


Fig. 432 (b).

7. *Effect of tilting.* The principal forces that act on an engine or a car as it passes around a curve are gravity, represented by W in Fig. 432 (b), the centrifugal force, represented by F, and the reactions exerted by the two rails. The forces W and F may be considered as applied at the center of

gravity g which may be taken as 7 ft. above the base of rail. The effect of the centrifugal force will be considered in the next chapter. The effect of tilting is to throw the line of action of W nearer to the inner rail than it is to the outer rail. The load on the inner rail will then be increased and that on the outer rail will be decreased as compared with the equal loads on the two rails when the track is straight.

1. *Eccentric loading on stringers.* Let ab and $a'b'$ in Fig. 433 (a) represent two stringers in the same panel, and dd an axis midway between the two stringers. Let the curve gg represent the horizontal projection of the point g in Fig. 432 (b) as this center of gravity swings around the curve. The curve gg lies between the center of the track and the inner rail. The eccentricity at any point of the curve is the perpendicular distance from that point to the axis dd. Assume that a locomotive is in the position in which it causes the maximum bending moment in the stringer ab. Disregard, for the present, the overturning effect of centrifugal force. The total load on any axle may be assumed to take effect at the point where that axle crosses the curve gg, and the eccentricity of the load is the distance from that point to the axis dd. Because of this eccentricity a greater portion of the total axle load will eventually take effect on stringer ab than on stringer $a'b'$. This statement holds true for all axle loads in the panel, but it is to be noted that the eccentricity differs for each axle load. The wheel loads on the stringer ab could be easily calculated. These calculations, however, would be based on assumptions that are not in accord with actual conditions. For example, the center of gravity was assumed in 432 : 7 to be 7 ft. above the base of rail and this assumption partly determines the curve gg, but the height of the center of gravity above the rail differs for different parts of the locomotive. Since the assumptions are only approximately in accord with actual conditions, refinements in calculating the distribution of loads between the two stringers are not justified. The following approximate method is sufficiently accurate: Determine the equivalent uniformly-distributed live load per linear foot of *track* that would cause the same maximum bending moment in the stringer ab as the locomotive loading. (405 : 1.) Determine the *average* eccentricity for the entire panel of the curve gg'. From the average eccentricity determine the proportion of the equivalent uniform load that is carried by the stringer that is the more heavily loaded, and from this uniform load on the stringer determine the maximum bending moment. The average eccentricity differs for different panels. It is usually economical to determine the bending moment and end shear for the most heavily loaded stringer, regardless of which panel it is in, and then, having designed this stringer and its connections, to make all other stringers like it.

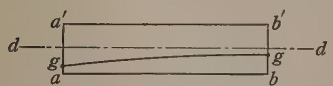


Fig. 433 (a).

2. *Note:* Centrifugal force produces an overturning effect that increases the loads on the outer rail and stringer and reduces the load on the inner rail and stringer, an effect opposite to that of eccentricity due to tilting and curvature. This will be explained in the next chapter.

3. *Eccentric loading on floor beams.* Not only is the live load distributed unequally between the stringers that are connected to a floor beam, but the stringer connections may be eccentric with respect to the center of the floor beam, particularly if the stringers are offset. The eccentricity *at* a floor beam is the distance from the center of the beam to the point in which the curve gg in Fig. 433 (a), representing the path of the assumed center of gravity, intersects the axis of the floor beam. This eccentricity differs for different floor beams. As in the case of stringers, refinements in calculating the distribution of the live load are not justified. There are several approximate methods of determining the bending moment for a floor beam. The following method is often used: Disregard, for the present, the overturning effect of centrifugal force. Determine the mean eccentricity e for the length of the curve gg extending one-half a panel length on each side of the floor beam. Determine the maximum total live load W_f that the stringers can bring to the floor beam. Proceed as if the track were straight and use one of the methods in 401 : 6 for determining W_f. Consider W_f applied at a distance e (mean eccentricity) from the center of the beam as shown in Fig. 433 (b), and determine the bending moments at the two points s and s' of stringer connections. M_B at $s = R_L \times a$ and M_B at $s' = R_R \times b$.

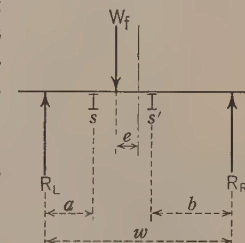


Fig. 433 (b).

If the track were straight each of the reactions would ordinarily be $\frac{W_f}{2}$.

In Fig. 433 (*b*), however, $R_L = \frac{W_f}{2} + W_f \frac{e}{w}$, and $R_R = \frac{W_f}{2} - W_f \frac{e}{w}$. The quantity $W_f \frac{e}{w}$ may be regarded as a correction for eccentricity that may be used in determining the reactions at the ends of a floor beam; it is added to $\frac{W_f}{2}$ to obtain the reaction at the end of the beam nearer to the applied force W_f, and subtracted from $\frac{W_f}{2}$ in obtaining the other reaction. The mean eccentricity differs for different floor beams. It is usually economical to design that floor beam for which the stresses are greatest, and then to make all other intermediate floor beams like it.

1. *Note:* Centrifugal force produces an overturning effect that affects the live loads on the floor beam. This will be explained in the next chapter.

2. *Eccentric loading on the trusses.* Let *aa* and *a′a′* in Fig. 434 represent the trusses of a six-panel, single-track bridge with a curved track. Let *dd* represent the axis of the bridge half-way between trusses, and let *gg* be the horizontal projection of the path of the center of gravity *g* (assumed in 432 : 7) as this point passes around the curve. The eccentricity

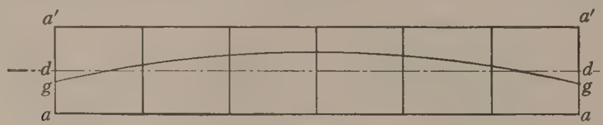


Fig. 434.

at any point of the curve is the distance from that point to the axis *dd*. Due to eccentricity the live loads brought to the two trusses by a single floor beam are unequal. (Disregard, for the present the overturning effect of centrifugal force.) Since the mean eccentricity differs for different floor beams, the live loads at the joints of the loaded chord of each truss are unequal. For reasons already given in connection with stringers and floor beams, these floor-beam concentrations on the trusses can be determined only approximately, and consequently refinements in the calculation of stresses are not justified. The following approximate method may be used: To calculate the stress in any member determine first the equivalent uniform live load *per track* for that member just as if the track were straight. (405 : 1.) Let W represent the total panel load *per track* found by multiplying the equivalent load per linear foot by the length of panel. Place a load of $\frac{W}{2}$ tentatively at each end of each floor beam that should be loaded (according to the conventional method) in order to obtain the maximum stress in the given member. At each loaded floor beam determine the mean eccentricity e, and add or subtract the corresponding correction of $W \frac{e}{w}$ to the load $\frac{W}{2}$ at each end of the beam, as explained in 433 : 3. Since the mean eccentricity differs for different floor beams, the correction will differ. The live loads at the loaded joints of each truss having thus been determined, the stress in the member is best calculated by the method of sections. Since the live loads on one truss differ from the corresponding loads on the other truss, the live-load stress in a member of one truss will differ from the live-load stress in the corresponding member of the other truss.

3. *Note:* Centrifugal force, producing an overturning effect, causes vertical loads downward on the truss that is outside of the curve and vertical forces upward on the truss that is inside. This will be explained in the next chapter.

4. The extent to which calculations should be carried in determining eccentric loadings on stringers, floor beams, and trusses, either by one of the approximate methods given here or by any other method, depends largely upon the curvature of the track. When the curvature is great, it may be desirable to determine the eccentric loadings as accurately as possible, but when the curvature is slight, the eccentricity may be too small to affect materially the distribution of the live load.

5. **Tables, Diagrams, and Formulas.** This chapter has been devoted to an explanation of the theoretical methods of determining stresses due to concentrated load systems in general. The work of computation in actual practice may be reduced greatly by the use of various tables, diagrams, and formulas. The tables mentioned in 407 : 2 as useful in the calculation of reactions, shears, and bending moments due to concentrated loads on beams may be used for trusses as well. In addition, various useful tables may be prepared especially for any type of truss and for any system of locomotive loading.

1. *Formulas.* When one can calculate stresses due to locomotive loadings *without* the use of formulas he is qualified to use them. As stated in 411 : 3, the algebraic equations in this chapter are not intended as formulas, but rather as expressions that indicate general methods of procedure. The experienced engineer, however, can and does use to advantage various formulas in the calculation of stresses, and the structural engineer should be constantly on the alert to adopt formulas and various other short cuts that are best adapted to his work.

2. **Systematic Methods of Procedure.** The methods of procedure given in this chapter have been for individual, typical members. In the calculation of stresses in *all* members of a truss, some systematic, efficient method of procedure should be followed, regardless of whether or not diagrams or tables are used. Some such method as that indicated in the illustrative problem on pages 366 and 367 is recommended. It is well to *indicate* the work of computation as completely as may be necessary before calculations are made, and to group like operations. In the determination of live-load stresses due to concentrated loads, the groups should include preliminary calculations, the calculations of bending moments for various centers, and the calculations of shears for different panels. The beginner will find it convenient to indicate criterions, shears, and bending moments by some such method as that used in this chapter. He can then read from the tables for shears and bending moments many of the quantities indicated, and finally carry out such calculations as may be necessary.

3. **Equivalent Uniform Live Load.** In addition to the simple equivalent live load as determined by the methods of 405 : 3 to 406 : 7, there is the combined equivalent load system which consists of a uniform load and one or more concentrated excess loads. An example of such a system is the loading for highway bridges described in 340 : 1. Equivalent live load is very useful in the calculation of stresses in trusses, particularly when the trusses are of the more complex type. Tables or diagrams of equivalent uniform live loads, such as those mentioned in 407 : 1, may be found in various structural engineers' handbooks and elsewhere.

4. **Locomotive Loading Versus Uniform Live Loads.** Ever since locomotive loadings for railroad bridges were first proposed, they have been criticized as a refinement not justified by actual conditions. Some system of locomotive loading, is, nevertheless, specified as the live load in practically every standard set of specifications for railroad bridges. Many of the leading structural engineers, however, advocate the use of equivalent uniform loads, particularly, in the determination of stresses in trusses. There is, perhaps, no important question in structural engineering on which there is such a difference of opinion among engineers. The discussion on the subject has extended over many years. (See assignment at the end of this chapter.)

5. Whatever may be the relative merits of the two systems of live load, concentrated and uniform, it is obvious that the stresses due to either system are not the actual stresses in a bridge when a train passes over it. The effect of a train in motion is to increase very materially the stresses that occur when the same train is assumed merely to stand on the bridge. The impact allowance in all standard specifications is intended to provide for this increase, but in spite of the careful experiments upon which empirical formulas for impact are based, the actual effect on a bridge of the movement of a train is still an uncertain quantity. In view of this, the number of places to which some tables are carried, and the number of apparently certain figures that are often given as the results of computations are not only misleading, but are a reflection upon the scientific ability of the engineer. (88 : 3.) One of the criticisms that may be made of systems of locomotive loadings is that they seem to be more accurate than they really are. When this leads to unjustifiable refinements in computations it also leads away from the true scientific method.

ASSIGNMENTS

(1) Report on the various discussions concerning live loads for highway bridges, particularly discussions in the *Transactions of the American Society of Civil Engineers.*

(2) Report on experiments made to determine the effect of the motion of locomotives and trains on railroad bridges.

(3) Report on experiments made to determine the correct impact allowance for highway bridges.

(4) Report on discussions concerning impact allowance.

(5) Report on the opinions of various engineers as to the relative merits of the different systems of loading for railroad bridges — locomotive loading versus uniform live load.

CHAPTER XXIV

HORIZONTAL FORCES ON BRIDGES

Horizontal external forces cause stresses in the bracing of a bridge. When such forces tend to overturn the bridge, they may also cause stresses in the main trusses and in members of the floor system. In this chapter are explained methods of determining the various stresses due to horizontal forces, particularly stresses in different forms of bracing.

1. **Lateral Forces.** A **lateral force** on a structure, in a literal sense, is one that acts on the side of the structure. Lateral forces on bridges, however, are generally understood to include all horizontal forces that may be assumed to act in a direction normal to the planes of the main trusses. The most important of these lateral forces are: (a) wind pressure on the exposed surface of the bridge; (b) wind pressure on the live load crossing the bridge; (c) lateral forces due to the movement of the live load, such, for example, as the lateral forces due to the " nosing," lurching, and swaying of engines and train; (d) lateral forces due to centrifugal force when a bridge is on a curve, particularly a railway bridge with curved track.

2. **Wind Pressure.** *The difficulties in estimating wind pressure* were explained in 133 : 1. Some of the more important questions which arise in connection with wind pressure on bridges are: (a) What is the actual area exposed? (b) What is the effect of the open spaces? (c) How much less is the average pressure on a large area than on a small area? (d) To what extent is the leeward truss sheltered by the windward truss and by the live load? (e) What is the effect of suction? (f) What should be assumed as the greatest *average* pressure? These questions render the whole subject of wind pressure on bridges complex and uncertain. The most that can be done is to assume wind loads that are approximately correct for given conditions and that are considered to be on the side of safety. Useless refinements should be avoided, both in the assumption of wind pressure and in the assumed distribution of wind loads.

3. For an unloaded bridge, the wind pressure generally assumed is 50 lbs. per square foot on the exposed area of the trusses and floor system. For a loaded bridge, the pressure is 30 lbs. per square foot on the exposed area of the bridge *and its load.* The pressure is assumed to be less on a loaded than on an unloaded bridge, partly because it is improbable that any live load will be on a bridge when the velocity of the wind is so great as to cause a pressure of 50 lbs. per square foot, and partly because when the live load is on the bridge the total exposed area is considerably greater than that of the unloaded bridge.

4. The wind pressure on the live load is assumed to be a moving load; formerly the wind pressure on the bridge itself was considered as a static load, but the trend in current practice is to assume that also as a moving load. There is some difference of opinion as to what extent the exposed area of the leeward truss is reduced by the sheltering effect of the windward truss and of the live load. Because of this and of other uncertainties previously mentioned, specifications differ with respect to the methods of determining wind loads.

5. **Methods of Specifying Lateral Forces.** *Wind pressure.* There are in general two methods of specifying wind pressure, namely (1) the pressure per *square* foot of the exposed area, and (2) the load per *linear* foot.

6. In the first method, the exposed area is assumed to be the area seen in elevation, i.e., the area of the various parts of the bridge projected on a vertical plane parallel to the axis of the bridge. For railroad bridges, this total exposed area has sometimes been specified as one and a half times

the projection of the structure on the vertical plane. In more recent specifications it is defined as follows: (a) The floor system projected on a vertical plane considered as a solid area; (b) the projection of each truss complete. In determining the projection of each truss, the area of each member may be found by multiplying its length by its average width, the open spaces in the member being disregarded. The total area obtained by adding the exposed areas of the members and connections is the exposed area for one truss. If there are two trusses, this area is multiplied by two, if there are three trusses, it is multiplied by three. The sheltering effect of one truss on another or of a train on a truss is thus disregarded. The wind force on the train is specified as 300 pounds per linear foot on one track, applied 8 ft. above the top of the rail. For a loaded bridge, the total wind force is 30 pounds per square foot of the exposed areas as defined in (a) and (b) plus the wind force on the train. For an unloaded bridge the total wind force is 50 pounds per square foot of the total exposed surface as defined in (a) and (b). If this wind force, as specified for the unloaded bridge, produces a greater stress in any member than the forces specified for the loaded bridge, that stress is used. It is usually stated that if the loads obtained from the specified pressure on the exposed areas of the floor systems and trusses are less than 200 pounds per linear foot for the loaded chord and 150 pounds per linear foot for the unloaded chord, these latter values should be used. All loads are moving loads.

1. The second method of specifying lateral forces is simpler since it requires no calculation of the exposed area. It is particularly adapted to highway bridges. It is illustrated from the following paragraph, taken from recent specifications for highway bridges: "Spans of 150 ft. and less shall be designed to resist a lateral force of 300 lbs. per lin. ft. on the loaded chord and 150 lbs. per lin. ft. on the unloaded chord. For spans of more than 150 ft., for each additional 30 ft. of span there shall be added 10 lbs. per lin. ft. for the loaded chord and 5 lbs. per lin. ft. for the unloaded chord. All lateral forces are to be considered as moving loads."

2. *Lateral forces due to the movement of the live load.* In order to provide for the "nosing" and lurching effect of the engines and train, an additional uniform moving load has sometimes been specified, as, for example, a load of "5 per cent of the specified live load on one track, but not more than 400 lbs. per linear foot, applied at the base of the rail." The assumption that there can be such a lateral force acting *in the same direction throughout the length of the train* is contrary to fundamental principles of mechanics. In more recent specifications it is stated that the lateral force to provide for the effect of the sway of the engines, in addition to the wind loads specified, shall be a moving concentrated load of 20,000 lbs. which may be applied at the base of the rail in either horizontal direction at any point of the span.

3. **Loads Due to Lateral Forces.** A portion of the wind pressure on a bridge is assumed to be transferred to the apices of the upper lateral system and the remainder to the apices of the lower lateral system. The wind pressure on the live load and the lateral forces due to the motion of that load are assumed to be transferred to the lateral system immediately below the floor system. For example, in a through railway bridge, the wind pressure on a train and the lateral forces at the base of the rail due to the motion of the train are assumed to be transferred by the stringers and floor beams to the apices of the lower lateral system; in a deck bridge, these same forces are assumed to be transferred to the upper lateral system.

4. In Fig. 438 (b) is shown a truss diagram of the leeward truss of a through bridge; the windward truss is assumed to be directly behind it. In Fig. 438 (a) is shown a top view of the upper lateral system and of the end posts and portal struts. Joints B, C, D, E, F, G, and H are the joints in the upper chord of the leeward truss, and joints B', C', D', E', F', G', and H' are the joints in the upper chord of the windward truss. These two upper chords of the main trusses serve also as the parallel chords of the upper lateral system. The wind loads carried by the upper lateral system are due partly to wind pressure on the windward truss and partly to wind pressure on the leeward truss, but in Fig. 438 (a) each panel load, represented by W, is shown as applied wholly on the windward side. This assumption, is on the side of safety and is sufficiently correct for practical purposes. (Some engineers prefer to assume one-half of the load on the windward and one-half on the leeward side.) The panel load W is equal to the wind pressure per linear foot specified for the unloaded chord multiplied by the panel length; if the wind pressure is specified per square foot of exposed area, it may be reduced to panel loads. Note that the load at each hip joint (at B' and at H') is assumed as a full panel load.

1. The line of action of the reaction at the left-hand end of the top lateral system is BB', and the line of action of the reaction at the right-hand end is HH'. The reaction at each end is exerted by the end posts and portal at that end.

2. In Fig. 438 (c) is shown a top view of the lower lateral system. Joints a, b, c, d, e, f, g, h, and i are joints in the lower chord of the leeward

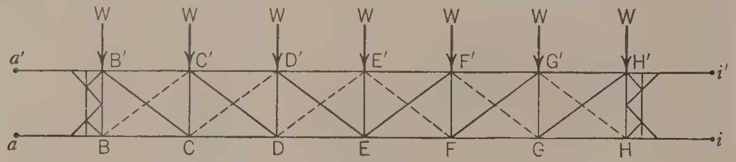


Fig. 438 (a).

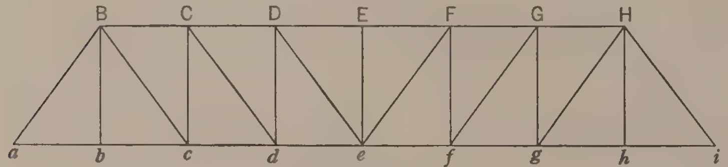


Fig. 438 (b).

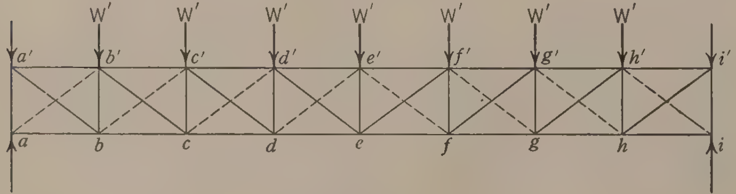


Fig. 438 (c).

main truss, and joints a', b', c', d', e', f', g', h', and i' are joints in the lower chord of the windward truss. These two lower chords of the main trusses serve also as the parallel chords of the lower lateral system. The entire lateral force carried by the lower lateral system is assumed as concentrated at panel points on the windward side. The panel load W' is calculated from the specified lateral force per linear foot on the loaded chord or from that portion of the wind pressure on the exposed area that is transmitted to the loaded chord; in the case of a railway bridge a panel load of wind pressure on the train is included in W'.

3. **Stresses in Lateral Systems.** When the laterals (diagonals) in a lateral system are designed as tension members, only one diagonal in each panel can act at a time. For the lateral forces indicated in Figs. 438 (a) and 438 (c), the laterals shown by full lines are in action and those shown by broken lines are not. When the lateral forces act in the opposite direction, the laterals shown by broken lines are in action and those shown by full lines are not. The stresses may be determined just as for any parallel-chord truss, either by the method of sections or by the method of coefficients. The latter method is usually the better. When loads are static, the method of coefficients explained in 215 : 5 to 223 : 14 may be used, but when loads are moving, as they are now generally assumed to be, the maximum stresses in web members are obtained by the method of coefficients explained in 317 : 1 to 320 : 1.

4. When the laterals are designed to take either tension or compression, the two laterals in each panel act simultaneously. The assumption usually made is that the shear for any panel is equally divided between the two laterals. For static loads the coefficients, based on this assumption, are first placed on all web members, and from these coefficients those for chord members may be determined. For moving loads, the coefficients for chord members are determined for a fully loaded truss, just as if the loads were static, and the coefficients for web members are determined by the method explained in 317 : 1 to 320 : 1, except that the coefficient on each of the two laterals in any panel, obtained from the maximum shear for that panel, is half as great as the coefficient would be for a single lateral in that panel.

5. *Note:* A common assumption is that the top laterals of a through bridge will take tension only but that the bottom laterals will take either tension or compression.

6. *When the top chords of the trusses in a bridge are inclined*, as, for example, in a Parker truss (photograph on page 102), the entire top lateral system may be treated as if it were flattened out into a horizontal plane. The panels in the horizontal truss thus formed will usually be unequal and slightly longer than the regular panel length, but the panel loads may be

determined from that panel length. The lateral stresses may then be determined by the usual method. Many members of the upper lateral system, including top chord members of the main truss, are not horizontal, and the lateral stresses in such members all have vertical components, consequently there are stresses in the main trusses which would not occur were the lateral system really in a horizontal plane. In bridges of ordinary spans, however, these stresses are negligible.

1. *In a deck bridge* the lateral forces are greater on the upper lateral system than on the lower, but otherwise conditions are similar to those in a through bridge, and the lateral stresses are determined by the same general methods.

2. *Lateral stresses due to the motion of the train.* In determining the maximum stress in any member of a lateral system due to the concentrated lateral force of 20,000 lbs. mentioned in 437 : 2, the load must first be placed in the position to cause that maximum stress. For a lateral it will be at one end of the panel that contains the lateral, and for a chord member its line of action will pass through the center of moments for that member. Note that the lateral force is normal to the axis of the lateral truss and may be applied in either direction.

3. **Action of End Posts and Portal.** In Fig. 439 is a true projection of the end posts and portal at the left-hand end of the truss previously shown in Fig. 438 (*b*). Assume that the end posts are hinged at the bottom at points a' and a. Let P represent the total horizontal force due to wind pressure that is brought to the portal strut by the upper lateral system. This force is equal in magnitude but opposite in direction to the reaction exerted on the lateral systems by the portal. (See 438 : 1 for the lines of action of this reaction.) Consider as the body in equilibrium the frame $a'B'Ba$ formed by the portal and the end posts. The force P causes reactions at a' and a. For the purpose of determining these reactions it is immaterial whether P is considered as applied wholly at B' or partly at B' and partly at B. The reactions at a' and a are indeterminate; whatever they are, they may be replaced by the components J', K', J, and K. The lines of action of J' and J coincide, respectively, with the axes of $a'B'$ and aB of the end posts. The forces K' and K are horizontal. In order to make the problem statically determinate, some assumption must be made. The most common assumption is that K' is equal to K and therefore, from $\Sigma H = 0$, $K' = K = \frac{1}{2}P$. Since the end posts are assumed to be hinged at joints a' and a, these joints are points of zero moments, and consequently the resultant moment of the external forces P, K', K, J', and J about either a' or a is zero. From $\Sigma M = 0$ about a, $J' = -P\frac{k}{w}$ (downward); from $\Sigma M = 0$ about a' (or from $\Sigma V = 0$), $J = P\frac{k}{w}$ (upward). The method of determining these components of the reactions is essentially the same as that used for a roof truss with knee braces. (258 : 3.)

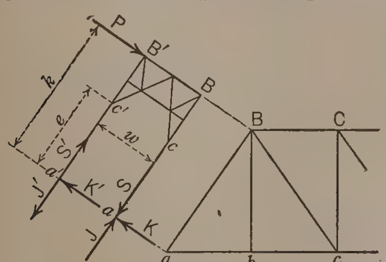


Fig. 439.

4. *Stresses in the end posts below the portal.* Let S' represent the *axial* stress in the end post $a'B'$ between a' and c' due to the external forces P, K', K, J', and J. From fundamental principle applied at a', $S' = J' = P\frac{k}{w}$, and since J' acts away from joint a', S' must also act away from a' and is, therefore, tension. Similarly, if S represents the axial stress in the end post between a and c, $S = J = P\frac{k}{w}$ compression.

The portion of each end post below the portal is subject to bending stress as well as to axial stress. These bending stresses are greatest at c' and c where the magnitude of the bending moment in each case is $\frac{1}{2}Pe$.

5. *The point of zero moments in an end post* depends upon the conditions at the foot of the post and upon the rigidity of the portal. If the end post is considered as fixed at the bottom and the portal is rigid, the point of zero moments is half-way between the bottom of the post and the bottom of the portal; if the post is partially fixed at the bottom and the portal is rigid, the point of zero moments is nearer to the bottom of the post than it is to the bottom of the portal. (257 : 8.)

6. When the point of zero moment is assumed a distance n above the bottom of the post the lever arms k and e in Fig. 439 will be reduced to $k - n$ and $e - n$. The components J' and J of the reactions will be correspondingly reduced, and consequently the axial stresses S' and S

in the end posts below the portal will be reduced. Likewise the bending moments at c' and c will be less and consequently the bending stresses will be less than when the points of zero moment are assumed at a' and a.

1. In most cases each end post is at least partially fixed at the bottom by the action of dead load and by other conditions. In all of the explanations that follow, however, the end posts will be assumed as hinged at the bottom. (It is evident from the preceding paragraph that this assumption is on the safe side.) The analyses for different types of portals will hold true, however, for any other point of zero moment, provided that $k - n$ and $e - n$ are substituted for k and e respectively.

2. *Action of forces at the foot of each end post.* The tensile stress S' in the end post at a' was determined without reference to the action of the end post $a'B'$ as a member of the windward vertical or main truss. In Fig. 440 (a) is shown the left-hand end of this truss. From the equilibrium equations $\Sigma V = 0$ applied at a', the vertical reaction at a' is $R' = -S'_V$. From 439 : 4, $S' = P\frac{k}{w}$, hence $S'_V = S'\frac{h}{k} = P\frac{h}{w}$. The reaction R' is equal to $P\frac{h}{w}$ and acts downward.

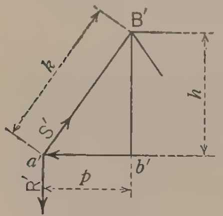


Fig. 440 (a).

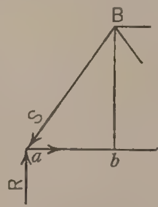


Fig. 440 (b).

3. From $\Sigma H = 0$ applied at a', the stress in $a'b' = -S'_H = -P\frac{p}{w}$. Note that this stress in $a'b'$ must act toward a' and is therefore compression. Hence the action of the two end posts and portal in resisting the wind load brought to the portal by the upper lateral system tends to cause compression in the lower chord member at the foot of the end post of the windward truss, and thus to make the tensile stress in that member less than the normal tension due to vertical loads.

4. It may be shown by methods similar to those just used that the vertical reaction R at a in Fig. 440 (b) is equal to $P\frac{h}{w}$ and acts upward, and that the stress in the lower chord member ab is equal to $P\frac{p}{w}$ and is tension. Hence the action of the two end posts and portal makes the tension in the lower chord member of the leeward truss greater than the normal tension due to vertical loads.

5. *Deck bridges.* When the end posts of a deck bridge are inclined in the direction shown in Fig. 439, the bracing between them may extend clear to the bottoms of the posts, but the general method of determining reactions explained in 439 : 3 for through trusses still holds good. When the end posts are vertical, these methods may easily be modified to apply to that condition. It is to be noted, however, that in all deck bridges the lateral forces on the top lateral system are much greater than those on the top lateral system of a through bridge.

6. **The External Forces on a Portal.** Let $B'Bcc'$ in Fig. 440 (c) represent any rigid portal between two end posts $B'a'$ and Ba. Assume that the end posts are hinged at the bottom at points a' and a. Let P represent the total horizontal force due to wind pressure that is brought to the portal by the upper lateral system. The external forces that hold the portal in equilibrium are P and the forces exerted on the portal by the end posts. The latter forces will now be determined.

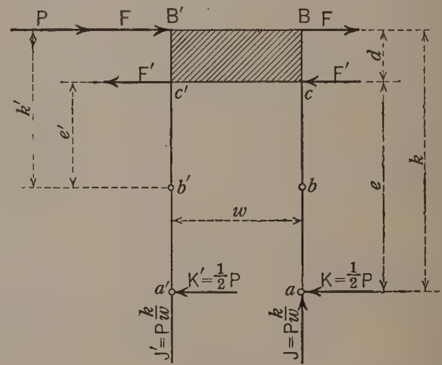


Fig. 440 (c).

7. The components of the reactions at a' and a due to the single force P were determined in 439 : 3.

$$-K' = -K = -\tfrac{1}{2}P \quad \text{and} \quad -J' = J = p\frac{k}{w}.$$

8. The axial stress in $a'c'$ is $P\frac{k}{w}$ tension (439 : 4), and therefore this portion of the end post exerts a force of $P\frac{k}{w}$ downward at c' along the line $c'a'$. Similarly ac the portion of the other end post below the portal exerts

an upward force at c also equal to $P\frac{k}{w}$. In addition, the end posts exert horizontal forces F and F on the portal at B' and B, and horizontal forces F' and F' on the portal at c' and c.

1. The horizontal forces F' and F may be determined by the method explained in 258 : 4 for a roof truss with knee braces. The horizontal force at c' is $F' = K'\frac{k}{d} = \frac{1}{2}P\frac{k}{d}$ to the left; the force F' at c is equal to that at c' and acts in the same direction. The horizontal force at B' is $F = K'\frac{e}{d} = \frac{1}{2}P\frac{e}{d}$ to the right; the force F at B is equal to that at B' and acts in the same direction.

2. When the portal alone is considered as the body in equilibrium, the four forces F', F, F', and F are among the external forces, but the reactions J', K', J, and K are not, since these four forces do not act directly on the portal. On the other hand, when the portal frame, which includes the end posts as well as the portal, is considered as the body in equilibrium, the reactions J', K', J, and K are among the external forces, but the forces F', F, F', and F are not, since these four forces are not external to the portal frame as a whole. It follows that a reaction at the foot of an end post and a force F' or F exerted on the portal by an end post should not appear in the same equilibrium equation.

3. *Note:* When the points of zero moments are assumed elsewhere than at the bottoms of the posts, as, for example, at points b' and b, the lever arm k' is used in place of k, and e' in place of e in determining the components of reactions and the forces F' and F. (439 : 6.)

4. **General Methods of Determining Stresses in Portals.** There are two general methods of determining stresses in portals. In the first method, the end posts are considered as parts of the body in equilibrium. The external forces in Fig. 440 (c) that act on this body (portal frame) are P, J', K', J, and K, determined by methods already explained. (Forces F', and F do not act on the frame as a whole and may be disregarded.) The stresses in the portal may be determined by the method of sections, the section in each case being vertical. The stresses in web members may be determined from the shear, and the stresses in the upper and lower chords may be determined from moment equations, using either segment as the body in equilibrium, just as for any truss.

5. In the second method, only the portal itself is considered as the body in equilibrium. The external forces in Fig. 440 (c) that act on the portal are P, F at B', F at B, F' at c', F' at c, the axial stress S' in the end post $a'c'$ acting downward at c', and the axial stress S in the end post ac acting upward at c. (440 : 6.) These external forces on the portal having been calculated by methods already explained, the stresses in the portal may be determined by the algebraic method of joints or by the method of sections or by a combination of the two methods. The graphic method of successive joints may also be used. The second method corresponds to that explained on page 259 for a roof truss with knee braces.

6. The application of the general methods to different types of portals will now be explained. It will be assumed in each case that the end posts are hinged at the bottom, and that the components of the reaction at the bottom of each post have been determined by the method of 439 : 3. The axial and bending stresses in the end posts below the portal are not affected by the type of portal, and hence they may be determined in each case by the methods explained in 439 : 4. No further reference to the method of determining these stresses will be necessary.

7. *Note:* When the points of zero moment are assumed elsewhere than at the bottoms of the posts, the stresses in the portal are smaller than when the end posts are assumed to be hinged at the bottom. The lever arms used in determining the external forces are changed as explained in 439 : 6, but once these external forces have been calculated, the general methods of determining stresses in the portals are essentially the same as those used when the posts are hinged at the bottom.

8. *Note:* When a portal is rectangular in form, there are generally brackets or knee braces between the bottom of the portal and the end posts, as shown in the photograph on page 112. The knee braces stiffen the connection of the portal to the end posts, and to some extent reduce the bending moment in each post, but they are generally disregarded in determining stresses in the portal.

9. *Stresses in an A-frame portal.* In the portal shown in Fig. 442 (a), the members $B'n$, no, and oB are secondary members that serve to stiffen the primary members $c'M$ and Mc. If the horizontal reactions at a' and at a are equal, there can be no stress in no and therefore no stress in $B'n$ or in oB. (Prove.) Consequently, in determining the stresses in the portal these supplementary members may be disregarded.

1. The reactions at a' and a are determined by the method explained in 439 : 3. The stress in each end post is $P\frac{k}{w}$, tension for ac' and compression for ac. (439:4.) The forces F' and F are determined as explained in 441 : 1. $F' = \frac{1}{2}P\frac{k}{d}$ and $F = \frac{1}{2}P\frac{e}{d}$. Assume the portal as the body in equilibrium, thus making use of the forces F' and F. (441 : 5.)

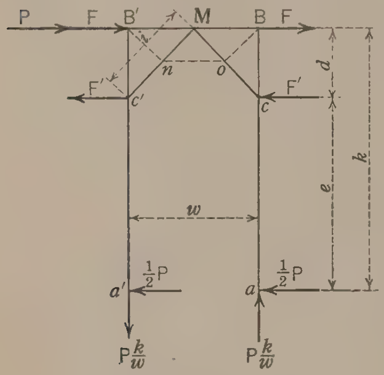


Fig. 442 (*a*).

From $\Sigma V = 0$ applied at B' and at B the stress in each of the members $B'c'$ and Bc is zero.

From $\Sigma V = 0$ applied at c' and at c, the stress in each of the members $c'M$ and Mc is $P\frac{k}{w} \times \frac{i}{d}$, tension in $c'M$, compression in Mc.

From $\Sigma H = 0$ applied at B', the stress in $B'M = P + F = P + \frac{Pe}{2d} = \frac{P}{2}\left(\frac{k}{d} + 1\right)$.

From $\Sigma H = 0$ applied at B, the stress in $MB = F = \frac{Pe}{2d} = \frac{P}{2}\left(\frac{k}{d} - 1\right)$.

2. *Exercise:* (a) Check the values for $B'M$ by the method of sections. (Disregard forces F' and F (441 : 4).) (b) Using the values derived for the stresses in the members that meet at joint M, show that $\Sigma H = 0$ applied at M holds true. (c) Show that $\Sigma H = 0$ holds true for the joint at c'.

3. *Stresses in a portal with diagonal bracing.* Let it be assumed that the diagonals of the portal in Fig. 442 (*b*) take tension only. The stresses in the portal will be determined by two methods.

4. First method. (441 : 4.) The portal frame is held in equilibrium by P and the H and V forces at a' and at a. For P acting at B' as shown, the diagonal $c'B$ is in action and the diagonal $B'c$ is not. Let s–s represent a section. Let the right-hand segment of the portal frame be the body in equilibrium, it having the smaller number of external forces.

From shear ($\Sigma V = 0$ applied to the segment) $c'B_V = P\frac{k}{w}$, and $c'B = P\frac{k}{w} \times \frac{i}{d}$ tension.

From $\Sigma M_{c'}$, $B'B = P + \frac{Pe}{2d}$ compression.

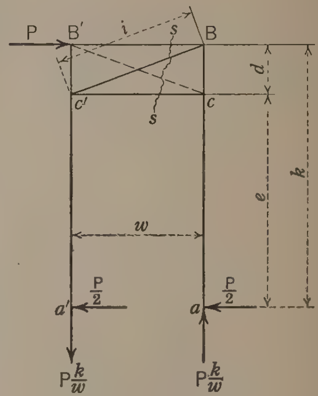


Fig. 442 (*b*).

From ΣM_B, $c'c = \frac{Pk}{2d}$ compression.

From $\Sigma V = 0$ at B', $B'c' = 0$.

From $\Sigma V = 0$ at B, $Bc = cB_V = P\frac{k}{w}$ compression.

5. Second method. (441 : 5.) The portal in Fig. 442 (*c*) is held in equilibrium by the forces P, F, F', F, F', and the axial stresses in the end posts at c' and c. From 441 : 1, $F' = \frac{1}{2}P\frac{k}{d}$ and $F = \frac{1}{2}P\frac{e}{d}$. The stresses in the portal may be determined as follows:

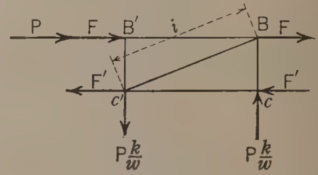


Fig. 442 (*c*).

From $\Sigma V = 0$ at B', $B'c' = 0$.

From $\Sigma H = 0$ at B', $B'B = P + F = P + \frac{Pe}{2d} = \frac{P}{2}\left(\frac{k}{2d} + 1\right)$ compression.

From $\Sigma V = 0$ at c, $Bc = P\frac{k}{w}$ compression.

From $\Sigma H = 0$ at c, $c'c = F' = \frac{Pk}{2d}$ compression.

From $\Sigma V = 0$ at c', $c'B_V = P\frac{k}{w}$, and $c'B = P\frac{ki}{wd}$ tension.

6. When the two diagonals $B'c$ and $c'B$ are designed to act simultaneously, each may be assumed to resist one-half of the shear, and the magnitude of the stress in each is $P\frac{ki}{2wd}$. Under these conditions and for a load P applied at B', the stress in $B'B$ is $\frac{P}{2}$ compression, and the stress in $c'c$ is zero.

1. *Exercise:* (a) Prove by the method of sections the statements made in the preceding paragraph. (200 : 6.) (b) Using the values for $B'B$ and $c'B$ given in the preceding paragraph, prove that $\Sigma H = 0$ holds true when applied to the joint at B.

2. *Stresses in a lattice portal.* The portal in Fig. 443 (a) is of the same general type as that shown in the photograph on page 92 and on page 111.

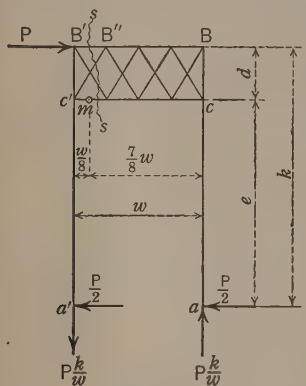


Fig. 443 (a).

A section s–s cuts two diagonal web members. It may be assumed that these two diagonals act simultaneously, one in tension, the other in compression, and that each resists one-half the shear. This shear is $P\frac{k}{w}$. If i represents the length of a diagonal, the magnitude of the stress is $P\frac{ki}{2wd}$. This is also the magnitude of the stress in every diagonal web member, since the shear is $P\frac{k}{w}$ for any section between B' and B.

3. To determine the stress in $B'B''$ assume a center of moments at m a distance from c' equal to $\frac{w}{8}$. The moment of the stress in one diagonal cut by the section will then cancel the moment of the stress in the other. Taking the right-hand segment of the portal frame as the body in equilibrium the moment equation is:

$$\Sigma M_m = -\frac{Pk}{w} \times \tfrac{7}{8}\, w + \frac{P}{2} \times e + B'B'' \times d$$

$$B'B'' = P\left(\frac{7k}{8d} - \frac{e}{2d}\right), \text{ and since } k = d + e,\ B'B'' = \frac{P}{8}\left(7 + 3\frac{e}{d}\right) \text{ (compression).}$$

The stress in any portion of the upper chord $B'B$ may be determined in a similar manner; the same general method may be used for calculating the stress in any portion of the lower chord $c'c$, the center of moments in each case being assumed in the upper chord $B'B$. It is usually sufficient, however, to determine only one chord stress, namely, that in $B'B''$. This is because, in designing the portal, it is customary to make the upper and lower chords alike, each capable of carrying the maximum compression that can occur in either, and the stress in $B'B''$ is this maximum compression.

4. *Exercise:* Check the stress in $B'B''$ by applying $\Sigma H = 0$ to the forces at B', including the force F determined in 441 : 1.

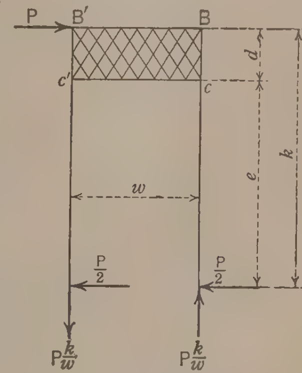


Fig. 443 (b).

5. The portal in Fig. 443 (b) is of the same general type as that shown in the photograph on page 98 and on page 100. A vertical section cuts four diagonal web members. If each diagonal is assumed to resist one-fourth of the shear, the stress in each is $P\frac{ki}{4wd}$. (The quantity i is the length of a web member between upper and lower chords.) The stress in any portion of the upper chord $B'B$ may be determined by the method explained in 443 : 2. The web members are so numerous, however, that the conditions are nearly the same as those for a portal with solid web, consequently the method of determining the maximum stress in the upper flange of a plate-girder portal, explained in the next article, may be used for determining the stress in $B'B$ without serious error.

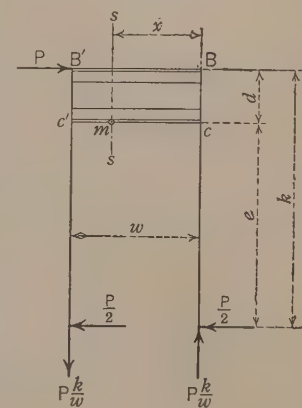


Fig. 443 (c).

6. *Stresses in a plate-girder portal.* The portal in Fig. 443 (c) is composed of a solid web with top and bottom flange angles and is therefore a small plate girder. Let s–s be any vertical section a distance x from B. The shear is constant for any section between B' and B and is equal to $P\frac{k}{w}$. It may be assumed that this shear is resisted by the web, and that the bending moment is resisted by the flange angles. The stress S in the upper

flange $B'B$ at the section s–s may be determined from $\Sigma M = 0$ about m as a center. Let the right-hand segment of the portal frame be the body in equilibrium.

$$\Sigma M = Sd - P\frac{kx}{w} + \frac{Pe}{2} = 0$$

$$S = \frac{P}{d}\left(\frac{kx}{w} - \frac{e}{2}\right)$$

This stress will be maximum compression when $x = w$.

Stress in $B'B$ at $B' = \frac{P}{d}\left(k - \frac{e}{2}\right) = \frac{P}{2d}(k + d) = P\left(1 + \frac{e}{2d}\right)$ (compression).

Stress in $B'B$ at $B = -\frac{Pe}{2d} = -\frac{P}{2d}(k - e)$ (tension).

By a similar method the stress in the lower flange at c' may be determined as $\frac{Pk}{2d}$ tension, and at c as $\frac{Pk}{2d}$ compression. The general equation for stress in the upper flange is that of the first degree, hence the stress in the flange for different values of x between B' and B varies as the ordinates to a straight line. A similar statement is true of the stress in the lower flange. In designing the portal the upper and lower flanges are made alike, with a cross-sectional area sufficient to carry the maximum stress that can occur in either.

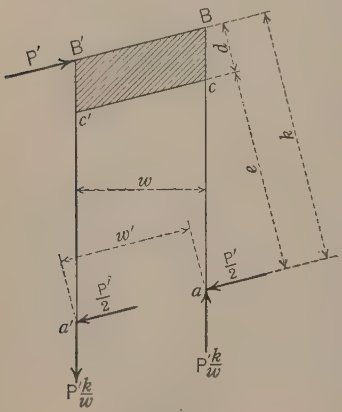


Fig. 444.

1. *Skew portals.* The most common case of a skew portal is that in which the end posts have equal slopes. In Fig. 444 is represented a skew portal frame *in its true shape.* The portion $B'Bcc'$ represents any rigid portal. The load P brought to the portal at B' by the upper lateral system is normal to the planes of the main vertical trusses, and therefore its line of action can not lie in the line $B'B$ as it does in a bridge which is not on a skew. This force P, however, may be resolved into two components, one of which, P', has a line of action that coincides with $B'B$. The other component is in the line of action of the top chord. The component P', equal to $P\frac{w'}{w}$, causes the stresses in the portal frame. (Note that both w and w' are measured in the plane of the portal.) This component having been determined, the reactions and stresses may be calculated by the methods used for ordinary portals.

2. **Stresses in Sway Bracing.** Different types of sway bracing were described in 112 : 4, and illustrated by photographs of bridges referred to in 113 : 1. A bridge with complete lateral systems and adequate portals is stable without sway bracing, therefore the stresses in sway bracing are indeterminate. It is sometimes assumed that the sway bracing at any panel point should be capable of transferring from the upper to the lower lateral system approximately one-half of the horizontal panel load due to the wind pressure on the upper system. This assumption is based on the fact that the upper lateral system is more rigid than the lower, and that therefore the latter will deflect horizontally more than the former, particularly when there is no live load on the bridge. The amount of load transferred, however, depends upon so many undetermined conditions that the assumption is purely arbitrary.

3. The most severe conditions to which sway bracing is subject occur when only one track of a double-track deck bridge is loaded. At any panel point, the floor-beam concentration due to live load is greater on the truss next to the loaded track than it is on the other truss. The sway bracing, which in a deck bridge extends the full depth of the main trusses, may be expected to distribute the load more evenly by transferring some of the excess load from the more heavily loaded truss to the other truss.

4. Let W represent the total live load on *one* track that is carried by a floor beam; w the width from the center of one main truss to the center of the other; g the distance from the center of the loaded track to the center of the nearer main truss. The load brought to the nearer truss by the floor beam is $\frac{W(w-g)}{w}$, and that brought to the other truss is $\frac{Wg}{w}$. The former load is larger than the latter by an amount equal to $W\left(1 - \frac{2g}{w}\right)$. To equalize the loads on the two trusses it would be necessary to transfer

one-half of this amount or $W\left(\frac{1}{2} - \frac{g}{w}\right)$ to the truss carrying the smaller load. If this could be done by the diagonal of the sway bracing, the stress in that diagonal would be $W\left(\frac{1}{2} - \frac{g}{w}\right) \times \frac{l}{h}$, in which l is the length of the diagonal and h the height of truss. This is, theoretically, the maximum stress that the eccentric loading can cause in the diagonal of the sway bracing; in reality it probably never does cause so great a stress. Some specifications require, however, that in designing sway bracing this maximum stress shall be included. In through bridges, the sway bracing, extending as it does only a small distance from the top of the bridge, cannot be counted on to transfer load from one main truss to the other.

1. **Overturning Effect of the Wind on a Train.** The wind pressure on a train may be taken as 300 lbs. per linear foot on one track, applied 8 ft. above the top of the rail. This pressure is almost great enough to overturn an empty freight car, and its effect is to transfer the greater part of the vertical load from one rail to the other. The floor beam will then carry a greater load to the leeward truss than to the windward truss. The horizontal reactions at the rails may be assumed to be transferred by the floor system to the lower lateral system, though the floor system itself may act to a considerable extent as a lateral system.

2. Let p represent the panel length, w the width center to center of main trusses, and h' the vertical distance from the plane of the lower lateral system to the plane of the assumed wind pressure. (This distance h' is usually equal to the distance from the bottom of the floor system to the top of the rails plus 8 ft.) The vertical load downward at a panel point of the leeward main truss is $300\,p\frac{h'}{w}$; there is an equal load *upward* at the corresponding panel point of the windward truss. The stresses in the main trusses due to these panel loads may be determined by the usual methods for vertical loads. The stress in any member of the leeward truss is the same in character as the stress in that member due to dead load or live load, but the stress in any member of the windward truss is opposite in character to the stress due to dead or live load. Thus the wind pressure on a train tends to cause reversal of stress in members of the windward truss.

3. The concentrated moving lateral force of 20,000 lbs. to provide for the effect of the sway of the engines, is assumed to be applied at the base of the rail. (437 : 2.) If d' is the depth from the base of the rail to the plane of the bottom lateral system, the vertical load on each main truss due to this single lateral force is $20{,}000\,\frac{d}{w}$ lbs., *downward* on the leeward truss, *upward* on the windward truss. This load may be applied at any panel point of the lower chord of either truss. Like the wind pressure on the train, this lateral force tends to cause reversal of stress in members of the windward truss.

4. **Centrifugal Force.** A train moving around a curve is deflected from continuing in a straight line by the forces exerted on the wheels by the curved rails. There is at the same time a tendency for the train to overturn which is partially counteracted by the superelevation of the outer rail. The deflecting forces exerted on the wheels by the rails are actual forces, as are the reactions of the wheels on the rails. There is, however, no inherent "centrifugal force" as represented by F in Fig. 445. This centrifugal force is really a fictitious force commonly used as one that applied at the center of gravity, produces the same overturning effect and the same lateral pressure on the rails as are produced by the actual forces. As thus understood the term "centrifugal force" is allowable.

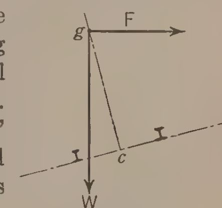


Fig. 445.

5. A bridge on a curve is designed to resist the centrifugal force of the live load applied 7 ft. above the base of the rail as computed by the following formula

$$F = \frac{0.067\, W V^2}{R}$$

in which F = horizontal centrifugal force; W = live load, including impact, which may be expressed in terms of axle loads, load per linear foot, or panel loads; V = speed in miles per hour, which may be taken as $60 - 2\frac{1}{2}$ times the degree of curvature; R = radius of curve, i.e., 5730 divided by the degree of curve. Another form of the equation is

$$F = 0.0000117\, D W V^2$$

in which D = degree of curvature.

1. *Note:* The two formulas just given may be derived from the general expression for centrifugal force $F = \frac{v^2}{gR} W$, in which v is the velocity in feet per second, g the acceleration of gravity equal to 32 feet-per-second per second, R the radius of curvature in feet, and W the weight of the body.

2. Centrifugal force causes stresses in the lateral system that is immediately below the floor system. The horizontal loads at panel points having been determined, the stresses in members of the system are found either by the method of coefficients or by the method of sections, exactly as for wind loads.

3. Centrifugal force also produces an overturning effect that may be treated exactly as the overturning effect due to wind. (445 : 1.) It should be noted that centrifugal force increases the vertical load on the outer rail and stringer, and decreases it on the inner rail and stringer, whereas the effect of tilting is the reverse of this. (432 : 7.) Hence in this respect the effect of centrifugal force and of tilting tend to neutralize each other. Similarly, centrifugal force increases the reaction at the connection of the outer row of stringers to a floor beam and decreases the reaction at the connection of the inner row. Finally, if C_p represents the horizontal centrifugal force for one panel, the overturning effect results in a vertical load downward at a panel point of the outer truss of $C_p \frac{h'}{w}$ and an equal load upward at a panel point of the inner truss, exactly as for wind pressure on the inner side of the train. (445 : 2.) The wind pressure, however, is assumed to act 8 ft. above the top of the rail whereas the centrifugal force is assumed to be 7 ft. above the base of rail, consequently h' for centrifugal force is about $1\frac{1}{2}$ ft. less than it is for wind pressure. When the eccentricity due to curvature and tilting is such as to increase the live load at a panel point of the inner truss (434 : 2), the effect of this eccentricity and the overturning effect of centrifugal force tend to neutralize each other at that panel point.

4. **Longitudinal Forces.** The starting or stopping of a train on a bridge causes longitudinal or tractive forces. The tractive force on each rail was formerly specified as 20 per cent of the live load on that rail throughout the length of the bridge. In more recent specifications the coefficient of friction is given as 20 per cent on engine drivers and 10 per cent on the remainder of the train. These tractive forces are transferred from the rails to the stringers, from the stringers to the floor beams, from the floor beams to the panel points of the loaded chords of the main trusses, and by these lower chords to the *fixed* end of the bridge. The forces transferred to any floor beam are those due to the total tractive force in one panel, and since these forces are applied through the stringers at right angles to the floor beam they cause bending stresses in the beam. These stresses may be reduced by connecting the lower laterals to the stringers. The corresponding horizontal, longitudinal reactions at the ends of the floor beams cause axial stresses in the chord members of the loaded chords of the main trusses. In proceeding from the free end of a truss toward the fixed end, there is an increment at each panel point of the loaded chord of one panel load per truss of longitudinal force, hence the corresponding chord stresses increase from the free to the fixed end. When the train is moving toward the fixed end these chord stresses are compression; when it is moving away from the fixed end they are tension.

5. The stresses due to longitudinal forces are ordinarily negligible in floor beams and laterals. They are also negligible in members of the main trusses except, perhaps, in the lower chord members of a through bridge. The compression due to longitudinal forces may unite with compression due to lateral forces to change the stresses in lower chord members from tension to compression, particularly in lower chord members near the ends of the bridge. (447 : 6 to 448 : 1.) It should be noted, however, that maximum stresses due to longitudinal forces and maximum stresses due to lateral forces are not likely to occur concurrently. In the determination of stresses in viaduct towers the horizontal forces at the top of a tower due to traction are included in the external forces.

6. **Stresses in the Main Trusses.** Some of the stresses in the main trusses of a bridge are due to vertical external forces and some to horizontal. In the main trusses of a through railway bridge, for example, there may be at least three kinds of stresses due to vertical forces and five kinds due to horizontal. Stresses in such a bridge caused primarily by *vertical* external forces are (a) dead-load stresses, (b) live-load stresses, and (c) impact stresses. Stresses caused primarily by *horizontal* external forces are (d) lateral stresses in the upper and lower chords due to wind pressure (438 : 3), (e) lateral stresses in the lower chords due to the horizon-

tal forces on the rails caused by the "nosing" or swaying of the engines (439 : 2), (f) stresses in the upper and lower chords due to the action of the entire portal frame (440 : 3 and 4), (g) stresses in all members due to the overturning effect of the wind on the train (445 : 1), and (h) stresses in all members due to the overturning effect of the horizontal forces on the rails caused by the nosing or swaying of the engines (445 : 3). In addition, there may be stresses in the loaded chords due to traction, and, if the bridge is on a curve, stresses in the main trusses due to centrifugal force (445 : 4).

1. *Combination of the stresses in the main trusses.* The accompanying table is for the purpose of indicating the possible combinations of stresses in the upper and lower chords of a parallel-chord truss, through railway bridge. Each "C" in the table represents a stress in compression and each "T" a stress in tension. The stresses above the double horizontal line are those due primarily to vertical external forces and the stresses below are those due primarily to horizontal external forces.

Stresses	Leeward Truss		Windward Truss	
	I Upper Chord	II Lower Chord	III Upper Chord	IV Lower Chord
(a) Dead load	C	T	C	T
(b) Live load	C	T	C	T
(c) Impact	C	T	C	T
(d) Lateral stresses due to wind pressure	T	T	C	C
(e) Lateral stresses due to "nosing" or swaying		T or C		C or T
(f) From the action of the portal frame	C	T	T	C
(g) From overturning effect of wind on train	C	T	T	C
(h) From overturning effect of swaying	C or T	T or C	T or C	C or T

In addition to the stresses given in the table there may be stresses due to traction, and, if the bridge is on a curve, stresses due to centrifugal force.

2. In combination I, all stresses except one are compression, and in II all stresses are tension. This means that the maximum combined stress in either an upper or a lower chord member will occur when the truss containing that member is the *leeward* truss

3. The sum of all the stresses in a chord member due to horizontal external forces, i.e., the stresses below the double horizontal line in the table, is small compared with the sum of the stresses in the same member due to dead load, live load, and impact — often not over 25 per cent as great. Moreover, the various kinds of maximum stresses due primarily to wind pressure are infrequent and seldom occur simultaneously. For these reasons, it is often specified that when the total maximum stress in a member is obtained by combining the stresses due to lateral and longitudinal forces with those due to dead load, live load, and impact, the unit stresses may be increased 25 per cent over those generally used. This is equivalent to saying that all stresses in a member due to lateral and longitudinal forces may be disregarded when their algebraic sum does not exceed 25 per cent of the sum of the stresses due to dead load, live load, and impact.

4. In combination III of the table, three stresses in an upper chord due to lateral forces are tension and their effect is merely to reduce the compression in the chord caused by dead load, live load, and impact. This reduction does not affect the design of the chord.

5. From what has preceded it is evident that in designing all upper chord members, stresses due to horizontal external forces may often be entirely disregarded, and that the same statement may be made concerning a lower chord member so far as the maximum tension in that member is concerned. From inspection of combination IV, however, it is evident that when a lower chord member is in the windward truss, the stresses in that member due to horizontal external forces are all compression; it may be possible that the sum of these stresses will exceed the total tension due to dead load, live load, and impact, in which case reversal of stress will occur. This combination, therefore, requires special consideration.

6. First let it be assumed that there is no live load on the bridge, but that the wind pressure on the bridge is at its maximum. The only stress above the double line in the table will be that due to dead load (a) and the only stresses below the double line are the lateral stress due to wind pressure (d) and the stress due to the action of the portal frame (f). The combined compression in a lower chord member from (d) and (f) may be greater than the tension from (a), in which case the wind pressure will change the stress in that chord member from tension to compression.

7. Second let it be assumed that a train of *empty* freight cars (weighing 1200 lbs. per linear foot) extends from end to end of bridge and is *stationary;* if there are two tracks, the train is assumed to be on the track toward the

leeward truss. Under these conditions, the live-load tension in a lower chord member of the windward truss will be as small as possible yet the compression in the same member due to wind pressure will be as great as possible. There will be no stress due to impact (c) or to swaying (e) or to the overturning effect of swaying (h). If the total compression in a lower chord member due to the stresses from (d), (f), and (g) is greater than the total tension due to dead load and the *reduced* live load, the stress in that chord member is changed from tension to compression.

1. If compression in a lower chord member is obtained under both the first and second assumptions, provision in designing the member is made for the greater of the two stresses. This reversal of stress in lower chord members is most likely to occur in panels near the ends of the bridge.

2. The action of the portal frames causes both bending and axial stresses in the end posts. The effect of lateral forces on other web members of a truss is usually small and negligible. Such members do not receive stresses from lateral systems except in the case of a bridge with inclined chords (438 : 6). Stresses in web members occur from the overturning effect of the wind on the train and from the overturning effect of the horizontal forces on the rails due to swaying, but these stresses are also usually negligible.

3. The effect of lateral forces on the stresses in the main trusses of a deck bridge is somewhat different from the effect on the stresses in trusses of a through bridge, but the general method of combining the various stresses is essentially the same and requires no special explanation.

4. **Practical Considerations.** The methods of determining stresses due to horizontal forces have been explained in this chapter largely from the theoretical point of view without regard to certain practical considerations that may affect the design of a bridge. It is not within the scope of this book to discuss in detail these considerations. It is well to note, however, that in many cases lateral bracing, sway bracing, and portals are designed in accordance with standard forms and sizes without determining stresses, particularly for bridges of ordinary length and standard type. Frequently the design of a member depends as much upon its relations to other members as upon the stress in it. For example, top laterals, such as those shown in the photograph on page 111, are made approximately equal in depth to the top chord, and the usual specification that the ratio of unsupported length to the least radius of gyration shall not exceed a certain quantity, such as 120 or 140, may determine largely the design of the lateral.

The possible combinations of stresses in members of the main trusses, such as those indicated in the table on page 447, should also be treated from the practical as well as the theoretical point of view. Certain combinations will occur infrequently if at all. Provision is usually made for such infrequent occurrence by an increase in the allowable unit stress such as that mentioned in 447 : 3. Any combination that merely decreases the stress in a member below its maximum may usually be disregarded, but any combination that results in a reversal of stress should be provided for in the design. Practical considerations should include first of all safety, then economy and durability. The aesthetic element in design may not be a practical consideration, but it is one that should not be ignored — frequently it is of great importance.

ASSIGNMENTS

Report on various specifications pertaining to longitudinal forces on railway bridges, including wind pressure, lateral forces due to swaying, tractive forces, and centrifugal force; note the essential differences in similar clauses pertaining to such forces in different standard specifications for railroad bridges. State the changes which have taken place in requirements pertaining to horizontal forces as shown by the various editions of the specifications of the American Railway Engineering Association. Report also on various specifications pertaining to horizontal forces on highway bridges.

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